Geometrical Optics for AO

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(slides originally from Claire Max)
Goals of this lecture

• Review of Geometrical Optics
  - Understand the tools used for optical design of AO systems
  - Understand what wavefront aberrations look like, and how to describe them
  - Characterization of the aberrations caused by turbulence in the Earth’s atmosphere

• Application to the layout of an AO system
Keck AO system optical layout: Why on earth does it look like this ??
Keck AO system optical layout:
Why on earth does it look like this??
Optical elements are portrayed as transmitting, for simplicity: they may be lenses or mirrors
What optics concepts are needed for AO?

- Design of AO system itself:
  - What determines the size and position of the deformable mirror? Of the wavefront sensor?
  - What does it mean to say that “the deformable mirror is conjugate to the telescope pupil”?
  - How do you fit an AO system onto a modest-sized optical bench, if it’s supposed to correct an 8-10m primary mirror?

- What are optical aberrations? How are aberrations induced by atmosphere related to those seen in lab?
Levels of models in optics

- Geometric optics - rays, reflection, refraction
- Physical optics (Fourier optics) - diffraction, scalar waves
- Electromagnetics - vector waves, polarization
- Quantum optics - photons, interaction with matter, lasers
Review of geometrical optics: lenses, mirrors, and imaging

- Rays and wavefronts
- Laws of refraction and reflection
- Imaging
  - Pinhole camera
  - Lenses
  - Mirrors
- Diffraction limit (a heuristic derivation)

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Rays and wavefronts

- A wavefront is a surface of constant phase
Rays and wavefronts

In homogeneous media, light propagates in straight lines.
Spherical waves and plane waves
Index of refraction: determines propagation speed in a medium

- Index of refraction \( n \)

- Phase velocity \( V_\phi = \frac{c}{n} \)
  - Speed of sinusoidal phase maxima

- In solid media like glass, \( n > 1 \) \( \Rightarrow \) \( V_\phi < c \)
## Examples of index of refraction in media

<table>
<thead>
<tr>
<th>Substance</th>
<th>Index of Refraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.00029</td>
</tr>
<tr>
<td>Water</td>
<td>1.31</td>
</tr>
<tr>
<td>Fused silica (SiO$_2$)</td>
<td>1.46</td>
</tr>
<tr>
<td>Crown glass</td>
<td>1.52</td>
</tr>
<tr>
<td>ZnSe (10.6 µm)</td>
<td>2.40</td>
</tr>
</tbody>
</table>

- Quite a large variation, even among common substances
Huygens’ Principle

• Every point in a wavefront acts as a little secondary light source, and emits a spherical wave

• The propagating wave-front is the result of superposing all these little spherical waves

• Destructive interference in all but the direction of propagation
Refraction at a surface: Snell’s Law

- Snell’s law: \( n \sin \theta = n' \sin \theta' \)

Medium 1, index of refraction \( n \)

Medium 2, index of refraction \( n' \)
The wave picture of refraction

- If $n_t > n_i$, phase velocity is slower in the transmitting medium.
- Distance propagated in time $\Delta t$ is shorter in transmitting medium.

\[ n_i \sin \theta_i = n_t \sin \theta_t \]

- Credit: Hecht, “Optics”
Reflection at a surface

- Angle of incidence equals angle of reflection
The wave picture of reflection

- Atoms at surface re-radiate the EM fields
- The re-radiated waves undergo destructive interference, except in direction where $\theta_i = \theta_r$
- Credit: Hecht
Concept Question

- You want to buy a full-length mirror for your bedroom, but they are all too expensive
- What is the length of the smallest vertical planar mirror in which you can see your entire standing body all at once?
- How should it be positioned?

Hint:
- Draw a picture, and use similar triangles
Concept Question

• You want to buy a full-length mirror for your bedroom, but they are all too expensive

• What is the length of the smallest vertical planar mirror in which you can see your entire standing body all at once?

• How should it be positioned?

A mirror half your height with its upper edge lowered by half the distance between your eye and the top of your head serves nicely.
Why are imaging systems needed?

- Every point in the object scatters an incident light into a spherical wave.
- The spherical waves from all the points on the object’s surface get mixed together as they propagate toward you.
- An imaging system reassigns (focuses) all the rays from a single point on the object onto another point in space (the “focal point”), so you can distinguish details of the object.
Pinhole camera is simplest imaging instrument

- Opaque screen with a pinhole blocks all but one ray per object point from reaching the image space.
- An image is formed (upside down). Good news.
- BUT most of the light is wasted (it is stopped by the opaque sheet)
- Also, diffraction of light as it passes through the small pinhole produces artifacts in the image.
Imaging with lenses: doesn’t throw away as much light as pinhole camera

Collects all rays that pass through solid-angle of lens
“Paraxial approximation” or “first order optics” or “Gaussian optics”

- Angle of rays with respect to optical axis is small
- First-order Taylor expansions:
  - $\sin \theta \sim \tan \theta \sim \theta$,  $\cos \theta \sim 1$,  $(1 + x)^{1/2} \sim 1 + x / 2$
Thin lenses, part 1

Definition: *f*-number: \( f / \# = f / D \)

D = lens diam.
Thin lenses, part 2

D = lens diam.

point image at 2nd FP

focal length f

plane wave (or parallel ray bundle);
object at infinity
Sign conventions for refraction

- Light travels from left to right
- A radius of curvature is positive if the surface is convex towards the left
- Longitudinal distances are positive if pointing to the right
- Lateral distances are positive if pointing up
- Ray angles are positive if the ray direction is obtained by rotating the $+z$ axis counterclockwise through an acute angle
Refraction and the Lens-users Equation

- Any ray that goes through the focal point on its way to the lens will come out parallel to the optical axis. (ray 1)

Credit: J. Holmes, Christian Brothers Univ.
- Any ray that starts out parallel to the optical axis must go through the focal point on the other side of the lens. (ray 2)
- Any ray that goes through the center of the lens must go essentially undeflected. (ray 3)
Refraction and the Lens-users Equation

- Note that a **real** image is formed (image is on opposite side of the lens from the object)
- Note that the image is **up-side-down**.
By looking at ray 3 alone, we can see by similar triangles that \[ M = \frac{h'}{h} = -\frac{i}{o} \]

Example: \( f = 10 \text{ cm}; \quad o = 40 \text{ cm}; \quad i = 13.3 \text{ cm} \): \[
M = \frac{-13.3}{40} = -0.33
\]

Note \( h' \) is up-side-down and so is \( i < 0 \)
The Thin Lens Equation

\[ \frac{1}{o} + \frac{1}{i} = \frac{1}{f} \]

- Memorize this equation and the above picture. It is almost all the information needed to do first order optics calculations.
Focal length of mirrors

- Focal length of spherical mirror is \( f_{sp} = - \frac{R}{2} \)
- Convention: \( f \) is positive if it is to the left of the mirror
- Near the optical axis, parabola and sphere are very similar, so that 
  \( f_{par} = - \frac{R}{2} \) as well.
Imaging with mirrors: spherical and parabolic mirrors

\[ f = -\frac{R}{2} \]

Spherical surface: in paraxial approx, focuses incoming parallel rays to (approx) a point

Parabolic surface: perfect focusing for parallel rays (e.g. satellite dish, radio telescope)
Mirror equations

- Imaging condition for spherical mirror
  \[ \frac{1}{s_0} + \frac{1}{s_1} = -\frac{2}{R} \]

- Focal length
  \[ f = -\frac{R}{2} \]

- Magnifications
  \[ M_{\text{transverse}} = -\frac{s_0}{s_1} \]
  \[ M_{\text{angle}} = -\frac{s_1}{s_0} \]
Sign conventions for reflection

• Light travels from left to right before reflection and from right to left after reflection
• A radius of curvature is positive if the surface is convex towards the left
• Longitudinal distances before reflection are positive if pointing to the right; longitudinal distances after reflection are positive if pointing to the left
• Longitudinal distances are positive if pointing up
• Ray angles are positive if the ray direction is obtained by rotating the +z axis counterclockwise through an acute angle
Refracting telescope: **two lenses whose focal points coincide**

- Main point of telescope: to gather more light than eye. Secondly, to magnify image of the object

- **Magnifying power** $M_{tot} = -\frac{f_{Objective}}{f_{Eyepiece}}$ so for high magnification, make $f_{Objective}$ as large as possible (long tube) and make $f_{Eyepiece}$ as short as possible

$$\frac{1}{f_{obj}} = \frac{1}{s_0} + \frac{1}{s_1} \approx \frac{1}{s_1} \quad \text{since } s_0 \to \infty$$

$$s_1 \approx f_{obj}$$
Optical invariant ( = Lagrange invariant)

\[ y_1 \theta_1 = y_2 \theta_2 \]
Lagrange invariant has important consequences for AO on large telescopes

- Deformable mirror is much smaller than primary mirror
- Hence angles within AO system are much larger
- Consequences: limitations on field of view; vignetting

From Don Gavel
Geometric optics for telescopes

Diameter = D

Focal length = F

F ratio = F/D

F gives the plate scale at the focal plane (ratio between physical dimension in focal plane and angle on the sky): \( \delta = \text{angle} \times F \)

F/D gives physical size of diffraction limit at the focal plane = \((F/D) \lambda\)

Afocal telescope (= beam reducer)

Pupil plane (entrance pupil)

Focal plane

Pupil plane (exit pupil)

Instrument
Concept Question

- What happens to the images of a telescope (hypothetically) if the scale of the whole system is doubled?

Make sketches to illustrate your reasoning
A look ahead to Fourier Optics: Heuristic derivation of the diffraction limit

Uncertainty principle

Photon momentum

\[ p = \frac{h}{\lambda} \]

\[ \Delta x \Delta p \geq \hbar \]

\[ \Delta x = \infty \]

\[ \Delta p = 0 \]

Law of diffraction

\[ \theta \equiv \frac{\Delta p}{p} = \frac{\lambda}{D} \]
Time for a short break

- Please get up and move around!
Lick Observatory’s 36” Refractor: one long telescope!
Reflective telescopes

Newton, 1668
Concept Question

- Why were early telescopes (when optics could not be as precisely fabricated) refractive, while modern telescopes are reflective?
Cassegrain reflecting telescope

- Hyperbolic secondary mirror:
  - shortens physical length of telescope.
  - Provides accessible focus
  - Used to improve off-axis aberrations
Aberrations

- In optical systems
- In atmosphere
- Description in terms of Zernike polynomials

- Based on slides by Brian Bauman, LLNL and UCSC, and Gary Chanan, UCI
Third order aberrations

- $sin \theta$ terms in Snell’s law can be expanded in power series
  \[ n \sin \theta = n' \sin \theta' \]
  \[ n \left( \theta - \theta^3/3! + \theta^5/5! + \ldots \right) = n' \left( \theta' - \theta'^3/3! + \theta'^5/5! + \ldots \right) \]

- Paraxial ray approximation: keep only $\theta$ terms (first order optics; rays propagate nearly along optical axis)
  - Piston, tilt, defocus

- Third order aberrations: result from adding $\theta^3$ terms
  - Spherical aberration, coma, astigmatism, …..
Different ways to illustrate optical aberrations

Side view of a fan of rays
(No aberrations)

“Spot diagram”: Image at different focus positions

Strike hypothetical detector
Spherical aberration

Rays from a spherically aberrated wavefront focus at different planes

Through-focus spot diagram for spherical aberration
Hubble Space Telescope suffered from Spherical Aberration

- In a Cassegrain telescope, the hyperboloid of the primary mirror must match the specs of the secondary mirror. For HST they didn’t match.
Profiles of HST f/30 planetary camera normalized to the same peak brightness for $\lambda = 0.57 \, \mu m$. The FWHM of the core is 0.1” in both cases, but only 15% is contained in the spherically aberrated image core.
Optical Images: Hubble space telescope, optics before and after repair

stars in the 30 Doradus region of the large Magellanic cloud

best image of earth bound telescopes

images of the HST before and after repair

point spread function with and without COSTAR optics

Source: http://www.seds.org/hst/; Optics and Photonics News, Vol.4 No.11 November 1993
Effect of a Parabolic Mirror

- Off-axis beam sees a “tilted” parabola.
- Resulting image looks like it has a “comet-like” tail.

[Image: www.telescope-optics.net]
Coma

- “Comet”-shaped spot
- Chief ray is at apex of coma pattern
- Centroid is shifted from chief ray!
- Centroid shifts with change in focus!

Wavefront
Coma

Rays from a comatic wavefront

Through-focus spot diagram for coma

Note that centroid shifts:
Effects of a title optic: Astigmatism

Figure 1.16 Astigmatism represented by sectional views

Credit: Melles-Griot
Astigmatism

Top view of rays

Side view of rays

Through-focus spot diagram for astigmatism
Wavefront for astigmatism
Where does astigmatism come from?

- When a cone of light enters a lens surface obliquely, it extends over more surface in the “y” direction than in the “x” direction.
- This will introduce more power in the “y” direction than in the “x” direction.
- The result is that the “y” or tangential ray fan will focus closer to the lens than the “x” or sagittal ray fan.
- This is astigmatism.

From Ian McLean, UCLA
Concept Question

• How do you suppose eyeglasses correct for astigmatism?
Zernike Polynomials

- Convenient basis set for expressing wavefront aberrations over a circular pupil
- Zernike polynomials are orthogonal to each other
- A few different ways to normalize - always check definitions!
Expansion of the Phase in Zernike Polynomials

An alternative characterization of the phase comes from expanding $\varphi$ in terms of a complete set of functions and then characterizing the coefficients of the expansion:

$$\varphi (r,\theta) = \Sigma a_{m,n} Z_{m,n}(r,\theta)$$

\[
\begin{align*}
Z_{0,0} &= 1 & \text{piston} \\
Z_{1,-1} &= 2 r \sin\theta & \text{tip/tilt} \\
Z_{1,1} &= 2 r \cos\theta & \\
Z_{2,-2} &= \sqrt{6} r^2 \sin2\theta & \text{astigmatism} \\
Z_{2,0} &= \sqrt{3} (2r^2 - 1) & \text{focus} \\
Z_{2,2} &= \sqrt{6} r^2 \cos2\theta & \text{astigmatism}
\end{align*}
\]
From G. Chanan

- Piston
- Tip-tilt
- $Z_{0,0}$
- $Z_{1,-1}$
- $Z_{1,1}$
Astigmatism (3rd order)

$Z_{2,-2}$

$Z_{2,2}$

Defocus

$Z_{2,0}$
Spherical

"Ashtray"

Z_{4,-4}

Z_{4,0}

Astigmatism (5th order)

Z_{4,-2}

Z_{4,2}
Random Zernikes
Tip-tilt is single biggest contributor

Focus, astigmatism, coma also big

Reference: Noll
Aberrations in two populations of 70 normal eyes for 7.5 mm pupil

$\text{Logio (Wavefront Variance)}$

$\text{microns}^2$

Diffraction limit
($\lambda = 0.6 \text{ um}$)

Zernike Order

References for Zernike Polynomials

• **Pivotal Paper:** Noll, R. J. 1976, “Zernike polynomials and atmospheric turbulence”, JOSA 66, page 207

• **Books:**
  - e.g. Hardy, Adaptive Optics, pages 95-96
Let’s get back to design of AO systems
Why on earth does it look like this ??
Considerations in the optical design of AO systems: pupil relays

Deformable mirror and Shack-Hartmann lenslet array should be “optically conjugate to the telescope pupil.”

What does this mean?
Define some terms

• “Optically conjugate” = “image of....”

• “Aperture stop” = the aperture that limits the bundle of rays accepted by the optical system

• “Pupil” = image of aperture stop
So now we can translate:

- “The deformable mirror should be optically conjugate to the telescope pupil”
  
  means

- The surface of the deformable mirror is an image of the telescope pupil
  
  where

- The pupil is an image of the aperture stop
  - In practice, the pupil is usually the primary mirror of the telescope
Considerations in the optical design of AO systems: “pupil relays”
Typical optical design of AO system

- Telescope
- Primary mirror
- Deformable mirror
- Collimated beam
- Pair of matched off-axis parabola mirrors
- Wavefront sensor (plus optics)
- Beamsplitter
- Science camera
More about off-axis parabolas

- Circular cut-out of a parabola, off optical axis
- Frequently used in matched pairs (each cancels out the off-axis aberrations of the other) to first collimate light and then refocus it
Concept Question: what elementary optical calculations would you have to do, to lay out this AO system? (Assume you know telescope parameters, DM size)
Review of important points

- Both lenses and mirrors can focus and collimate light
- Equations for system focal lengths, magnifications are quite similar for lenses and for mirrors
- Telescopes are combinations of two or more optical elements
  - Main function: to gather lots of light
- Aberrations occur both due to your local instrument’s optics and to the atmosphere or eye
  - Can describe both with Zernike polynomials
- Location of pupils is important to AO system design