Lecture 4: Momentum, Energy, Tides, and the Scientific Method

Claire Max
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Outline of this lecture

- Newton’s laws of Motion
- Angular momentum
- Types of energy, and conservation laws
- Tides
- The “scientific method” and what is science

Please remind me to take a break at 12:45 pm
What are Newton’s three laws of motion?

Newton’s first law of motion:
An object moves at constant velocity unless a net force acts to change its speed or direction.

Example: A spaceship needs no fuel to keep moving in space.
Newton’s second law of motion

Force = mass acceleration
\[ \vec{F} = m\vec{a} \]

Example: A baseball accelerates as the pitcher applies a force by moving his arm. (Once the ball is released, the force from the pitcher’s arm ceases, and the ball’s path changes only because of the forces of gravity and air resistance.)

- The arrow above the symbols means that they are vectors: quantities that have both a magnitude and a direction.
- Vector quantities are sometimes indicated by a bold font.
**Newton’s third law of motion:**

For every force, there is an *equal and opposite* reaction force.

*Example:* A rocket is propelled upward by a force equal and opposite to the force with which gas is expelled out its back.
**Newton’s Universal Law of Gravitation**

1. Every mass attracts every other mass.

2. Attraction is *directly* proportional to the product of their masses.

3. Attraction is *inversely* proportional to the square of the distance between their centers.

\[ F_g = G \frac{M_1 M_2}{d^2} \]
Consequence of Newton’s 2nd Law

\[ \vec{F} = m\vec{a} \]

- If there’s no force, there’s no acceleration
- Rate of change of velocity = 0
- Implies velocity = constant
- “A body in motion will stay in motion” = concept of inertia. Newton’s first law follows from his 2\textsuperscript{nd} law!
Newton’s second law re-phrased in term of momentum conservation

• Definition: momentum = mass x velocity
  - Symbol for momentum = p (a vector)
  - $p = m \, v$
  - momentum has a direction because $v$ does

• Newton’s second law:

  Force = mass x acceleration = rate of change of momentum
  $F = m \, a = mass \times rate \ of \ change \ of \ velocity$
  If mass = constant, $F = rate \ of \ change \ of \ (mv)$
  Define momentum as mass times velocity = $mv$

• If force = 0, momentum = $mv = constant$
Conservation of Angular Momentum

angular momentum = mass \times velocity \times radius

- Angular momentum conservation:

- The angular momentum of an object cannot change unless an external twisting force (torque) is acting on it

- Earth experiences no twisting force as it orbits the Sun, so its rotation and orbit will continue indefinitely
Angular momentum conservation explains why objects rotate faster as they shrink in radius

\[ m \times v \times r = m \times v_0 \times r_0 = \text{constant}, \]

\[ v = \frac{v_0 r_0}{r} \] if mass is conserved

Angular velocity (rate of spin):

\[ \Omega \equiv \frac{v}{r} \quad \text{Units: angle/sec} \]

\[ \Omega = \frac{v}{r} = \frac{1}{r} \times \left( \frac{v_0 r_0}{r} \right) = \left( \frac{v_0 r_0}{r^2} \right) \propto \frac{1}{r^2} \]
Centrifugal force

The inward force along the string keeps the ball moving in a circle.

\[ F = ma \]
Without the string, the ball would just keep moving in a straight line

If the string breaks, the inward force is gone...

...so the ball moves with constant velocity from the point of the break.
For a planet in orbit, gravity from the Sun takes the place of the string.

If the string breaks, the inward force is gone...

...so the ball moves with constant velocity from the point of the break.
Concept Question

- A cloud of interstellar gas is collapsing under the force of its own gravity.

- As it collapses, its rotational speed
  
  A. Depends on its mass  
  B. Increases  
  C. Decreases  
  D. Is independent of its initial rotation
What does this imply about the rotation rates of newly born stars?

- Initial big gas cloud rotates slowly - perhaps just taking part in the overall rotation of the Galaxy

- As it collapses to form a star, its angular velocity increases

- Hence newly formed stars frequently have high rotation rates (they spin rapidly)
Next topic: Energy

- **Energy:**
  - The capacity to make matter move, or to “do work”

- **Energy comes in different forms**
  - Kinetic energy, potential energy (gravity), radiative energy, energy in atoms & molecules, electrical energy, mass energy, ....
  - Energy can change from one form to another

- **But total energy is always conserved**
“Follow the energy” is a good rule in astronomy

• Why does something take place?
  - Ask where its energy comes from

• Examples:
  - Heat from the Sun (nuclear reactions at its core)
  - Weather on Earth (heat from the Sun)
  - Orbits of planets (determined by kinetic energy and gravitational potential energy)
Units of energy (in Metric system)

\[
[\text{energy}] = \left[ \text{mass} \times \text{velocity}^2 \right] = \text{grams} \times \left( \frac{\text{cm}}{\text{sec}} \right)^2 = \text{grams} \times \frac{\text{cm}^2}{\text{sec}^2}
\]

or

\[
[\text{energy}] = \text{kg} \times \frac{m^2}{\text{sec}^2} \equiv \text{joules}
\]

- “cgs units” : grams and cm
- “mks units” : kg and meters
- Completely equivalent (choose which one to use)
### Table 4.1 Energy Comparisons

<table>
<thead>
<tr>
<th>Item</th>
<th>Energy (joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average daytime solar energy striking Earth, per m² per second</td>
<td>$1.3 \times 10^3$</td>
</tr>
<tr>
<td>Energy released by metabolism of one average candy bar</td>
<td>$1 \times 10^6$</td>
</tr>
<tr>
<td>Energy needed for 1 hour of walking (adult)</td>
<td>$1 \times 10^6$</td>
</tr>
<tr>
<td>Kinetic energy of average car traveling at 60 mi/hr</td>
<td>$1 \times 10^6$</td>
</tr>
<tr>
<td>Daily energy needs of average adult</td>
<td>$1 \times 10^7$</td>
</tr>
<tr>
<td>Energy released by burning 1 liter of oil</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>Energy released by fission of 1 kg of uranium-235</td>
<td>$5.6 \times 10^{13}$</td>
</tr>
<tr>
<td>Energy released by fusion of hydrogen in 1 liter of water</td>
<td>$7 \times 10^{13}$</td>
</tr>
<tr>
<td>Energy released by 1-megaton H-bomb</td>
<td>$5 \times 10^{15}$</td>
</tr>
<tr>
<td>Energy released by major earthquake (magnitude 8.0)</td>
<td>$2.5 \times 10^{16}$</td>
</tr>
<tr>
<td>U.S. annual energy consumption</td>
<td>$10^{20}$</td>
</tr>
<tr>
<td>Annual energy generation from the Sun</td>
<td>$10^{34}$</td>
</tr>
<tr>
<td>Energy released by supernova (explosion of a star)</td>
<td>$10^{44} - 10^{46}$</td>
</tr>
</tbody>
</table>
Kinetic energy

• Energy of motion

• Kinetic energy $KE = \frac{1}{2} m v^2$
  - $m$ is mass, $v$ is velocity

• Units: if mass is in kilograms, velocity is in meters/sec, then energy is in joules
  - 1 joule = 1 kg (m / sec)$^2$ = 1 kg m$^2$ / sec$^2$
Kinetic energy, continued

- Kinetic energy is proportional to mass
  - more massive objects have more KE
- Kinetic energy is proportional to the square of the velocity
  - If you double your speed (e.g. from 30 to 60 mph), your kinetic energy goes up by factor of four
  - Auto accidents: front part of cars are made to absorb energy, crumple up (to avoid squishing the passenger compartment and hence you).
    » Must absorb 4 X more energy at 60 mph than at 30 mph
    » If it isn’t able to do so, passengers get hurt
**Potential energy**

- Energy that is available by virtue of an object’s position
- Most common example is gravitational potential energy
- If you stand at top of diving board, you have the potential to turn your gravitational potential energy into kinetic energy of motion
Size of gravitational potential energy

Potential energy on surface of a big planet or moon:

- \( \text{PE} = -mgh \)
  
  mass \( \times \) (gravitational acceleration) \( \times \) height

- **Units are same as kinetic energy**
  - \( \text{kg} \times (\text{meters/ sec}^2) \times \text{meters} = \text{kg m}^2/\text{sec}^2 \)

- **Increases with mass, height, gravitational acceleration** \( g \)
Conservation of energy

- Kinetic Energy + Potential Energy = const = E

- At surface of a big planet or moon:

\[
\frac{1}{2}mv^2 + mgh = \text{constant} = E
\]

- Implications:
  - Initial state: \( v = 0 \), total energy = \( mgh \)
  - Final state: \( h = 0 \), total energy = \( \frac{1}{2}mv_{\text{final}}^2 \)
  - \( mgh = \frac{1}{2}mv_{\text{final}}^2 \)
  - Solve for \( v_{\text{final}} \): \( v_{\text{final}} = (2gh)^{1/2} \)
Implications, continued

- $v_{\text{final}} = (2gh)^{1/2}$

- If you fall from a higher place (h large), your final velocity will be higher

- If you fall on the Moon (g small), your final velocity will be lower than if you fall on Earth

- Note that final velocity is independent of mass
  - Galileo’s famous experiment at leaning tower of Pisa
  - Dropped heavy object and light object; they hit ground at same time
Concept Question

- Can you think of examples where gravitational potential energy is converted to kinetic energy?
  - In our daily lives here on Earth?
  - In the Solar System?
Examples where potential energy is converted to kinetic energy
More examples of potential energy converting to kinetic energy
More examples of potential energy converting to kinetic energy

- Pendulum
More examples of potential energy converting to kinetic energy

- Skiing
Waterfall: what are roles of potential and kinetic energy here?
Gravitational Potential Energy

• On Earth, depends on:
  - object’s mass \((m)\)
  - strength of gravity \((g)\)
  - distance object could potentially fall
Gravitational Potential Energy

- In space, an object or gas cloud has more gravitational energy when it is spread out than when it contracts.

- A contracting cloud converts gravitational potential energy to thermal energy.
Energy can do “work”

The ram of a pile-driver possesses mechanical energy – the ability to do work. When held at a height, it possesses mechanical energy in the form of potential energy. As it falls, it possesses mechanical energy in the form of kinetic energy. As it strikes the spike, it applies a force to displace the spike – i.e., it does work on the spike.

Work = Force x Distance

( a physicist’s definition of work)
General expression for gravitational potential energy

- **PE = m g h**
  - only applies on the surface of a big planet or moon

- **General expression (holds everywhere):**

\[
\text{Gravitational Potential Energy} = - \frac{G m_1 m_2}{r}
\]

where \( G = \text{Gravitational Constant} = 6.7 \times 10^{-11} \frac{m^3}{kg \text{ sec}^2} \)
More implications: For a planet in orbit around a star

\[ \frac{1}{2} m_1 v^2 + \text{potential energy} = \frac{1}{2} m_1 v^2 - \frac{G m_1 m_2}{r} = \text{constant} = K \]

\[ \frac{1}{2} m_1 v^2 = \frac{G m_1 m_2}{r} + K \]

- \( r = \text{distance between } m_1 \text{ and } m_2 \)
- \( \text{Gravitational PE is negative} \)
- \( \text{Speed of planet is largest when it is closest to star:} \)
- \( r \text{ is small, so } \frac{G m_1 m_2}{r} \text{ is large} \)
**Concept Question**

- Imagine a straight shaft bored from the Earth’s surface, thru the center of the Earth, and out the other side
- Drop a baseball down this shaft
- What is the baseball’s motion
  - At the start
  - At the center of the Earth
  - Just as it comes out the other side
- At each of these points, describe the **acceleration** and the **velocity** of the baseball
- Will the ball’s motion be periodic?
Temperature

- Temperature measures average kinetic energy of all the particles in a region

 Longer arrows mean higher average speed.
Temperature scales: Kelvin, Celsius, Fahrenheit

- 373.15 K, 100°C, 212°F: water boils
- 273.15 K, 0°C, 32°F: water freezes
- 0 K, -273.15°C, -459.67°F: absolute zero

Kelvin | Celsius | Fahrenheit
--- | --- | ---
Sidebar: what are the units of energy? (metric system)

- Consider kinetic energy: KE = (1/2) m v^2

- Units: kg (meters/ sec )^2

- Definition: 1 Joule = 1 kg m^2 / sec^2

- If you prefer to use cm and gm instead of meters and kg:

- 1 erg = 1 gm cm^2 / sec^2 = 10^{-7} Joule
Thermal energy

- Thermal energy = $N \ k \ T$,  \( N = \text{no. of particles} \)
- \( k = \text{Boltzmann’s const.} = 1.4 \times 10^{-23} \text{ J / deg K} \)
Thermal energy is a measure of the total kinetic energy of all the particles in a substance.

It therefore depends both on temperature AND density.
Difference between temperature and heat flow

- Heat flow: rate of spontaneous transfer of thermal energy from a higher T system to a lower T system

  Heat content or thermal energy \( Q \) joules per \( m^3 \)

  Heat flow \( \dot{Q} \) = rate of change of \((NkT)\) with time

- Heat can flow via conduction, convection, or radiation

- Example from Bennett: much faster heat flow if you stick your hand in boiling water than if you stick your arm in a hot oven.
  - Why?
Other forms of energy

- Energy in atoms and molecules
- Radiative energy
- Mass energy
Atomic structure: energy in atoms and molecules

Ten million atoms could fit end to end across this dot.

The nucleus is nearly 100,000 times smaller than the atom but contains nearly all of its mass.

Nucleus: Contains positively charged protons (red) and neutral neutrons (gray).

Atom: Electrons are “smeared out” in a cloud around the nucleus.

$10^{-10}$ meter
Energy in atoms (in this case, in the electrons surrounding the nucleus)

ground state  excited state  ionization
**Discrete energy levels**

- Electrons inside atoms can take on only discrete energy levels

- Analogy to ladder with specific steps
Energy levels in hydrogen atom

1 eV = 1 electron volt = $1.6 \times 10^{-19}$ joule
Radiative energy

- Energy carried by light

- Atoms radiate light when their electrons make transitions from one energy level to another

- Hot matter radiates light, transfers heat to surrounding cooler matter
Mass energy

- Albert Einstein: \( E = m c^2 \)

- Energy = mass \( \times \) (speed of light)^2

- Examples where mass is actually converted into other forms of energy:
  - In core of Sun and hydrogen bombs (nuclear fusion)
  - In nuclear reactors (nuclear fission)

- Example from Bennett: the mass-energy of a 1 kg rock represents \( 7.5 \times 10^9 \) more energy than burning a barrel of oil!
Conservation of Energy

- Energy can be neither created nor destroyed.
- It *can* change form or be exchanged between objects.
- The total energy content of the Universe was determined in the Big Bang and remains the same today.
What have we learned?

• Why do objects move at constant velocity if no force acts on them?
  - Conservation of momentum

• What keeps a planet rotating and orbiting the Sun?
  - Conservation of angular momentum

• Where do objects get their energy?
  - Conservation of energy: energy cannot be created or destroyed but only transformed from one type to another.
  - Energy comes in three basic types: kinetic, potential, radiative.
  - Energy sources: heat flow, radiation flow (light), nuclear fission and fusion, gravitational potential energy, ...
Tides

• Tides are due to the difference between the force of gravity on opposite sides of a planet or moon
  - Tides can have far-reaching effects on planets and their moons
The physics behind tides

- Gravitational force is strongest on side of the Earth closest to Moon, weakest on other side
How strong is tidal force?

\[ F_2 - F_1 \propto \frac{x_2 - x_1}{r^3} \]

- Tidal forces fall off like \( 1 / r^3 \)
- “Regular” gravitational force falls off like \( 1 / r^2 \)
What is the effect on the Moon?

On the Earth?

- Moon pulls backward on Earth’s tidal bulge, slows rotation rate of Earth. Day gets longer (very slowly).
- Tidal bulge pulls Moon ahead in its orbit, makes it spiral outwards away from Earth (very slowly)
Force on Moon depends strongly on distance between Earth and Moon

Mass of Earth's tidal bulge $\equiv \Delta m \propto$ tidal force

$$\Delta m \propto F_2 - F_1 \propto \frac{x_2 - x_1}{r^3}$$

Force of Earth's tidal bulge on Moon $\propto \frac{\Delta m}{r^3} \propto \frac{1}{r^6}$

- Tidal recession of Moon was very fast when Earth and Moon were close together; is slower now
Lengthening of Earth’s day

• Earth’s day: Evidence from growth bands in fossil bivalve shells and corals
  - There were \( \sim 400 \) days per solar year about 350 million years ago. So an Earth-day was shorter.

• Historical records of eclipses imply day is slightly longer now than it was \( \sim 2000 \) years ago
Tidal origin of Moon’s synchronous rotation

- Just as tides on Earth slow Earth’s day, tides on Moon slow Moon’s rotation rate
  - Yes, rock bulges a bit, forming “tides” on Moon

- Moon’s “day” slowed down so much that now it only rotates once a month
  - Called Synchronous Rotation

- Once that happened, Moon’s tidal bulge always pointed toward Earth, so Moon’s day won’t slow down still more
Synchronous rotation elsewhere in Solar System: Pluto and Charon

- Pluto-Charon: Each spins on its axis in same length of time they orbit around each other
  - Same hemisphere of Pluto always faces Charon
  - Same hemisphere of Charon always faces Pluto
Tidal forces elsewhere in Solar System

- Most inner moons of giant planets rotate synchronously

- Mercury’s rotation was slowed by tides from Sun
  - Now after two Mercury orbits around Sun, planet has rotated on its axis three times
  - Called an “orbital resonance”
Tidal heating: Io is best example

- Io is Jupiter’s closest moon
- Io’s orbit is kept non-circular by Europa, another of Jupiter’s moons
- Continued flexing and bulging produces internal motions of Io’s rocks, friction, internal heating
- Hot enough inside Io to melt rock, form molten lava
- Erupts to surface in > 200 volcanoes
- Total heat flow several trillion watts (!)
Concept Question

- Science fiction stories like to describe what would happen to you if your space ship accidentally came too close to a black hole
  - For the purposes of this question, consider a black hole a very small region of space where gravity is extremely intense.

- If your space ship flies too close to the black hole, it is stated that “you’ll be torn apart by tidal forces”

- Draw a stick-figure of yourself, and show in a diagram how these “tidal forces” might “tear you apart.” Show the differences in strength of the gravitational force on the different parts of your body.
The Scientific Method

- What is a scientific theory?
- How can we distinguish science from non-science?
What is a scientific theory?

- The word “theory” has a somewhat different meaning in science than in everyday life.

- A scientific theory must:
  - Explain a wide variety of observations with a few simple principles,
  - Be supported by a large, compelling body of evidence,
  - Must not have failed crucial tests of its validity,
  - Must be amenable to modification if new data require this.

- Newton’s laws of gravitation are a good example
  - They explain a wide body of observations, have lots of evidence, but under some (very unusual) circumstances they require modification.
  - Near black holes and neutron stars, gravity is so strong that Einstein’s theory of General Relativity applies, instead of Newton’s laws.
How can we distinguish science from non-science?

- Defining science can be surprisingly difficult.

- *Science* from the Latin *scientia*, meaning “knowledge.”

- But not all knowledge comes from science...
The idealized scientific method

- Based on proposing and testing hypotheses
- Hypothesis = educated guess
But science doesn’t always proceed in this idealized way

• Sometimes we start by “just looking” and then coming up with possible explanations.

• Sometimes we follow our intuition rather than a particular line of hard evidence.

• There are frequently several blind alleys that don’t work out, before a successful theory is developed and tested.
Hallmarks of science

- Useful criteria to decide whether an argument is scientific or not

Seeks explanations for observed phenomena that rely solely on natural causes.

Progresses through creation and testing of models of nature that explain the observations as simply as possible.

Science

Makes testable predictions about natural phenomena. If predictions do not agree with observations, model must be revised or abandoned.
Hallmarks of Science: #1

- In ancient times, actions of the gods were invoked as explanations for things that were hard to understand.

- But modern science seeks explanations for observed phenomena that rely solely on natural causes.

- Other kinds of explanations don’t come under the heading “science”, but rather are different kinds of discussions.
Hallmarks of Science: #2

- Science progresses through the creation and testing of models of nature that explain the observations as simply as possible.

- Example: By early 1600s, there were several competing models of planetary motion (Ptolemy, Copernicus, Kepler, ...) Kepler’s gained acceptance because it worked the best.
Hallmarks of Science: #3

• A scientific model should make testable predictions about natural phenomena.

• If subsequent tests don’t agree with the predictions, a scientist would be willing (even eager) to revise or even abandon his/her model.

• If someone, in the face of data that contradict his/her model, isn’t willing to revise or abandon it, they are not using the scientific method.
Issues for Planetary Science

- Planets and their moons are hugely varied

- For example: We aren’t advanced enough to have an *a priori* theory that would predict what a newly discovered moon of Jupiter or Saturn should be like

- “Retrodiction” or “postdiction” rather than “prediction”
  - Try to understand new observations using overarching principles based on previous body of data
Scientific Method: Main Points

• How can we distinguish science from non-science?
  - Science: seeks explanations that rely solely on natural causes; progresses through the creation and testing of models of nature; models must make testable predictions

• What is a scientific theory?
  - A model that explains a wide variety of observations in terms of a few general principles and that has survived repeated and varied testing
What about astrology?

- How is astrology different from astronomy?
- Is astrology a scientific theory?
- Does astrology have scientific validity?
Astrology asks a different type of question than astronomy

- **Astronomy** is a science focused on learning about how stars, planets, and other celestial objects work.

- **Astrology** is a search for hidden influences on human lives based on the positions of planets and stars in the sky.
Does astrology have scientific validity?

- In principle the stars might influence human affairs.
- How do we know whether they do or not?
- Scientific tests consistently show that astrological predictions are no more accurate than we should expect from pure chance.
- Proponents of astrology say that the act of doing controlled experiments ruins the “aura” and that’s why predictions aren’t accurate when tested in a lab environment.
- In my opinion this means that astrology doesn’t come under the heading “science”, since it can’t make testable predictions.
What have we learned?

- A scientific theory should:
  - Explain wide variety of observations with a few simple principles,
  - Be supported by a large, compelling body of evidence,
  - Must not have failed crucial tests of its validity,
  - Be amenable to modification if new data require this.

- Astrology
  - Search for hidden influences on human lives based on the positions of planets and stars
  - Thus far scientific tests show that astrological predictions are no more accurate than we should expect from pure chance