Astro 289—Adaptive Optics and its Applications – Winter 2016
Homework 3: Kolmogorov turbulence and its effects

Due Tuesday February 9th before class.

Optical effects of atmospheric turbulence: Many of the derivations that we did in class went by quickly. In these problems I'm asking you to go over the derivations and "make them your own."

1) In an equation and, equally importantly, in qualitatively descriptive words, define each of the following. Please use as many words as you'd like, including your own musings and questions about the meaning and utility of each quantity. Draw a sketch illustrating each one. How are they related to each other?

a) the structure function

b) the correlation function

c) the covariance function

d) the spatial coherence function

2) Derive the relationship between the structure function and the covariance function:

$$D_\phi(\vec{r}) = 2\left[B_\phi(0) - B_\phi(\vec{r})\right]$$

where $D_\phi(\vec{r})$ is the structure function and $B_\phi(\vec{r})$ is the covariance function.

Show all the steps in your derivation. Hint: expand the product in the definition of $D_\phi(r)$ and assume homogeneity to take the averages.

3) I'd like you to have for your future reference a complete derivation of the famous expression for the Fried parameter r_0. Please go back and review the notes by Quirrenbach and the slides in Lecture 5.

Based on the slides and on the derivations in the reading, write up for yourself a full derivation of r_0 with ***all*** the intermediate steps filled in and ***all*** the assumptions explicitly stated. You should feel free to add your own editorial comments, such as "now Claire assumes *blah blah* but this seems arbitrary to me." This exercise will serve two purposes: it will tell me which parts I didn't explain well enough, and it will serve as a useful future reference for you.

4) Evaluate the coherence length r_0 and the isoplanatic angle θ_0 for the following three turbulence profiles. For each turbulence profile, show r_0 and θ_0 for two wavelengths: 0.5 μm and 2 μm. Be sure to state your units at each step of your derivations.

a) From h=0 to h=1 km, $C_N^2(h) = 5 \times 10^{-16}\,\mathrm{m}^{-2/3}$. From h =1 km to top of atmosphere, $C_N^2(h) = 0$. sec $\zeta = 1$ (zenith angle of 0).

b) From h=0 to h=1 km, $C_N^2(h) = 2 \times 10^{-16}\,\mathrm{m}^{-2/3}$. From h =1 km to 12 km, $C_N^2(h) = 10^{-19}\,\mathrm{m}^{-2/3}$. sec $\zeta = 1$ (zenith angle of 0).

c) Same as b) but zenith angle of 50 deg (elevation angle of 40 deg above horizon).

For what telescope diameter D would λ / r_0 just equal the diffraction limit in each of the above three cases?

Notes:
1) Be careful. The usual unit for r_0 is cm. The usual unit for C_N^2 is $\mathrm{m}^{-2/3}$. The usual unit for altitude is km. The usual unit for θ_0 is arc seconds. So there's plenty of room for units-confusion here. Give your final answers for r_0 in cm and for θ_0 in seconds of arc on the sky.
2) Remember that these are real integrals, so not just sums.