

ASTRONOMY 2 — Overview of the Universe
Second Practice Problem Set — Solutions

Problem: 1. Consider a $2 M_{\odot}$ neutron star. The mass of a neutron is 1.67×10^{-24} g, and $1 M_{\odot} = 2 \times 10^{33}$ g.

- (i) How many neutrons are in this neutron star? (1 point)

Solution:

$$N_{neutrons} = M_*/m_{neutron}$$

$$N_{neutrons} = 2 \times 2 \times 10^{33} \text{g} / 1.67 \times 10^{-24} \text{g}$$

$$N_{neutrons} = (4/1.67) \times 10^{57} = 2.4 \times 10^{57} \text{ neutrons, D}$$

- A. 6.0×10^{23} neutrons
- B. 1.2×10^{23} neutrons
- C. 1.2×10^{57} neutrons
- D. 2.4×10^{57} neutrons
- E. 6.7×10^9 neutrons

Problem:

- (ii) Assuming the energy released during core bounce and supernova phase of this neutron star was 2 MeV per neutron, calculate the total energy output of the supernova in ergs. Note that $1 \text{ MeV} = 1.6 \times 10^{-6}$ ergs. (2 points)

Solution:

$$E = (\text{Number of neutrons}) \times (\text{Energy per neutron})$$

$$E = 2.4 \times 10^{57} \text{ neutrons} \times 2 \text{ MeV/neutron}$$

$$E = 2.4 \times 10^{57} \times 2 \text{ MeV} \times 1.6 \times 10^{-6} \text{ ergs/MeV}$$

$$E = 2.4 \times 2 \times 1.6 \times 10^{51} = 7.7 \times 10^{51} \text{ ergs, C}$$

- A. 1.5×10^{63} ergs
- B. 3.8×10^{51} ergs
- C. 7.7×10^{51} ergs
- D. 7.7×10^{57} ergs
- E. 4.8×10^{57} ergs

2. Galaxy A is observed to have a velocity of $v_{\text{obs}} = -350$ km/s and is known to have a peculiar velocity of $v_{\text{pec}} = -500$ km/s.

Problem

- (i) Use the formula: $v_{\text{obs}} = v_{\text{Hubble}} + v_{\text{pec}}$, to calculate the Hubble expansion velocity v_{Hubble} for this galaxy. (1 point)

Solution:

$$v_{\text{obs}} = v_{\text{Hubble}} + v_{\text{pec}}$$

$$v_{\text{Hubble}} = v_{\text{obs}} - v_{\text{pec}}$$

$$v_{\text{Hubble}} = -350 \text{ km/s} - (-500) \text{ km/s} = 150 \text{ km/s, A}$$

- A. 150 km/s
- B. -150 km/s
- C. -850 km/s
- D. 850 km/s
- E. π km/s

Problem:

- (ii) Galaxy B has a Hubble expansion velocity of $v_{\text{Hubble}} = 2.8$ km/s. Using Hubble's Law $v_{\text{Hubble}} = H_0 d$ where $H_0 = 70$ km/s/Mpc, determine the distance d to this galaxy. (2 points)

Solution:

$$v_{\text{Hubble}} = H_0 d$$

$$d = v_{\text{Hubble}}/H_0$$

$$d = (2.8 \text{ km/s})/(70 \text{ km/s/Mpc}) = 0.04 \text{ Mpc, B}$$

- A. -196 Mpc
- B. 0.04 Mpc
- C. 196. Mpc
- D. 25 Mpc
- E. -0.04 Mpc

Problem:

- (iii) What is the most likely identity of galaxy B? (1 point)

Solution:

From Arny, p. 490: Large Magellanic Cloud is about 150,000 light years away.

From Seeds, p.257: Andromeda is about 2 million light years away

(see also Problem 3 below).

$$0.04 \text{ Mpc} \times 3.26 \times 10^6 \text{ light years/Mpc} = 1.3 \times 10^5 \text{ light years,}$$

B Large Magellanic Cloud

- A. Milky Way galaxy
- B. Large Magellanic Cloud galaxy
- C. Andromeda galaxy
- D. The most distant known galaxy
- E. The most distant known quasar

Problem: 3. Our neighbor galaxy, Andromeda, has a luminosity (intrinsic brightness) 3 times as bright as the Luminosity of our Milky Way galaxy, $L_{\text{Andromeda}} = 3L_{\text{MW}}$, and is at a distance of $d_{\text{Andromeda}} = 0.7$ Mpc. The flux we observe from the Andromeda galaxy (apparent brightness) is 10,000 times brighter than the flux observed from a distant quasar, $f_{\text{Andromeda}} = 10^4 f_{\text{quasar}}$. This quasar has a luminosity that is 1000 times the luminosity of our Milky Way galaxy, $L_{\text{quasar}} = 10^3 L_{\text{MW}}$. What is the distance d to the quasar? (4 points)

[**HINT:** Use the formula: $f \propto L/d^2$. Write two relations, one for Andromeda and another for the quasar. Divide one relation by the other.]

Solution:

$$f_{\text{Andromeda}}/f_{\text{quasar}} = (L_{\text{Andromeda}}/L_{\text{quasar}})(d_{\text{quasar}}^2/d_{\text{Andromeda}}^2)$$

$$d_{\text{quasar}}^2/d_{\text{Andromeda}}^2 = (f_{\text{Andromeda}}/f_{\text{quasar}})(L_{\text{quasar}}/L_{\text{Andromeda}})$$

$$d_{\text{quasar}}/d_{\text{Andromeda}} = \sqrt{(f_{\text{Andromeda}}/f_{\text{quasar}})(L_{\text{quasar}}/L_{\text{Andromeda}})}$$

$$d_{\text{quasar}}/d_{\text{Andromeda}} = \sqrt{(10^4)(10^3 L_{\text{MW}}/3L_{\text{MW}})}$$

$$d_{\text{quasar}} = \sqrt{10^7/3} \times 0.7 \text{ Mpc} = 1826 \times 0.7 \text{ Mpc} = 1.28 \times 10^3 \text{ Mpc}, \text{ C}$$

- A. 1.28×10^{-1} Mpc
- B. 3.85 Mpc
- C. 1.28×10^3 Mpc
- D. 2.36×10^6 Mpc
- E. 3.85×10^{12} Mpc

Problem: 4. An astronomer observes a bright star (Altair) that has a parallax angle of $p = 0.20$ arcseconds. The flux f from Altair is approximately 9.4×10^{-12} times the flux from the Sun. The distance d from the Earth to the Sun is $(1/206265)$ pc.

- (a) What is the distance d to Altair star in units of parsecs (pc)? (2 points)

[**HINT**— Use the formula: $d = 1/p$, with distance d in units of pc and parallax p in units of arcseconds.]

Solution:

$$d = 1/p = 1/0.20 = 5 \text{ pc, D}$$

- A. 0.20 pc
- B. 2.06 pc
- C. 3.14 pc
- D. 5 pc
- E. 50 pc

Problem:

- (b) What is the luminosity L of Altair in units of the solar luminosity L_{\odot} ? (4 points)

[**HINT**— Use the formula: $f \propto \frac{L}{d^2}$ or $f = C \frac{L}{d^2}$, where C is a constant. Write one equation for Altair and one for the sun. Divide one equation by the other.]

Solution:

$$f_{\text{Altair}}/f_{\odot} = (L_{\text{Altair}}/L_{\odot})(d_{\odot}^2/d_{\text{Altair}}^2)$$

$$L_{\text{Altair}}/L_{\odot} = (f_{\text{Altair}}/f_{\odot})(d_{\text{Altair}}^2/d_{\odot}^2)$$

$$L_{\text{Altair}} = 9.4 \times 10^{-12} \times (5 \text{ pc})^2 / (1/206265 \text{ pc})^2 L_{\odot}$$

$$L_{\text{Altair}} = 9.4 \times 10^{-12} \times 25 / 2.4 \times 10^{-11} L_{\odot} = 0.94 \times 25 / 2.4 L_{\odot} = 10 L_{\odot}, \mathbf{E}$$

- A. $2.35 \times 10^{-10} L_{\odot}$
- B. $0.016 L_{\odot}$
- C. $0.40 L_{\odot}$
- D. $1.00 L_{\odot}$
- E. $10.00 L_{\odot}$

Problem: 5. A distant quasar is observed to have a redshift $v/c = 0.15$, where v is the recession velocity of the quasar, and $c = 300,000$ km/s is the speed of light.

(a) What is the recession velocity v of the quasar in units of km/s? (2 points)

Solution:

$$v/c = 0.15$$

$$v = 0.15c = 0.15 \times 3 \times 10^5 \text{ km/s} = 45,000 \text{ km/s, B}$$

- A. 7.47×10^{-5} km/s
- B. 45,000 km/s
- C. 0.059 km/s
- D. 1.43 km/s
- E. 1.97×10^6 km/s

Problem:

(b) Using the Hubble expansion formula: $v = H_0d$, where the Hubble constant $H_0 = 70$ km/s/Mpc, calculate the distance d to the quasar in units of Mpc? (3 points)

Solution:

$$v = H_0d$$

$$d = v/H_0$$

$$d = (45 \times 10^4 \text{ km/s})/(70 \text{ km/s/Mpc}) = 642.9 \text{ Mpc, A}$$

- A. 642.9 Mpc
- B. 0.038 Mpc
- C. 4.77×10^{-12} Mpc
- D. 6.77 Mpc
- E. 5.35×10^{10} Mpc

Problem:

(c) How long ago was the light we are now seeing from the quasar emitted? Note, 1 Mpc = 3.26 million light years. (3 points)

Solution:

$$642.9 \text{ Mpc} \times 3.26 \text{ million light years/Mpc} = 2.096 \times 10^3 \text{ million years, C}$$

- A. 2.96×10^{-14} million years
- B. 0.00032 million years
- C. 2.096×10^3 million years
- D. 1.66×10^{-4} million years
- E. 9.27 million years

Problem: 6. Kepler's Third Law can be written as $P^2 = CR^3/M$, where C is a universal constant and M is the mass of the central object. The Moon orbits the Earth with a period of $P = 27.32$ days, at a distance of $R = 384,400$ km. (7 points total)

- (a) The mass of Saturn is 95 times the mass of the Earth. If a moon of Saturn is in orbit around it with a period of $P = 234.01$ days. What is the radius of the orbit of Saturn's moon? (4 points)

[**HINT:** Write one equation for the Earth-Moon system and another equation for Saturn and its moon. Divide one equation by the other.]

Solution:

$$(P_{\text{Saturn's moon}}/P_{\text{Moon}})^2 = (R_{\text{Saturn's moon}}/R_{\text{Moon}})^3(M_{\text{Earth}}/M_{\text{Saturn}})$$

$$(R_{\text{Saturn's moon}}/R_{\text{Moon}})^3 = (P_{\text{Saturn's moon}}/P_{\text{Moon}})^2(M_{\text{Saturn}}/M_{\text{Earth}})$$

$$(R_{\text{Saturn's moon}}/R_{\text{Moon}}) = (P_{\text{Saturn's moon}}/P_{\text{Moon}})^{2/3}(M_{\text{Saturn}}/M_{\text{Earth}})^{1/3}$$

$$(R_{\text{Saturn's moon}}/R_{\text{Moon}}) = (234.01 \text{ days}/27.32 \text{ days})^{2/3}(95)^{1/3}$$

$$(R_{\text{Saturn's moon}}/R_{\text{Moon}}) = (8.6)^{2/3}(95)^{1/3} = 4.2 \times 4.6$$

$$R_{\text{Saturn's moon}} = 4.2 \times 4.6 R_{\text{Moon}} = 19.1 \times 3.844 \times 10^5 \text{ km} = 7.34 \times 10^6 \text{ km}, \mathbf{B}$$

- A. 6.78×10^2 km
- B. 7.34×10^6 km
- C. 4.63×10^{19} km
- D. 3.13×10^8 km
- E. 3.19×10^{-9} km

Problem:

- (b) If the Earth's mass were tripled, while keeping the Earth-Moon distance the same, what would the period of the Earth's Moon's orbit now be? (3 points)

[**HINT:** Write two equations for the Earth-Moon system, one for the 'normal' Earth and another for the case where the Earth mass is tripled. Divide one equation by the other.]

Solution:

$$(P_3/P_1)^2 = M_1/M_3$$

$$P_3^2 = (M_1/M_3)P_1^2$$

$$P_3 = \sqrt{M_1/M_3}P_1 = \sqrt{1/3} \times 27.32 \text{ days} = 15.77 \text{ days}, \mathbf{D}$$

- A. 2.24×10^3 days
- B. 47.32 days
- C. 0.109 days
- D. 15.77 days
- E. 81.96 days