ASTRONOMY 2 — Overview of the Universe
First Practice Problem Set

1. Calculate the number of stars per unit volume in a certain elliptical galaxy. The number of stars in the elliptical galaxy is 10^{11}, and the galaxy can be approximated as a sphere of radius $r = 15$ kpc. Recall that the volume of a sphere is given by $V = 4\pi r^3/3$, $\pi = 3.14$, and 1 kpc$=3.09 \times 10^{21}$ cm.

$r = 15 \text{ kpc} = 15 \times 10^{21} \times 3.09 \text{ cm} = 4.635 \times 10^{22} \text{ cm}$

$V = \frac{4\pi}{3} (4.635 \times 10^{22} \text{ cm})^3 = \frac{4}{3} \times 3.14 \times 9.96 \times 10^{67} \text{ cm}^3 = 4.17 \times 10^{68} \text{ cm}^3$

NUMBER OF STARS PER UNIT VOLUME $= 10^{11} / 4.17 \times 10^{68} \text{ cm}^3 = \boxed{2.4 \times 10^{-58} / \text{cm}^3}$ ANSWER

2. Recall that Kepler's Third Law can be written as $M = Kv^2R$ where $K = 1.5 \times 10^7$ g s^2/cm^3; 1 $M_\odot = 2 \times 10^{33}$ g; and 1 AU$= 1.5 \times 10^{13}$ cm.

(i) How fast would a body orbit a 10^6 $M_\odot$ black hole at a distance of 1 AU?

$M = Kv^2R$, $v^2 = \frac{M}{KR}$, $v = \sqrt{\frac{M}{KR}}$

$v = \sqrt{\frac{10^6 \times 2 \times 10^{33} \text{ gm}}{1.5 \times 10^7 \frac{\text{gm s}^2}{\text{cm}^3} \times 1.5 \times 10^{13} \text{ cm}}} = \sqrt{8.89 \times 10^{18} \text{ cm}^2/\text{s}^2} = \boxed{2.98 \times 10^9 \text{ cm/s}}$ ANSWER

(ii) At what orbital radius from this black hole will the orbital velocity reach the speed of light, $c = 3 \times 10^{10}$ cm/s? This radius is known as the Schwartzchild radius.

$M = Kv^2R$, $R = \frac{M}{Kv^2}$

$R = \frac{10^6 \times 2 \times 10^{33} \text{ gm}}{1.5 \times 10^7 \frac{\text{gm s}^2}{\text{cm}^3} \times (3 \times 10^{10} \text{ cm/s})^2} = \boxed{1.48 \times 10^{11} \text{ cm}}$ ANSWER

3. The Sun has a luminosity of 3.9×10^{33} ergs/s, a radius of 7×10^{10} cm, and a surface temperature of 5800 K. Use the blackbody formula $L = 4\pi\sigma R^2T^4$ to answer the following questions.

(i) A star has a luminosity 0.16 times that of the sun and a temperature of 4000 K. What is its radius?

$L_\odot = 4\pi\sigma R_\odot^2 T_\odot^4$ — ①

$L_\star = 4\pi\sigma R_\star^2 T_\star^4$ — ②

②/① → $\frac{R_\star^2}{R_\odot^2} \times \frac{T_\star^4}{T_\odot^4} = \frac{L_\star}{L_\odot}$

→ $\frac{R_\star}{R_\odot} = \left(\frac{T_\odot}{T_\star}\right)^2 \times \sqrt{\frac{L_\star}{L_\odot}}$

$R_\star = 7 \times 10^{10} \text{ cm} \left(\frac{5800 \text{ K}}{4000 \text{ K}}\right)^2 \sqrt{0.16} = \boxed{5.89 \times 10^{10} \text{ cm}}$ ANSWER

(ii) Another star has a luminosity 500,000 times that of the sun and a radius 18 times that of the sun. What is its temperature?

$\left(\frac{T_\star}{T_\odot}\right)^4 \left(\frac{R_\star}{R_\odot}\right)^2 = \frac{L_\star}{L_\odot}$

$T_\star = \sqrt[4]{\frac{L_\star}{L_\odot}} \times \sqrt{\frac{R_\odot}{R_\star}} \times T_\odot$

→ $T_\star = \sqrt[4]{500{,}000} \times \sqrt{\frac{1}{18}} \times 5800$

$= 26.59 \times 0.2357 \times 5800 \text{ K}$

$= \boxed{3.64 \times 10^4 \text{ K}}$ ANSWER