

1. **Time scales:** Assume, as empirically determined for binary star systems, that the radius of main sequence stars scales approximately as $M^{2/3}$. Also assume that the mass-luminosity relation found on the main sequence, i.e., $L \propto M^3$, is valid for stars somewhat above the main sequence experiencing their Kelvin-Helmholtz evolution. How does the Kelvin-Helmholtz time scale for pre-main sequence stars vary with their mass during their last bit of contraction to the main sequence? A proportionality is all that is needed though you could normalize to the known Kelvin-Helmholtz time scale for the sun. Low mass stars may still be in their Kelvin-Helmholtz phase while more massive stars have already formed and died.

2. **Virial Theorem:** Prialnik 3.2 page 44

Assuming a uniform value of $\beta = P_{\text{gas}}/P_{\text{tot}}$ throughout the star (i.e., an $n = 3$ polytrope) and defining

$$U = \int (u_{\text{gas}} + u_{\text{rad}}) dm, \quad (1)$$

show that the Virial theorem (as written in eq. 2.23 of Prialnik) leads to

$$E = U + \Omega = \frac{\beta}{2}\Omega = \frac{\beta}{2 - \beta}U \quad (2)$$

where E is the total energy. Discuss the implications for $\beta = 0$ (radiation dominated) and 1 (gas dominated) in terms of a) the Virial theorem we obtained in class, $E_{\text{tot}} = \Omega/2$, and b) the stability of stars supported entirely by radiation pressure.

3. **Equation of state:** Ideal gas. Calculate the pressure and internal energy per gram of an ideal gas composed of 35% hydrogen and 65% helium by mass at a temperature of 1.5×10^7 K and density of 150 g cm^{-3} (i.e., close to the present solar center)? What is μ and what is Y_e ? What is the electron number density, n_e ? What is $1-\beta$, the ratio of radiation pressure to total pressure?
4. **Equation of state:** Neutrons also obey Fermi statistics and can become degenerate. Repeat the derivation for complete non-relativistic degeneracy that we did for electrons but for neutrons, obtaining a number times a density to some power. What is the ratio of neutron degeneracy pressure to electron degeneracy pressure at a given density? At what density would the neutrons become *relativistically* degenerate? Do you think this ever happens in nature?

5. **Radiation transport:** similar to Pals 5.1 but different numbers.

- (a) Estimate a typical value for the temperature gradient dT/dr deep inside the sun by dividing its central temperature, 15.7×10^6 K by its radius, 6.96×10^{10} cm. Assume the mean free path of photons in the sun is 0.1 cm. What is the average change in temperature across this distance? The anisotropy of radiation in the stellar interior is very small. This is why radiation in the solar interior is close to that of a black body.
- (b) How much energy leaves each face of 1 cm^3 cube with a temperature equal to that in the solar interior? How many cubic centimeters emitting, each in isolation, would add up to the solar luminosity, $3.8 \times 10^{33} \text{ erg s}^{-1}$ (each cube has 6 emitting surfaces)? Compare this with the volume of the inner 10% of the radius of the sun - $1.4 \times 10^{30} \text{ cm}^3$. Why is this a nonsensical model for how heat emerges from the solar core?
- (c) Show that the minute deviation from isotropy between two surfaces in the solar interior one photon mean free path apart that you estimated in a) evaluated e.g., at $r = R_{\odot}/10$ and $T = 10^7 \text{ K}$, is sufficient for the transfer of energy that results in the luminosity of the sun. Given the approximate nature of the question you should not expect to get exactly L_{\odot} .

6. **Radiation transport** What are the 3 major kinds of opacity in stars (a 4th, H^- will be discussed later)? Which are absorptive and which are scattering? If $\kappa \propto \rho^a T^b$, what are a and b for each? Which dominates at very high temperature? In general how will κ change for stars of lower metallicity?

7. **Polytropes:** A neutron star is roughly describable as a polytrope of index 1. The radius of a typical neutron star of mass 1.4 solar masses is 10 km. What is its *central* density? The density of the atomic nucleus is $2.4 \times 10^{14} \text{ g cm}^{-3}$. Compare the value you computed to this number. Neutron stars are laboratories for exploring physics under conditions not attainable on the earth.

8. **Polytropes:** Helium burning: We shall see later that after they finish central and shell hydrogen burning, many stars, including the sun, go on to ignite helium burning in their centers at a temperature $\sim 1.5 \times 10^8 \text{ K}$. (aside: This temperature does not vary very much due to the extreme temperature sensitivity of the helium burning reaction rate to temperature - to be discussed. The entire range in stars of all masses is 1 to $3 \times 10^8 \text{ K}$). Helium ignites in a core that is embedded in, but little affected by a low density envelope of hydrogen and helium - the red giant - that surrounds it. It thus

evolves approximately, like a separate little helium star. In low mass stars like the sun the helium core has a mass of about 0.45 solar masses and is quite degenerate when it first ignites helium. In fact, it resembles a white dwarf made of helium.

- (a) Assuming that non-relativistic degeneracy pressure supports the dwarf what would be the best polytropic index to describe the $0.45 M_{\odot}$ core?
- (b) Taking a value of Y_e appropriate for pure helium, what would be the radius, and then, the average density of the helium core?
- (c) What would be its central density (remember the polytropic index gives $\rho_c/\bar{\rho}$)?
- (d) Locate the point $\rho_c, T = 1.5 \times 10^8$ K on a plot of dominant pressures given in class and confirm that it is indeed degenerate.

After the flash, the helium core heats up expands and becomes non-degenerate, i.e., an ideal gas, and later ignites helium a second time. This time it is more describable as an $n = 3$ polytrope (Eddington model). To solve this part, you may need equations, that while in the notes, were not derived in class. Start with the definition $M = -4\pi\alpha^3\rho_c\zeta_1^2 \left(\frac{d\theta}{d\zeta}\right)_{\zeta_1}$ and assume $n = 3, \beta \approx 1$. to get T_c a) as a function of $M, \mu,$ and ρ_c and b) as a function of $M, \mu,$ and R .

- (e) Using your results and a μ appropriate to helium and assuming $\beta \approx 1, M = 0.45 M_{\odot}$ what is the central density now at 1.5×10^8 K?
- (f) What is the average density and the radius of the helium core now? This describes the core of a horizontal branch star.