Astronomy 112: Physics of Stars

Problem set 1 - due April 15

1. **Magnitudes**
   The absolute bolometric magnitude, M, of the Sun is 4.75. a) Show that that the absolute magnitude of a star with luminosity L is given by
   $$M = 4.75 - 2.5\,\log\left(\frac{\mathrm{L}}{\mathrm{L}_\odot}\right). \tag{1}$$
   b) Now solve this equation for $\mathrm{L}/\mathrm{L}_\odot$ given M. c) Hipparcos measures a parallax of a star of 0.01 arc sec. Its apparent magnitude is 8.0. Ignoring bolometric corrections, and using equations in your notes, what is the absolute bolometric magnitude of the star? d) what is the luminosity of the star in units of solar luminosities?

2. **Blackbody radiation**
   A spherical planet orbits an F-v star with luminosity 1.5×10^{34} erg s^{-1}. The orbit is circular with a radius from the star of 2 AU. Assume that the planet is rapidly rotating, has an atmosphere and reflects 20% of the light that falls on it, but absorbs the other 80% and, in the assumed steady state, radiates it as a blackbody. a)Ignoring any greenhouse effect, what is the temperature of the planet? b) Does the radius of the planet matter? Why or why not? c) At what wavelength does the planet emit most of its radiation? d) What would the temperature on the bright side be if the planet was tidally locked and kept the same face pointed at the star, had no atmosphere, and absorbed 100% of the incident light? Express your answer in Centigrade (C).

3. **Measuring Stellar Masses Using Spectroscopic Binaries.**
   For spectroscopic binaries, we can directly observe the maximum line-of-sight velocities $v_{1,\mathrm{LOS}}$ and $v_{2,\mathrm{LOS}}$ of the two stars, and their orbital period P. Given this information, we want to calculate the masses of the two stars, M_1 and M_2. For simplicity we will assume that the orbits are circular around the center of mass. Star 1 has radius r_1 and star 2 has radius r_2. Their total separation is thus also constant a $= r_1 + r_2$. The orbital plane of the binary is inclined, however, at an unknown angle i relative to the plane of the sky, where $i = 0$ corresponds to an orbit that is perfectly face-on and $i = 90°$ to one that is perfectly edge-on.

   (a) In terms of M_1, M_2, and a, calculate the velocities v_1 and v_2 of the two stars about their common center of mass.

(b) Calculate the orbital period P in terms of M_1, M_2, and a.

(c) In terms of v_1, v_2, and i, what is the largest component of each star's velocity that will lie along our line of sight? We will call these $v_{1,\mathrm{LOS}}$ and $v_{2,\mathrm{LOS}}$. You may neglect the constant offset to $v_{1,\mathrm{LOS}}$ and $v_{2,\mathrm{LOS}}$ that comes from the motion of the binary's center of mass relative to Earth.

(d) Use your answers to the previous parts to calculate M_1 and M_2 in terms of the observed quantities and i.

(e) Some spectroscopic binary star systems have eclipses, where one stars blocks the light of the other. Why would these systems particularly useful for measuring stellar masses?

4. **Hydrostatic Equilibrium and the Virial Theorem.**
Suppose that a star of mass M and radius R has a density distribution $\rho(r) = \rho_c(1 - r/R)$, where ρ_c is the density at the center of the star. (This isn't a particularly realistic density distribution, but for this calculation that doesn't matter.)

(a) Calculate ρ_c in terms of M and R. For all the remaining parts of the problem, express your answer in terms of M and R rather than ρ_c.

(b) Calculate the mass $m(r)$ interior to radius r.

(c) Calculate the total gravitational binding energy of the star.

(d) Using hydrostatic equilibrium, calculate the pressure $P(r)$ at radius r. You may assume that the $P(R) = 0$.

(e) Assume that the material in the star is an ideal gas. Calculate the total internal energy of the star from $P(r)$, and show that the Virial theorem is satisfied.

5. **Powering Jupiter by Gravity.**
The sun is in thermal equilibrium because its thermal timescale is short compared to its age. However, smaller objects need not be in thermal equilibrium, and their radiation can be powered entirely by gravity.

(a) Jupiter radiates more energy than it receives from the Sun by $8.7 \times 10^{-10}\ L_\odot$. Jupiter's radius is 7.0×10^4 km and its mass is 1.9×10^{27} kg. Compute its Kelvin-Helmholtz timescale. Could gravitational contraction power this luminosity for Jupiter's entire lifetime of 4.5 Gyr?

(b) Use conservation of energy to estimate the rate at which Jupiter's radius is shrinking to power this radiation. You may ignore the factor of order unity that arises from Jupiter's unknown density distribution.

6. **The thirsty professor - not required, but could be substituted, at the time of submission, for one of the above**
The professor likes to drink wine and prefers it chilled to 10 degrees C (283 K). The wine and bottle have a mass, m, of 1000 gm and a surface area of 700 cm^2. The heat capacity of the wine and bottle together is $C_P = 4 \times 10^7$ erg gm^{-1} K^{-1}. The total heat content of the bottle of wine at temperature, T, is assumed to be C_Pm T. The wine and bottle are good conductors and maintain a uniform temperature throughout. Initially the wine is at room temperature, 20 degrees C (293 K). a) The professor dreams he is an astronaut in space and puts the wine out on the dark side of the space craft where it is in a vacuum at nearly 0 K. Calculate how long it takes the wine to cool to 10 C. (an integral will be necessary for an accurate answer). b) The professor wakes up, but is still thirsty, so he puts the same wine, initially at 20 C, in a freezer where the temperature is -10 C (263 K). How how long does it take the wine to cool down to +10 C (283 K)? An approximate answer will get substantial credit, but an accurate one requires a quartic integral (dx/(x^4 - a^4)) that can be evaluated at mathportal.org (http://www.mathportal.org/calculators/calculus/integral-calculator.php). If evaluating an arctan on your calculator, be sure it is set to radians mode and not degrees. [postscript: the professor has done many such experiments and finds that the cooling time is less than 30 minutes in the freezer. Apparently conduction and convection, neglected here, are important.]