Homework 4

1) Homology

a) a) $L \propto \frac{R^4 T^4}{\kappa M}$, b) $T \propto \frac{\mu M}{R}$

c) $\kappa \propto \rho T^{-7/2}$ d) $L \propto M \epsilon_0 \rho T^{\nu}$ e) $\rho \propto M/R^3$

a+c+e $\Rightarrow$ $L \propto \frac{R^4 T^4}{M} \frac{T^{7/2}}{\rho} \propto \frac{R^4 T^{15/2}}{M \; M} R^3$

$$\propto \frac{R^7}{M^2} \left(\frac{\mu M}{R}\right)^{15/2} = \frac{\mu^{15/2} M^{11/2}}{R^{1/2}}$$

$$\boxed{L \propto \frac{\mu^{7.5} M^{5.5}}{R^{0.5}}}$$

b) Add d)

$$L \propto \epsilon M \propto M \rho T^{\nu} \propto M \frac{M}{R^3} \left(\frac{\mu M}{R}\right)^{\nu} \propto \frac{\mu^{15/2} M^{11/2}}{R^{1/2}}$$

$$\frac{M^{2+\nu} \mu^{\nu}}{R^{3+\nu}} \propto \frac{\mu^{15/2} M^{11/2}}{R^{1/2}}$$

$$R^{3+\nu-1/2} \propto M^{2+\nu-11/2} \mu^{\nu-15/2}$$

$$R \propto \mu^{\nu-15/2/(5/2+\nu)} M^{\nu-7/2/(5/2+\nu)}$$

$\nu = 4$

$$R \propto \mu^{(4-15/2)/13/2} M^{\frac{1}{2}/13/2}$$

$$\boxed{R \propto \mu^{-7/13} M^{1/13}}$$

c) $L \propto \mu^{7.5 + 7/26} M^{5.5 - 1/26} = \boxed{\mu^{7.769} M^{5.462}}$

2) $L = 2.3 \times 10^5 L_\odot \left(\frac{M_c}{M_\odot}\right)^6$

a) $$\boxed{\frac{dM_c}{dt} = \frac{L}{q}}$$ $q = 4.4 \times 10^{18}$ erg/gm

$$\frac{dM_c}{dt} = \frac{(2.3\times10^5)(3.84\times10^{33})}{4.4\times10^{18}} \left(\frac{M_c}{M_\odot}\right)^6 \text{ gm/s}$$

$$\frac{d(M_c/M_\odot)}{dt} = \frac{(2.3\times10^5)(3.84\times10^{33})}{(4.4\times10^{18})(1.989\times10^{33})} \left(\frac{M_c}{M_\odot}\right)^6 \text{ s}^{-1}$$

b) $$= 1.00\times10^{-13} \left(\frac{M_c}{M_\odot}\right)^6 \text{ s}^{-1}$$

$$\boxed{\frac{dM_c/M_\odot}{dt} = 3.19\times10^{-6} (M_c/M_\odot)^6 \text{ yr}^{-1}}$$

let $x = M_c/M_\odot$

$$\int_{0.2}^{0.45} \frac{dx}{x^6} = 3.2\times10^{-6} \int_0^\tau dt$$

$$\tau = \left(\frac{1}{3.2\times10^{-6}}\right) \left(\frac{-1}{5x^5}\right)\Bigg|_{0.2}^{0.45}$$

$$= \frac{3.1\times10^5 \text{ yr}}{5} \left(\frac{1}{.2^5} - \frac{1}{(.45)^5}\right)$$

$$= \frac{3.1\times10^5 \text{ yr}}{5} (3125 - 54.2)$$

c) $$= \boxed{1.9\times10^8 \text{ yr}}$$

d) It ignites helium with a flash

3) White dwarf cooling:

$0.6 M_\odot$ 50% C 50% O by mass

a)

$$N = \int n\, dV = \int \rho N_A \sum_i Y_i\, dV = N_A \sum Y_i \int \rho\, dV$$

$$= N_A M \sum_i Y_i = N_A M \left(\frac{0.5}{12} + \frac{0.5}{16} \right)$$

$$= (0.6)(1.989 \times 10^{33})(6.02 \times 10^{23})(7.29 \times 10^{-2})$$

$$= \boxed{2.99 \times 10^{55} \text{ nuclei}} \quad 5.24 \times 10^{55}$$

b)

$$U = 3/2\, N k T = (1.5)(1.38 \times 10^{-16})(2.99 \times 10^{55})(10^8)$$

$$= \boxed{~~4.13 \times 10^{47}~~ \text{ erg}} \quad \boxed{1.08 \times 10^{48}}$$

c)

$$\tau = U/L = \frac{1.08 \times 10^{48} \;\; ~~4.13 \times 10^{47}~~}{(0.01)(3.84 \times 10^{33})} = 2.82 \times 10^{16} \text{ sec}$$

$$= \boxed{8.9 \times 10^{8} \text{ yr.}}$$

Actually a) cools down from 10^8 K quickly to 10^7 K

b) L typically 10^{-3} or 10^{-4} $L_\odot$

4) Nucleosynthesis

^{12}C - made by the 3α reaction during helium burning in all stars over $0.5 M_\odot$

Principally made in intermediate mass stars and ejected in red giant winds and planetary nebulae

^{14}N - made by the CNO cycle in second generation stars that contain C and O to start with.

Chiefly made in intermediate mass stars

^{16}O made by helium burning in stars above 0.5 $M_\odot$. Chiefly made in massive stars and ejected in supernovae

^{24}Mg made by carbon burning in massive stars and ejected in supernovae

^{28}Si made by oxygen burning in massive stars and ejected in supernovae.

s-process made during helium burning by side reactions that produce neutrons. $^{13}C(\alpha,n)^{16}O$ and $^{22}Ne(\alpha,n)^{25}Mg$. Ejected in intermediate mass stars by red giant winds and planetary nebulae. Ejected in high mass stars by supernovae.

5) Type Ia energetics

50% C and 50% O in 0.8 $M_\odot$ → ^{56}Ni

$$q = 9.65\times10^{17}\ \text{erg/gm}\ \sum(\delta Y_i)(BE_i)$$

$$= (9.65\times10^{17})\left(\left(\frac{1}{56}\right)(484.003) - \frac{0.5}{12}(92.163) - \frac{0.5}{16}(127.621)\right)$$

$$= (9.65\times10^{17})(0.8146) = 7.86\times10^{17}\ \text{erg/gm}$$

$$E_{nuc} = qM = (7.86\times10^{17})(0.8)(1.989\times10^{33})$$

$$\boxed{= 1.25\times10^{51}\ \text{erg}}$$

Net energy of SN $= |\Omega| - E_{nuc} = (1.25 - 0.5)\times10^{51}$

$$= \boxed{7.5\times10^{50}\ \text{erg}}$$

5) continued

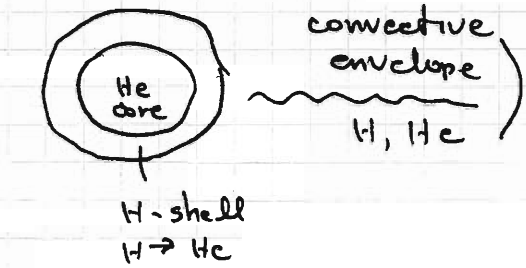
$$KE = \frac{1}{2} M v^2 = 7.5 \times 10^{50} \qquad M = 1.38\ M_{\odot}$$

$$v^2 = \frac{(2)(7.5 \times 10^{50})}{(1.38)(1.989 \times 10^{33})} = 5.46 \times 10^{17}$$

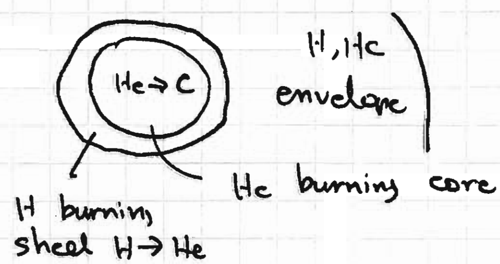
$$v = 7.4 \times 10^{8}\ \text{cm/s} \qquad \boxed{7400\ \text{km/s}}$$

6)

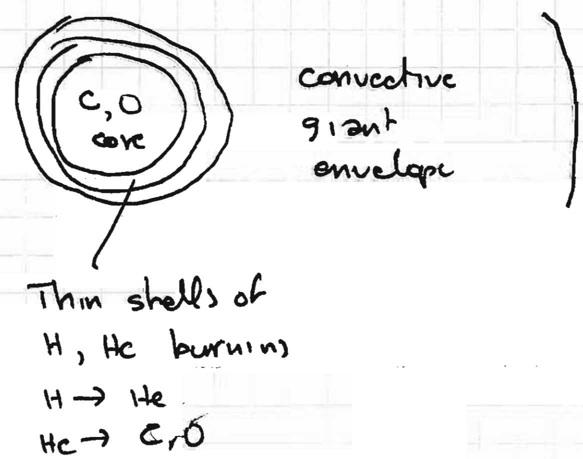
a) red giant



b) horizontal branch



c) AGB star



(7) Supernova

$L = 4\pi R^2 \sigma T^4 \qquad R = 3\times10^{15}\ \text{cm} \qquad T = 5500\,K$

$L = (4\pi)(9\times10^{30})(5.67\times10^{-5})(5500^4)$

$= \boxed{5.86\times10^{42}\ \text{erg/s}}$

such a large radius places it in the extreme upper right of the HR diagram, but its temperature is just a bit cooler than the sun.

(8) carbon burns at 8×10^8 K liberates 1×10^{17} ergs/gm
oxygen " " 1.8×10^8 K " 4×10^{17} ergs/gm

a) $\tau_c \sim \dfrac{10^{17}}{\epsilon_\nu} = \dfrac{10^{17}\ (0.2)}{(5.\cancel{x}\ 10^7)(0.8)^{14}}$

$= 9.09\times10^9\ s = \boxed{288\ \text{years}}$

b) $\tau_o = \dfrac{4\times10^{17}\ (2)}{(5\times10^7)(1.8)^{14}}$

$= 4.27\times10^6\ s = \boxed{0.14\ \text{years}}$