Astronomy 112: Physics of Stars

Problem set 3: Due May 22

1. **Nuclear binding energy:**

   For nuclei with equal numbers of neutrons and protons and ignoring shell and pairing effects, the binding energy of a nucleus with mass A is given by the liquid drop model BE = a_1A - $a_2 A^{2/3}$ - a_3 ($Z^2/A^{1/3}$) with a_1 = 15.68 MeV, a_2 = 18.56 MeV, and a_3 = 0.717 MeV.

   (a) What is the physical meaning of each of the three terms and why do they have the signs that they do?

   (b) At what A would the *binding energy per nucleon*, BE/A, have a maximum? Note that, by assumption, Z = A/2. Solve for this by taking the derivative of the equation given. Don't just state the answer.

   (c) What is the meaning of that nucleus, with that particular A, in terms of possible *energy release* by the fusion of other nuclei with the same total number of nucleons, for example, the fusion of A/2 + A/2 into A? What does it mean for the fission of heavier nuclei into some multiple of A, conserving A, for example, a nucleus of 2A fissioning into A + A.

   (d) Actually the most tightly bound nucleus with Z = N = A/2 is ^{56}Ni. Why didn't you get that answer?

2. **Energy from nuclear burning:**

   What is the total energy released (in erg g^{-1}) if a composition consisting initially of pure ^{12}C burns to pure ^{24}Mg. The binding energies of ^{12}C and ^{24}Mg are 92.162 MeV MeV and 198.258 MeV respectively.

3. **Reaction rates**

Helium burning occurs at a temperature of about 1.5×10^8 K. Two reactions are important, 3α, and ${}^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$. This problem is about the second of these two reactions.

(a) At what center of mass alpha-particle energy would most reactions (${}^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$) take place at the given temperature? Express the energy in MeV. Compare this with kT in MeV at the same temperature. The ions that are reacting are far out on the tails of their Maxwell-Boltzmann distributions.

(b) The S-factor for this reaction is slowly varying in the energy range of interest. Its value at the energy you just calculated is 0.17 MeV barns. What is the reaction rate factor $\lambda_{\alpha,\gamma}(^{12}C)$ at 1.5×10^8 K?

(c) At a density of 10^4 g cm^{-3} and an alpha particle mass fraction of 1, what would be the lifetime of a carbon nucleus against alpha-capture at that temperature? (Hint: The destruction term in the rate equation for ^{12}C is $dY_{12}/dt = -Y_{12}Y_\alpha\rho\lambda_{\alpha\gamma}$ so $\tau_{\alpha\gamma} = (1/Y_{12}\ dY_{12}/dt)^{-1} = (\rho Y_\alpha \lambda_{\alpha\gamma})^{-1}$).

(d) Assuming a slowly varying S-factor, to what power of the temperature would this reaction be sensitive. $\lambda_{\alpha,\gamma}(^{12}C) \propto T^n$. What is n?

4. **Star Formation**

In a region of a molecular cloud consisting of material with solar composition, but unionized and with the hydrogen in the form of molecules ($\mu = 2.33$; see Glatzmaier and Krumholz notes 15.1.B) the number density is 10^4 cm^{-3} (note tat this is not mass density but number density) and the temperature 20 K. Set constants such as α, the multiplier on the gravitational binding energy, that are of order unity equal to 1.

(a) What is the Jeans mass for these conditions

(b) If, for these conditions, a Jeans mass collapsed to form a star approximately how long would it take? Is the relevant time scale here the nuclear, hydrodynamic, diffusion or Kelvin Helmholtz time?

(c) In fact it is unlikely that a single star of this mass forms, but instead many stars form because the collapsing cloud fragments. Consider a 1 solar mass fragment, also collapsing on the time scale you just estimated. Say the inner 0.5 solar masses attains hydrostatic equilibrium which allows it to maintain a radius of 5 solar radii while the remainder falls freely and accretes on it. Estimate the accretion rate and the accretion luminosity. Compare the latter with the present solar luminosity.

(d) For this luminosity and radius, if the radiation were emitted as a black body what would be the effective emission temperature?

(e) At what wavelength would most of the light come out? Is this a shorter wavelength or longer than optical light (roughly 4000 - 7000 Angstroms)?

(f) After the accretion phase, the proto-sun will experience a Kelvin Helmholtz evolution. How long does that take? Is the relevant time scale here the nuclear, hydrodynamic, diffusion or Kelvin Helmholtz time?

(g) Eventually nuclear reactions begin in the sun and it is a main sequence star. How long does it remain a main sequence star? Is the relevant time scale here the nuclear, hydrodynamic, diffusion or Kelvin Helmholtz time?

5. **Minimum mass for ignition:** (see also Pols 8.1.3)

We derived in class an expression for the central pressure vs central density of a polytrope with mass M, $P_c = C_n G M^{2/3} \rho_c^{4/3}$. Assume a polytrope of index 1.5 (as is appropriate for low mass completely convective stars. Then $C_n = 0.48$.)

(a) Assuming the dominance of ideal gas pressure with $\mu = 0.60$ derive a relation between the central temperature, T_c, and ρ_c for a given mass M.

(b) Now assume a star supported by non-relativistic electron degeneracy pressure, also with mass M and $Y_e = 0.88$. This is also a polytrope of index n = 1.5. Derive a relation between the central density and mass. The temperature does not appear because the gas is degenerate.

(c) Sketch qualitatively how these two curves would appear in a plot of log density (x-axis) vs log T for a given M. See Pols Fig 8.2 for guidance. Not the existence of an intersection for the two lines derived in a) and b), and therefore a maximum temperature for the contracting mass M. At this maximum temperature ideal gas and degeneracy pressure each contribute roughly 1/2 the pressure needed for hydrostatic equilibrium.

(d) Equating your two results derive an expression for the maximum temperature reached for a given mass M with the assumed composition. This is not very precise because the gas cannot be both degenerate and ideal at the same time, but you are assuming it has comparable components of each. That is you are to assume that each pressure, ideal and degenerate, contributes 1/2 of what the polytropic equation for central pressure says is needed.

(e) From homology, to be covered in class soon, the central temperature for a star powered by nuclear reactions is $T_c \propto \mu M^{0.57}$ for stars powered by the pp-chain. In fact this expression is for constant opacity and Kramer's opacity would be better for low mass stars, but use this expression and normalizing to the sun with $T_c = 15.7$ M K write an equation for the central temperature required to ignite hydrogen burning in a star of mass M (μ is still 0.6).

(f) Now using your previous expression for the maximum temperature reached by a contracting star of polytrope n = 1.5 and mass M, estimate the minimum mass star capable of igniting hydrogen burning and becoming a main sequence star. Your answer will not be exact. We neglected convection and took a very simple EOS and opacity.

(g) Using your expression for the maximum temperature reached by a core of mass M as it contracts, estimate the minimum mass required to reach 1.0×10^8 K, i.e., the critical mass for helium ignition.

6. **Solar neutrinos:**

The solar luminosity is 3.84×10^{33} erg s^{-1}. The sun is powered by the fusion of hydrogen to helium. The production of each helium liberates roughly 28 MeV (the binding energy of ^{4}He) and is accompanied by the emission of two neutrinos.

(a) What is the flux of neutrinos from the sun (neutrinos cm^{-2} s^{-1}) at the Earth?

(b) Would the flux be the same on the day and night side of the Earth?