Star Formation

Glatzmaier and Krumholz 15
Pols 9
Prialnik 9.1

THE INTERSTELLAR MEDIUM

- Total mass ~ 5 to 10 x 10^9 solar masses of about 5 – 10% of the mass of the Milky Way Galaxy interior to the sun's orbit
- Average density overall about 0.5 atoms/cm^3 or ~10^{-24} g cm^{-3}, but large variations are seen
- Composition - essentially the same as the surfaces of Population I stars, but the gas may be ionized, neutral, or in molecules (or dust)

H I – neutral atomic hydrogen
H_2 - molecular hydrogen
H II – ionized hydrogen
He I – neutral helium
Carbon, nitrogen, oxygen, dust, molecules, etc.

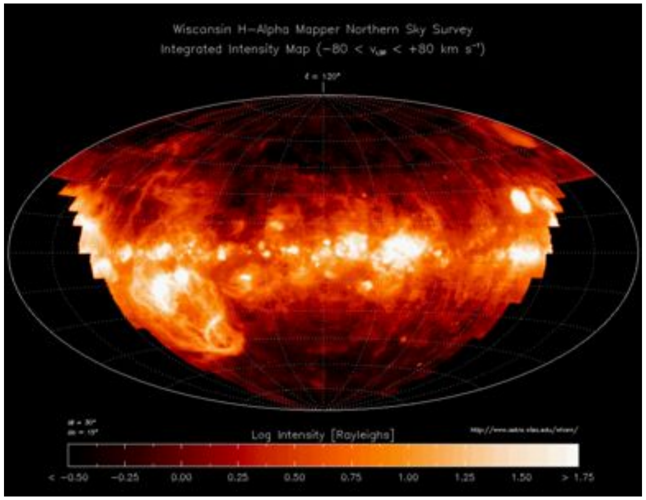
PHASES OF THE INTERSTELLAR MEDIUM

Component	Fractional volume	Scale Height (pc)	Temperature	Density	State of Hydrogen	Observational Technique
Molecular Clouds	< 1%	70	10 - 20	10^2 - 10^6	H_2	Radio and infrared (molecules)
Warm Neutral Medium (WNM)	10 - 20%	300 - 400	5000-8000	0.2 - 0.5	H I	21 cm
Warm Ionized Medium (WIM)	20 - 50%	1000	6000 - 12000	0.2 - 0.5	H II	H_α pulsar (n_e)
H II Regions	<1%	70	8000	10^2 - 10^4	H II	H_α
Coronal Gas (Hot Ionized Medium (HIM)	30 - 70%	1000 - 3000	10^6 - 10^7	10^{-4} - 10^{-2}	H II metals also ionized	x-ray ultraviolet

THE NEUTRAL AND WEAKLY IONIZED MEDIA
(about one-half the mass and volume of the ISM)

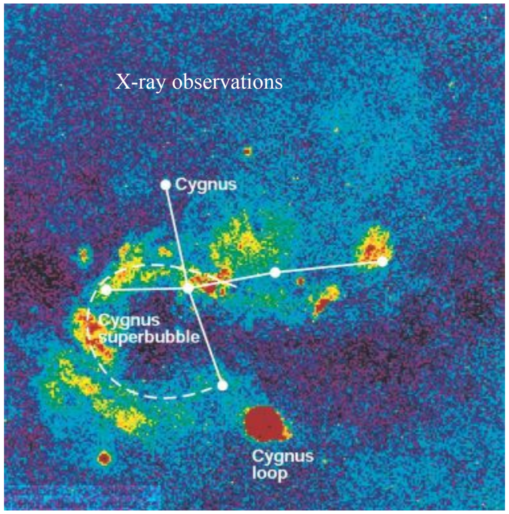
- Neutral (H I) and partially ionized hydrogen
- Study with 21 cm (H I) and emission lines (H I + H II)
- Scale height greater for hotter gas – 100 – 1000 pc
- Cooler gas often found in clouds. Not actively forming stars. Rough pressure equilibrium.
- Peaks 8 – 13 kpc from galactic center, i.e. outside the sun's orbit

WARM IONIZED MEDIUM



Distribution of ionized hydrogen (H II) in the local vicinity as viewed in Balmer alpha. Warm partly ionized medium.

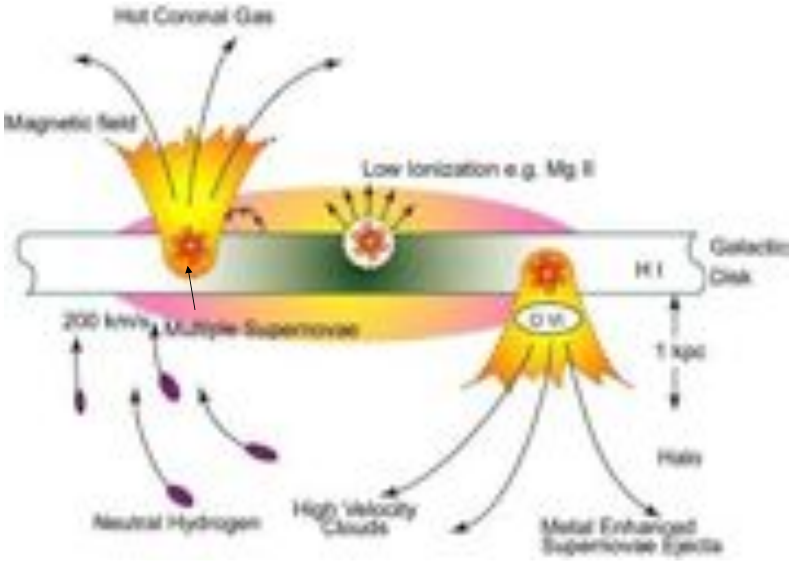
CORONAL GAS

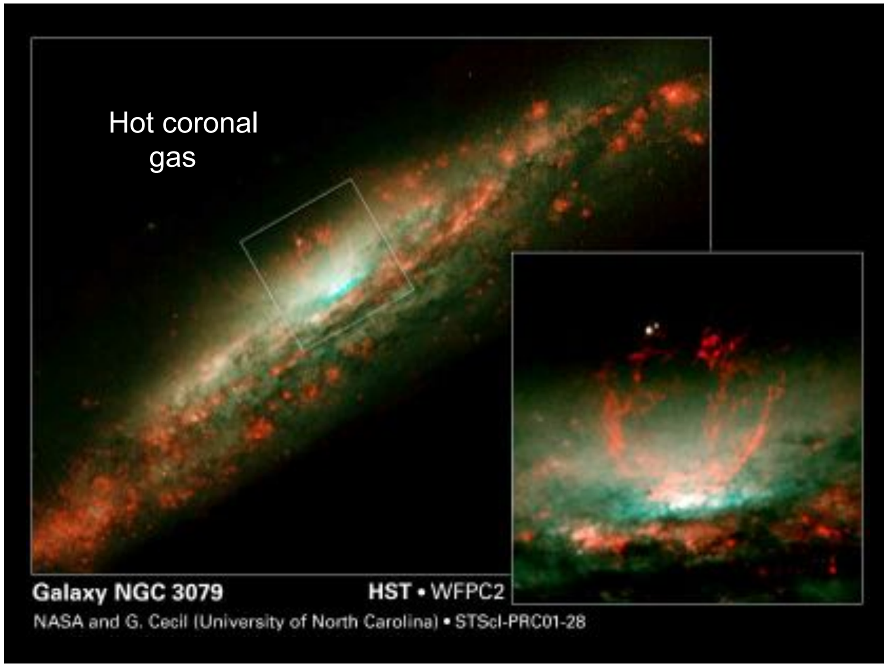


Called “coronal gas” because of its similarity to solar coronal gas, but very different origin.

Probably originates from supernova explosions and winds from very hot stars

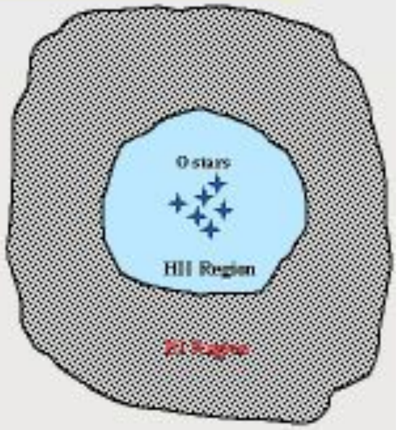
Also recall the “Local Bubble”





HII Regions

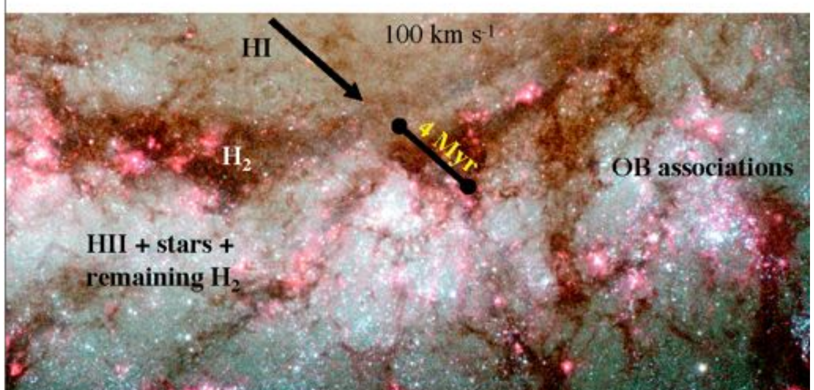
- Interstellar matter excited and ionized by ultraviolet photons from hot O and B (earlier than B3) stars
- Spectrum is emission
- HII region surrounded by cooler, denser, HI region



H II regions

Messier 51

Spiral Arm Section of M51



3.40 x 1.65 kpc: Elmegreen 2007 ApJ, 668, 1064

Lagoon Nebula – in Sagitarius – 5000 ly away – spans 90 x 40 arc min and 130 by 60 light years. Another H II region on the boundary of a molecular cloud (like Orion)

The Great Nebula in Orion. An illuminated portion of a nearby (1300 ly) giant molecular cloud. The field of view here is 32 arc min. Each arc min at this distance is about 0.4 ly.

A better resolved image ofthe Trapezium from the Hubble Space Telescope.

John Balley et al (1997)

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MOLECULAR CLOUDS

- Colliding flows or density waves produce regions of locally high density.
- In the cool dense gas, dust forms and accumulates icy mantles
- This dust shields molecules from destruction by uv light
- Molecules emit radio and the dust emits IR, making the cloud cooler.
- About 40% of the mass of the ISM is molecular clouds (but a small fraction of the volume)

$10^3 - 10^7\ M_\odot$

$n = 10^2 - 10^6\ \text{cm}^{-3}$

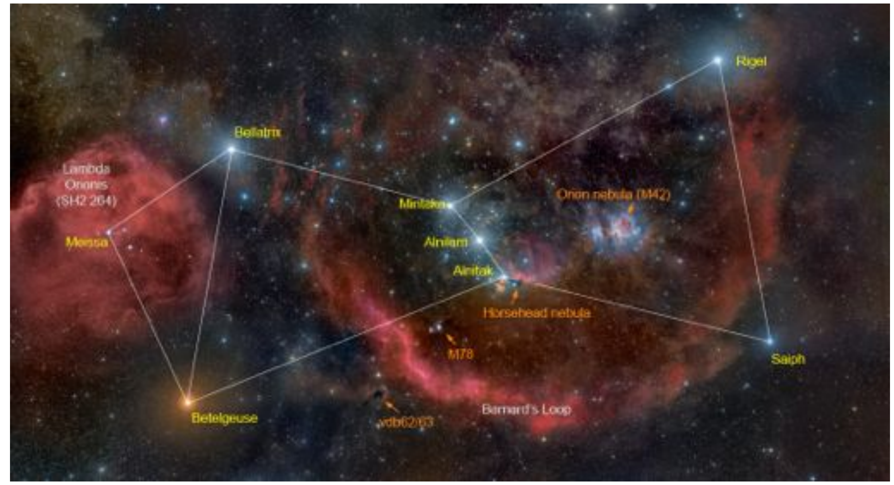
$T = 10 - 20\ K$

scale height in disk 50 - 75 pc

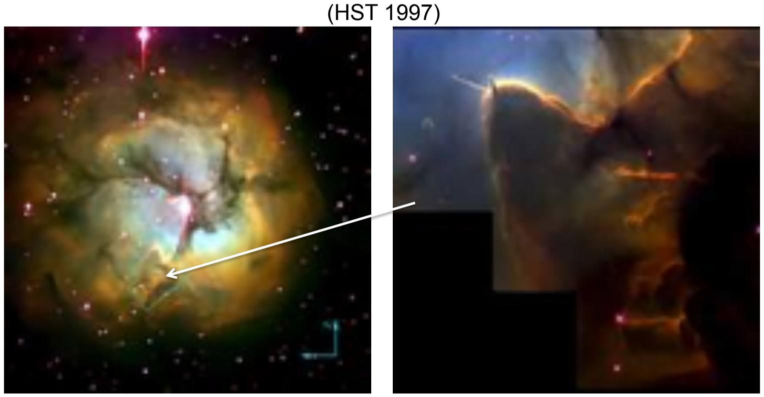
Part of the molecular cloud complex in Orion. The belt of Orion is clearly visible.

Total extent of the cloud complex is several hundred light years. 1600 light years away and several degrees across.

Total mass a few times 10^5 solar masses. Many young O and B stars and perhaps 1000 lower mass stars and proto-stars. Several million years old.



(HST 1997)



about the angular size of the moon. roughly 40 ly across

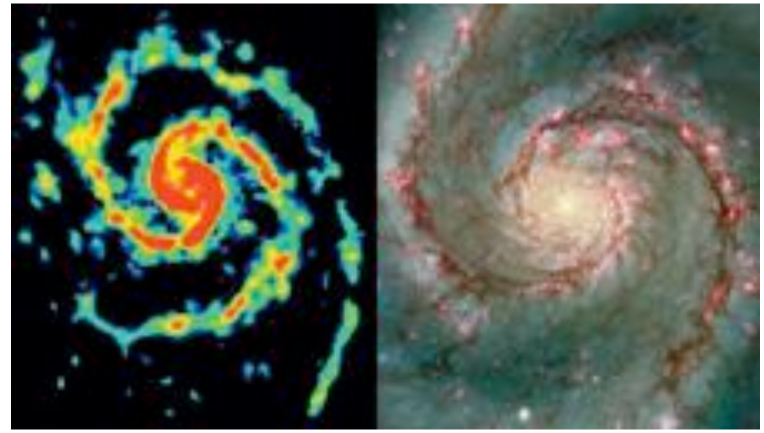
The *Trifid Nebula,* 1600 parsecs away in the constellation Sagitarius, is a molecular cloud where new stars are being born. Here the bright emission of the central stars is eroding the surroundings of several nearby stars about 8 light years away. Note the nebula is quite dusty. The stalk has survived because at its tip there is still gas that is dense enough to resist being boiled away by the nearby bright stars.

Star birth in the Eagle Nebula, 7000 light years away in the constellation Serpens.

This is a column of cool molecular hydrogen and dust that is an incubator for new stars. Each finger-like protrusion is larger than our solar system.

This "pillar of creation" is being slowly eroded away by the ultraviolet light of nearby young stars.

Andromeda galaxy in an infrared
image taken by the Spitzer Space Telescope

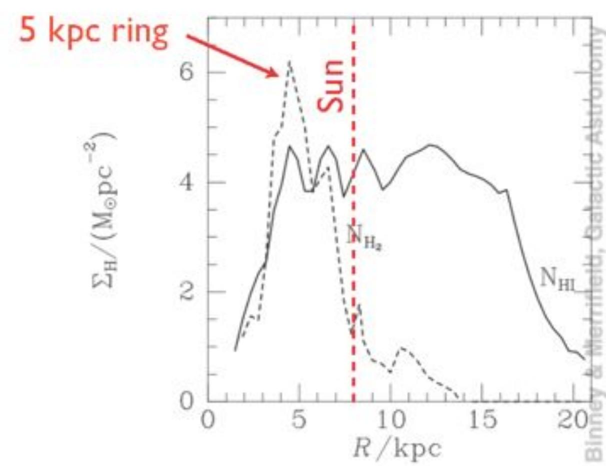


M51 in CO emission (left) and optical (right) (Koda et al., 2009)

Molecular Clouds in our Galaxy seen in ^{12}CO (1-0)

Clouds are composed primarily out of H_2 and He. However, in cold clouds, these molecules don't emit. We map molecular with dust extinction and a variety of molecules that do emit such as CO (tracers).

CO in our Galaxy



The Composition of Molecular Clouds

H_2
He (25% of mass)
dust (1% mass of mass)
CO (10^{-4} by number),
CS ($\sim10^{-9}$ by number)
NH_3 (10^{-9} by number)
N_2H^+ (10^{-10} by number)

and many other molecules with low abundances.

The Jean's Mass

There are several approaches to calculating the properties of a gas cloud necessary for it to collapse, but here we follow Glatzmaier and Krumholz and assume the cloud is initially spherical and in hydrostatic equilibrium so it obeys the Virial Theorem and

$$U_{gas} = \frac{3}{2}\frac{P}{\rho} = \frac{3N_A k\bar{T} M}{2\mu} = -\Omega/2 = \frac{\alpha G M^2}{2R}$$

$$M = \frac{3N_A k\bar{T}}{\mu}\frac{R}{\alpha G}$$

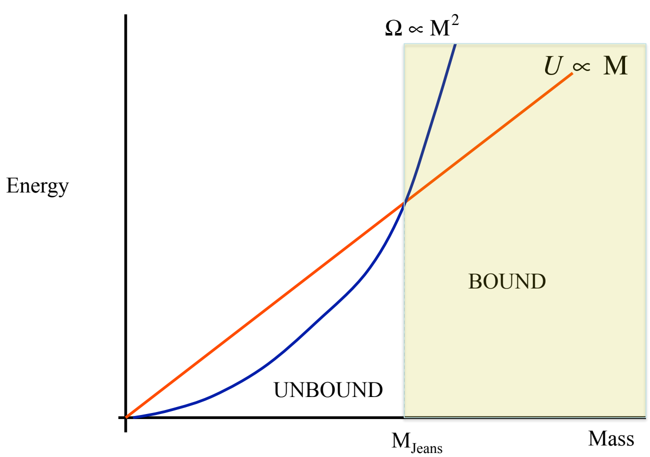
from which it follows for a spherical cloud $R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$

Assume constant density, constant temperature

$$M^{1/3}M^{2/3} = \frac{3N_A k\bar{T}}{\alpha\mu G}\left(\frac{3}{4\pi\rho}\right)^{1/3} M^{1/3}$$

$$M_J = \left(\frac{3N_A k\bar{T}}{\alpha\mu G}\right)^{3/2}\left(\frac{3}{4\pi\rho}\right)^{1/2} = \boxed{\frac{9}{2\sqrt{\pi\alpha^3}}\left(\frac{N_A k}{\mu G}\right)^{3/2}\left(\frac{T^3}{\rho}\right)^{1/2}}$$

For masses larger than the Jean's Mass, in a medium with nearly constant T and ρ, gravitational binding energy exceeds internal energy



The Jean's mass is the minimum mass cloud that, under the force of its own gravity is unstable to collapse. It is unstable because as the density rises efficient cooling keeps the temperature nearly constant, $\tau_{cool} << \tau_{HD}$ quite unlike a star where

$$\tau_{HD} << \tau_{cool}$$

$$M_J = \frac{9}{2\sqrt{\pi\alpha^3}}\left(\frac{N_A k}{\mu G}\right)^{3/2}\left(\frac{T^3}{\rho}\right)^{1/2} = 1.1\times10^{23}\left(\frac{T^3}{\alpha^3\mu^3\rho}\right)^{1/2} \text{ gm}$$

Using n= $\mu^{-1}N_A\rho$ and taking $\alpha \approx$ 1 this becomes

$$M_J = \frac{8.7\times10^{34}}{\mu^2}\left(\frac{T^3}{n}\right)^{1/2}$$ gm where n is the number density (atoms or molecules per cm^3.

$$\boxed{M_J = \frac{44 M_\odot}{\mu^2}\left(\frac{T^3}{n}\right)^{1/2}}$$

The cooling time for temperatures above about 10 K is of order 10 years. Kippenhalhn, Weigert, and Weiss. The collapse time scale is of order 10^5 years.

Detail

In fact, more detailed considerations that do not ignore the pressure at the clouds boundary and attempt to evaluate a proper value of α for the binding energy give a somewhat reduced mass called the "Bonner-Ebert" mass

$$M_{BE} = 1.18\left(\frac{N_A k}{\mu}\right)^2 \frac{T^2}{G^{3/2}} P_*^{-1/2}\ M_\odot$$

$$\approx \frac{20 M_\odot}{\mu^2}\left(\frac{T^3}{n}\right)^{1/2} \quad \text{if } P_* \approx nkT$$

but for our purposes the two are equal (use M_J in problems).

A key difference between clouds and stars is that the former are optically thin, at least at some wavelengths and cool as they collapse. So once unstable, their collapse accelerates.

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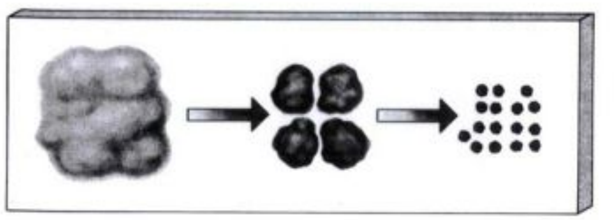
By this criterion, only molecular clouds are unstable to collapse.

A question on the homework allows you to estimate the Jeans mass and collapse time scale for typical molecular cloud conditions.

It is interesting that, for tyical molecular cloud conditions, the Jeans mass turns out to be some solar masses (not 0.1, not 10^3), so to some extent star masses are set by the initial conditions that exist in the star forming regions. In the early universe with no metals, cooling may have been less effeicient and the Jeans mass may have been much bigger.

That is not all however, because once the collapse starts it is subject to fragmentation and also magnetic fields, turbulence, and accretion physics all play important roles.

Fragmentation



$\tau_{HD} \approx 1000/\sqrt{\rho}$ sec (GK give $\tau_{ff} = 2100/\sqrt{\rho}$ sec;

$\approx 24\ My/\sqrt{\mu n}$ but they also give $\tau_{HD} = 2680/\sqrt{\rho}$ sec)

$\sim 10^5 - 10^6$ y for molecular cloud densities

Complications:

Rotation

Magnetic fields

Since the Jeans mass depends on $\rho^{-1/2}$ at constant temperature, a growing density makes the critical mass smaller. If the compression were adiabatic, with T rising as $\rho^{2/3}$, then the Jean's mass would actually rise with increasing density as $\rho^{+1/2}$ and fragmentation would be impossible. It is the efficient cooling that keeps T low and constant that allows collapse and fragmentation to proceed once the initial Jeans mass is exceeded.

Radius of the initial Jean's mass $\quad n = \mu^{-1}\rho N_A$

$$M_J = \frac{8.7\times10^{34}}{\mu^2}\left(\frac{T^3}{n}\right)^{1/2} \qquad R = \left(\frac{3M_J}{4\pi\rho}\right)^{1/3} = \left(\frac{3N_A M_J}{4\pi\mu n}\right)^{1/3}$$

$$R = 2.3\times10^{19}\frac{1}{\mu}\left(\frac{T}{n}\right)^{1/2}\ \text{cm}$$

For n $=10^4$; $T=10$; $\;R\sim 1pc$, but this rapidly fragments into smaller regions.

The luminosity during the initial collapse is quite low but grows as the object shrinks (initially at least, all energy from contraction is radiated)

$$L\sim\frac{d\Omega}{dt}\approx\frac{\Omega}{\tau_{HD}}\approx\frac{\alpha G M_J^2}{R\tau_{HD}} \qquad \alpha=\frac{3}{n-5}=\frac{6}{7}\ \text{if fully convective}$$

$$\tau_{HD}=\left(\frac{2\pi}{3}G\bar{\rho}\right)^{-1/2}=2680./\sqrt{\bar{\rho}}=2680\left(\frac{4\pi R^3}{3M}\right)^{1/2}\ \text{sec}$$

$$L\sim7\times10^{31}\left(\frac{M}{M_\odot}\right)^{5/2}\left(\frac{1000\ \text{AU}}{R}\right)^{5/2}$$

1 pc = 206,262 AU

The temperature and pressure will go up within a collapsing core once $\kappa\,\bar{\rho}$ R > 1 and radiation is trapped in the gas. For $\kappa\sim1$ and $\bar{\rho}$ for the current sun = 1.4 g cm^{-3}, this gives ρ~10^{-14} g cm^{-3};R < 2000 AU. So about the time the luminosity becomes appreciable the radiation becomes trapped and the collapsing cloud heats up.

Old View – due to Hayashi – pre 1970

Once the radiation is trapped the collapsing cloud core becomes hot enough (few thousand K) that H_2 is dissociated and a short time later (10,000 K) hydrogen is ionized. This takes about 1.6 x 10^{13} erg g^{-1}(i.e., 13.6 eV * N_A). Contraction during this phase is almost free fall because "ionization" of H_2 keeps γ below 4/3. The energy goes into ionization, not heat.

The rapid contraction, which is optically thick during its potentially high luminosity phase stops when the ionization is complete at which time the radius is approximately

$$\frac{6}{7}\frac{GM^2}{2R}=1.3\times10^{13}\ M$$

$$R=60\ R_\odot\ (M/M_\odot)$$

This simple picture, due to Hayashi, thus predicts a common starting point for star formation in the upper right of the HR diagram. These protostars would be fully convective (TBD)

This picture today is thought to be largely incorrect because the collapse is not homologous, but it is still worth exploring its consequences.

Non-homologous Collapse:

$$L_{cloud} \sim 7\times 10^{31}\left(\frac{M}{M_\odot}\right)^{5/2}\left(\frac{1000\text{ AU}}{R}\right)^{5/2}$$

But how small can M be?

A limit to fragmentation comes from setting this luminosity equal to the maximum value that the cloud can radiate without substantially increasing its temperature. One limit is a black body.

$$L_{BB} = f4\pi R^2\sigma T^4 \qquad f<1$$

$$\frac{L_{cloud}}{L_{BB}} \propto \frac{M^{5/2}}{R^{5/2}} / R^2 \propto \frac{M^{5/2}}{R^{9/2}} \text{ at const T}$$

but $R \propto M^{1/3}$ so $\frac{L_{cloud}}{L_{BB}} \propto M$

As M shrinks this ratio gets smaller

$$7\times 10^{31}\left(\frac{M}{M_\odot}\right)^{5/2}\left(\frac{1000\text{ AU}}{R}\right)^{5/2} = f4\pi R^2\sigma T^4$$

for the smallest possible fragment with T constant inside ~ 10 K

$$6.6\times 10^{31}\left(\frac{M}{M_\odot}\right)^{5/2}\left(\frac{1000\text{ AU}}{R}\right)^{5/2} < f4\pi R^2\sigma T^4$$

Kippenhahn, Weigert and Weiss p 308

but R = $\left(\frac{3M}{4\pi\rho}\right)^{1/3}$ and $M_J = \frac{9}{2\sqrt{\pi\alpha^3}}\left(\frac{N_A k}{\mu G}\right)^{3/2}\left(\frac{T^3}{\rho}\right)^{1/2}$

Combining equations one can get M as a function of just T

$$\left(\frac{M}{R}\right)^{5/2} \propto T^4R^2 \Rightarrow M^{5/2} \propto R^{9/2}T^4 \propto \left(\frac{M}{\rho}\right)^{3/2} T^4$$

$$M^{5/2} \propto M^{3/2}T^4M^3 / T^{9/2} \propto M^{9/2} / T^{1/2}$$

$$M^2 \propto T^{1/2} \quad so \quad M > KT^{1/4}$$

Evaluating the constants gives M > 0.003 $M_\odot \frac{T^{1/4}}{f^{1/2}}$

Jean's masses smaller than this have insufficient surface area, at the given temperature, T, to radiate their gravitational binding energy on a hydrodynamic time scale. These small masses can grow by accretion however, and are similar to the smallest masses found in simulations.

Non-homologous Collapse

As Larson and others pointed out during the late 60's, the collapse does not proceed overall as the simple contraction of a sphere in which all regions move in with a speed proportional to their radius (homologous collapse). Instead the inner regions collapse faster, and, as the density there grows, they collapse faster still. A small inner region eventually becomes optically thick and heats up, the pressure resisting further contraction. The smallest core is initially of the size just estimated using the blackbody limit.

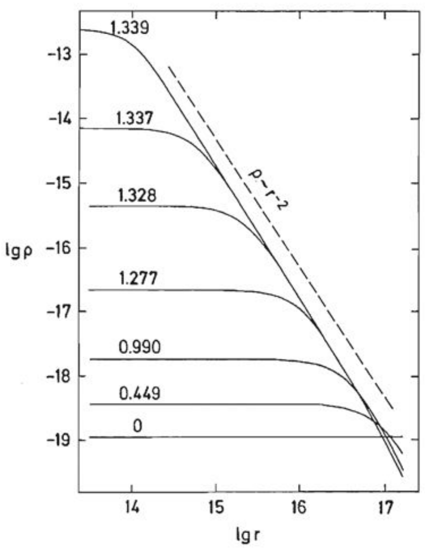
Once a small core in hydrostatic equilibrium has formed in the center, the remainder of the star accretes onto it. The accretion time scale is much shorter than the Kelvin-Helmholtz time scale of the core so it is simply compressed by the accretion. Most of the luminosity in this stage comes from accretion.

$L_{acc} \approx \frac{GM}{R}\frac{dM}{dt}$ that is, at a given R, the power radiated is about equal to the rate at which gravitational binding energy, in the form of increased mass is being added.

Well outside the homologous core, $\frac{dM}{dt} = 4\pi r^2\rho v$ is a constant and $\rho \propto \frac{1}{r^2 v}$. If the speed is the escape speed $\approx$ free fall speed, v = $\sqrt{\frac{2GM}{r}}$ and $\rho \propto r^{-3/2}$ but calculations that include the pressure give v $\approx$ constant and $\rho \propto r^{-2}$

note earlier typos

Kippenhahn, Weigert and Weiss Fig 27.1

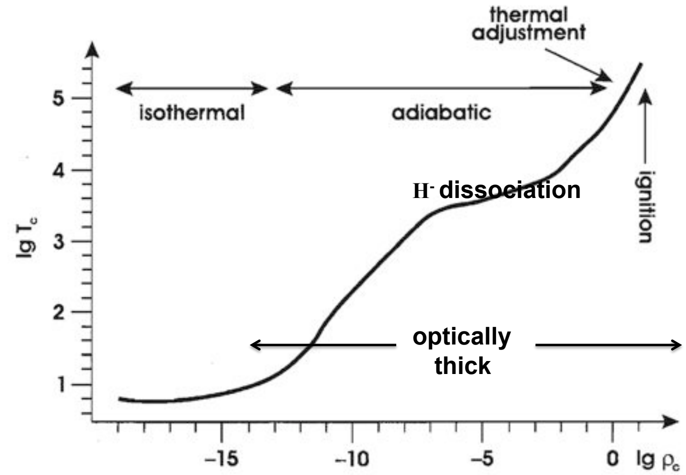


Log_{10} of the central density vs distance from the center in a collapsing cloud. Different times in the simulation measured in units of 10^{13} s are shown. Regions of homologous contraction are nearly flat and regions that are falling freely have density proportional to r^{-2}.

A smaller and smaller homologous core collapses on an increasingly rapid time scale

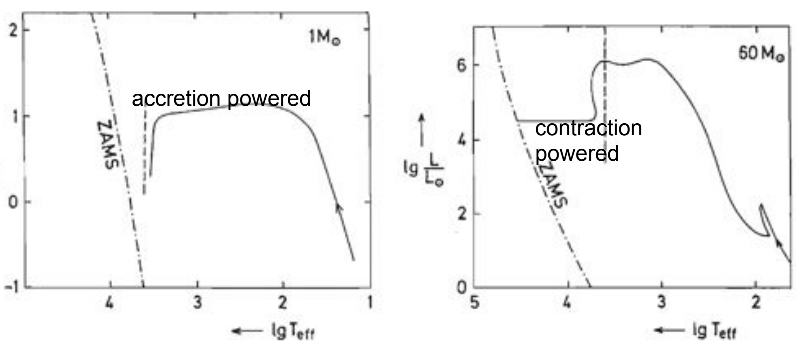
After Larson (1969).

Kippenhahn, Weigert and Weiss Fig 27.3 after Masunaga and Inutsuka (2000)



The central evolution of a one solar mass cloud from isothermal collapse until hydrogen ignition. Once H^- is dissociated, the opacity goes down and convection stops in the center. The core enters a radiative phase.

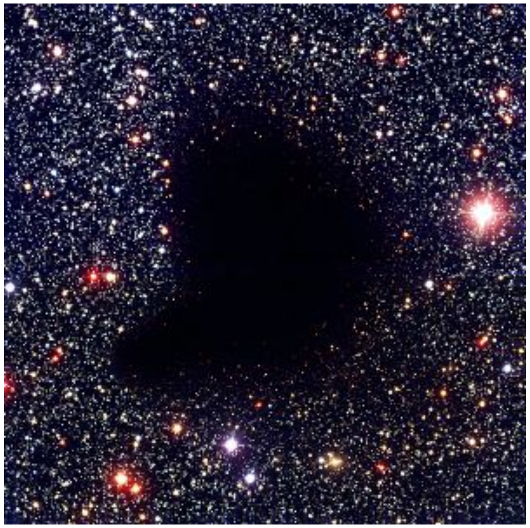
Kippenhahn, Weigert and Weiss, Fig 27.5 after Tscharnuter



Evolution in the HR diagram for two proto-stars of masses 1 and 60 solar masses. The vertical line shows the location of the Hayashi strip (TBD). The proto-stars initially are quite cold and have low luminosity. The formation of an appreciable core in which the H^- ion has been ionized truncates the downward evolution, especially in high mass stars. The 60 solar mass model lost mass along the way and ended up with 17 solar masses. The dashed vertical line is the “Hayashi strip”

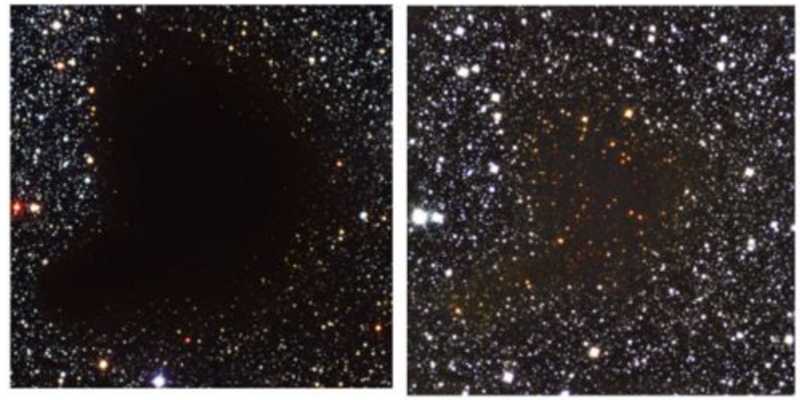
Summary

- Stars are born in molecular clouds when regions larger than the Jeans mass reach such high density or low temperature that they are unstable
- The collapse proceeds on a hydrodynamic time scale with fragmentation occurring down to a scale ~ 0.01 Msun
- These small fragments grow by accretion over a period of ~10^5 yr to stellar mass dimensions. During this time they emit in the infrared
- The luminosity of proto-stars comes from a combination of accretion energy and gravitational contraction. The latter dominates at late times. They can be quite bright and have larger radii than main sequence stars.
- During their most luminous phase, the lower mass protostars are fully convective (high opacity; large L). When a substantial part of the core no longer has H^- it ceases being convective



Barnard 68 is a Bok globule 410 ly away (one of the closest)
size about 12,000 AU (similar to Oort cloud in our solar system)
T = 16 K, about two solar masses. Coldest matter in universe.
Definitely forming stars, but not yet

Barnard 68



Visible Light Infrared Light

https://www.eso.org/public/images/eso0102c/

Interlude: H^- Opacity (Pols p 61)

A free electron in the vicinity of a H atom can induce a dipole moment on it and become loosely (0.754 eV) bound to it. The abundance of H^- is given by the Saha equation

$$\frac{n_H n_e}{n_{H^-}} = \frac{(2\pi mkT)^{3/2}}{h^3}\frac{2g_H}{g_{H^-}} e^{-0.754\ \text{eV}/kT} \qquad g_H = 2;\ g_{H^-} = 1$$

So

$$\frac{n_{H^-}}{n_H} = \frac{n_e h^3}{4(2\pi mkT)^{3/2}} e^{+0.754\ \text{eV}/kT}$$

by itself this implies an opacity that goes down as T $\uparrow$

H^- Opacity (Pols p 61)

As the temperature rises at a given n_e there is less H^-. This might lead one think that H^- opacity would decrease with increasing temperature (and eventually it does at high T). But n_e here does not come from H^- or H I ionization but from easily ionized metals like Ca and Na which obey their own Saha equations and have exponential factors that dominate $e^{0.754/kT}$. E.g., for Ca as we saw some time ago, n(Ca II) $\propto e^{-6.11/kT}$. As a result, n_e in the temperature range of interest (3000 - 8000 K) is a strongly increasing function of T and so, consequently is κ_{H^-}.

$$\kappa_{H^-} \approx 2.5\times10^{-31}\left(\frac{Z}{0.02}\right)\rho^{1/2}T^9 \ \ \text{cm}^2\ \text{g}^{-1} \qquad T = 3000 - 6000$$

nb positive power
of T; unlike Kramers

Pols 5.34

The Hayashi track

Stars and protostars with high luminosty and high opacity will be completely convective. Consider their properties. This will be relevant to both star formation and red giant evolution.

From our discussion of convection, the dimensionless adabatic temperature gradient for a star supported by ideal gas pressure is

$$\nabla_{ad} = \frac{d \log T}{d \log P} = 0.4$$

which is to say T=const • $P^{2/5}$

but P = $\frac{N_A k \rho T}{\mu} \Rightarrow P^{3/5} \propto \rho \;\Rightarrow\; P = K\rho^{5/3}$

which impies that completely convective stars are polytropes of index n = 1.5 and K is given by the polyropic equations

K= $N_{3/2}\, GM^{1/3}R$ where $N_{3/2} = 0.4242$

The photosphere is defined by

$$\tau = 2/3 = \int_{R_{ph}}^{\infty} \kappa\rho\, dr \approx \kappa_{ph} \int_{R_{ph}}^{\infty} \rho\, dr$$

and $\frac{dP}{dr} = -\frac{GM}{R^2}\rho \Rightarrow P_{ph} = \frac{2}{3}\frac{GM}{\kappa_{ph}R_{ph}^2}$ (1)

Also we have L = 4 $\pi R_{ph}^2 \sigma T_{eff}^4$ (2)

The opacity will depend on the temperature and density $\kappa_{ph} = \kappa_0 \rho_{ph}^a T_{eff}^b$ (3)

and finally we also have $P_{ph} = \frac{N_A k \rho_{ph} T_{eff}}{\mu}$ (4)

and $P_{ph} = 0.424\, GM^{1/3} R_{ph} \rho_{ph}^{5/3}$ (5)

That is we have (for each M) 5 equations in 6 unknowns, $P_{ph}, \rho_{ph}, \kappa_{ph}, T_{eff}, L$ *and* R_{ph}. We can thus form a relation between any two of them, e.g. L and T_{eff}

Detail:
See Pols p 127
GK 15.2.B

Ignoring all constants and droping subscript "ph"
using the symbol for the log and dropping Mdepndence

$P = -k - 2R$
$L = 2R + 4T$
$k = a\rho + bT$
$P = \rho + T$
$P = R + 5/3\rho$

$R + 5/3\rho = -k - 2R$
$R = -1/3k - 5/9\rho$
$\rho + T = -k - 2R$
$R = -1/2\rho - 1/2T - 1/2(a\rho + bT) = -1/3(a\rho + bT) - 5/9\rho$
$\rho(1/2 + 1/2a - 1/3a - 5/9) + T(1/2 + 1/2b - 1/3b) = 0$
$\rho(1/6a - 1/18) + T(1/6b + 1/2) = 0$
$\rho = \frac{(1/6b + 1/2)}{(1/18 - 1/6a)}T = \frac{(3b+9)}{(1-3a)}T$
$R = -1/3(a\frac{(3b+9)}{(1-3a)}T + bT) - 5/9\frac{(3b+9)}{(1-3a)}T$
$L = -2/3(a\frac{(3b+9)}{(1-3a)}T + bT) - 10/9\frac{(3b+9)}{(1-3a)}T + 4T$

if a = 1

L = $1/3(\frac{(3b+9)}{1}T + bT) + 5/9\frac{(3b+9)}{1}T + 4T$
$= (b + 3 + b/3 + 5/3b + 5 + 4)T = (2b + 12)T$

After some algebra (Pols p. 127)

$$\log T_{eff} = \frac{\frac{3}{2}a - \frac{1}{2}}{9a + 2b + 3}\log L + \frac{a+3}{9a+2b+3}\log M + \text{constant}$$

If a = 0.5 and b = 9 (H^-opacity)

$$\log T_{eff} = \frac{1}{102}\log L + \frac{7}{50}\log M + \text{constant}$$

For a given M, log L= 102 log T_{eff}. This is an almost vertical line in the HR diagram. Note that stars of larger mass have a greater L at a given T_{eff}

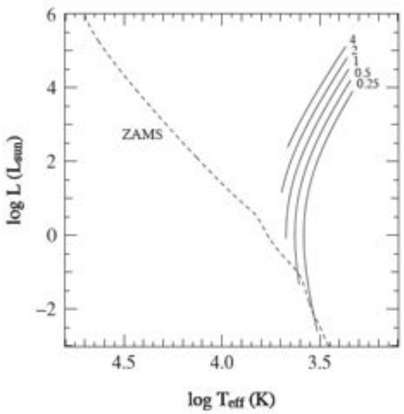
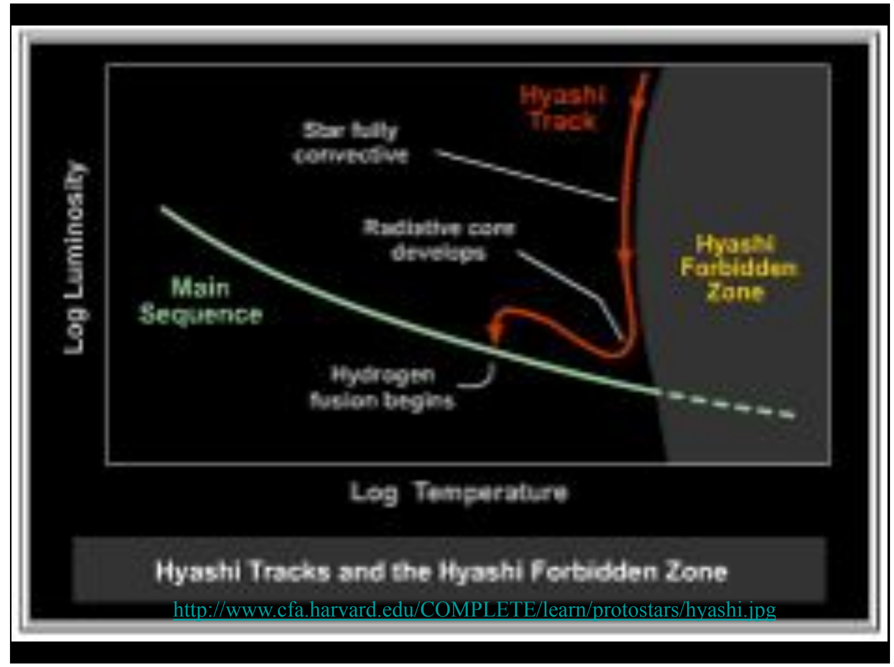
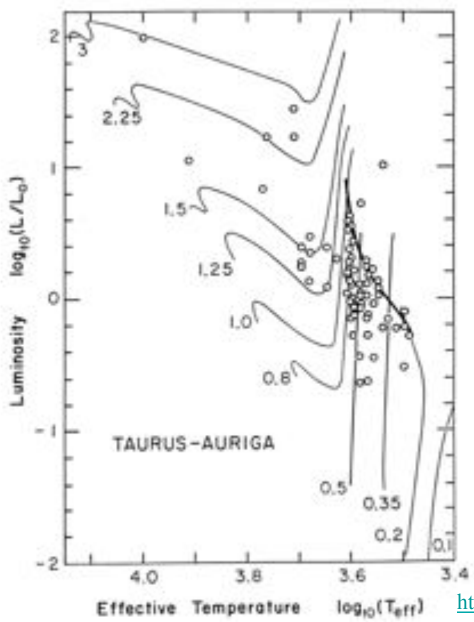


Figure 9.3. The position of the Hayashi lines in the H-R diagram for masses $M = 0.25, 0.5, 1.0, 2.0$ and $4.0\,M_\odot$ as indicated. The lines are analytic fits to detailed models computed for composition $X = 0.7, Z = 0.02$. The zero-age main sequence (ZAMS) for the same composition is shown as a dashed line, for comparison.
Note that the Hayashi lines do not have a constant slope, as expected from the simple analysis, but have a convex shape where the constant A (eq. 9.12) changes sign and becomes negative for high luminosities. The main reason is our neglect of ionization zones (where $\nabla_{ad} < 0.4$) and the non-zero superadiabaticity in the outer layers, both of which have a larger effect in more extended stars.



http://www.cfa.harvard.edu/COMPLETE/learn/protostars/hyashi.jpg



Cores of T-Tauri stars in a young stellar association.

The numbers give the masses for the theoretical curves

Stars above about 10 solar masses skip the Hayashi phase

http://en.wikipedia.org/wiki/Hayashi_track

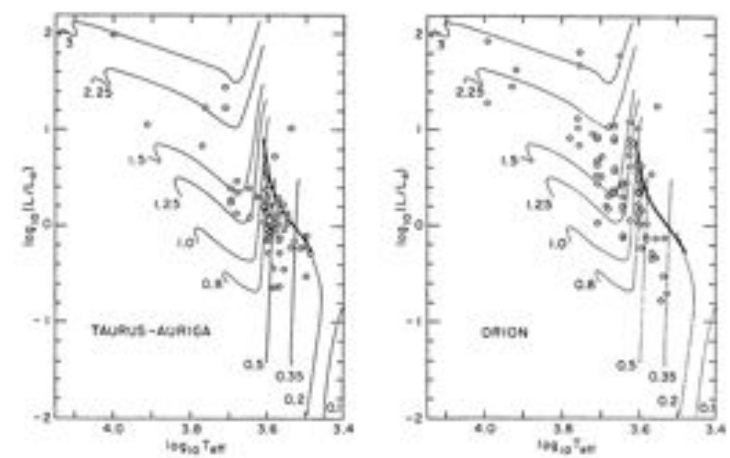


Fig. 7.3. Theoretical tracks for the pre-main sequence contraction phase for several different masses (as indicated). Overdrawn are observed temperatures and luminosities for pre-main sequence stars in two star-forming regions with rather different properties. In both, stars first appear along a very similar "birth line" (indicated with the thick line).

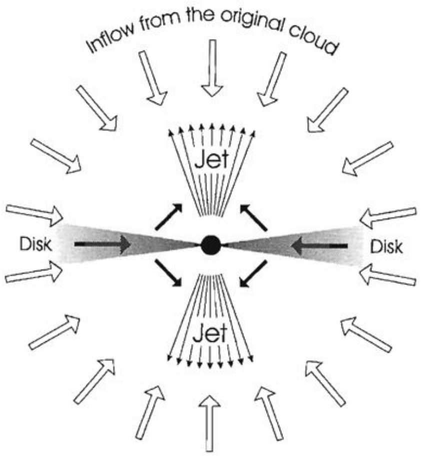
http://www.astro.utoronto.ca/~mhvk/AST320/notes56789.pdf

T-Tauri Stars

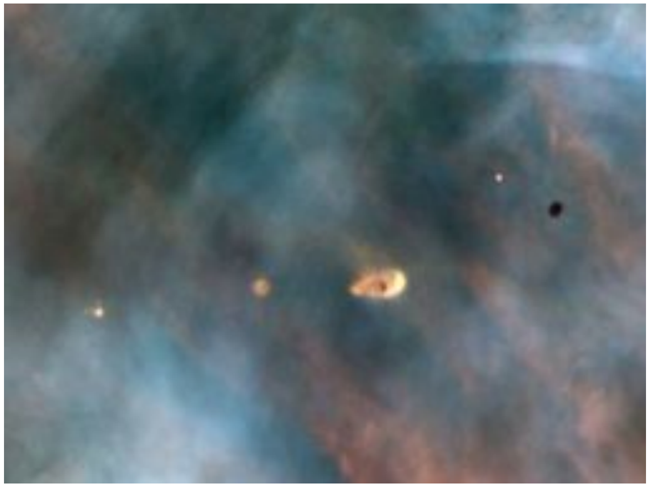
- Short lived phase in life of stars under 2 solar masses. Heavier stars evolve quicker and start burning by the time the star is visible. Above 2 solar masses the objects evolve rapidly and are rarely seen - “Herbig Ae and Be stars”.
- Accretion disks and jets are common features
- Emission and absorption lines
- Powered by gravitational contraction, not nuclear burning
- May be forming planetary systems
- High lithium abundance
- Embedded in dense, dusty regions
- Can be highly variable

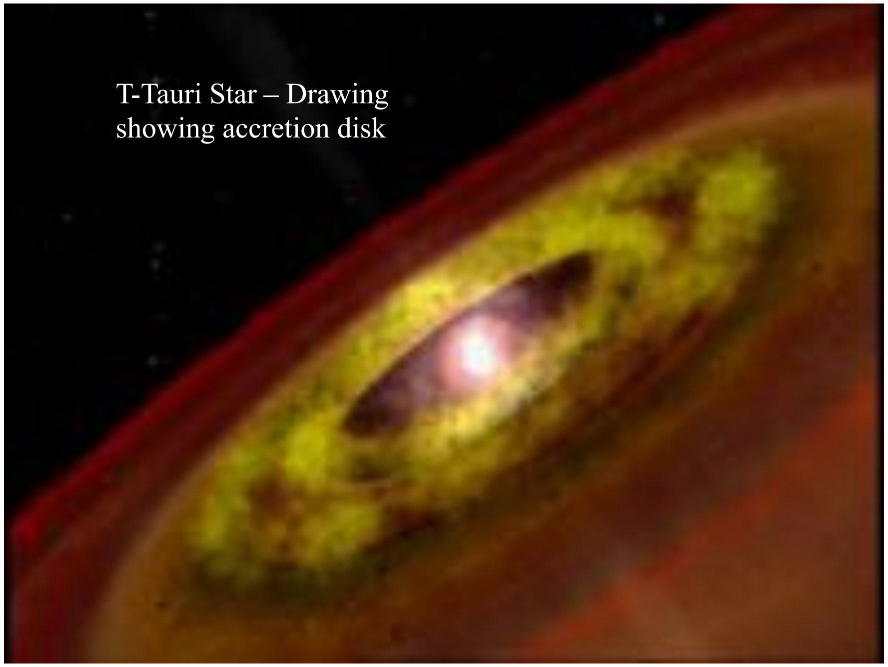
T-Tauri - about 400 ly away at the edge of a molecular cloud.
FOV here is 4 ly at the distance of T-Tauri http://apod.nasa.gov/apod/ap071213.htm

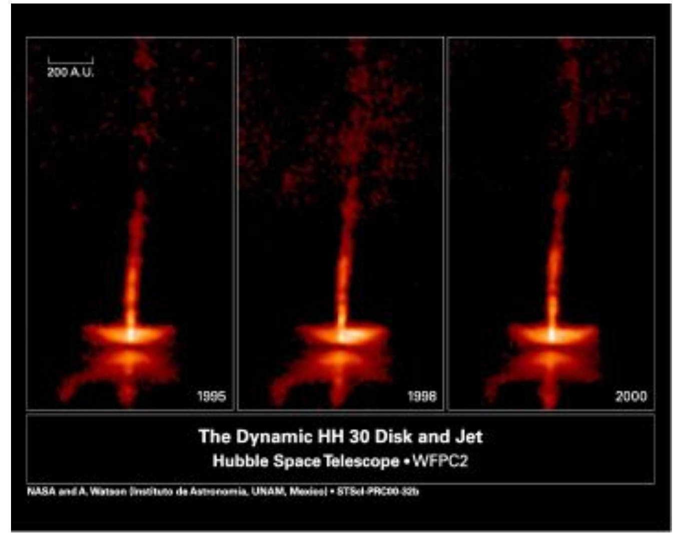
In fact, angular momentum cannot be ignored



Protoplanetary disks orbit over half of T-Tauri stars.
This shows 5 such stars in the constellation Orion.
Picture using HST - field is about 0.14 ly across
http://en.wikipedia.org/wiki/T_Tauri_star







30” west of the brightest point in Hind’ s nebula is a disk-jet system, Herbig-Haro 30. At the center of this is probably a T-Tauri like star.

It is thought that the disks in T-Tauri stars dissipate in low mass stars like the sun before the star ignites on the main sequence.

For higher mass stars, the Kelvin Helmholtz time is shorter (see homework) and the disks may still exist on the main sequence.