

Homework 2: Optics

**Problem 1) A Derivation of Snell's Law:** In Figure 1 the wavefronts in the incident medium match the wavefronts in the transmitting medium everywhere on the interface between the two media. This concept is known as "wavefront continuity." Write expressions for the number of waves per unit length along the interface in terms of  $\theta_i$  and  $\lambda_i$  for the incident wave, and in terms of  $\theta_t$  and  $\lambda_t$  for the transmitted wave. Use these two expressions to derive Snell's law of refraction.

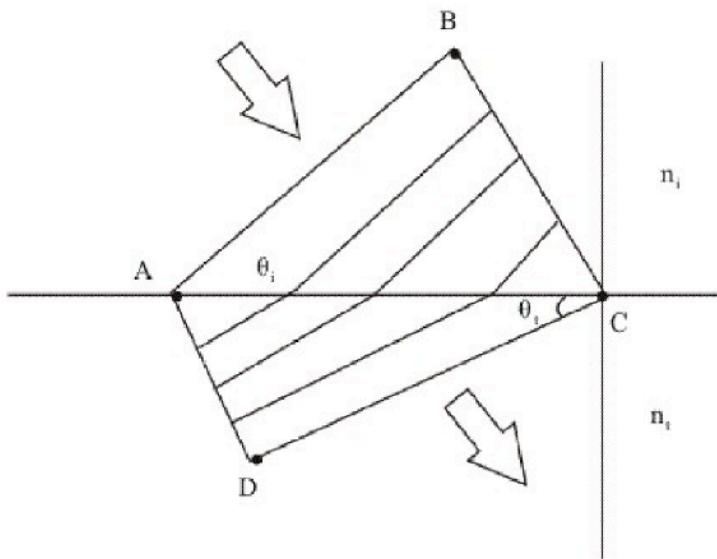


Figure 1: Incident and transmitted plane wavefronts at a flat interface between media with different indices of refraction  $n_i$  and  $n_t$ . The angles of incidence and of refraction are  $\theta_i$  and  $\theta_t$ .

**Problem 2) Swimming Pool:** Prove that to someone looking straight down into a swimming pool, any object in the water will appear to be at  $3/4$  of its true depth.

**Problem 3) An application to the human eye:** A device used to measure the radius of curvature of the cornea of the eye is called a keratometer. This is useful information for fitting contact lenses, for example. An illuminated object is placed a known distance from the eye, and the instrument observes the image as it is reflected back off the cornea. The instrument measures the size of this (virtual) image, and compares it with the size of the original image to obtain a magnification. If the measured magnification is a factor of 0.037 when the object distance is 100mm, what is the radius of curvature of the cornea?

**Problem 4) The Moon:** A ground-based telescope has a concave spherical mirror with radius of curvature of 8 meters. Find the location and diameter of the image of the Moon formed by this mirror. Draw a sketch to illustrate your result.

**Problem 5) The Moon and Jupiter:**

- a) Using the Rayleigh criterion (spatial resolution in radians =  $1.22 \lambda / D$ ), estimate the angular resolution of the human eye at a wavelength of 500 nm. Assume that the diameter of the pupil is 5 mm.
- b) Compare your answer in part a) to the angular diameters of the Moon and Jupiter seen from Earth.
- c) What can you conclude about the ability of the human eye to resolve the Moon's disk and Jupiter's disk with the unaided eye?

**Problem 6) The Keck Telescopes:**

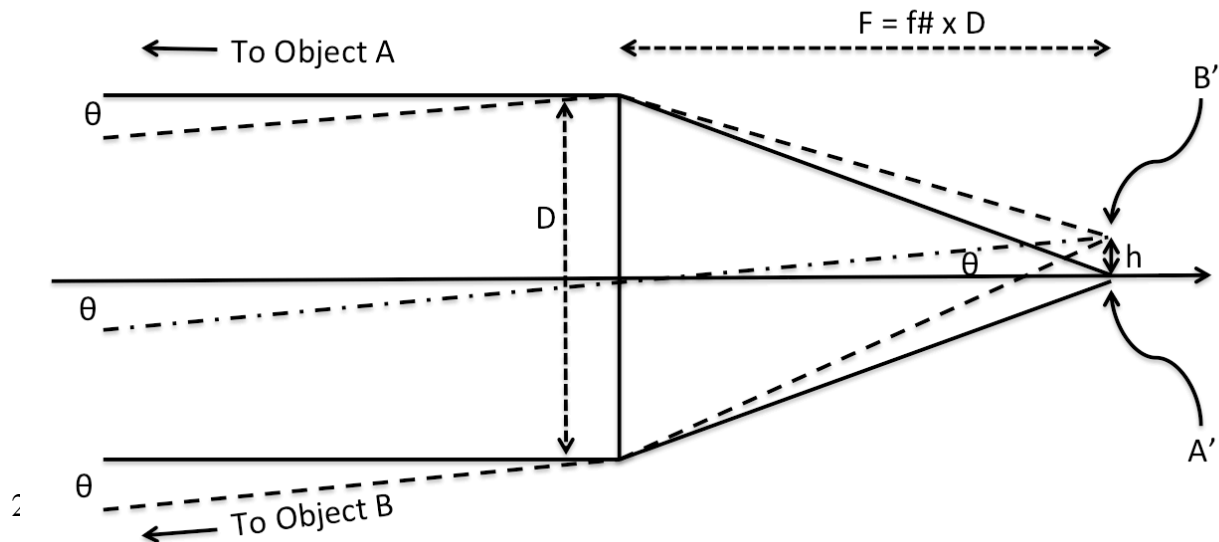
The two Keck Telescopes are each hexagonal, 10 meters in diameter, and have 17.5 meter focal length.

Because they have hexagonal primary mirrors, we need to be careful how we define the mirror "diameter". So let's simplify by approximating the hexagons by circular mirrors of diameter 10 meters.

Cassegrain telescopes have both a primary mirror and a secondary mirror, each with different shapes and different focal lengths. But it is possible to define an "equivalent focal length" or an "equivalent f-number" that describes the optical system as a whole.

This problem refers to the most commonly used secondary mirror at Keck, for which the "equivalent f-number" of the optical system as a whole is  $f/15$ .

Consider the following diagram (not to scale):



a) Plate Scale:

i) Using the geometry shown in the Figure, write an expression for the ratio of  $\theta$ , the angular separation of objects A and B on the sky, to the height  $h$  of their image in the focal plane. In other words what is  $\theta / h$ , where the units of  $\theta$  are radians and the units of  $h$  are meters? You may assume that  $\theta$  is small, so that  $\sin \theta \sim \theta$  and  $\cos \theta \sim 1$ .

ii) What is the ratio  $\theta / h$  where the units of  $\theta$  are seconds of arc and the units of  $h$  are mm? [Since a full circle is  $2\pi$  radians = 360 deg, one arc second  $\approx 4.85$  microradians.]

b) At an observing wavelength of 2 microns, how many resolution elements ( $\lambda / D$ ) would there be within one second of arc?

**Problem 7) Diffraction:**

A 1-mm-diameter hole is illuminated by plane waves of 546-nm light. According to the diffraction criteria, which techniques (near-field, Fresnel, Fraunhofer, far-field, etc.) may be applied to the diffraction problem when the detector is at 50 cm, 1 m, and 5 m from the aperture?

Credit: many of these problems are based on those in Hecht's "Optics" textbook, 4th edition