Lecture 12
Part 2: AO System Optimization

Claire Max
Astro 289, UC Santa Cruz
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Optimization of AO systems

• If you are designing a new AO system:
  - How many actuators?
  - What kind of deformable mirror?
  - What type of wavefront sensor?
  - How fast a sampling rate and control bandwidth (peak capacity)?

• If you are using an existing AO system:
  - How long should you integrate on the wavefront sensor? How fast should the control loop run?
  - Is it better to use a bright guide star far away, or a dimmer star close by?
  - What wavelength should you use to observe?
Issues for designer of astronomical AO systems

• **Performance goals:**
  - Sky coverage fraction, observing wavelength, degree of image compensation needed for science program

• **Parameters of the observatory:**
  - Turbulence characteristics (mean and variability), telescope and instrument optical errors, availability of laser guide stars

• **AO parameters chosen in the design phase:**
  - Number of actuators, wavefront sensor type and sample rate, servo bandwidth, laser characteristics

• **AO parameters adjusted by user:** integration time on wavefront sensor, wavelength, guide star mag. & offset
Example: Keck Observatory AO “Blue Book”

- Made scientific case for Keck adaptive optics system
- Laid out the technical tradeoffs
- Presented performance estimates for realistic conditions
- First draft of design requirements

The basis for obtaining funding commitment from the user community and observatory
What is in the Keck AO Blue Book?

- Chapter titles:
  1. Introduction
  2. Scientific Rationale and Objectives
  4. Limitations and Expected Performance of Adaptive Optics at Keck
  5. Facility Design Requirements

- Appendices: Technical details and overall error budget
Other telescope projects have similar “Books”

- Keck Telescope (10 m):
  - Had a “Blue Book” for the telescope concept itself

- Thirty Meter Telescope:

These documents are the kick-off point for work on the “Preliminary Design”
First, look at individual terms in error budget one by one

- Error budget terms
  - Fitting error
  - WFS measurement error
  - Anisoplanatism
  - Temporal error

- Figures of merit
  - Strehl ratio
  - FWHM
  - Encircled energy
  - Strehl ratio
Deformable mirror fitting error only

\[ S = \exp\left(-\sigma^2_\phi\right) = \exp\left[-0.28 \left(\frac{d}{r_0}\right)^{5/3}\right] \]

\[ r_0(\lambda) = r_0(\lambda = 0.5 \, \mu m) \left(\frac{\lambda}{0.5 \, \mu m}\right)^{6/5} \]

\[ S = \exp\left[-0.28 \left(\frac{d}{r_0(\lambda = 0.5 \, \mu m)}\right)^{5/3} \left(\frac{0.5 \, \mu m}{\lambda}\right)^2\right] \]

- Assume very bright natural guide star
- No meas’t error or anisoplanatism or bandwidth error

Strehl increases for smaller subapertures and shorter observing wavelengths
Strehl increases for smaller subapertures and longer observing wavelengths

- Assume very bright natural guide star
- No meas’t error or anisoplanatism or band-width error

Deformable mirror fitting error only
Strehl increases for longer $\lambda$ and better seeing (larger $r_0$)

- Assume very bright natural guide star
- No meas’t error or anisoplanatism or bandwidth error

Deformable mirror fitting error only
Wavefront sensor measurement error: Strehl vs $\lambda$ and guide star magnitude

Assumes no DM fitting error or other error terms

$$\sigma_{S-H}^2 \approx \left( \frac{6.3}{SNR} \right)^2$$

$$S = \exp(-\sigma_{S-H}^2) = \exp \left[ -\left( \frac{6.3}{SNR} \right)^2 \right]$$

$SNR$ increases as flux from guide star increases

**Strehl increases for brighter guide stars**

But: $SNR$ will decrease as you use more and more subapertures, because each one will gather less light
**Strehl increases for brighter guide stars**

Assumes no DM fitting error or other error terms

Keck 10 m, d = 66 cm (271 act. continuous face sheet hexagonal)
r0 = 20 cm, theta0 = 20 ur, fg = 50 Hz
natural guide star

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Decreasing measurement error
\[ S = \exp\left[-\sigma_{iso}^2\right] = \exp\left[-\left(\frac{\theta}{\theta_0}\right)^{5/3}\right], \quad \theta_0 = \frac{r_0}{h} \propto \lambda^{6/5} \]

\[ S = \exp\left[-\left(\frac{\theta}{\theta_0(0.5 \mu m)}\right)^{5/3} \left(\frac{0.5 \mu m}{\lambda}\right)^2\right] \]

Strehl increases for smaller angular offsets and longer observing wavelengths
Strehl increases for smaller angular offsets and longer observing wavelengths

Keck 10 m, d = 66 cm (271 act. continuous face sheet hexgeom.)

r₀ = 20 cm, θ₀ = 20 ur, f₀ = 50 Hz

n₀ = 4 natural guide star

Want to learn more? Visit [CfA](http://www.cfa.harvard.edu).
PSF with bright guide star: more degrees of freedom ⇒ more energy in core

Point Spread Function very bright star, $\lambda = 2.2$ $\mu$m, $D / r_0 = 8.5$
What matters for spectroscopy is “Encircled Energy”

Fraction of light encircled within diameter of xx arc sec

Keck 10 meter, 271 actuators (d=66 cm)
seeing = 0.5 arcsec (r0 = 20 cm at 0.5 microns)

Encircled Energy Fraction

Diameter (arcsec)

Encircled Energy Fraction

Diffraction limited
Overall system optimization

- Concept of error budget
  - Independent contributions to wavefront error from many sources

- Minimize overall error with respect to a parameter such as integration time or subaperture size
Error model: mean square wavefront error is sum of squares of component errors

- Mean square error in wavefront phase

\[ \sigma_p^2 = \sigma_{Meas}^2 + \sigma_{BW}^2 + \sigma_{DM}^2 + \sigma_{iso}^2 + \sigma_{T-T}^2 + \ldots \]

\[ \sigma_{Meast}^2 = \sigma_{S-H}^2 \approx \left[ \frac{3.5 \theta_b}{SNR} \frac{d}{\lambda} \right]^2 \]
Signal to Noise Ratio for a fast CCD detector

\[ SNR = \frac{Flux \times T_{\text{int}}}{\text{Noise}} = \frac{Flux \times T_{\text{int}}}{\sigma_{\text{PhotonNoise}}^2 + \sigma_{\text{SkyBkgnd}}^2 + \sigma_{\text{DarkCurrent}}^2 + \sigma_{\text{ReadNoise}}^2}^{1/2} \]

- **Flux** is the average photon flux (detected photons/sec)
- **\( T_{\text{int}} \)** is the integration time of the measurement,
- **Sky background** is due to OH lines and thermal emission
- **Dark current** is detector noise per sec even in absence of light (usually due to thermal effects)
- **Read noise** is due to the on-chip amplifier that reads out the charge after each exposure
Short readout times needed for wavefront sensor ⇒ read noise is usually dominant

• **Read-noise dominated**: read noise >> all other noise sources

• In this case SNR is

\[
SNR_{RN} = \frac{Flux \times T_{int}}{\left[ R^2 n_{pix} \right]^{1/2}} = \frac{Flux \times T_{int}}{R \sqrt{n_{pix}}}
\]

where \( T_{int} \) is the integration time, \( n_{pix} \) is the number of pixels in a subaperture, \( R \) is the read noise/px/frame
Now, back to calculating measurement error for Shack-Hartmann sensor

\[ \sigma_{S-H}^2 \cong \left[ \frac{\pi}{4\sqrt{2}} \frac{1}{SNR} \vartheta_b \frac{2\pi d}{\lambda} \right]^2 \text{rad}^2 \]

- Assume the WFS is read-noise limited. Then

\[ SNR_{S-H} = \frac{Flux \times T_{\text{int}}}{R\sqrt{n_{\text{pix}}}} \text{ and} \]

\[ \sigma_{S-H}^2 = \left[ \frac{3.5\vartheta_b d}{SNR \lambda} \right]^2 = \left[ \frac{3.5\vartheta_b d}{\lambda} \right]^2 \frac{R^2n_{\text{pix}}}{(Flux \times T_{\text{int}})^2} \]
Error model: mean square wavefront error is sum of squares of component errors

\[ \sigma_p^2 = \sigma_{Meast}^2 + \sigma_{BW}^2 + \sigma_{DM}^2 + \sigma_{iso}^2 + \ldots \]

\[ \sigma_p^2 = \left[ 3.5 \theta_b \frac{d}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(Flux \times T_{int})^2} + \left( \frac{T_{control}}{\tau_0} \right)^{5/3} + \mu \left( \frac{d}{r_0} \right)^{5/3} + \left( \frac{\theta}{\theta_0} \right)^{5/3} + \ldots \]

**Flux** in a subaperture will increase with subap. area \( d^2 \)

\( T_{control} \) is the closed-loop control timescale, typically \( \sim 10 \) times the integration time \( T_{int} \) (control loop gain isn’t unity, so must sample many times in order to converge)
Integration time trades temporal error against measurement error

\[ \sigma_p^2 \approx \left[ 3.5 \theta_b \frac{d}{\lambda} \right]^2 \left( \frac{R^2 n_{\text{pix}}}{(\text{Flux} \times T_{\text{int}})^2} \right) + \left( \frac{10 T_{\text{int}}}{\tau_0} \right)^{5/3} \]

From Hardy, Fig. 9.23
**First exercise in optimization: Choose optimum integration time**

- Minimize the sum of read-noise and temporal errors by finding optimal integration time

\[
\sigma_p^2 = \left[ 3.5 \theta_b \frac{d}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(Flux \times T_{int})^2} + \left( \frac{10T_{int}}{\tau_0} \right)^{5/3}
\]

\[
\frac{d\sigma_p^2}{dT_{int}} = 0 = -2 \left[ 3.5 \theta_b \frac{d}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(Flux^2 T_{int}^3)} + \frac{5}{3} \left( \frac{10}{\tau_0} \right)^{5/3} T_{int}^{2/3}
\]

\[
T_{int}^{opt} = \left[ 0.32 \tau_0^{5/3} \left( \frac{\theta_b d}{\lambda} \right)^2 \frac{R^2 n_{pix}}{(Flux^2)} \right]^{3/11}
\]

- Sanity check: optimum $T_{int}$ larger for long $\tau_0$, larger read noise $R$, and lower photon $Flux$
Similarly, subaperture size $d$ trades fitting error against measurement error

\[ \sigma^2 = \left[ 3.5 \theta_b \frac{d}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(Flux \times T_{int})^2} + \mu \left( \frac{d}{r_0} \right)^{5/3} \]

Flux = $I \times area = I \times d^2$ where intensity $I$ = photons/(sec cm$^2$)

\[ \sigma^2 = \left[ 3.5 \theta_b \frac{d^2}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(I \times d^2 \times T_{int})^2} + \mu \left( \frac{d}{r_0} \right)^{5/3} \]

\[ \sigma^2 = \left[ 3.5 \theta_b \frac{1}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(I \times T_{int})^2 d^2} + \mu \left( \frac{d}{r_0} \right)^{5/3} \]

Hardy, Figure 9.25

- Smaller $d$: better fitting error, worse measurement error
Solve for optimum subaperture size \( d \)

\[
\sigma_\phi^2 = \left[ 3.5 \theta_b \frac{d}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(Flux \times T_{int})^2} + \mu \left( \frac{d}{r_0} \right)^{5/3}
\]

\( Flux = I \times area = I \times d^2 \) where intensity \( I = \) detected photons/(sec cm\(^2\))

\[
\sigma_\phi^2 = \left[ 3.5 \theta_b \frac{d}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(I \times d^2 \times T_{int})^2} + \mu \left( \frac{d}{r_0} \right)^{5/3} = \left[ 3.5 \theta_b \frac{1}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(I \times T_{int})^2 d^2} + \mu \left( \frac{d}{r_0} \right)^{5/3}
\]

Set derivative of \( \sigma_\phi^2 \) with respect to subaperture size \( d \) equal to zero:

\[
d_{opt} = \left\{ \frac{6}{5} \frac{r_0^{5/3}}{\mu} \left[ \frac{3.5 \theta_b}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(I \times T_{int})^2} \right\}^{3/11}
\]

\( d_{opt} \) is larger if \( r_0, \) read noise, and \( n_{pix} \) are larger, and if \( T_{int} \) and \( I \) are smaller
# Keck 2 AO error budget example

(bright TT star)

<table>
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<th>LGS (10th mag TT star) Case</th>
<th>Wavefront Error (nm)</th>
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<td>Atmospheric Fitting Error</td>
<td>110</td>
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<tr>
<td>Bandwidth Error</td>
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<td>High-order Measurement Error</td>
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<td>18</td>
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<tr>
<td>Residual Tel Pointing Jitter (1-axis)</td>
<td>96</td>
</tr>
</tbody>
</table>

Total Wavefront Error (nm) = 370
Strehl at K-band = 0.3

### Assumptions

- Zenith angle (deg) 10
- Guide star magnitude 10
- HO WFS Rate (Hz) 500
- TT Rate (Hz) 1000
- Laser power (W) 16
- WFS camera CCD39
- Subaperture diameter (m) 0.5625
- Science Instrument NIRC2
- Amplitude of vibrations (arcsec) 0.2
- d0 (m) 2.41
- TT sensor STRAP APDs
Summary: What can you optimize when?

• Once telescope is built on a particular site, you don’t have control over $\tau_0, \theta_0, r_0$

• But when you build your AO system, you CAN optimize choice of subaperture size $d$, maximum AO system speed, range of observing wavelengths, sky coverage, etc.

• Even when you are observing with an existing AO system, you can optimize:
  - wavelength of observations (changes fitting error)
  - integration time of wavefront sensor $T_{int}$
  - tip-tilt bandwidth
  - brightness and angular offset of guide star