Ay 112 Final Review
(Final at 8:00 AM Wednesday 6/11)

In general: a) Study the class notes on the web. The test will be mostly based upon these. The test covers the entire quarter. b) Study the solutions to the four homework sets and the midterm, Part B, all posted on the web. c) Review the midterm review notes. 1/3 of the test will come from the first half of the quarter. Study the midterm itself. Some of the questions may appear again. And d) study these notes which are -

A brief incomplete summary of the second half of the course :

Lecture 9 – Introductory Nuclear Physics – Nuclear Mass Law and Stability

The nuclear force is one of 4 fundamental forces – the strong force, electric force, weak force and gravity. The nuclear force is short ranged acting over distances less than a few times 10^{-13} cm (= few Fermis (fm)). It has a complex nature and is a tensor force, not just scalar. It thus depends on the orientation and spins of the interacting nucleons, not just their separation. On scales of a fm or so the nuclear force is strongly attractive. On shorter scales though, less than about 0.5 fm, t becomes strongly repulsive.

Nucleons, n and p, are made from the bound grouping of three quarks. Quarks are Fermions and have spin ½ and so too do the neutron and proton. Quarks are bound together by the strong or "color" force. The binding is such that it is not possible to have a free quark. Mesons are bound pairs of quarks and antiquarks and thus have spin 0 or 1. They are thus bosons and can be used as an exchange particle to produce a force. Historically the nuclear force has been described as resulting from the exchange between n and p of mesons, the lightest of which is the pi-meson. Other mesons can be exchanged at shorter range though and this gives the strong force its unusual character of being attractive at longer range and repulsive at very short range.

The nuclear force is chiefly responsible for keeping n and p together in a very small region of space where their Fermi energy is large. The nucleons in a nucleus constitute a degenerate gas and the Fermi energy is larger than the Coulomb repulsion. The Coulomb repulsion is in fact negligible for light nuclei but becomes more important for heavier nuclei since it grows as Z^2. The nucleons in the nucleus move at a substantial fraction of the speed of light (~10%) because of their degeneracy.

Because of the short range of the nuclear force, nucleons interact only with their nearest neighbors. The repulsive component of the nuclear force and the fact that they are Fermions keeps them from getting too close together. The volume of the nucleus thus grows linearly with A and the radius of the nucleus is proportional to

$A^{1/3}$. The nuclear binding energy, to first order is also linear in A and not quadratic like gravitational or electrical energy. The nuclear density is nearly constant (A/R^3) is constant and has a value 2.4×10^{14} g cm^{-3}

Two nucleons with their spins aligned are more tightly bound than two with their spins antiparallel. This is why the deuteron (1p1n with spins aligned) is bound and the diproton (1p1p with spins counter-aligned) is not.

The binding energy of a nucleus is the energy required to disperse its neutrons and protons to infinity. The binding energy can be approximated that of a liquid drop with correction terms for surface area, Coulomb energy, symmetry energy, and shell and pairing. $BE = a_1A - a_2A^{2/3} - a_3(Z^2/A^{1/3}) - a_4 (Z-N)^2/A - \delta(A) - S(A)$ where the first term is the "volume term", the attraction of each nucleon for nucleons around it; the term in $A^{2/3}$ is term subtracted for the surface area (nucleons at the surface lack partners on one side); the Z^2 term is the electrical repulsion energy (Z^2/R); the $(Z-N)^2/A$ term is the symmetry energy (it takes less energy to put equal numbers of neutrons and protons in two Fermi gases than to put an excess of either in higher energy states); $\delta(A)$ is the pairing energy (an odd number of n or p requires that the last n or p be put in a higher energy state); and S(A) is a correction term for being close to a closed shell). The liquid drop model so stated is thus a combination of analogues to classical physics (the liquid drop), bulk degeneracy properties, and quantum mechanical shell effects.

The binding energy per nucleon has a maximum in the iron group, i.e., for $A \sim 50$. Nuclei lighter than this can release energy by fusing. Heavier nuclei can release energy by fission. In order to obtain greater precision and especially to account for existence of closed shells in a quantum mechanical description of the nucleus. Uisng a realistic nuclear potential and accounting for spin-spin and spin-orbit interactions, there are closed shells when either Z or N = 2, 8, 20, 28, 50, 82, or 126 (know the first 4). As a result the nucleus ^{56}Ni (Z = N = 28) is especially tightly bound and is indeed the most tightly bound nucleus (per nucleon) with Z = N. This has important implications for nucleosynthesis and supernova light curves. Other tightly bound nuclei with Z not equal to N are ^{56}Fe, the most tightly bound iron nucleus, and ^{62}Ni, the most tightly bound nucleus of all (for any Z and N).

Because of the symmetry term, the most abundant isotope of any evenly charged nucleus up to Ca has equal numbers of neutrons and protons – e.g., ^{4}He (Z=N=2), ^{12}C (Z=N=6), ^{16}O (Z=N=8), ^{28}Si (Z=N=14), ^{40}Ca (Z=N=20).

The energy yield from any nuclear reaction is the sum of the binding energies of the products times their abundances minus the binding energies of the reactants times their abundances. $q = 9.65x10^{17} \sum(\delta Y_i)(BE_i)$ minus a small correction term for neutrino losses and protons turning into neutrons by weak interactions.

Only a small fraction of all collections of neutrons and protons are stable. Many are unstable to break up on a very short time – those that have too many neutrons or protons to stick together. These are said to be beyond the neutron or proton "drip line", e.g., 26 protons and 100 neutrons do not constitute a bound nucleus. The extra neutrons would go into so high energy states that they would be immediately ejected. Some nuclei, like uranium, emit alpha particles. The nucleus ^{8}Be immediately splits into two alpha particles (He nuclei). Of those nuclei that are bound, the vast majority are still unstable to decay by the weak interaction. Two kinds of weak interaction – electron capture and positron decay – turn protons into neutrons; one – beta decay – turns neutrons into protons. Each time a weak decay occurs a neutrino or anti neutrino is emitted to conserve lepton number. As a result for each A there is only one or a few isotopes that are stable – e.g., ^{56}Fe (Z = 26, N = 30), but not ^{56}Co (Z = 27, N =29) or ^{56}Mn (Z = 25, N = 31). The latter two decay to ^{56}Fe.

Lecture 10 – Nuclear Physics – Reaction Rates and Stellar Burning Processes

The nuclear reactions of interest in stellar astrophysics are usually of the binary fusion variety $I + j \rightarrow k + L$ or $\mathrm{I(j,k)L}$. The key quantity needed to evaluate the rate at which these fusion reactions occur is the cross section $\sigma_{jk}(I)$ which has units of area (the basic unit is the barn = 10^{-24} cm^2) and gives the number of reactions per nucleus I that happen per second for a given flux of species j. The ions are always an ideal gas so the reaction rate is the average over a Maxwell Boltzman distribution of velocities of σ times v. Then a typical term in a differential equation describing the time evolution of species I is $\frac{dY_I}{dt} = -\rho Y_I Y_j N_A \left\langle \sigma_{jk}(I) v \right\rangle + \ldots$ The cross section itself has several major dependences - a quantum mechanical area factor out front, $\pi \lambdabar^2 \propto 1/E$; a quantum mechanical barrier penetration factor $P \propto e^{-2\pi\eta}$ with $\eta = 0.1576\, Z_I Z_j \sqrt{\hat{A}/E_{MeV}}$ and $\hat{\mathrm{A}} = \mathrm{A}_I A_j / (A_I + A_j)$ and a structure factor X, which for non-resonant reactions is nearly constant. The appearance of the product $Z_I Z_j$ in the exponential makes the reaction rate for the fusion of nuclei of higher charge much smaller at a given temperature than for lighter charge, or equivalently means a higher temperature is required to fuse nuclei with bigger charge. Carbon burning requires a higher temperature than hydrogen burning, etc.

The cross section σ is measured in the laboratory when possible, or else comes from theory. It often turns out that, because of the barrier penetration exponential, the cross section can only be directly determined at a higher energy than is appropriate for the reaction in stars and an extrapolation is required. It is thus useful to note that for charged particles, the factor $S = \sigma\ E\ e^{2\pi\eta}$ is approximately constant and can be more accurately extrapolated than the cross section itself.

Given that the rate factor involves an integral over energy for an exponentially declining energy distribution (the Maxwell Boltzmann distribution) and an exponentially increasing barrier penetration factor, the integrand can be approximated as a Gaussian and analytically integrated. In the substitution the energy of the center of the Gaussian, called the Gamow energy, $0.122\left(Z_I^2 Z_j^2 \hat{A} T_9^2\right)^{1/3}$ MeV and the width of the Gaussian $0.237\left(Z_I^2 Z_j^2 \hat{A} T_9^5\right)^{1/6}$ MeV are derived by matching first and second derivatives. The Gamow energy is the center of mass energy where most of the reactions occur in the star at temperature T_9 measured in billions of K and Δ is the range of energies of interest. The S factor is evaluated at E_0 and pulled out of the integral. The reaction rate factor

$$\lambda_{jk}(I) = N_A \left\langle \sigma_{jk}(I)\, v \right\rangle = \frac{4.34 \times 10^8}{Z_I Z_j \hat{A}}\, S\left(E_0\right)\tau^2 e^{-\tau} \quad \text{where } \tau = \frac{3E_0}{kT} = 4.248\left(\frac{Z_I^2 Z_j^2 \hat{A}}{T_9}\right)^{1/3} .$$

Given the S factor from the laboratory or other calculations, one can then evaluate the rate at a given temperature. It can also be shown that $\lambda \propto T^n$ with $n=\left(\tau - 2\right)/3$.

Hydrogen can burn to helium in stars by a variety of processes. In the sun and other low mass stars, the pp-cycles dominate. The simplest (pp1) cycle that accounts for 85% of the sun's present luminosity is $p(p,e^+\nu)^2H(p,\gamma)^3He(^3He,2p)^4He$ where the first two reactions happen twice in order to make two ^{3}He's. As a result 4 protons disappear, two positrons are ejected which annihilate with electrons to give back their rest mass energy, two neutrinos escape, and one ^{4}He is created. There are two other ways to complete the pp chain. $^3He(\alpha,\gamma)^7Be(e^-,\nu)^7Li(p,\alpha)^4He$ and $^3He(\alpha,\gamma)^7Be(p,\gamma)^8B(e^-,\nu)^8Be \rightarrow 2\,^4He$. These side reactions are mostly of interest because of the higher energy neutrinos they emit which were historically easier to detect in terrestrial neutrino observatories. [You need to memorize pp1, not the other two].

Hydrogen can also burn, at higher temperatures, by the CNO cycle. This requires the presence of some initial carbon and oxygen in the star to catalyze the reaction sequence $^{12}C(p,\gamma)^{13}N(e^-,\nu)^{13}C(p,\gamma)^{14}N(p,\gamma)^{15}O(e^-,\nu)^{15}N(p,\alpha)^{12}C$. [you do not have to memorize this chain for the exam]. Here α denotes a helium nucleus and once again 4 protons go in and two neutrinos come out in addition to the helium. There are a couple of side branches that feed into this main loop that feed ^{16}O into the chain. The slowest reaction is $^{14}N(p,\gamma)^{15}O$ [you do need to know that] so besides turning H into He, the CNO cycle turns C and O into N and that is the origin of ^{14}N in nature. The CNO cycle dominates for temperatures above 18 million K and depends on T^{18} power (while the pp cycle depends on T^4) and so, while unimportant in the present sun, the CNO cycle dominates in stars above 2 solar masses and also during hydrogen shell burning for all stars.

The energy yields of the pp cycles and the CNO cycles are similar, around 26 MeV for each helium made, with a small difference due to the energies of the neutrinos that are emitted. This implies about 4 x 10^{18} erg/gm is liberated when a composition of 70% H, 30% He burns to helium. Energy generation rates can be evaluated given the known S-factors for the critical reactions (pp and $^{14}N(p,\gamma)$) and need not be memorized (but know the powers of T upon which each depends). Using these rates one finds that the present luminosity of the sun and its expected lifetime are well reproduced.

Helium burning is a quite different sort of process in that it involves the almost simultaneous reaction of 3 helium nuclei (it is called the "3-alpha process") and it can produce two major products, ^{12}C and ^{16}O. The first is made by $^4He(\alpha,\gamma)^8Be^*(\alpha,\gamma)^{12}C$ where the * on ^8Be indicates that it has a very short existence before becoming unbound to two helium nuclei. It takes a high density for ^8Be to capture another helium before splitting in two and that is why no heavy elements are made in the Big Bang (the density in the Big Bang when it is cool enough for neutrons and protons to stick together and make nuclei is about 10^{-5} g cm^{-3}). After appreciable carbon has accumulated the reaction $^{12}C(\alpha,\gamma)^{16}O$ can convert some of the carbon t oxygen and this is where carbon and oxygen come from in nature. No weak interactions are involved and no neutrinos are emitted. The reaction sequence is very temperature sensitive going as T^{20} for typical conditions (2 x 10^8 K). The energy yield from helium burning is about 8 x 10^{17} erg gm, depending on the final ratio of C and O. This is about 20% as much energy as hydrogen burning gives (for solar composition) and due to the higher luminosities and smaller yield, helium burning is a shorter lived stage in the life of stars.

The discussion in class was mostly about "non-resonant reactions" (which also includes the tails of very broad resonances) where the S-factor is slowly varying. There are many reactions where the presence of a few narrow resonances causes the nuclear structure factor, X, to vary rapidly and so S is not a constant. These "resonant reactions" require a different formalism discussed in Clayton. This will not be a topic on the test though.

Lecture 11 – Star Formation and the Interstellar Medium

The interstellar medium is complex and multi-phased. Broadly speaking it can be ruled into regions that are hot, warm, and cold. These are characterized by the state of hydrogen, whether it is fully ionized (H II – the hot medium), neutral (H I – the warm), or molecules (H_2 – cold). The hot medium, also called the hot coronal gas may have a temperature of millions of K and a very low density, ~ 0.001 ions per cm^{-3}. It is heated by supernova explosions, uv light from young massive stars, and cosmic rays. It comprises a substantial fraction of the volume of the ISM, but not much of the mass. A special case of the hot medium is the “H II regions” surrounding young massive stars. These are pinkish in appearance due to H_α emission and illuminate regions of star formation, especially along spiral arms. The warm medium has temperatures ~5,000 - 10,000 K and densities typical of the average ISM, about 0.1 to 1 atom cm^{-3}. This medium can be studied by 21 cm emission in the radio and accounts for a substantial fraction of both the volume and mass of the ISM. The total mass of the ISM is about 10^{10} solar masses.

The cold part of the ISM is chiefly found in molecular clouds which, while spread out through the disk of the galaxy, have a large concentration in a ring about 5 kpc from the center of our galaxy. Molecular clouds contain a sunstantial fraction of the ISM mass but have small volume. They are continuously being formed and collapsing into new stars. Molecular clouds are very cold (10 – 40 K is typical; water freezes at 273) and dense (100 – 10^4 molecules cm^{-3}). Because they are so cold a lot of the heavy elements have condensed into dust. The dust shields the cloud from star light and helps keep the interior cold. Clouds cool by molecular radiation and infra-red radiation from the dust. They are thus best observed in the radio and infrared. Carbon monoxide is the best tracer of molecular clouds. The emission of H_2 itself is hard to see.

The nearest example of a molecular cloud is the Orion nebula complex, a cluster of clouds in the constellation of Orion spread out over several degrees in the sky, almost as big as the constellation itself. This complex has a mass of several hundred thousand solar masses and is several hundred light years across. It is several million years old (as judged by its collapse time scale and the young stars we see in it). Quite prominent in the dagger of Orion is the H II region ionized by the recently born massive stars in the Trapezium. Star formation is very clearly going on in the Orion nebula. We see 1000’s of young stars and protostars.

Using the Virial theorem we can estimate the minimum mass a cloud has to have in order that its gravitational binding energy substantially exceed its internal energy in the form of heat. The result is called the “Jean’s mass”, $M_J = 44 M_\odot \left(\frac{T^3}{\mu^4 n} \right)^{1/2}$ where n is the number density of atoms or molecules per cm^3 and μ is, as usual, the mean molecular weight. For a given temperature and density, clouds with mass bigger than M_J are unstable to collapse and star formation. Those smaller are not. By this criteria only the cold ISM, i.e., the molecular clouds, is unstable to forming stars. An important difference between a collapsing cloud and a star is that the former is optically thin and light can escape from it, so as the cloud collapses and its density goes up, its temperature does not. The cloud thus remains unstable to further collapse and to fragmentation. Because of fragmentation the collapse of an initially large cloud can result in the formation of many low mass stars. There is a limit though of ~ 0.01 solar masses on the smallest fragment that can collapse. Smaller fragments would have to radiate more than the blackbody limit would allow on a hydrodynamic timescale.

A cloud of the Jeans mass collapses on a hydrodynamic time scale – typically hundreds of thousands of years. Eventually when the radius becomes a few thousand AU, it becomes optically thick and can no longer cool effectively. It begins to heat up, first dissociating its molecules and then ionizing hydrogen when the temperature approaches 10,000 K. The energy spent on the ionization keeps the cloud unstable to collapse. but once the temperature exceeds about 10,000 K in the inner part of the star, continued contraction leads to additional heating. The collapse stops in the inner core and hydrostatic equilibrium is achieved. The rest of the cloud continues to accrete on that core. The luminosity of the accretion and contraction of the core is very large and the opacity is quite high due to the H-minus ion. The star is thus fully convective. Such stars occupy the "Hayashi strip" in the HR diagram with a nearly constant photospheric temperature around 3500 – 4000 K and a variable luminosity. Low mass stars follow the Hayashi strip straight down (in the HR diagram) to the main sequence, since such stars are convective on the main sequence anyway. More massive stars eventually develop a radiative central core after H-minus is destroyed and evolve to the left onto the main sequence.

T-Tauri stars are young stars that may still be in their Kelvin-Helmholtz stage approaching the main sequence. They show optical variability, both emission and absorption lines, large lithium abundance, and evidence for accretion disks and jets (polar outflows).

Lecture 12 – Overview and Critical Masses

So long as the pressure at the center of a star remains non-degenerate, its evolution is governed by the Virial theorem. It contracts keeping $T_c \propto M^{2/3}\rho_c^{1/3}$. When it reaches a temperature where a given fuel can burn it pauses, sometimes for a very long time, and burns that fuel away, and then resumes its Kelvin Helmholtz contraction. If along the way the density becomes so high that the core becomes degenerate, then the contraction halts and the temperature no longer rises. Unless burning in a shell adds sufficient mass to cause the core to resume its contraction, the core remains degenerate and is permanently stable – held up by degeneracy pressure which does not depend on the temperature. It becomes a white dwarf of some kind.

This general idea gives rise to critical masses. For each mass of contracting protostar less than the Chandrasekhar mass (and assuming no mass is added), the Kelvin Helmholtz contraction produces a maximum temperature and then cools off as it becomes degenerate. This maximum temperature is $5.7\times10^7\ \text{K}\left(\frac{M}{M_\odot}\right)^{4/3}$ for hydrogenic stars and $3.2\times10^8\ \mathrm{K}\left(\frac{M}{M_\odot}\right)^{4/3}$ for helium stars. There is also a temperature that must be reached in order for nuclear reactions to provide the necessary energy generation rate for a star of a given mass to stably burn hydrogen, helium or carbon. That can be estimated for the sun from its luminosity to mass ratio, about 2 erg g^{-1} s^{-1} . Actually since the energy is generated in the center and not spread evenly throughout the mass of the star, the required energy is ~10 times larger. For other stars we may assume that $L \propto M^3$ and for helium stars we can use Eddingtons quartic model. Setting the energy generation rates derived in a previous lecture for ε_{pp}, ε_{CNO}, and $\varepsilon_{3\alpha}$ to these values required to power stars of various compositions we can derive "ignition curves" (central temperature vs central density) for hydrogen and helium burning. We then see for what masses, the highest temperature obtained before becoming degenerate exceeds the values needed for ignition.

As a result we find the minimum mass needed for hydrogen ignition is 0.08 solar masses and the minimum mass needed for helium ignition is 0.45 solar masses. Taking 8 x 10^8 K as the temperature needed for carbon ignition, it turns out that a core of 1.06 solar masses is needed. Know these masses.

Actually for carbon, oxygen and silicon burning, the ignition condition is given, not by any observed requirement or by an n = 3 polytrope, but by the necessity that energy generation balance neutrino losses by the pair process (see lecture 15). Because of the large temperature sensitivity of the reaction rates, the burning happens at a nearly unique set of temperatures, 0.8, 1.8 and 3.5 billion K for carbon, oxygen and silicon burning respectively. As already mentioned this gives 1.06 solar masses as the critical carbon-oxygen core mass required to ignite carbon burning. This size carbon oxygen core is developed only in stars bigger than 8 solar masses on the main sequence, hence the defining branch point between stars that make white dwarfs and those that make planetary nebulae is 8 solar masses on the main sequence (not 1.06). The necessary mass to ignite oxygen and silicon burning is close to the Chandrasekhar mass. Generally stars that ignite carbon burning will go on to ignite oxygen burning, though there is a narrow range of main sequence masses between 8 and 10 solar masses that give rise to neon-oxygen white dwarfs – stars that have burned carbon, not oxygen.

Along the evolutionary path other instabilities may occur, including the electron-positron pair instability for main sequence masses above 100 solar masses, and photodisintegration that occurs following silicon burning in all stars between 8 and 100 solar masses.

Lecture 13 - The Main Sequence and Homology

Two stars are homologous to each other if one is simply a geometric magnification of the other. This is true if the ratio of the radius containing a given fraction of the stars mass to the radius of the star is the same for both stars. If one star can be expanded into the other keeping $\rho \propto r^{-3}$ at all points in the star, the initial and final stars are also homologous. A sphere where every Lagrangian mass shell expends at a constant speed also generates a homologous set of spheres. The expanding universe is an example. Polytropes of a given index are another. Given the homology assumption, the 4 stellar structure equations and the equations of state, stellar energy generation and opacity can all be expressed as simple algrebraic power laws rather than differential equations. Combining these expressions gives useful scaling rules that describe how the properties of main sequence stars or helium cores scale with their mass. Examples are $L \propto \frac{\mu^4 M^3}{\kappa}$ for constant opacity and

and $L \propto \frac{\mu^{7.5} M^{5.5}}{R^{1/2}}$ for Kramer's opacity. In order to find a radius-mass relation it is necessary to specify the temperature and density dependence of the nuclear energy generation rate. For Kramer's opacity and pp-cycle burning, $L \propto \mu^{7.77} M^{5.46}$, $R \propto \mu^{-0.54} M^{0.0769}$. Many other relations can be obtained, that might be probed on Part B of the exam, but some interesting general results for main sequence stars are:

a) The radius depends on a fractional power of the mass (less than linear) but slowly increases with mass, consistent with observations

b) Stars of higher mass have lower central density and higher central temperature. They will thus tend to be dominated by electron scattering opacity in their interiors and burn hydrogen by the CNO cycle (above 2 solar masses). Given the high power of temperature to which energy generation is sensitive (ν = 18) massive stars will have convective cores. Given their higher luminosities and effective temperatures their surfaces will be radiative.

c) Conversely lower mass stars will have cooler surfaces (because of their lower luminosities) and will tend to be dominated by Kramers opacity and have convective surfaces. With energy generation from the pp cycle (ν = 4), they will have radiative cores until the mass is very low (0.5 solar masses).

d) The mass luminosity relation will be linear in M for very massive stars, propotional to M^3 for intermediate mass stars, proportional to M^5 around the mass of the sun and proportional to a lower power for masses less than the sun (our simple homology treatment did not include fully convective stars)

e) Main sequence stars of lower metallicity (lower opacity) will be bluer and have a smaller radius.

As the sun evolves, its central hydrogen abundance goes down, its central temperature and density go up, its luminosity and radius also increase. During its 9 Gy lifetime on the main sequence the sun's luminosity varies by about a factor of 2.

Neutrinos have been observed coming from the sun by 4 detectors. In each case the detectors are far underground to reduce background and very large because the cross sections for stopping neutrinos are small. The largest detectors search for radiation from recoil electrons in big tanks of water or heavy water. Other smaller detectors look for inverse weak interactions, turning ^{37}Cl into ^{37}Ar for example and watching the Ar decay back again. The solar neutrinos are clearly seen by all 4 experiments, but in all but one case the flux is about 1/3 to ½ of what is predicted by the models. The exception is the Sudbury Neutrino Observatory that can observe all three flavors of neutrinos, ν_e, ν_μ, ν_τ . When all three fluxes are added together the agreement with predictions is very good. The central temperature of the sun is measured to high accuracy to be 15.7 million K. However, the neutrinos initially produced solely as ν_e have "mixed" into ν_μ and ν_τ as well. This mixing requires that the neutrino have a small rest mass. The neutrino mass states are not pure flavor states. This denotes physics beyond the standard particle physics models (where the neutrino has no rest mass).

After leaving the main sequence, stars continue to burn hydrogen in a thick shell around a hydrogen depleted (helium) core that is initially non-degenerate. Lacking any internal source of energy and kept hot by the surrounding hydrogen burning shell, the helium core becomes isothermal and does not immediately experience a Kelvin-Helmholtz contraction. As mass is added to the helium core by shell burning, it reaches a critical mass called the "Schonberg Chandrasekhar mass" where it can no longer support the rest of the overlying star and remain isothermal at the same time. The SC mass is $0.37\left(\mu_{env} / \mu_{core}\right)^2$, or about 8% of the mass of the star. Once the helium core starts contracting it can become degenerate in stars below 2 solar masses.

Lecture 14 – Post-main Sequence for Low and Intermediate Mass Stars

The contraction of the helium core causes the hydrogen burning shell to move into a smaller radius where the gravitational potential is higher. The properties of the burning shell are set by the helium core upon which it rests and not the larger envelope mass above it. The hydrogen shell becomes thinner in mass and more luminous, its luminosity increasing as the sixth power of the helium core mass. For stars lighter than 2 solar masses, the thin shell gradually increases the degenerate helium core until it reaches 0.45 solar masses, the mass necessary for helium ignition. During this time, the luminosity of the star grows by a large factor. If the star was lighter than the sun and already convective, it moves almost straight up in the HR diagram, staying on the Hayashi strip around 4000 K. For higher mass stars, the increased power first goes into expanding the star at near constant luminosity and, once it develops a convective envelope, raising its luminosity.

Once the helium core reaches 0.45 solar masses, it ignites helium burning degenerately in a series of violent flashes. In the sun these take about 1.5 M y before the sun finally ignites helium stably in its center. The power developed in these flashes is very large but never reaches the surface of the star. Instead, as a result of the flashes, both helium core burning and hydrogen shell burning are put out and the star becomes fainter. When it stabilizes again as a star powered by helium core burning its luminosity has gone down and its radius shrunk. The sun (and other stars below 2 solar masses) burn helium on the "horizontal branch". The sun and other solar metallicity stars of similar mass burn helium on the right hand (red) end of the horizontal branch. Such stars are known as "red clump" stars and there are a lot of them. HB stars with lower metallicity and lower envelope mass lie more blueward in the HR diagram. Since all HB stars have helium cores near 0.45 solar masses, their luminosity dos not vary greatly – about 20 - 50 $L_{\odot}$. The lifetime on the HB is also roughly constant, about 120 My. The horizontal branch is quite striking in the HR diagram of globular clusters where one finds many bright bluish stars (owing to the lower metallicity and star mass). When helium is exhausted in the center of the helium core it leaves the horizontal branch and burns both hydrogen and helium in thin shells atop a degenerate core of carbon and oxygen. The thin shells are once again very luminous and the star ascends the red giant branch again, becoming an asymptotic giant branch (AGB) star.

Stars more massive than 2 solar masses (but less than 8) also become AGB stars, but have a very different evolution up to that point. When they first deplete hydrogen in their centers, they experience a period of thick hydrogen shell burning (as do the lower mass stars), but their luminosity does not rise immediately. Because they begin with radiative envelopes, they experience a phase of rapid expansion at near constant luminosity. This rapid motion to the right in the HR diagram creates what is called the "Hertzsprung gap". They then develop convective envelopes and thin hydrogen shells and their luminosity rises as they become fully developed red giants. They ignite helium burning non-degenerately on the first try and never become horizontal branch stars. Their helium core mass is already bigger than 0.45 solar masses when helium ignites in the center. They thus stay red giant stars, but may experience "blue loops" in the HR diagram while burning helium. During these blue loops they may be Cepheid Variables. While burning helium they are much more luminous than HB stars.

Shell burning in AGB stars is complicated because of the thin shell instability. Hydrogen shell burning leads to the accumulation of a thin helium shell that burns in a flash temporarily, putting out the hydrogen shell. Outside the shells the star is convective and convective dredge up extends into the ashes of the helium shell flash mixing its composition (rich in C, N, O, and s-process) to the surface of the star where it can be lost to a wind. After a period of dredge up, the hydrogen shell reignites and builds another thin helium shell and the process continues. As a result of all these shell flashes the mass of the degenerate carbon-oxygen core gradually grows, but never becomes big enough to ignite carbon burning. The radius of the star and its luminosity increase dramatically as the carbon-oxygen core grows and the helium shell flashes may aid in ejecting what is already a very loosely bound envelope. In any case, before the carbon-oxygen core ever becomes big enough to ignite carbon fusion (1.06 solar masses), the entre envelope is low. Towards the end the mass loss becomes very rapid and a planetary nebula is produced.

The helium shell flashes during the AGB phase also produce some interesting nucleosynthesis. Side reactions during the helium shell flash produce a small abundance of free neutrons $(^{13}C(\alpha,n)^{16}O \text{ and } ^{22}Ne(\alpha,n)^{25}Mg)$. [The ^{13}C is there because mixing a trace of hydrogen into the helium shell turns some ^{12}C into ^{13}C; the ^{22}Ne is there because of two α-captures on ^{14}N left over from the CNO cycle but you won't need to remember that]. These neutrons add onto existing iron (and other elements) to produce heavier nuclei by a process known as "slow neutron capture" or the "s-process". For each element neutrons capture one by one on a time scale of years until, for a given Z, a neutron number is reached where the isotope has a short lifetime to beta-decay. The decay increases the proton number to Z+1. Generally that new isotope is stable and the process of neutron capture continues. This s-process continues on up to bismuth (Z = 83) where alpha-decay finally terminates the s-process. Along the way, more than half of the isotopes heavier than the iron group are produced. The s-process is mixed to the surface by convection in between shell flashes where it is ejected either in a wind or during the planetary nebula phase.

Planetary nebulae are brief (~10,000 yr) phases in the lives of stars below 8 solar masses that mark the transition from being an AGB star to being a white dwarf. These colorful displays are relatively rare because of their short lives and often have peculiar symmetries, perhaps due to binary star wind interactions or the effects of magnetic fields. They have emission lines, e.g., of oxygen, and show evidence for freshly synthesized elements. They glow because of the ultraviolet energy input by the newly revealed hot carbon-oxygen core which, while some day to become a faint white dwarf, may initially have an effective emission temperature ~100,000 K and a luminosity much greater than the sun.

After the planetary nebula disperses, what is left behind is a carbon-oxygen white dwarf, a hot ember, not really a star, supported by electron degeneracy pressure. Due to efficient electron conduction, the degenerate core (most of the mass) of the white dwarf becomes isothermal. The initial core temperature is well over 10^8 K but this rapidly cools down to a few x 10^7 K as the white dwarf grows fainter. The degenerate layer is capped by a thin, low mass envelope through which the trapped heat slowly diffuses. The source of the white dwarf's luminosity is the heat capacity of the ions (C and O) in its core. While the electrons are degenerate, the ions are not and they are still capable of storing enough heat to keep the white dwar glowing faintly for billions of years. The sun will make a white dwarf of about 0.7 solar masses. 0.6 solar masses is the typical average mass of white dwarfs. Typical luminosities are 0.0001 to 0.001 solar luminosities (.01% to 1%) and radii are around 5000 km. The coolest white dwarfs today have surface temperatures of 4000 K, but most are much hotter and emit in the ultraviolet. Very old, very cold white dwarfs can eventually become solids – a crystal lattice of sorts.

Lecture 15 – The Advanced Evolution of Massive Stars, Supernovae, and Nucleosynthesis

Stars more massive than 8 $M_\odot$ have different lives and very different deaths chiefly because they can ignite and burn carbon and other heavier elements non-degenerately. To begin with they have short lives and high luminosities. They are blue and white hot and found in regions of recent star formation. Their cores are convective on the main sequence where they are powered by the CNO cycle and electron scattering opacity dominates in their interiors. Their evolution in the HR diagram is similar to the intermediate mass stars (as typified by the 5 solar mass model) but they are more luminous at each stage. After hydrogen depletion they move rapidly to the right in the HR diagram and become red supergiants. They ignite helium non-degenerately and some may experience blue loops. They never become horizontal branch stars or AGB stars because their helium and carbon cores are never degenerate. They do not have thin helium shell flashes. They do have a lot of mass loss during the red supergiant phase though. A 25 $M_\odot$ star may have only 12 $M_\odot$ when it dies. A 35 $M_\odot$ star may lose its entire hydrogen envelope before dying and heavier stars even lose part of their helium cores [these massive hydrogen stripped stars are observed and are called "Wolf-Rayet" stars but we did not discuss them].

The evolution of stars above $8M_\odot$ after carbon ignition is very rapid because of neutrino losses by the electron-positron pair process. For temperatures above 0.5 billion K, the blackbody radiation energy distribution has sufficiently energetic photons in its tail to maintain an appreciable steady state abundance of electron-positron pairs. Usually these annihilate to produce gamma-rays but occasionally a weak interaction occurs and $e^- + e^+ \to \nu + \bar{\nu}$. The neutrinos leave the star at the speed of light and constitute a growing sink of energy as the temperature rises. At very high T, the neutrino loss rate scales as T^9 but at lower T as appropriate for carbon and oxygen burning it is even more T-sensitive, going more like T^{14}. As a result, the burning of heavier fuels is rapidly accelerated and the bigger the charges involved the higher the T needed to enable Coulomb barrier penetration and the shorter the lifetime. Carbon burns in a few centuries ($T_9 \sim 0.8$; $T_9 = T/10^9$ K), oxygen in a few months ($T_9 \sim 1.8$) and silicon in a few days ($T_9 \sim 3.5$).

Each of these fuels burns to a variety of species, but the chief products are for a) carbon burning: Ne and Mg plus traces of Na and Al (but these traces are important because the abundances of these elements are small. This is where they are made); b) oxygen burning: Si and S plus traces of Cl, Ar, K, and Ca; and c) silicon burning: iron and nickel plus traces of Sc, Ti, V, Cr, Mn, and Co. The energy release for each phase is $\sim 10^{17}$ erg g^{-1} (somewhat more for oxygen burning) so the lifetime grows very short due to the neutrino losses. After carbon ignition the neutrino losses from the star greatly exceed the radiation in the form of light from its surface. The radius and effective temperature of the giant star do not change after carbon ignition (until the star explodes). At the end of silicon burning, the star has a complex structure due to the activity of numerous burning shells. Most of the volume is the red supergiant (convective) envelope composed of unburned hydrogen and helium. Inside, capped by a hydrogen burning shell, is the helium core with a radius comparable to the sun and a mass about ¼ - 1/3 of the original main sequence mass. Inside the helium core are carbon, oxygen, and silicon burning shells and, a the center, roughly a Chandrasekhar mass sized ball of semi-degenerate iron with a radius like that of the Earth. The central temperature and density for this starting "presupernova model" are both near 10^{10} (K and g cm^{-3}).

The iron core is unstable. No further energy can be released by nuclear fusion reactions. On the contrary, increased contraction and heating leads to "photodisintegration" – a tearing apart of the iron nuclei into helium that saps the core of energy. In addition the electrons have such high Fermi energy that they begin to capture on iron nuclei making them more neutron rich and robbing the core of the very electrons that were holding it up. The neutrino losses are also prodigious. So the iron core collapses, first slowly then rapidly. From the time of the presupernova model (when the contraction speed first reaches 1000 km/s), the remainder of the collapse and explosion of the core will all transpire in less than a second.

As the iron core collapses, it becomes so dense that neutrinos are trapped (at $\rho \sim 10^{11}\ \mathrm{g\ cm^{-3}}$) and must diffuse out. As the density approaches 10^{14} g cm^{-3} the nuclei touch and begin to merge into one gigantic stellar-sized nucleus. For an instant, the nucleons feel a strong attraction to each other due to the attractive component of the nuclear force, but as the density shoots up to several times terrestrial nuclear density (2.4×10^{14} g cm^{-3}) the short range repulsive component of the nuclear force comes into play, halting the collapse in roughly the inner half of the collapsing core. The rest of the iron core continues to rain down on this stagnant inner part and a bounce occurs, an outward moving shock wave is created, but this is quickly swallowed up by the infalling material and dissipated by neutrino emission and photodisintegration. After a ms or so, one is left with a hot proto-neutron star with radius about 50 km onto which the rest of the star is falling.

Over the next few seconds, the proto-neutron star experiences its Kelvin-Helmholtz evolution finally shrinking down to a final radius of about 10 km and emitting its binding energy, 2 – 3 x 10^{53} erg, as neutrinos. The mass of the neutron star decreases by ~ 15% due to the loss of so much binding energy. What happens during the first few tenths of a second after the proto-neutron star forms is critical and uncertain. If a small fraction, a percent of so, of the neutrinos deposit their energy outside where the density and gravitational binding are less, a supernova shock of ~10^{51} erg is generated that blows away all the star outside the neutronized core. If not then some combination of rotation and magnetic fields must blow up the star. This is possible, but computationally challenging. Or the star may collapse to a black hole. It must explode some of the time, because we are here, but it must also collapse to a black hole some of the time, because stellar mass black holes are observed to exist.

Given a 10^{51} erg explosion started at the center of a red supergiant, the outcome is predictable. As the shock wave moves through the star matter is compressed and heated, some to such high temperature that "explosive nucleosynthesis" occurs. This happens when the temperature is high enough that the nuclear time scale becomes shorter than or comparable to the hydrodynamical time scale $\sim 446/\rho^{1/2}$, or about a second. Above 5 billion degrees Si and O burn to ^{56}Ni and other species in the iron group. Above 3 billion. Oxygen burns explosively to Si and S; above 2 billion carbon burns explosively to Mg and Ne – the same products as previously discussed for burning in hydrostatic equilibrium, but on such a short time scale that there is no time for weak interactions. The product nuclei mostly have Z = N then and many of the familiar elements of nature are made as radioactive progenitors, e.g., ^{56}Fe is made as ^{56}Ni; ^{48}Ti is made as ^{48}Cr; etc. About 0.1 solar masses of ^{56}Ni is made in the explosion and this has an important effect on the late time light curves of Type II supernovae and the peak light emitted by Type Ib and Ic supernovae. In a typical 25 $M_\odot$ model the shock wave modifies the composition of the inner 3 $M_\odot$. The rest of the star is ejected without its composition being modified. Some additional s-processing goes on prior to the explosion in the helium shell and this is responsible for making the s-process nuclei between A = 60 and 90. AGB stars make the s-process from A = 90 to 208.

A special case of explosive nucleosynthesis is the r-process which may go on either in the deepest layers ejected in a core-collapse supernova, or in merging neutron stars – or both. What is required is an enormous flux of neutrons, roughly 10^{14} times greater than in the s-process. Synthesis of the most neutron-rich isotopes from Ni (Z = 28) up to uranium (Z = 92) then goes on in a hydrodynamical time scale of less than one second. In both the merging neutron star and neutron-star (neutrino-powered) wind case, the ejected matter begins as mostly neutrons with a few protons. During the expansion and cooling, a few neutrons and all the protons fuse to form helium and some of the helium burns to iron group elements. The ratio of neutrons per seed iron nucleus is then very large as the material cools down. Very neutron rich, unstable progenitor isotopes are formed that later decay back to the stable elements. Gold and platinum, as well as uranium, are among the elements predominantly made by the r-process.

Lecture 16 – Supernovae

A supernova is the explosive death of a star. Except in rare instances it only happens once. Observationally there are two types – Type I shows no evidence for hydrogen in the spectrum at peak light; Type II shows hydrogen. Model-wise there are also two kinds of (common) supernovae, but they do not necessarily line up with the two observational classes. Iron core collapse in massive stars produces Type II supernovae (from massive Population I stars), but also makes Type Ib and Ic if the massive star has lost its hydrogen envelope before dying (the distinction between Ib and Ic is not great – one spectral line, and we will not distinguish models for them). The thermonuclear explosion of a degenerate carbon-oxygen white dwarf star, on the other hand, makes Type Ia supernovae.

The most common kind of supernova is Type II-plateau (IIp). These come from the explosion following iron core collapse in a red supergiant star. There are three phases to the light curve: a) Shock breakout ($T_{eff} \sim$ few x 10^5 K for an hour followed by rapid cool down the first day); b) a plateau of nearly constant luminosity as hydrogen recombines in the expanding envelope at R $\sim 10^{15}$ cm and T = 5500 K; and c) a radioactive “tail” of slow decline as the ^{56}Co made in the explosion decays (the ^{56}Co is made initially as ^{56}Ni, but that decays to Co in the first week). Most of the light comes out during the plateau which lasts about 100 days. The total energy radiated is about 10^{49} erg. Thus the explosion energy is 99.5% neutrinos (3 x 10^{53} erg); 0.5% kinetic energy (10^{51} erg); and, bright as they are, only 0.005% light (10^{49} erg). The neutrino emission was detected in one case (SN 1987A) and agreed with the model predictions. The uv-emission immediately following break out has been seen in at least one case and the plateau and tail have been seen in very many cases. Type II supernovae happen about every 50 years in our Galaxy although many historically went undetected because of dust in the disk. Type II SN are concentrated in the Galactic disk and are associated with regions of star formation – molecular clouds, H II regions, spiral arms, spiral and irregular galaxies, but never elliptical galaxies. Core collapse supernovae (SN IIp, Ib, Ic) always leave neutron stars or black holes behind.

About two dozen presupernova stars have been identified shortly before they exploded as Type II supernovae. From the luminosity of the presupernova star and a stellar model we can infer its mass. In all cases where small error bars exist the mass is between 8 and 20 $M_{\odot}$ suggesting that more massive stars either fail to explode or that their progenitors are systematically harder to observe (dusty region?). In any case, theory too suggests that above some mass, the star becomes hard to explode. The gravitational binding energy of matter outside the collapsing iron core is just too great and the infall has too much momentum for the neutrinos to turn around. Presumably such stars make black holes when they die. Whether the black hole can be formed without any bright transient being produced is an interesting question.

If the neutron star left by a core collapse supernova a) is rapidly rotating, b) has a strong magnetic field (10^{12} Gauss or so), and c) has its rotational axis not aligned with its magnetic axis, it can be a pulsar. Pulsars are found with periods of ms to seconds and light up some remnants of recent supernovae like the Crab Nebula. They emit recurrent pulses of electromagnetic radiation, typically across the entire spectral range from radio to gamma-rays. Their periods are very precise closes (16 figures) though they do gradually grow longer as the pulsar spins down. Most of the energy radiated goes into accelerating electrons which leave the pulsar and energize the nebula they are in when they are young. A part of the energy is emitted as synchrotron radiation, which is beamed. We see a pulse of light as this beam sweeps across our line of sight like a lighthouse. Pulsars accelerate charge because the rotation gives a locally varying magnetic field and a time varying magnetic field gives rise to an electric field (actually $\nabla \times \mathrm{E}$) which accelerates electrons and positrons (if there are any). The electrodynamics of pulsar emission is complicated and somewhat controversial in detail. We only skimmed the surface. The emission may not be a pencil beam but a conical beam, the location where different wavelengths are emitted and the exact physical processes are uncertain, some pulsars show interpulse pulses at certain wavelengths, others don't, etc.

Pulsars have high velocities presumably acquired during the supernova and many leave the galaxy. Some are found at high Galactic latitude. From their speed and location we can infer that pulsars turn off after $\sim 10^7$ years (SN remnants like the Crab have an even shorter life ~10,000 years or we would see more of them). Many pulsars are found in the disk of the Galaxy though reflecting their recent birth from Population I star death. Besides gradually slowing down, pulsars occasionally experience discontinuous "glitches" in their period. This is attributed to neutron-star "quakes". When centrifugal force declines owing to a gradual slowing of rotation, the rigid crust lacks sufficient support and eventually snaps, changing the radius of the neutron star by perhaps 1 cm. Conservation of angular momentum then results in an abrupt increase in the pulsar rotation rate and a shorter period. The fastest pulsars (1 ms) are found in old binary systems where they have presumably been "rejuvenated" – spun up by accretion from a companion star through a disk. These ms pulsars also have atypically small magnetic fields $\sim 10^8$-10^9 Gauss. Pulsars also provide an accurate laboratory for measuring the masses of neutron stars if the neutron star finds itself in a binary. From such measurements and other less accurate measurements of x-ray sources involving neutron stars (see below) we know that neutron stars have a mass near 1.4 $M_\odot$ and a maximum mass near 2.0 $M_\odot$. No neutron star below 1.1 $M_\odot$ has been seen. Note that the neutron star is lighter by 10 – 20% than the iron core that collapsed to make it because of all the neutrino losses. Neutron stars all have about the same radius 10 – 12 km.

Type I supernova light curves are entirely due to the decay of radioactive ^{56}Ni and ^{56}Co, nevertheless, they can be quite bright. Typically they are very luminous for just a few weeks – briefer than SN IIp, but brighter. At peak a SN Ia reaches about 2 x 10^{43} erg s^{-1}, or about the luminosity of a bright galaxy. Because they are so bright and regular in appearance, SN Ia are useful standard candles for cosmology and were used to show the first indication of an accelerating expansion of the universe. SN Ib and Ic are about 4 times fainter than Ia (because they produce less ^{56}Ni). Type Ia supernovae have higher velocities and smaller masses than SN IIp and are not associated with star forming regions. They are observed in elliptical galaxies as well spirals and irregulars.

The observed properties of SN Ia - fast, bright, radioactive, high velocity, standard spectrum and light curve, association with an old stellar population – are consistent with the favored model – the thermonuclear explosion of an accreting carbon-oxygen white dwarf in some sort of binary system. The regularity could, e.g., be a consequence, of usually happening in white dwarfs that have all approached the same (e.g., Chandrasekhar) mass prior to exploding. A major problem in SN Ia modeling is that, unlike SN IIp, no progenitor of a SN Ia has ever been identified. There are limits on the luminosities of any progenitor system however, and these tend to rule out some of the models that involve rapid accretion from a companion which would make the progenitor bright. Current SN Ia models are all based on the explosion of a CO dwarf leaving no bound remnant behind. These include: a) accretion to the Chandrasekhar mass as mentioned ; b) less rapid accretion and explosion short of the Chandrasekhar mass (the "sub-Chandra models") and c) merging white dwarfs. Each has problems and sorting all this out is a current frontier research area. Perhaps SN Ia are more than one thing? In any case though, observationally we know that they make a lot of iron group elements (from the light curev we infer about 0.7 $M_\odot$ of ^{56}Ni produced, making them the chief source in nature of iron. They also (spectroscopically) make a few tenths $M_\odot$ of Si, S, Ar, Ca and thus contribute in a non-trivial way to the production of these species.

Models based e.g., on the explosion of Chandrasekhar mass white dwarfs can give good agreement with both the light curves and spectra of observed SN Ia and can even reproduce the observed width luminosity relation, but the lack of observed presupernova counterparts is problematic for this model. The other two kinds of models (sub-Chandra and merging dwarfs) can also, under some circumstances give results that look like observations but they also predict supernovae that are quite different. Why are those different events not observed?

Lecture 17 – High Energy Astrophysics

The live of a star can be greatly altered if it is in a close binary. The most massive star becomes a giant first and it its outer radius extends beyond the Roche radius (the place where centrifugal force plus gravity balance for the two stars), matter may flow from the (formerly) more massive star to the lighter one, thus influencing the subsequent evolution of both of them. If the matter tries to flow over faster than the companion star can accept it (about the Eddington luminosity) the matter may pile up and swirl around both stars creating a "common envelope". The friction from orbiting inside this dense matter takes energy from the orbit and brings the two stars closer together, meanwhile ejecting the common envelope. The (initially) more massive star goes ahead and completes its evolution creating a white dwarf, neutron star, or black hole in close orbit with the (initially) less massive star. This can give rise to several observable phenomena. Removing the envelope from a highly evolved massive star (e.g., 12 – 15 solar masses) leaves behind a helium and heavy element core which, when it explodes, is a Type Ib or Ic supernova. This is the most common channel for creating these events. If the close binary contains a neutron star, white dwarf r black hole, then the evolution of the lighter companion eventually creates circumstances where the compact object (WD, n*, or BH) can accrete. In the case of the white dwarf, and if the accretion rate can be sustained at a high value (10^{-8}- 10^{-7} solar masses per year), this eventually leads to a Type Ia supernova. In other situations involving slower accretion, a nova or binary x-ray source can be created.

A classical nova occurs when a white dwarf in a close binary accretes hydrogen-rich material at a rate of 10^{-10} – 10^{-8} $M_\odot\ y^{-1}$. For this low accretion rate, the H and He do not burn stably to C and O like in a SN Ia model but instead pile up in an unburned layer on top of the WD. When the mass of the layer reaches 10^{-5} – 10^{-4} solar masses and has a density of its base of $\sim 10^4$ g cm^{-3}, hydrogen burning ignites unstably. Initially the matter is degenerate and the runaway is violent but even after some expansion, the hydrogen shell remains unstable to the thin shell instability and continues to burn. Burning temperatures reach ~200 M K and the luminosity rises above the Eddington value. Matter is ejected at high speed from the WD with speeds ~100 – 1000 km/s. The energy released by burning a gram of H is greater than 10 times the gravitational binding energy per gram at the surface of the white dwarf so burning just a bit of the hydrogen (e.g., 10%) results in the ejection of all the matter that had accreted. Due to convective mixing during the runaway come of the carbon and oxygen are dredged up and ejected as well, so the mass of the WD actually decreases a bit. The ejected material glows from the energy radiated by hydrogen shell burning and emits a spectrum rich in H, He, C, O and other heavier elements (emission lines not absorption). After the $\sim 10^{38}$ erg/s transient is over the WD cools down again and becomes faint again. Accretion resumes and 1000 – 10,000 yrs later the outburst recurs. This goes on as long as the accretion is maintained. The high temperatures of the hydrogen burning, much hotter than the center of main sequence stars, gives rise to unusual nucleosynthesis by a variation of the CNO cycle called the "hot CNO cycle" (not described in detail in class). The rare isotopes ^{15}N and ^{17}O, which are not created either in supernovae or AGB stars, are thought to come from novae.

If instead of a white dwarf one has a neutron star or black hole, one can produce a "binary x-ray source" by similar evolution. There are two classes of x-ray binaries - low mass and high mass – and both contain neutron stars or black holes as the x-ray emitting object. In the high mass systems, the mass donor has a mass > 5 $M_{\odot}$. In the low mass systems, the donor is lighter than the sun. The high mass systems occur naturally as the product of the binary evolution of two massive stars. The low mass systems which are seen in the Galactic center and in globular clusters, may form when a neutron star or black hole captures a low mass star in a dense stellar environment. There are about 200 of each kind seen presently in our Galaxy. They have also been detected in other galaxies such as the LMC, SMC and Andromeda.

With an accretion rate of about ~10^{-9} $M_{\odot}\ y^{-1}$, a neutron star will radiate x-rays with a luminosity of 10^{37} – 10^{38} erg s^{-1}. This is just $(GM/R)\frac{dM}{dt}$ for the neutron star. Blackbody radiation, for R = 10^6 cm, then implies an emission temperature ~10^7 K and Wien's law gives a wavelength ~10 A, in the x-ray band. Some of the x-ray binaries are eclipsing systems and we can also learn orbital parameters from studying the Doppler shift of the spectrum of the mass donating star. From such studies we learn that many such systems have x-ray emitting compact objects with masses over several solar masses. This means that they must be black holes.

A black hole can also emit x-rays in an x-ray binary. The emission does not come from inside the event horizon but from a hot accretion disk. As matter with angular momentum spirals into the small hole, it must rid itself of excess rotation before going to small radius. It this piles up in a centrifugally supported, differentially rotating disk. The inner part of the disk goes round faster than the outer part and friction in the disk causes matter at smaller radius to move slower while speeding up material orbiting at larger radius. This way angular momentum is transported and matter is able to accrete (unlike Saturn's rings where there is no friction and hence no accretion). The friction also makes the disk glow and radiate x-rays, much like the neutron star's surface would have done had there been a neutron star.

Gamma-ray bursts were discovered in the 1960's by satellites orbiting the earth (the Vela satellite system). The satellites were launched by the Department of Defense to look for clandestine atmospheric (or even space-based) nuclear weapons tests. Their discovery was announced in the early 70's. Gamma-ray bursts are a diverse phenomena, but a typical one lasts about 20 s, is very bright in the gamma-ray part of the spectrum, and then never recurs. Some are as bright as the planet Venus (if we were in space and had gamma-ray sensitive eyes). But gamma-rays can only be seen by satellites (they don't make it through the Earth's atmosphere) and gamma-ray detectors don't have very good angular resolution, so for decades astronomers were very puzzled as to what gamma-ray bursts (GRBs) really were.

In the 1990's the Compton Gamma-Ray Observatory discovered that GRBs a)come in two classes – short with harder spectra (duration about 0.3 s on the average) and long with softer spectra (about 20 s duration) and b) that the distribution of both classes was highly isotropic on the sky.

Gamma-ray bursts are the brightest explosions in the universe. During the ~20 s a bright burst lasts, its luminosity is greater than ten billion supernovae, but their true nature was not known for a long time. Discovered in the late 60's by the Vela satellites, the actual distance to and counterparts of GRBs were not known for a long time. Today be know that they are typically billions of light years away and come in two classes – short hard (0.3 s average duration) and long soft (20 s average duration). The long soft bursts are associated with the deaths of massive stars. They happen in star forming regions of distant galaxies and are sometimes accompanied by supernovae. The short hard bursts are not clearly associated with massive stars and are thought to form a separate class of phenomena. The typical long GRB is a redshift $z > 2$. If one happened in our Galaxy, even on the far side, it would be as bright (in gamma-rays) as the sun (is in optical light).

The energy of a long GRB is thus stupendous, approaching the rest mass-energy of the sun (~10^{54} erg if the radiation is isotropic – going equally to all angles in the sky). However, there is evidence, both from observations and models, that gamma-ray bursts are "beamed", that they come from jets of matter accelerated to close to the speed of light and that we only see one when we are in the opening angle of the jet. There are thus 100's of GRBs that we don't see for every one that we do, but the energy requirement is correspondingly reduced to a few x 10^{51} erg. There are two models for long GRBs – the collapsar, which is based upon very rapid accretion into a black hole formed when the supernova initially fails to explode, and the millisecond magnetar – a pumped up version of ordinary pulsars with 1000 times the magnetic field strength and a rotation period of only 1 ms. The rotational energy of a neutron star rotating with a period of 1 ms is 2 x 10^{52} erg and, given the large field strength, this energy might be deposited in a few tens of seconds after the pulsar is born. Both models produce relativistic jets of matter that come out the rotational axes and burrow through the star before escaping and making the GRB.

Short hard bursts, while still originating from distant galaxies, are not associated with star forming regions or supernovae and are thought to have a different origin – from merging neutron stars. The two neutron stars merge, following a period of gravitational radiation that brings them together, forming a black hole and a disk of a few tenths of a solar mass around the hole. The smaller mass of the disk compared with a collapsar and the lack of an overlying star make the burst shorter and less energetic, but otherwise the physics is similar. Merging neutron stars may thus be responsible for both r-process nucleosynthesis and short gamma-ray bursts. The decay of the radioactive progenitors of the r-process produces a short transient somewhat like a supernova but much fainter. These transients are being searched for, in conjunction with short GRBS.

Unless a GRB happened very close to the earth, say within a kpc, it would not be terribly catastrophic. One closer than that could destroy all life. Fortunately the odds of that are very small, even in the entire lifetime of the earth, much smaller, e.g., than the odds of an impact with a large asteroid.