Practice problems

1. The theory of the inflation explains only **one** of the following facts — which one? (*2 points total*)

A. The Hubble expansion of the Universe where galaxies recede away from each other with a speed proportional to their distance

B. The uniformity of the Cosmic Microwave Background Radiation in all directions even though opposite parts are outside each other's horizons

C. (Re)combination of electrons and protons into neutral hydrogen atoms and the decoupling of matter from radiation

D. Pair production and annihilation leading to Hawking radiation from the vicinity of the Schwarzschild radius of a black hole

2. Consider a 2 $M_\odot$ neutron star. The mass of a neutron is 1.67×10^{-24} g, and 1 $M_\odot = 2 \times 10^{33}$ g.

(i) How many neutrons are in this neutron star? (*2 points*)

A. 6.0×10^{23} neutrons
B. 1.2×10^{23} neutrons
C. 1.2×10^{57} neutrons
D. 2.4×10^{57} neutrons
E. 6.7×10^{9} neutrons

(ii) Assume that the energy released during the gravitational collapse process that led to the formation of this neutron star was 2 MeV per neutron. About 10% of this energy is converted into the kinetic energy of a supernova explosion. Calculate the total kinetic energy output of the supernova in ergs. Note, 1 MeV = 1.6×10^{-6} ergs. (*2 points*)

A. 9.7×10^{23} ergs
B. 4.1×10^{23} ergs
C. 7.7×10^{50} ergs
D. 7.8×10^{57} ergs
E. 7.3×10^{9} ergs

3. The number of stars in the disk of the Milky Way is about 10^{11}, and the shape of the Milky Way's disk can be approximated as a cylinder of radius $r = 15$ kpc and height $h = 1$ kpc.

(i) Recall that the volume of a cylinder is given by $V = \pi r^2 h$, where $\pi = 3.14$, and 1 kpc=3.09×10^{21} cm. What is the volume of the Milky Way's disk in units of cm^3? (*3 points*)

A. $1.5 \times 10^{-56}\,\mathrm{cm}^3$
B. $7.2 \times 10^{-56}\,\mathrm{cm}^3$
C. $4.8 \times 10^{-57}\,\mathrm{cm}^3$
D. $1.4 \times 10^{66}\,\mathrm{cm}^3$
E. $2.1 \times 10^{67}\,\mathrm{cm}^3$

(ii) What is the density ρ of stars in the Milky Way in units of stars/cm^3? (*2 points*)

A. 1.5×10^{-56} stars/cm^3
B. 7.2×10^{-56} stars/cm^3
C. 4.8×10^{-57} stars/cm^3
D. 1.4×10^{66} stars/cm^3
E. 2.1×10^{67} stars/cm^3

4. Calculate the number of stars per unit volume in a certain elliptical galaxy. The number of stars in the elliptical galaxy is 10^{11}, and the galaxy can be approximated as a sphere of radius $r = 15$ kpc. Recall that the volume of a sphere is given by $V = 4\pi r^3/3$, $\pi = 3.14$, and 1 kpc$=3.09 \times 10^{21}$ cm. (*3 points*)

A. 4.8×10^{-57} stars/cm^3
B. 2.4×10^{-58} stars/cm^3
C. 7.2×10^{-56} stars/cm^3
D. 1.4×10^{66} stars/cm^3
E. 2.1×10^{67} stars/cm^3

5. Let us assume that there is a star called Star X whose mass is 0.8 $M_\odot$. Star X has two planets, Q and W, in orbit around it.

(i) Planet Q has a period of $P = 0.2$ years. What is the orbital radius R of Planet Q? Use Kepler's Third Law, $P^2 = CR^3/M$, where $C = 1$, P is the period in years, R is the orbital radius in AU, and M is the mass of the star being orbited in $M_\odot$. (*3 points*)

A. 0.32 AU
B. 0.74 AU
C. 0.17 AU
D. 0.54 AU
E. 0.98 AU

(ii) Planet W receives the same flux f from Star X as the Earth receives from the Sun. The luminosity L of Star X is 40% of the sun's luminosity. What is Planet W's orbital period P? Recall that flux $f = L/(4\pi R^2)$, and use this formula to calculate Planet W's orbital radius R. Then use this value of R in the formula in part (i) above to calculate its orbital period P. (*5 points*)

A. 0.05 years
B. 0.18 years
C. 0.32 years
D. 0.50 years
E. 0.96 years

(iii) Suppose the peak wavelength $\lambda_{\max}$ at which the sun radiates were to increase by a factor of 1.6 during a certain phase of the sun's evolution. If the original surface temperature T of the sun was 6,400 K, what is its new temperature? Recall Wien's Law: $\lambda_{\max} T = \text{constant}$. (*2 points*)

A. 4000 K
B. 3570 K
C. 5000 K
D. 6470 K
E. 2000 K

(iv) At a certain stage of its evolutionary cycle, Star X becomes a red giant star. In the process, its luminosity L increases by a factor of 40,000, while its temperature T drops by a factor of 1.4. If the original radius r of Star X (before it became a red giant) was 0.004 AU, what is its new radius in the red giant phase? Recall that $L \propto r^2 T^4$. It is important to make a distinction between the radius r of the star and the orbital radius R of the planets referred to in the earlier parts of this problem — don't confuse these two different quantities r vs R! (*4 points*)

A. 1.0×10^{-5} AU
B. 3.9×10^{-5} AU
C. 0.4 AU
D. 1.57 AU
E. 114 AU

(v) Of the two planets Q and W, which planet(s) is (are) absorbed by Star X in the course of its becoming a red giant star? (*1 point*)

A. Neither
B. Both Q and W
C. Q
D. W

5. Let us assume that there is a star called Star X whose mass is 0.8 $M_\odot$. Star X has two planets, Q and W, in orbit around it.

(i) Planet Q has a orbital period of $P = 0.2$ years as it moves around Star X. What is the orbital radius R of Planet Q? (*4 points*)

[HINT: Use Kepler's Third Law, $P^2 = CR^3/M$, where P is the period in years, R is the orbital radius in AU, and M is the mass of the star being orbited in $M_\odot$. Write the formula for the orbit of the Earth around the Sun ($R = 1$ AU, $P = 1$ year). Then write the formula again for the orbit of Planet Q around Star X. Divide one formula by the other and solve for the radius of the orbit of Planet Q.]

A. 0.32 AU
B. 0.74 AU
C. 0.17 AU
D. 0.54 AU
E. 0.98 AU

(ii) Planet W receives the same flux f from Star X as the Earth receives from the Sun. The luminosity L of Star X is 40% of the sun's luminosity. What is Planet W's orbital period P? (*8 points*)

[HINT: Recall that flux $f = L/(4\pi R^2)$. Write this formula twice, once for the Earth-Sun system and again for the Star X-Planet W system. Divide one formula by the other to solve for radius R of the orbit of Planet W. Then write Kepler's Third Law for the Earth-Sun system and again for the Star X-Planet W system. Divide one formula by the other and solve for the period of the orbit of Planet Q.]

A. 0.05 years
B. 0.18 years
C. 0.32 years
D. 0.50 years
E. 0.96 years

(iii) Suppose the peak wavelength λ_{max} at which the sun radiates were to increase by a factor of 1.6 during a certain phase of the sun's evolution. If the original surface temperature T of the sun was 6,400 K, what is its new temperature? (*5 points*)

[HINT: Recall Wien's Law: $\lambda_{max}T$ = constant. Write this formula once for before and once for after the temperature increase. Divide one formula by the other to solve for the new temperature.]

A. 4000 K
B. 3570 K
C. 5000 K
D. 6470 K
E. 2000 K

(iv) At a certain stage of its evolutionary cycle, Star X becomes a red giant star. In the process, its luminosity L increases by a factor of 40,000, while its temperature T drops by a factor of 1.4. If the original radius r of Star X (before it became a red giant) was 0.004 AU, what is its new radius in the red giant phase? (*8 points*)

[HINT: Recall that $L = Kr^2T^4$, where L is the luminosity of the star, r is its radius, and T its temperature. It is important to make a distinction between the radius r of the star and the orbital radius R of the planets referred to in the earlier parts of this problem — don't confuse these two different quantities r vs R! Write this formula twice, once before and once after the transition of the star to a red giant. Divide one formula by the other to solve for the new radius.]

A. 1.0×10^{-5} AU
B. 3.9×10^{-5} AU
C. 0.4 AU
D. 1.57 AU
E. 114 AU

(v) Of the two planets Q and W, which planet(s) is (are) absorbed by Star X in the course of its becoming a red giant star? (*1 point*)

[HINT: Compare the final radius r of Star X to the orbital radii R of the two planets.]

A. Neither
B. Both Q and W
C. Q
D. W

6. Kepler's Third Law can be written as $P^2 \propto R^3/M$. The moon orbits the earth with a period of 27.32 days, at a distance of 384,400 km (measured from the center of the Earth). The mass of the earth is 5.97×10^{27} g, and its radius is 6380 km.

(a) A satellite in geostationary orbit remains above the same point on the surface of the earth at all times. What is the orbital period of such a satellite? (*1 point*)

A. 1 hour
B. 12 hours
C. 5 minutes
D. 27.32 days
E. 1 day

(b) How far above the surface of the earth does a geostationary satellite orbit? (*4 points*)

7.A galaxy is observed to have a velocity of -300 km/s and is known to have a peculiar velocity of -350 km/s.

(i) Use the formula: $v_{\text{true}} = v_{\text{H}} + v_{\text{pec}}$, to calculate the recession velocity v_{H} for this galaxy. (*1 point*)

A. -650 km/s
B. 650 km/s
C. 50 km/s
D. -50 km/s
E. π km/s

(ii) Using Hubble's Law $v_{\text{H}} = H_0\, d$ where $H_0 = 68$ km/s/Mpc, determine the distance to this galaxy. (*2 points*)

A. 0.7 Mpc
A. 9.6 Mpc
A. 0.05 Mpc
A. -0.7 Mpc
A. -9.6 Mpc

(iii) What galaxy is this? (*1 point*)

A. The Milky Way
B. The Large Magellanic Cloud
C. Andromeda
D. M87
E. 3C273