Optical effects of atmospheric turbulence: Many of the derivations that we did in class went by quickly. In these problems I’m asking you to go over the derivations and “make them your own.”

1) In an equation and, equally importantly, in qualitatively descriptive words, define each of the following. Please use as many words as you’d like, including your own musings and questions about the meaning and utility of each quantity. Draw a sketch illustrating each one. How are they related to each other?
   a) the structure function
   b) the correlation function
   c) the covariance function
   d) the spatial coherence function

2) Derive the relationship between the structure function and the covariance function:

   \[ D_\phi(\vec{r}) = 2\left[ B_\phi(0) - B_\phi(\vec{r}) \right] \]

   where \( D_\phi(\vec{r}) \) is the structure function and \( B_\phi(\vec{r}) \) is the covariance function.

   Show all the steps in your derivation. **Hint:** expand the product in the definition of \( D_\phi(r) \) and assume homogeneity to take the averages.

3) I’d like you to have for your future reference a complete derivation of the famous expression for the Fried parameter \( r_0 \). Please go back and review the notes by Quirrenbach and the slides in Lecture 5.

   Based on the slides and on the derivations in the reading, write up for yourself a full derivation of \( r_0 \), with all the intermediate steps filled in and all the assumptions explicitly stated. You should feel free to add your own editorial comments, such as “now Claire assumes blah blah but this seems arbitrary to me.” This exercise will serve two purposes: it will tell me which parts I didn’t explain well enough, and it will serve as a useful future reference for you.
4) Evaluate the coherence length $r_0$ and the isoplanatic angle $\theta_0$ for the following three turbulence profiles. For each turbulence profile, show $r_0$ and $\theta_0$ for two wavelengths: 0.5 $\mu$m and 2 $\mu$m. Be sure to state your units at each step of your derivations.

a) From h=0 to h=1 km, $C_N^2(h) = 5 \times 10^{-16} \text{ m}^{-2/3}$. From h =1 km to top of atmosphere, $C_N^2(h) = 0$. sec $\zeta = 1$ (zenith angle of 0).

b) From h=0 to h=1 km, $C_N^2(h) = 2 \times 10^{-16} \text{ m}^{-2/3}$. From h =1 km to 12 km, $C_N^2(h) = 10^{-19} \text{ m}^{-2/3}$. sec $\zeta = 1$ (zenith angle of 0).

c) Same as b) but zenith angle of 50 deg (elevation angle of 40 deg above horizon).

For what telescope diameter $D$ would $\lambda / r_0$ just equal the diffraction limit in each of the above three cases?

Notes:
1) Be careful. The usual unit for $r_0$ is cm. The usual unit for $C_N^2$ is $\text{ m}^{-2/3}$. The usual unit for altitude is km. The usual unit for $\theta_0$ is arc seconds. So there’s plenty of room for units-confusion here. Give your final answers for $r_0$ in cm and for $\theta_0$ in seconds of arc on the sky.
2) Remember that these are real integrals, so not just sums.