Astronomy 112: Physics of Stars

Problem set 4: Due June 5 (no extensions)

1. **Homology**

   (Pols 7.4 d) Show that, if we replace the assumption of a constant opacity with a Kramer's opacity law, $\kappa = \kappa_0 \rho T^{-7/2}$,

   (a) The mass-luminosity-radius relation becomes

   $$L \propto \frac{\mu^{7.5} M^{5.5}}{R^{0.5}} \tag{1}$$

   Note that in this case the mass luminosity relation retains some dependence on the radius, and hence the nuclear reactions responsible for energy generation.

   (b) Further show that with Kramers opacity, stars shining by the pp cycle with Kramers opacity will have

   $$R \propto \mu^{-7/13} M^{1/13} \tag{2}$$

   (c) So that

   $$L \propto \mu^{7.769} M^{5.462} \tag{3}$$

2. **Core mass-luminosity relation for RGB stars:** From Pols problem 10.4

   Low-mass stars on the red giant branch (RGB) obey a core mass-luminosity relation, which is approximately given by Pols eq. (10.2). The luminosity is provided by hydrogen shell burning.

   $$L = 2.3 \times 10^5 \ \mathrm{L}_\odot \left(\frac{M_{core}}{\mathrm{M}_\odot} \right)^6 \tag{4}$$

   (a) Derive relation between luminosity L and the rate at which the core grows $\mathrm{dM}_{core}/\mathrm{dt}$. Use the energy released per gram in hydrogen shell burning - 4.4×10^{18} erg g^{-1}.

   (b) Derive how the core mass evolves in time, i.e, $\mathrm{M}_c = \mathrm{M}_c(\mathrm{t})$.

(c) Assume that the sun arrives on the RGB when its core mass is 0.2 $M_\odot$ and that it leaves the RGB when the core mass is 0.45 $M_\odot$. Calculate the total time it spends on the RGB. Compare this with the suns 10^{10} year lifetime on the main sequence.

(d) What happens when the helium core mass reaches 0.45 $M_\odot$?

3. **White dwarf cooling:**

A white dwarf shines because of the residual heat left over from when it was once a star. That heat is retained in the ions which remain an ideal gas even though the electrons are degenerate. The heat capacity of the ions is 3/2 N k erg g^{-1} K^{-1} where N is the total number of ions in the isothermal core. Assume a white dwarf temperature of 10^8 K, a mass of 0.6 $M_\odot$, and a composition of 50% by mass each of ^{12}C and ^{16}O.

(a) What is the total number, N, of carbon plus oxygen nuclei in the white dwarf? Do not count electrons because they are degenerate and don't contribute to the heat capacity.

(b) What is the tota heat content in the ions?

(c) How long, approximately, could the white dwarf shine at 0.01 $L_\odot$ using this heat reserve as its sole source of power? How long at 0.001 $L_\odot$?

4. **Nucleosynthesis**

Where in nature, in what kind of star or explosion, during what burning phase, and by what reactions are each of the following made: a) ^{12}C, b) ^{14}N, c) ^{16}O, d)^{24}Mg, e) ^{28}Si, and f) the s-process?

5. **Type Ia supernova energetics**

Consider a model for a Type Ia supernova to be a 1.38 $M_\odot$ white dwarf composed of 50 each (by mass) ^{12}C and ^{16}O. The initial net binding energy (internal energy plus gravitational potential) is -5.0 × 10^{50} erg. How much energy, erg/gm, is released when carbon and oxygen burn to ^{56}Ni? Suppose 0.8 $M_\odot$ of the star burns to ^{56}Ni (for simplicity, we will neglect the synthesis of intermediate mass elements here). What will then be the net energy of the white dwarf. If it expands to infinity, what would be a typical velocity? [Binding energies: ^{12}C 92.163 MeV; ^{16}O 127.621 MeV ^{56}Ni 484.003 MeV]

6. **Post-main sequence structure**

   Sketch the interior structure of a low-mass star during each of the following stages: red giant branch star, horizontal branch star, asymptotic giant branch star. In each diagram, indicate a) where fusion is occurring (core and/or shell) and the fusion reaction taking place there (e.g., H → He) and b) the dominant element present in each non-fusing region.Do not worry about drawing your diagrams to scale.

7. **Supernovae**

   Big and bright as a supernova is, it still radiates like a blackbody with an effective temperature not much hotter than the solar photosphere. A value of 5500 K is typical for a Type IIp supernova. But the supernova, at its brightest, has an enormous radius, $\sim 3 \times 10^{15}$ cm (i.e., 200 AU). Assuming this value for the temperature (as measured e.g., by Wien's Law) and the given radius, calculate the luminosity of the supernova in erg s^{-1} and in solar luminosities. In what part of the HR diagram would supernovae be found (upper left, upper right, lower left, or lower right?). Incidentally, this makes Type IIp supernovae useful standard candles. We don't know their radius directly but can calculate it from the age of the supernova and its expansion rate.

8. **Advanced burning stages in massive stars**

   Carbon and oxygen burning in a massive star occur in a state of approximately balanced power between neutrino losses and energy generation, but because of Coulomb barriers, burning heavier fuels requires a higher temperature. Carbon burns at 0.8 GK (1 GK $= 10^9$ K) and oxygen burns at 1.8 GK. The density continues to scale roughly as T^3 so carbon burns at 2×10^5 g cm^{-3} and oxygen at 2×10^6 g cm^{-3}. In the relevant temperature range neutrino losses, chiefly by the pair process, are $\sim 5 \times 10^7 (10^6/\rho) T_9^{14}$ erg g^{-1} s^{-1}. Carbon burning liberates 1×10^{17} erg g^{-1} and oxygen burning, 4×10^{17} erg g^{-1} (because there is more oxygen to burn). Estimate the lifetime of carbon burning and oxygen burning in a massive star. The actual lifetimes are changed somewhat by convection bringing in fresh fuel to the burning region and by the fact that the temperature varies with radius in the star.