

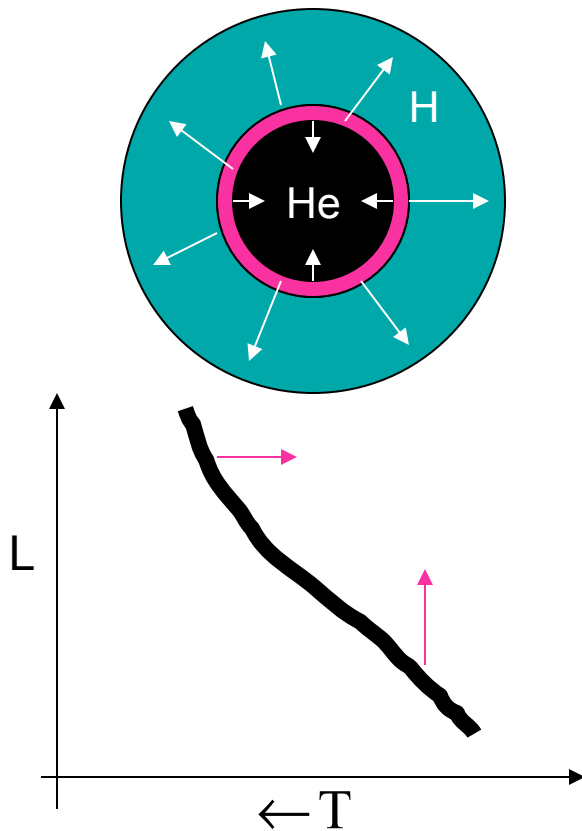
*Post-Main Sequence
Evolution – Low and
Intermediate
Mass Stars*

Pols 10, 11

Prialnik 9

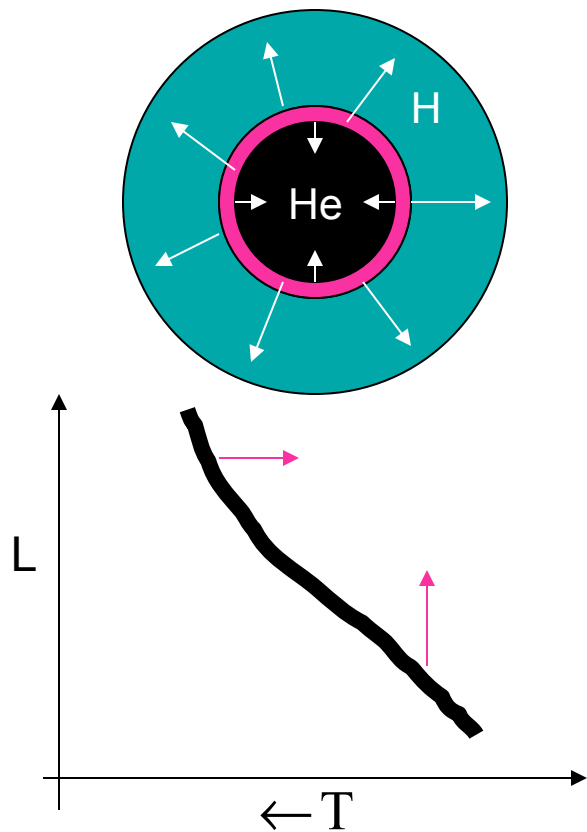
Glatzmaier and Krumholz 16

What happens when
the sun runs out of
hydrogen in its center?

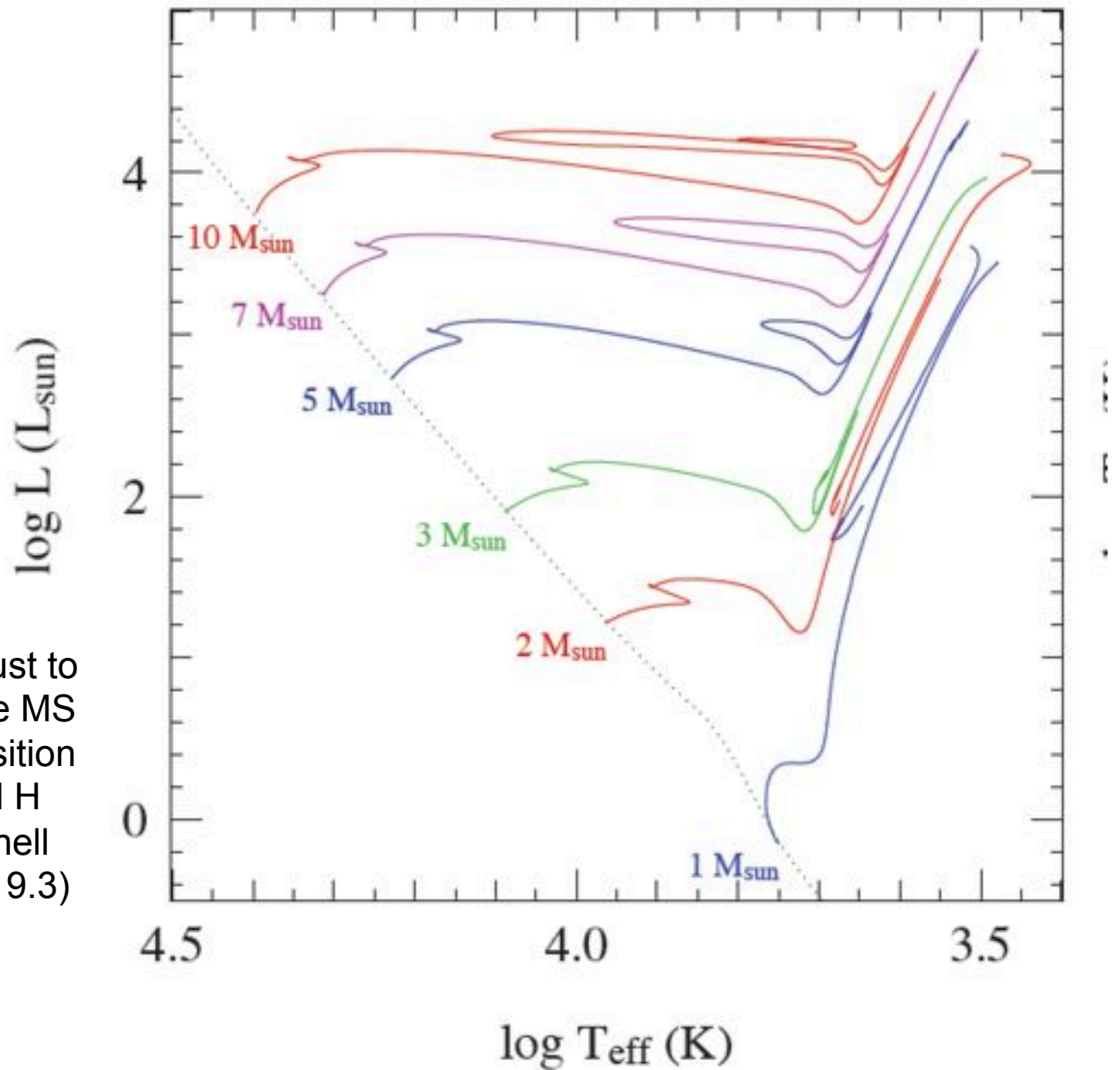


- Once hydrogen is exhausted in the inner part of the star, it is no longer a main sequence star. It continues to burn hydrogen in a thick shell around the helium core, and actually grows more luminous.
- Once the SC mass is exceeded, the contraction of the H-depleted core takes the hydrogen burning shell to greater depth and higher temperature. it burns very vigorously with a luminosity set more by the properties of the helium core than the unburned “envelope” of the star. The shell becomes thin in response to a reduced pressure scale height at the core’s edge because of its high gravity.
- The evolution differs for stars below about 2 solar masses and above. The helium core becomes degenerate as it contracts for the lower mass stars

What happens when
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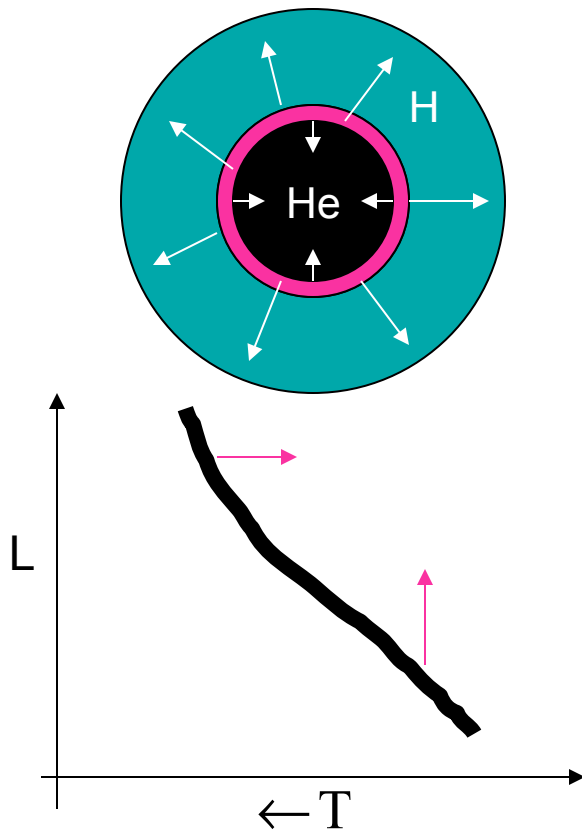


- H shell burning is by the CNO cycle and therefore very temperature sensitive.
- In stars lighter than the sun where a large fraction of the outer mass was already convective on the main sequence, the increased luminosity is transported quickly to the surface and the star's luminosity rises at nearly constant photospheric temperature (i.e., the star is already on the Hayashi strip)
- For more massive stars, the surface remains radiative for a time and the luminosity continues to be set by the mass of the star (i.e., is constant). The increased power from the H shell drives expansion. The star moves to larger radii (lower T) at nearly constant L, until it reaches the Hayashi strip



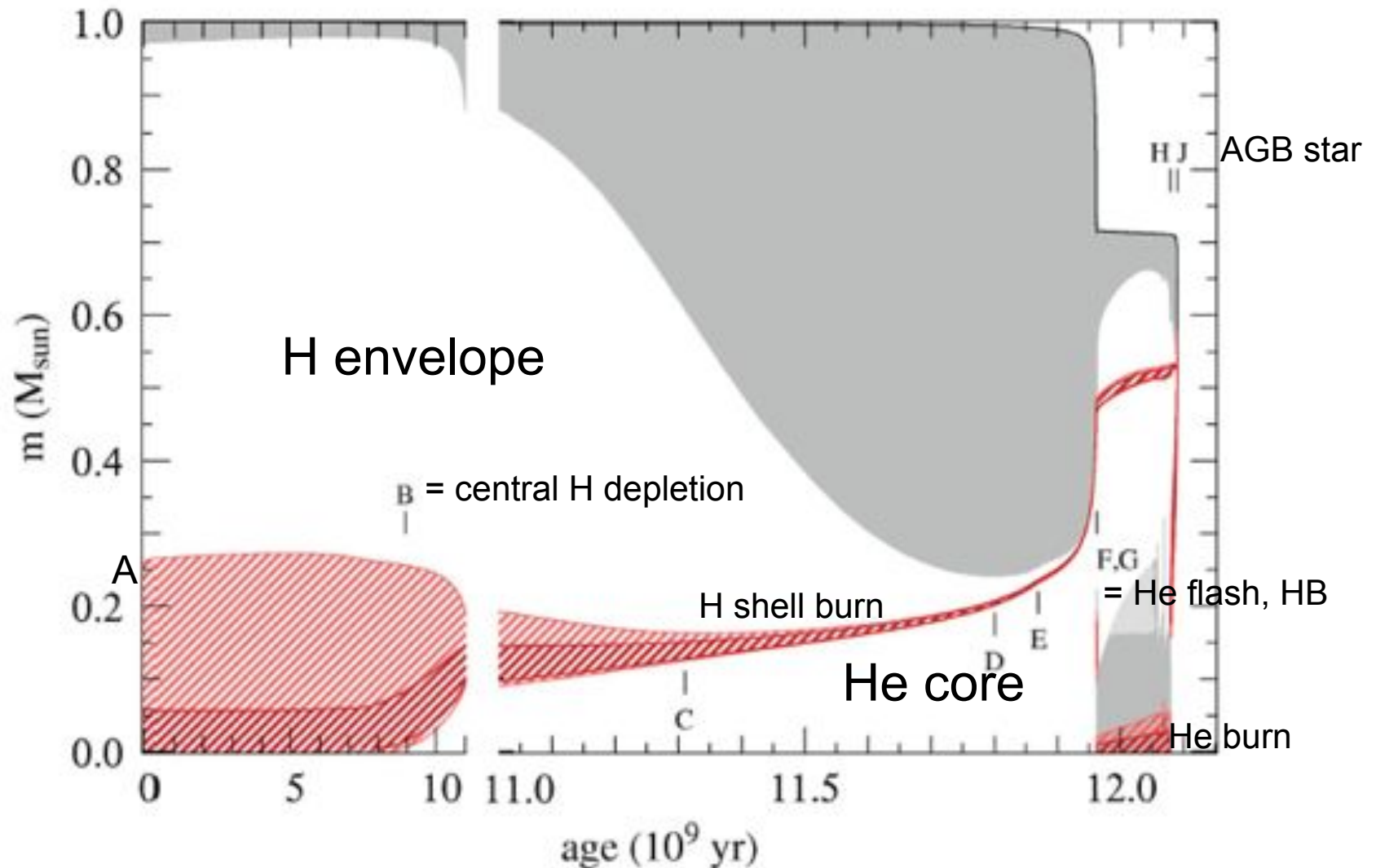
The “hooks” just to the right of the MS mark the transition from central H burning to shell burning (Pols 9.3)

What happens when
the sun runs out of
hydrogen in its center?



- All stars eventually converge on the Hayashi strip and have an effective temperature ~ 4000 K
- The time spent by the massive stars expanding is short – the Kelvin-Helmholtz time scale for the envelope, hence there are few stars in the region where they are evolving at constant L. This is called the “Hertzsprung gap”
- Massive stars cross the Cepheid instability strip on the way to becoming red giants, but spend too short a time there to explain the numbers that are observed.

Pols Fig 10.5



The history of the sun (typical of $M < 2$). Red is energy generation. Dark is $> 5 L_{\odot} / M_{\odot}$ pink is $> 1 L_{\odot} / M_{\odot}$. Grey indicates convective regions. Letters correspond to points on the HR diagram on the following page.

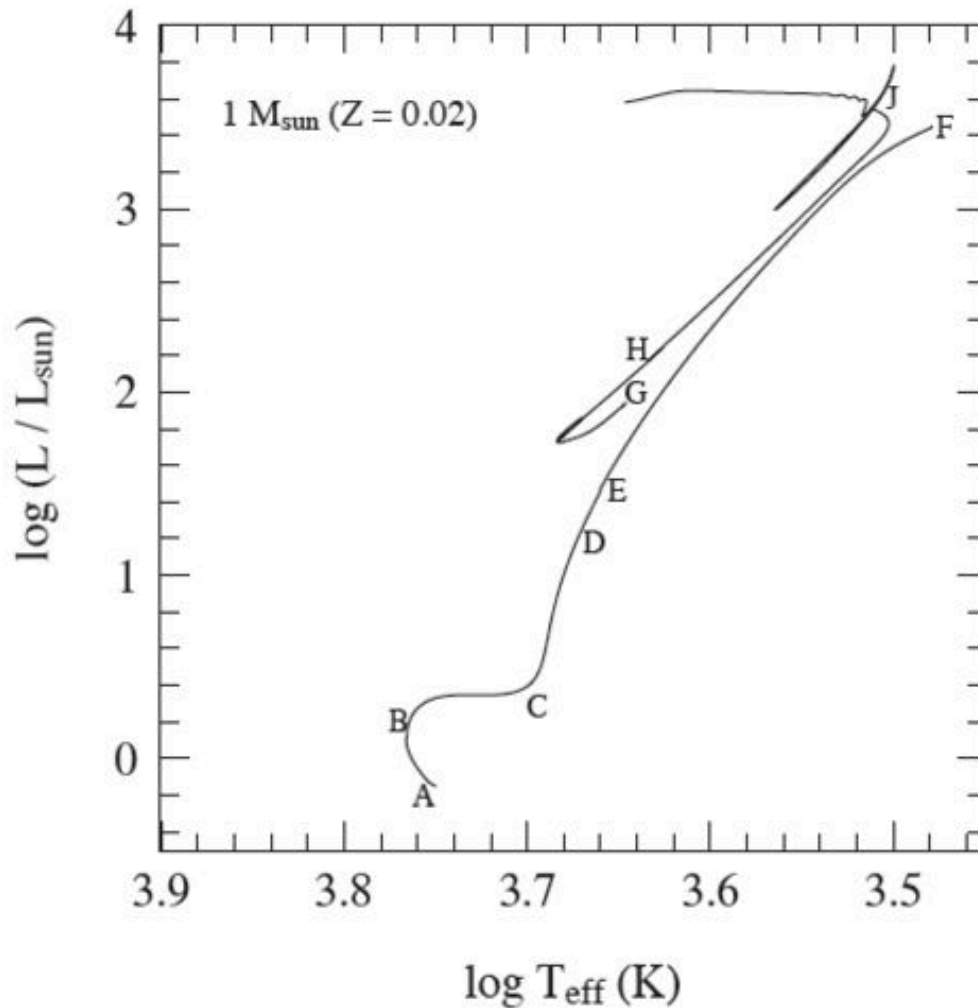


Fig 10.5 Pols

3.7 = 5000 K

3.6 = 4000 K

3.5 = 3000 K

Between points B and C the helium core becomes degenerate and hydrogen burns in a thinner shell. Point D is when the surface convection zone is its deepest. Products of the CNO cycle are mixed to the surface. At E the H shell reaches the mass where the deepest convection (D) extended and there is a small change in L (see Fig 10.6 Pols). At F, helium ignites with a flash. The helium core mass then is 0.45 M_{sun} .

Core Luminosity Relation

(for low mass stars that develop a degenerate He core)
(Paczynski (1970); Tuchman et al (1983))

Because of the steep density gradient just outside the helium core, the conditions in the hydrogen burning shell depend sensitively upon the properties of the degenerate helium core upon which it rests. The more massive the core, the smaller its radius and stronger its gravitational potential. This makes the temperature in HE higher which gives a greater luminosity by the CNO cycle. Empirically, from models, and later analytically.

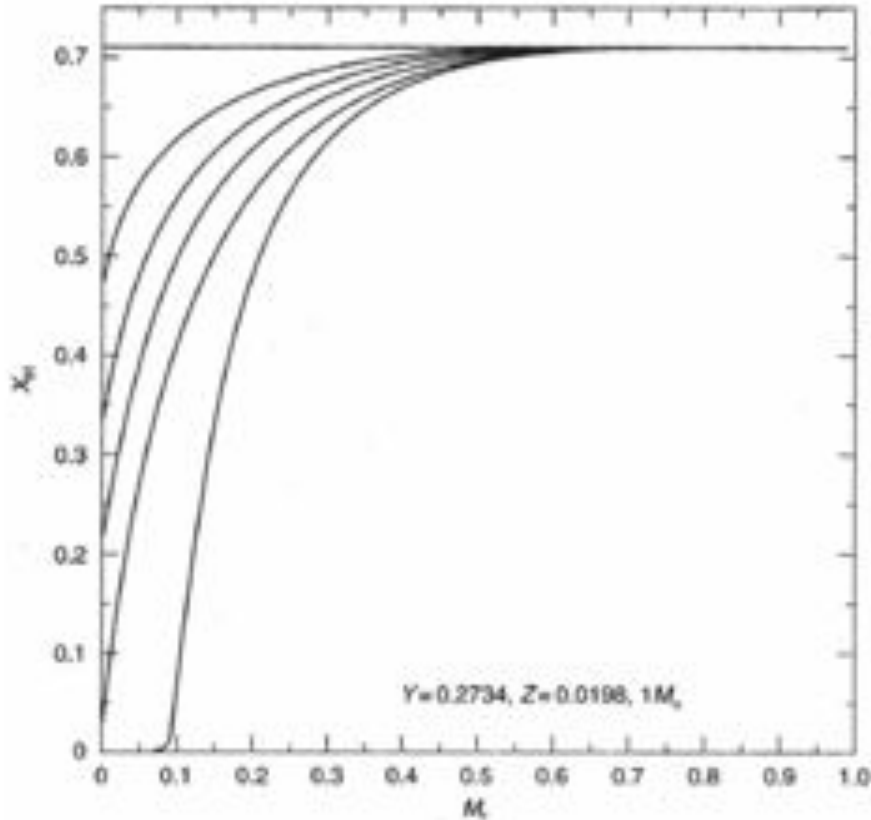
$$L \approx 2.3 \times 10^5 \left(\frac{M_c}{M_\odot} \right)^6 \text{ erg s}^{-1}$$

<http://adsabs.harvard.edu/abs/1983ApJ...268..356T>

As the helium core mass grows due to shell H burning adding ash, the luminosity goes up. Starting from about $0.1 M_{\odot}$ for the sun and growing to $0.45 M_{\odot}$, the luminosity climbs from roughly L_{\odot} to $2000 L_{\odot}$ at the time of the helium core flash. Because the larger luminosity means more hydrogen is being added to the core per unit time, the evolution accelerates at the end.

Hydrogen depletion

$1 M_{\odot}$



$5 M_{\odot}$

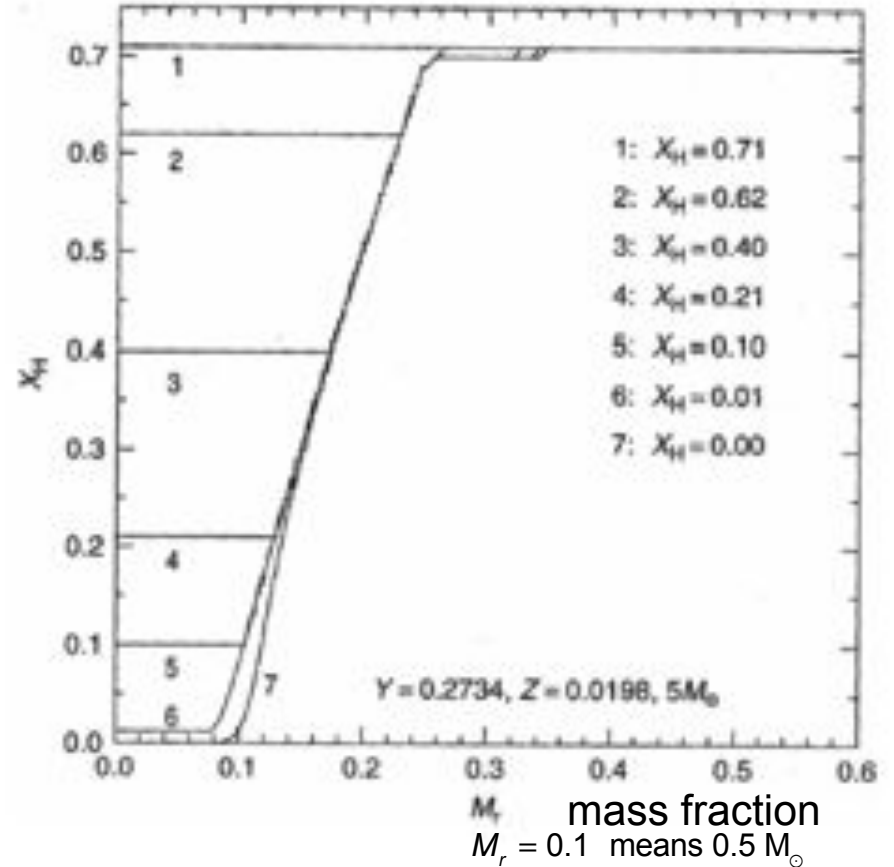
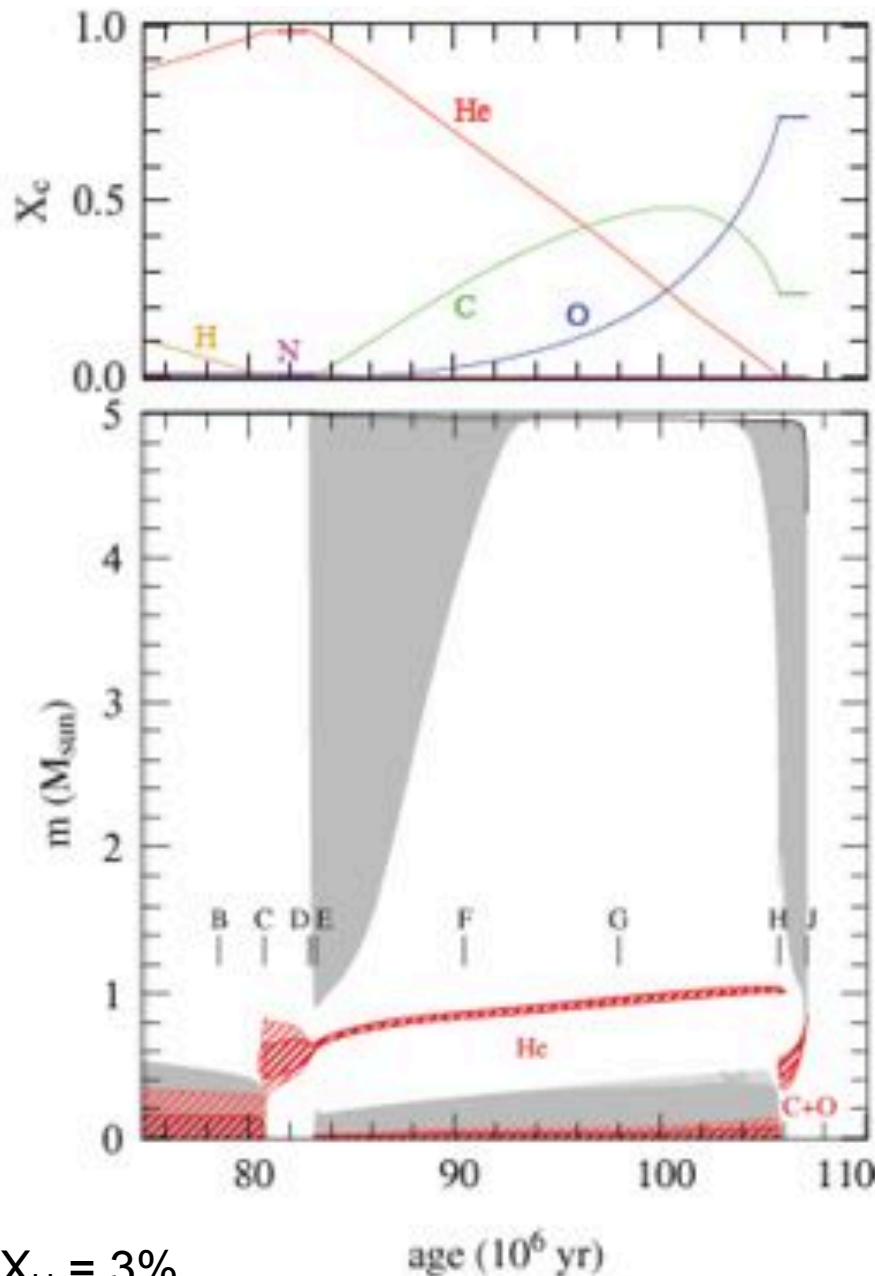


Figure 9.10. Hydrogen abundance profiles at different stages of evolution for a $1 M_{\odot}$ star (left panel) and a $5 M_{\odot}$ star (right panel) at quasi-solar composition. Figures reproduced from SALARIS & CASSISI.

5 M_{\odot} star

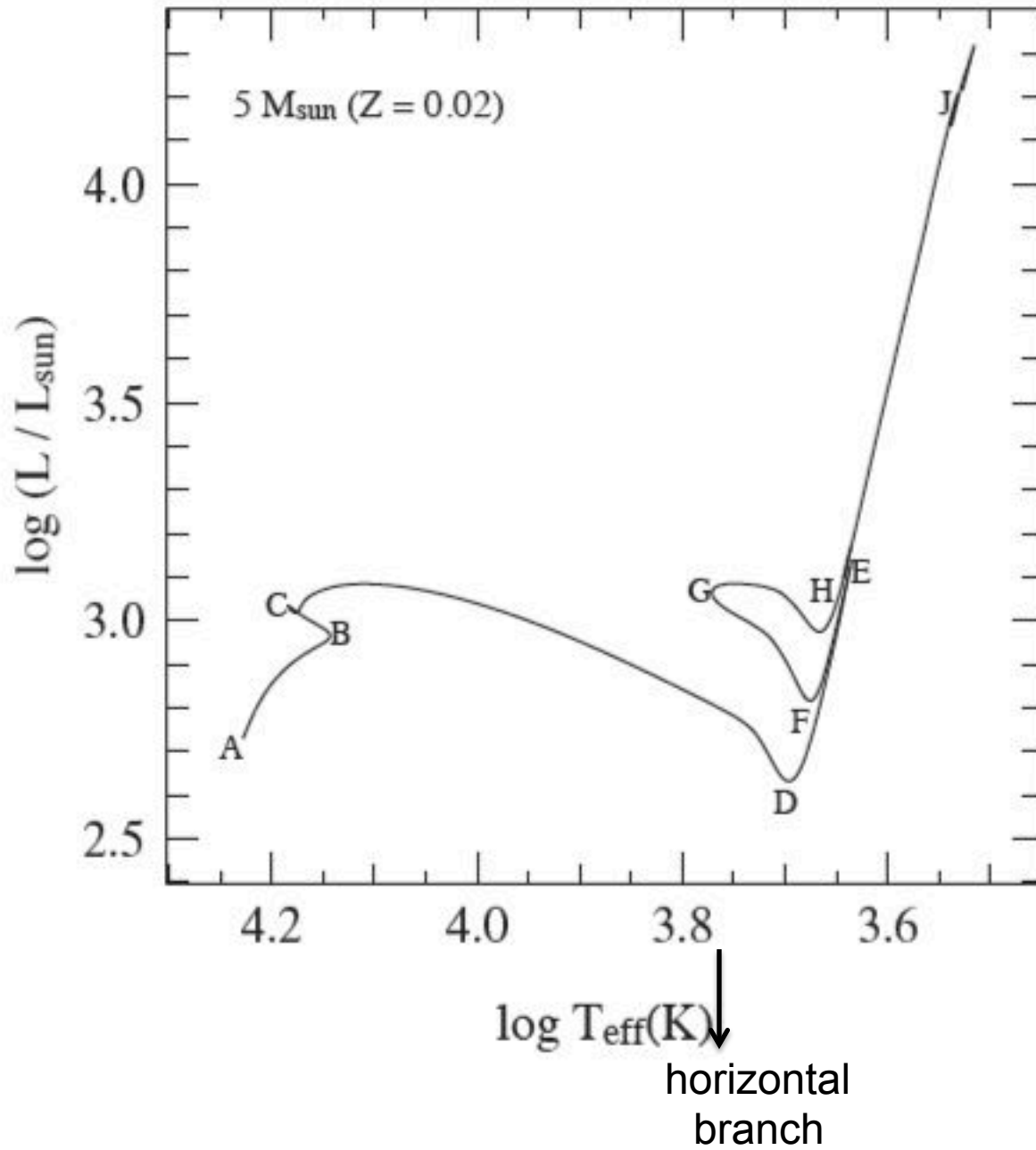


at B, $X_H = 3\%$

Fig 10.3 Pols

- Point C is central H exhaustion
Exhausted core has not yet reached the SC mass at C so the stage of thick H shell burning (CD) is relatively long, but most of it occurs near the main sequence.
 $\log T_e > 4.0$. The transition to a red giant is quite fast
- At D, the envelope becomes fully convective and L rises
- At E helium ignites non-degenerately in the center
- .During FGH the star exhibits a “blue loop”, spending part of that time as a Cepheid

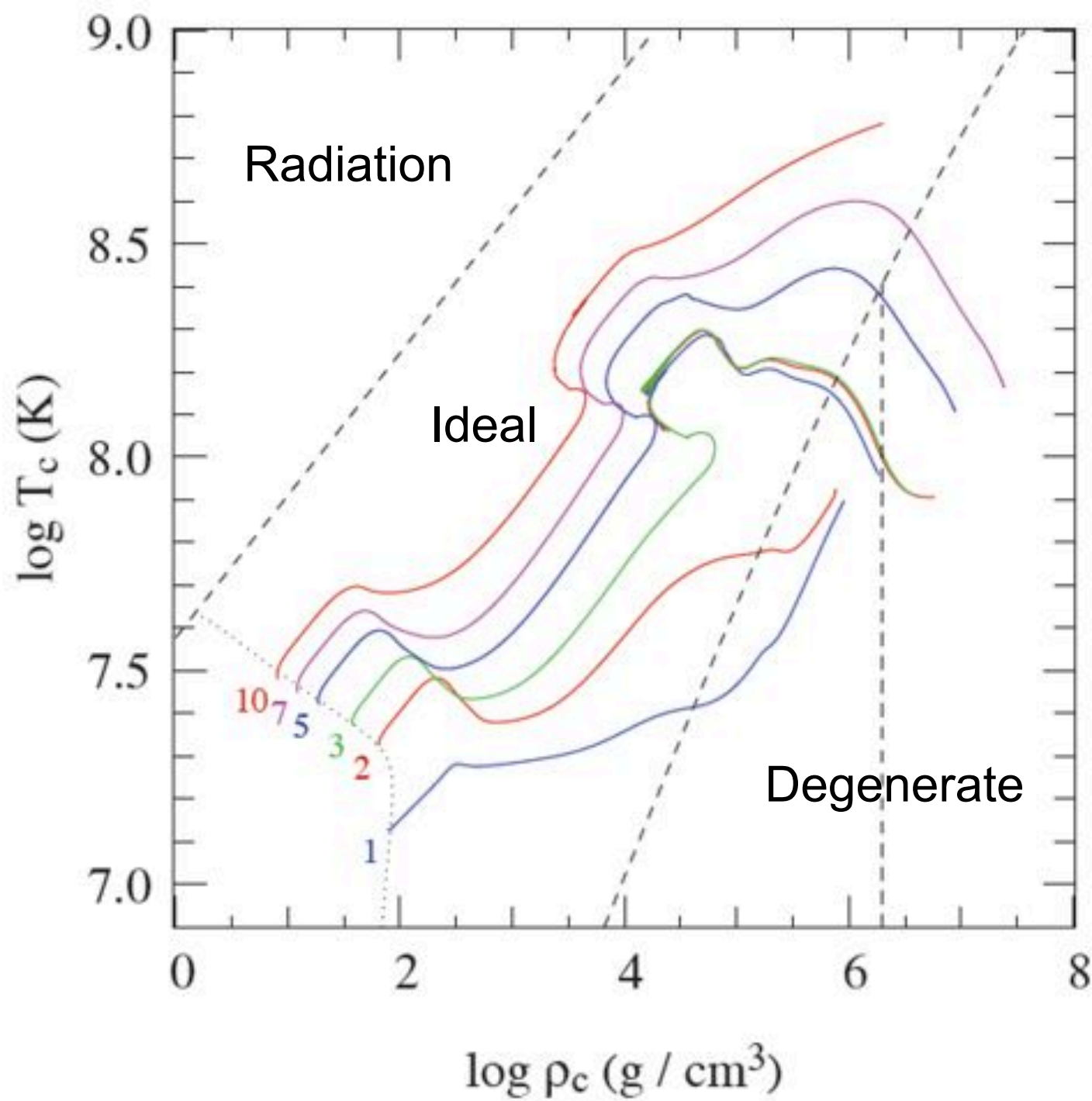
Fig 10.2 Pols

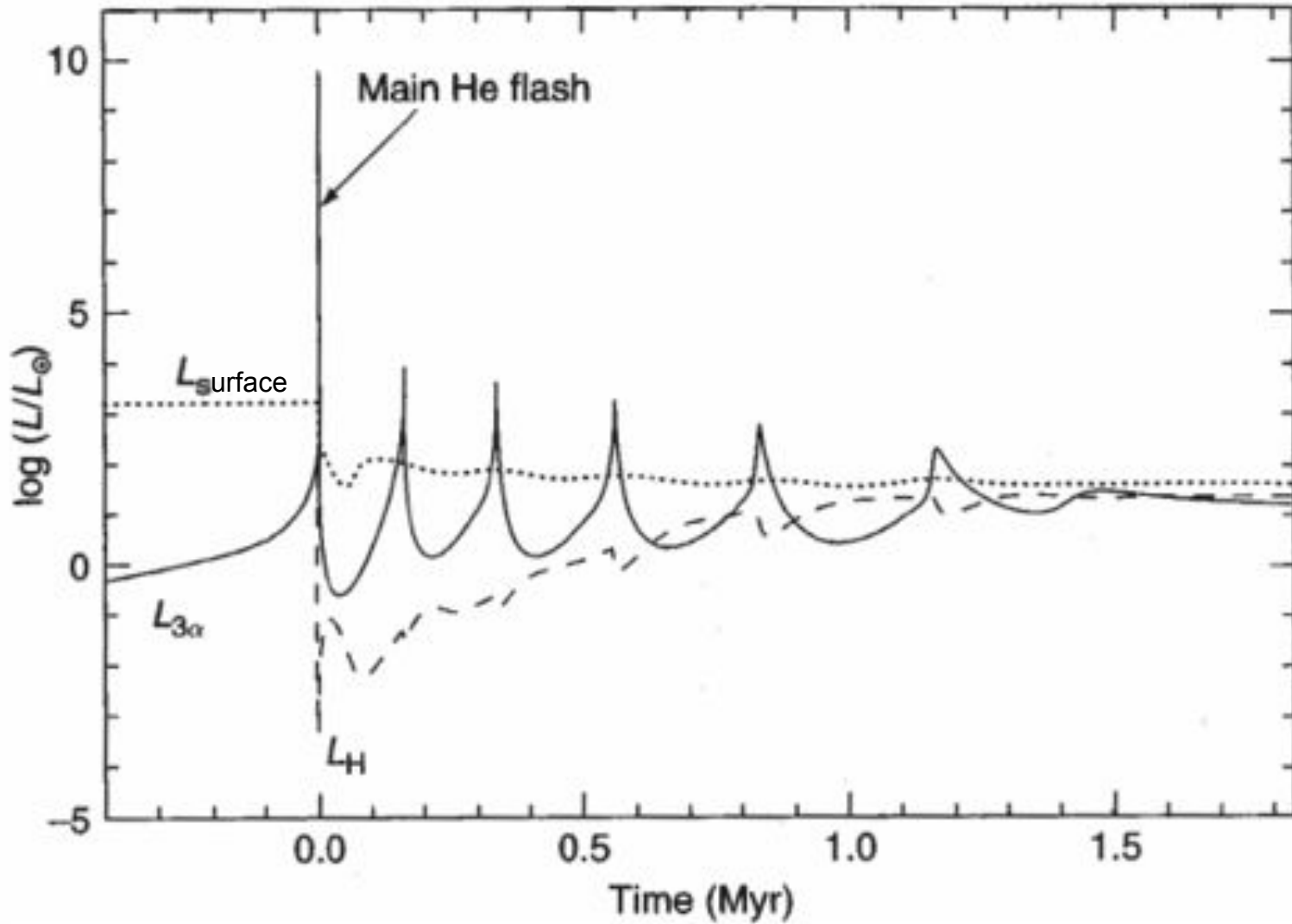


Helium flash in stars below 2 solar masses

Helium burning in stars below 2 solar masses develop degenerate helium cores and evolve differently from heavier stars.

- The stars all ignite helium burning with essentially the same mass – 0.45 – 0.50 solar masses. The core structure of all such stars is thus similar and that is why there is a horizontal branch of near constant luminosity
- Helium ignites degenerately. In the sun, for example, helium burning ignites at a temperature of about 1×10^8 K and a density about 10^6 g cm^{-3} . The pressure for this condition is given chiefly by degenerate electrons and the ignition leads to a brief thermonuclear runaway

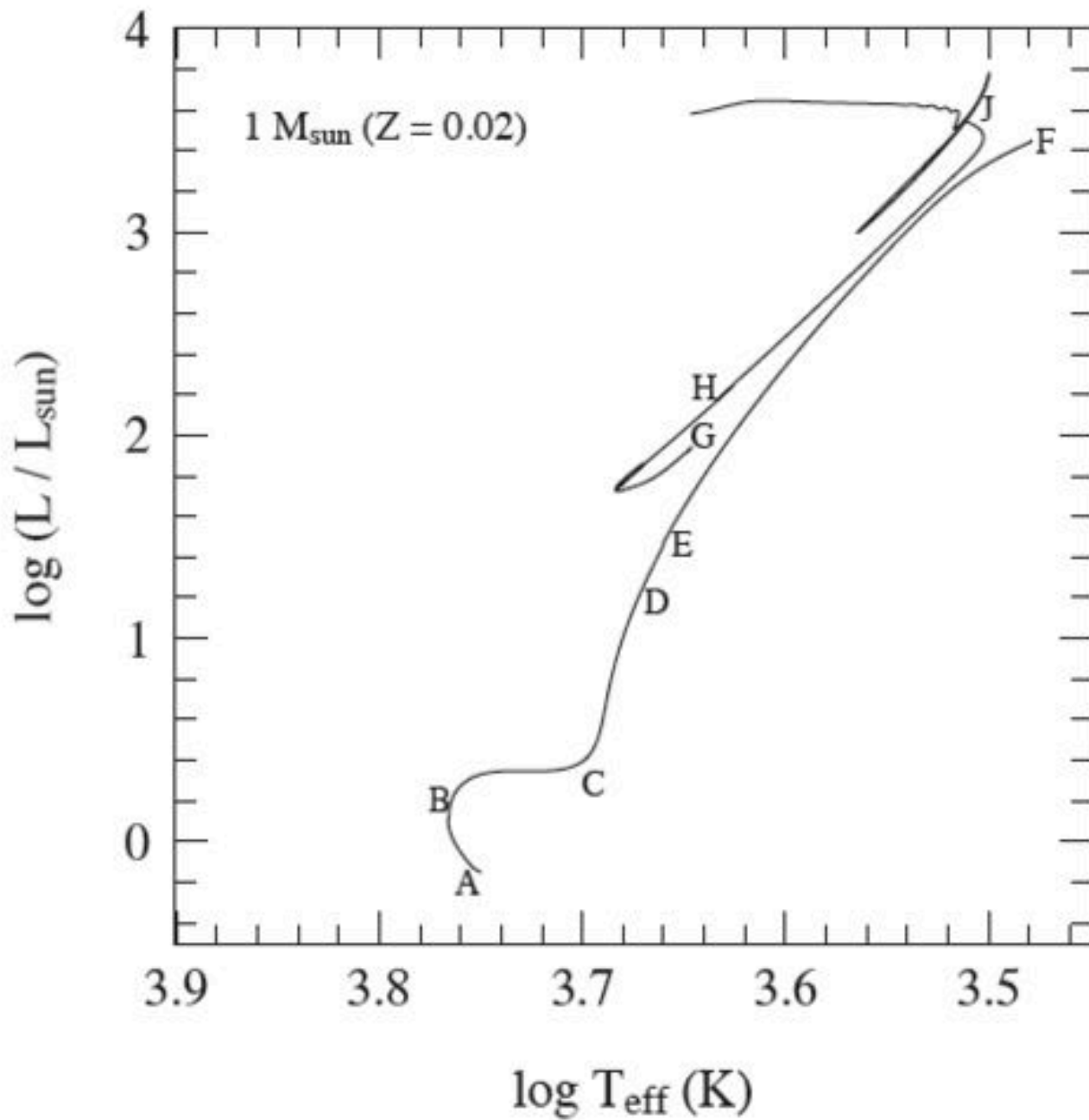




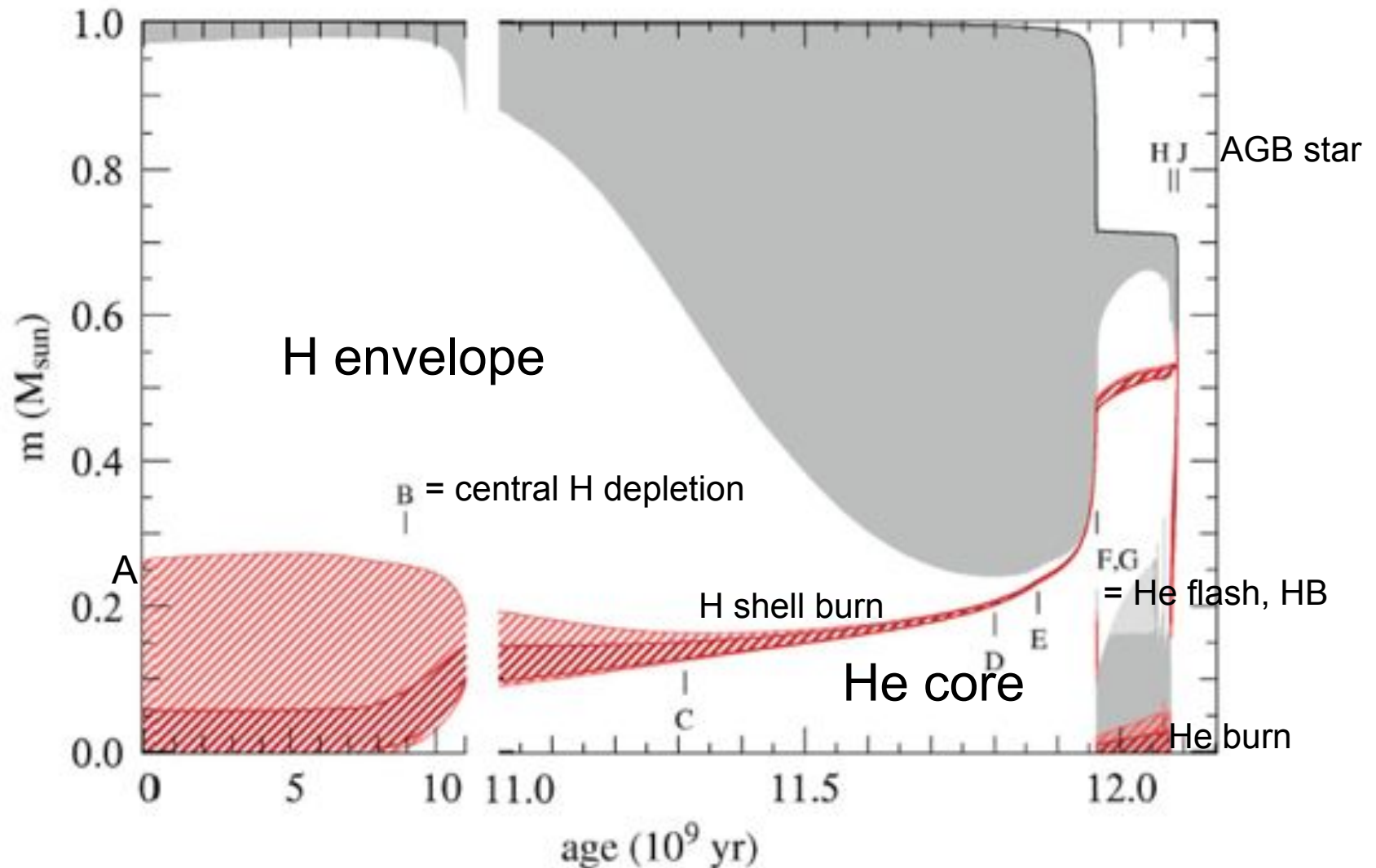
Pols 10.7 – helium core flash in a star of about 1 solar mass

At maximum the energy generation from helium burning during the flash reaches about 10^{10} solar luminosities, equivalent to a small galaxy. But very little helium burns before expansion puts out the runaway after only a few minutes (a factor of a few times the sound crossing time for the helium core).

This large luminosity does not arrive at the surface but instead goes into expanding both the helium core and the envelope outside it. The hydrogen shell goes out. The helium core evolves on its Kelvin-Helmholtz time and may undergo several more weaker flashes before finally settling down to burn helium stably. For the sun this takes a fairly brief time, ~ 1.5 My.



Pols Fig 10.5



The history of the sun (typical of $M < 2$). Red is energy generation. Dark is $> 5 L_{\odot} / M_{\odot}$ pink is $> 1 L_{\odot} / M_{\odot}$. Grey indicates convective regions. Letters correspond to points on the HR diagram on the following page.

Mass loss

Because of their high luminosity and weak surface gravity, red giants, and especially AGB stars (which have very large radii) lose a lot of mass to winds. A rough approximation to the mass loss is given by the Reimer's (empirical) relation

$$\dot{M} \approx -4 \times 10^{-13} \eta \frac{L}{L_{\odot}} \frac{R}{R_{\odot}} \frac{M_{\odot}}{M} M_{\odot} \text{ y}^{-1} \quad \text{Pols p 150}$$

$$\eta \sim 1$$

Many other, more complicated expressions exist.

The mass loss occurs because of the momentum deposited by the outgoing light in the atoms near the star's surface that absorb it. In red giants, grains may also form and be pushed out by the light.

Mass loss

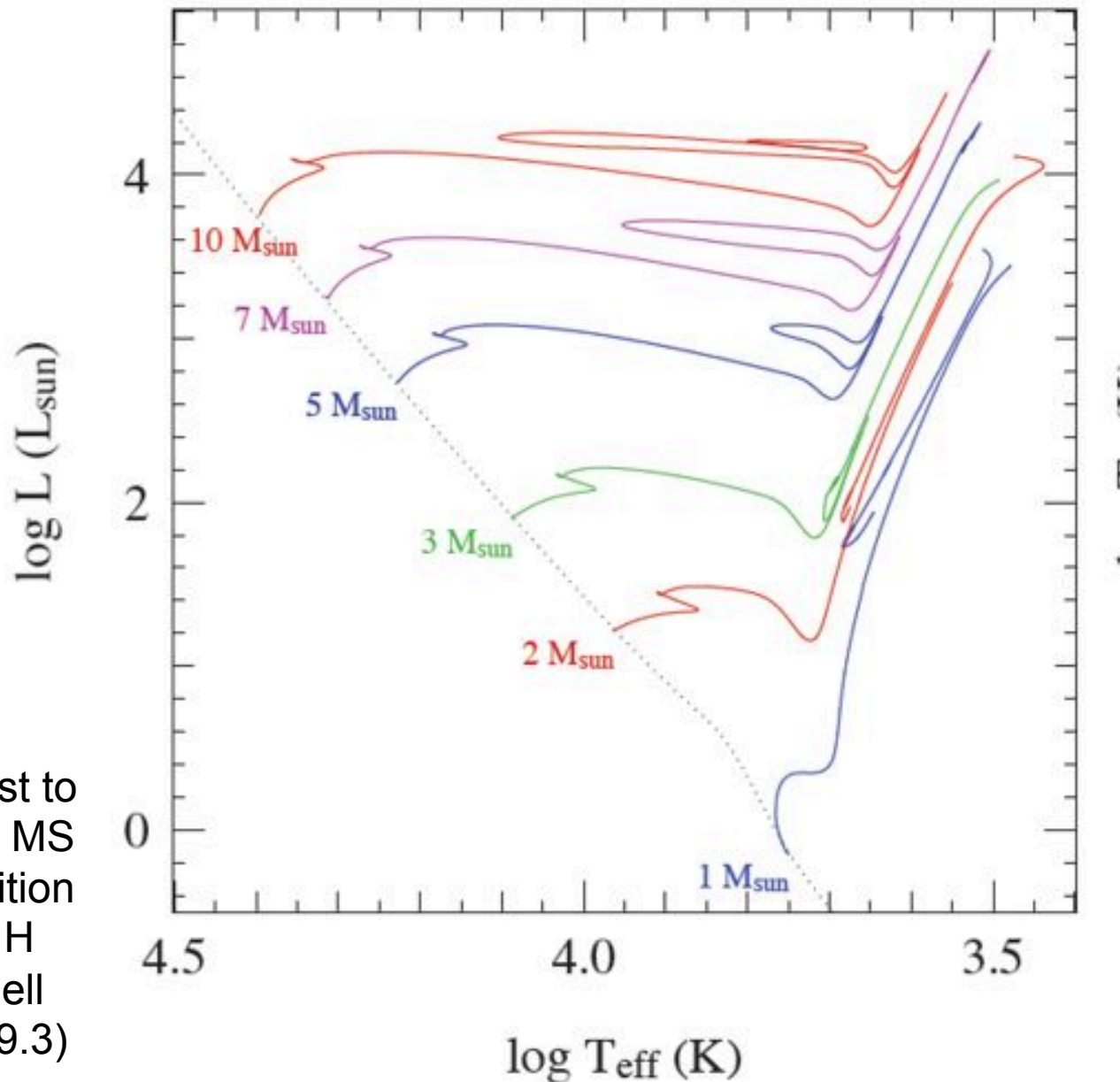
It is thought that the sun will lose roughly 0.3 solar masses by the time it finishes helium burning.

Since grains require heavy elements to form and since much of the momentum deposition involves interactions with heavy elements, especially iron, stars with lower metallicity have smaller mass loss rates than stars with solar metallicity.

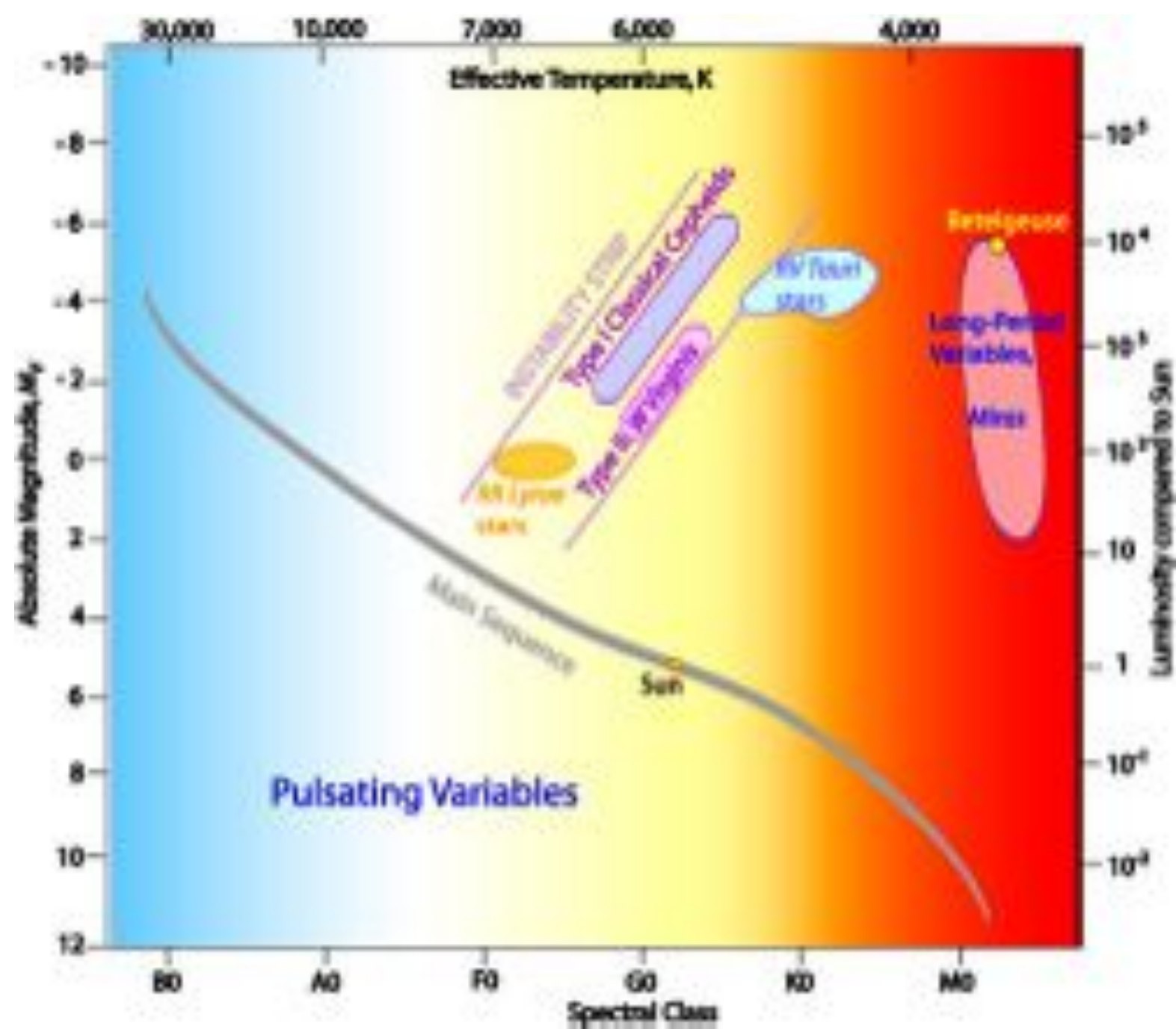
Low metal stars or all sorts may eventually die then with masses larger than their modern day counterparts.

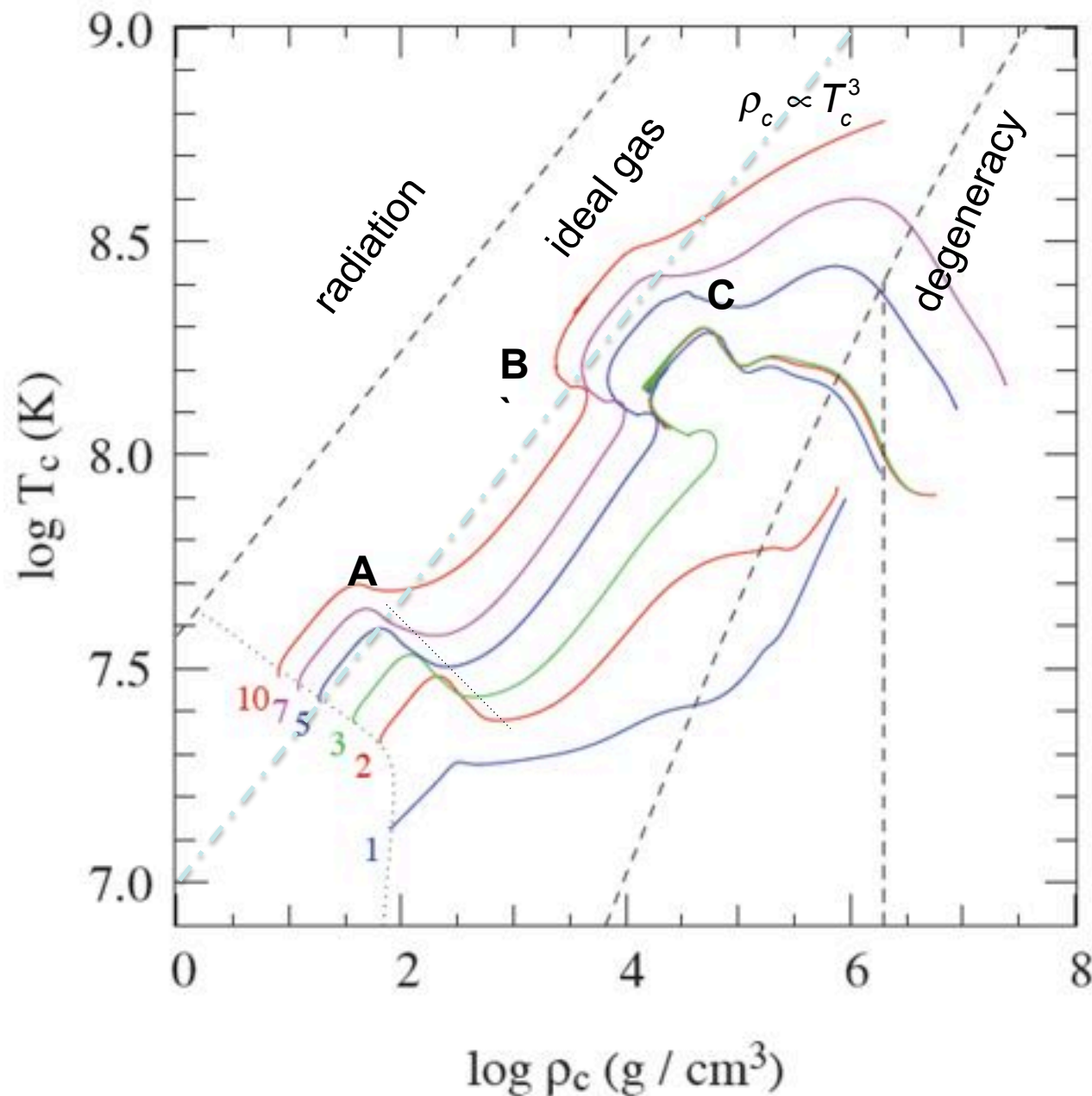
Except for very massive stars, most mass loss occurs during the red giant phase.

“Blue Loops” (during helium core burning)



The “hooks” just to the right of the MS mark the transition from central H burning to shell burning (Pols 9.3)





A) Red giant formation. The helium core evolves as if it were a star of reduced mass.

$$T_c \propto M^{2/3} \rho_c^{1/3}$$

B) Helium ignition. A new source of energy generation is found that is highly centrally concentrated due to the high temperature sensitivity of the 3-alpha reaction. The core becomes convective and the polytropic index decreases.

C) Helium burning over. Kelvin-Helmholtz contraction. Onset of degeneracy.

Horizontal Branch Stars

Only stars below 2 Msun make HB stars

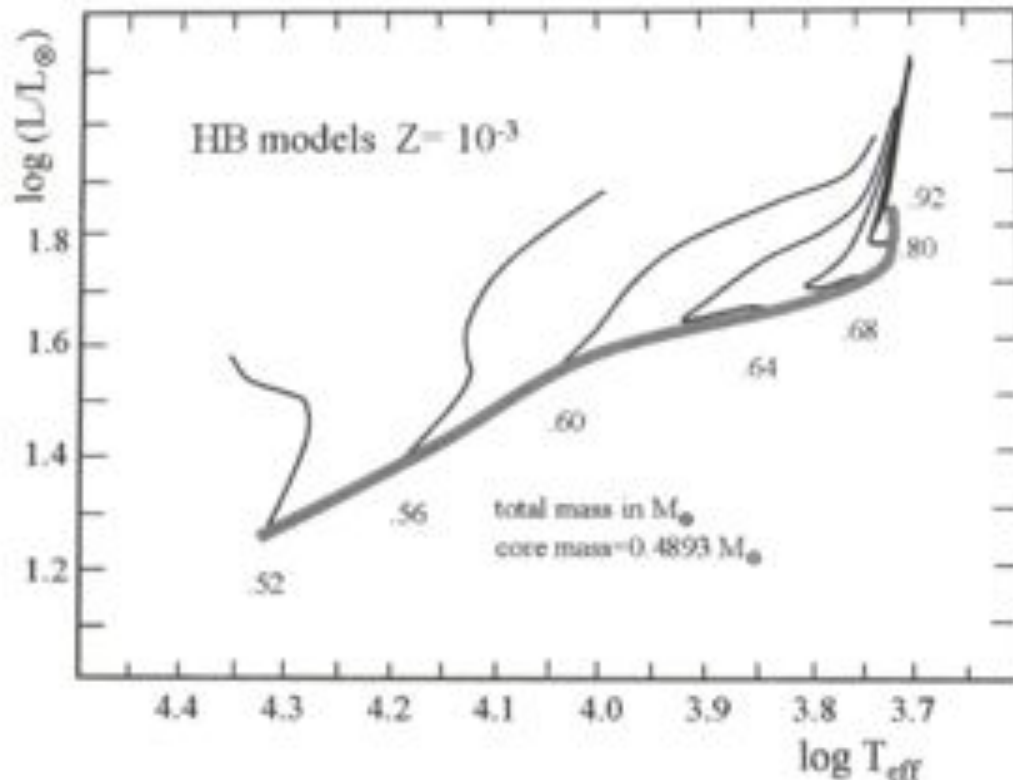


Figure 10.9. Location of the zero-age horizontal branch (think gray line) for a metallicity $Z = 0.001$ typical of globular clusters. These models have the same core mass ($0.489 M_{\odot}$) but varying total (i.e. envelope) mass, which determines their position in the H-R diagram. Evolution tracks during the HB for several total mass values are shown as thin solid lines. Figure from MAEDER.

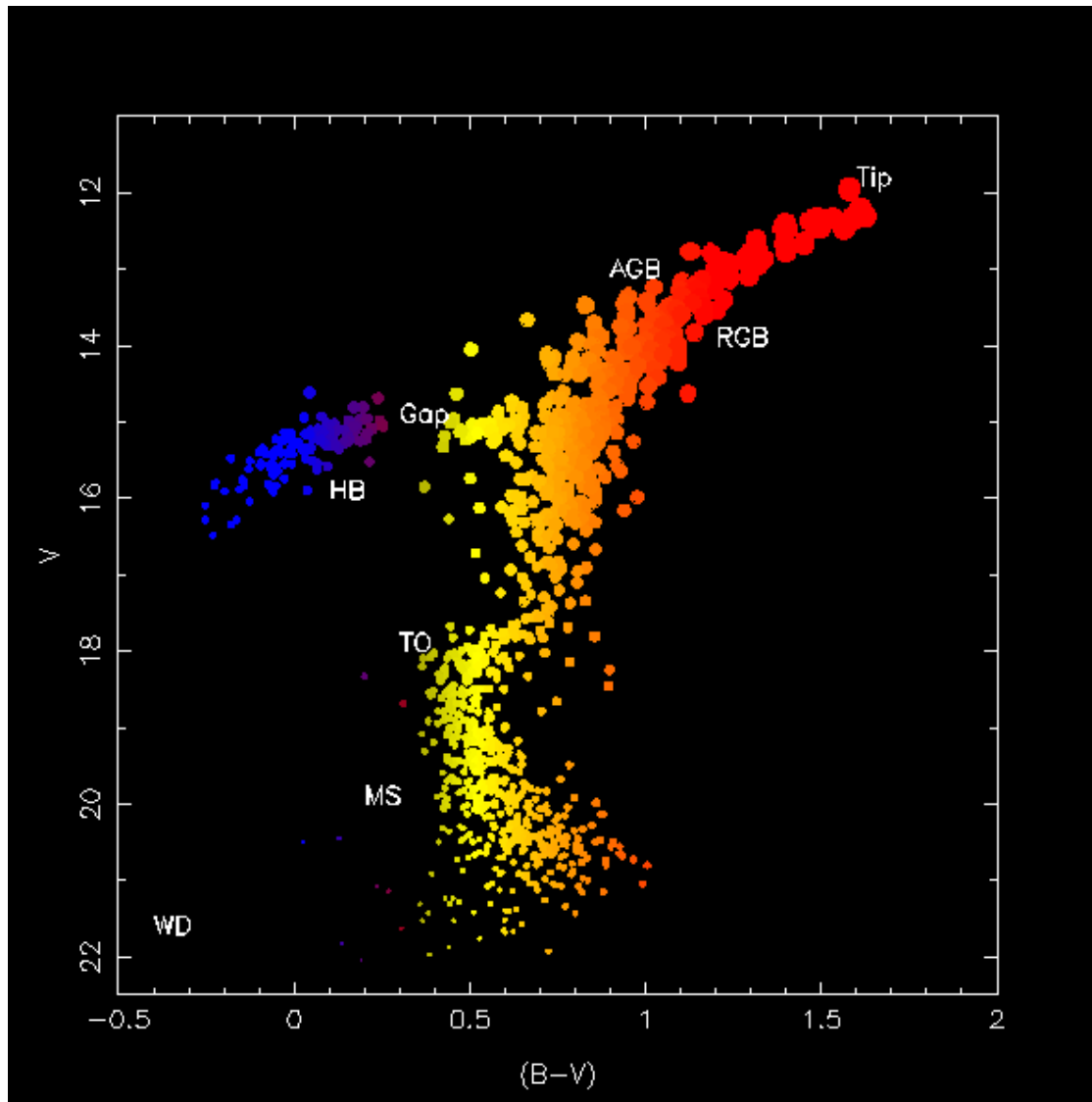
Lifetime on the HB is roughly constant at 120 My due to the nearly constant He core size, $0.45 - 0.50 M_{\odot}$. The H-shell also contributes to the luminosity though, especially when the envelope mass is large. The He flash was not included in these calculations. These calculations start at what was point “G” in the other figures.

Horizontal Branch Stars

- For solitary solar metallicity stars, there is a pile up on the right hand side of the HB called the “red clump”. This is because all such stars have substantial H envelopes relative to their 0.45 solar mass helium cores
- Stars to the left (blue) on the HB generally have smaller envelope masses, i.e., the helium core is a larger fraction of the star’s mass.
- But lower metallicity can also cause a blueward extension of the HB – hence its prominence in HR diagrams of globular clusters
- And - there is some evolution back and forth across the HB and there is an instability strip (RR-Lyrae stars)

Globular Cluster M5

http://www.dur.ac.uk/ian.smail/gcCm/gcCm_intro.html

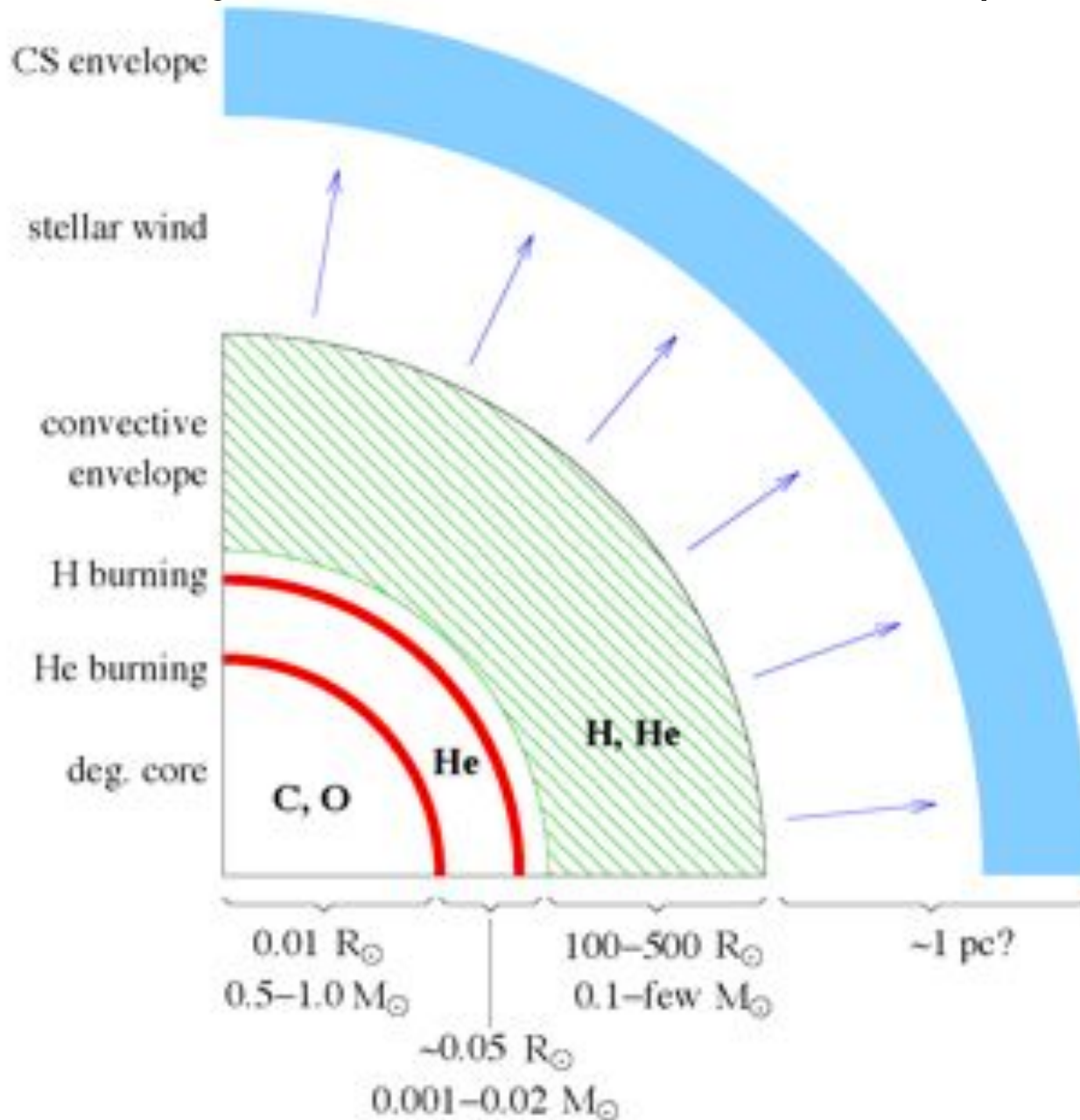


TO = “turn off mass”; HB = “horizontal branch”;
“Gap” is a region of atmospheric instability



The globular cluster M10. The bright yellow and orange stars are red giants burning hydrogen or helium in a shell, but the bright blue stars are “horizontal branch” stars, burning helium in their centers. Both kinds of stars are more massive and brighter than the low mass main sequence stars in M10.

Asymptotic Giant Branch (AGB) Stars



$5 M_{\odot}$

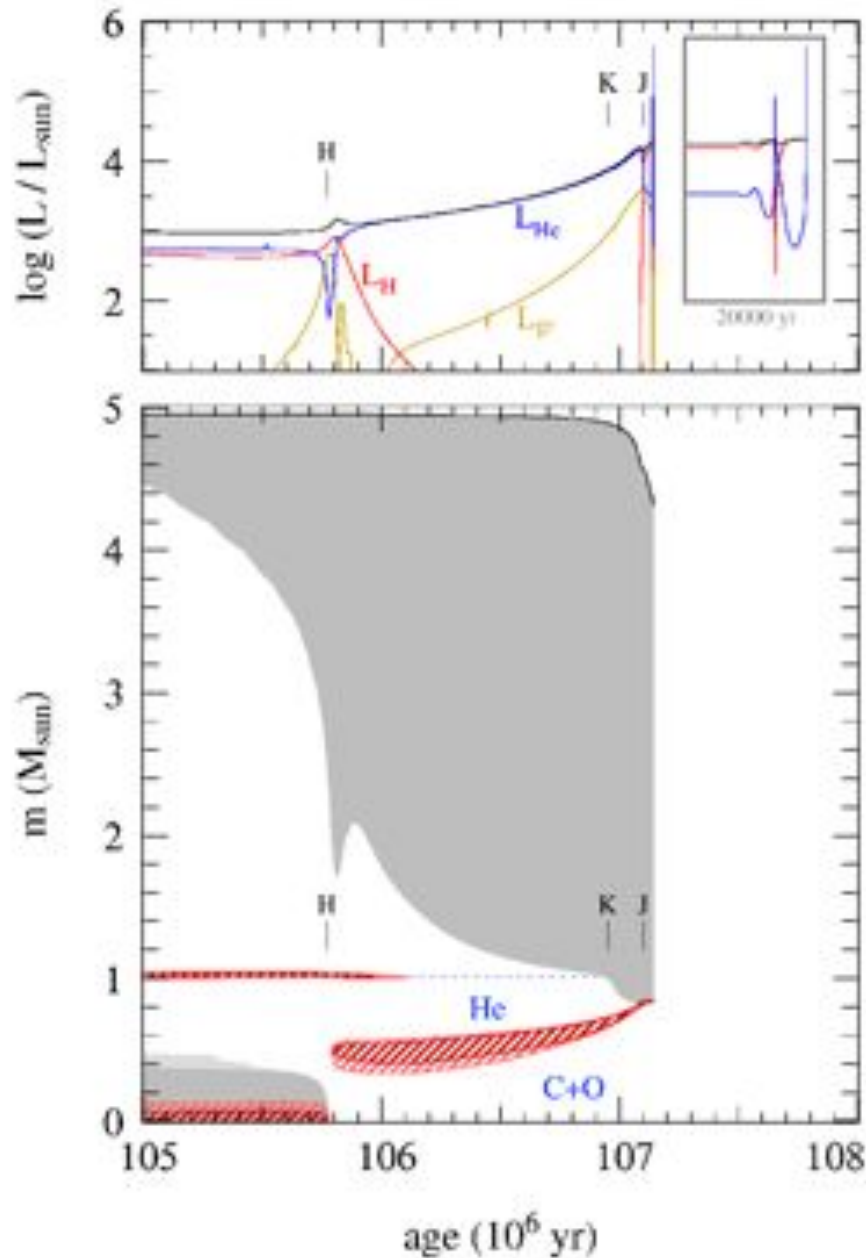
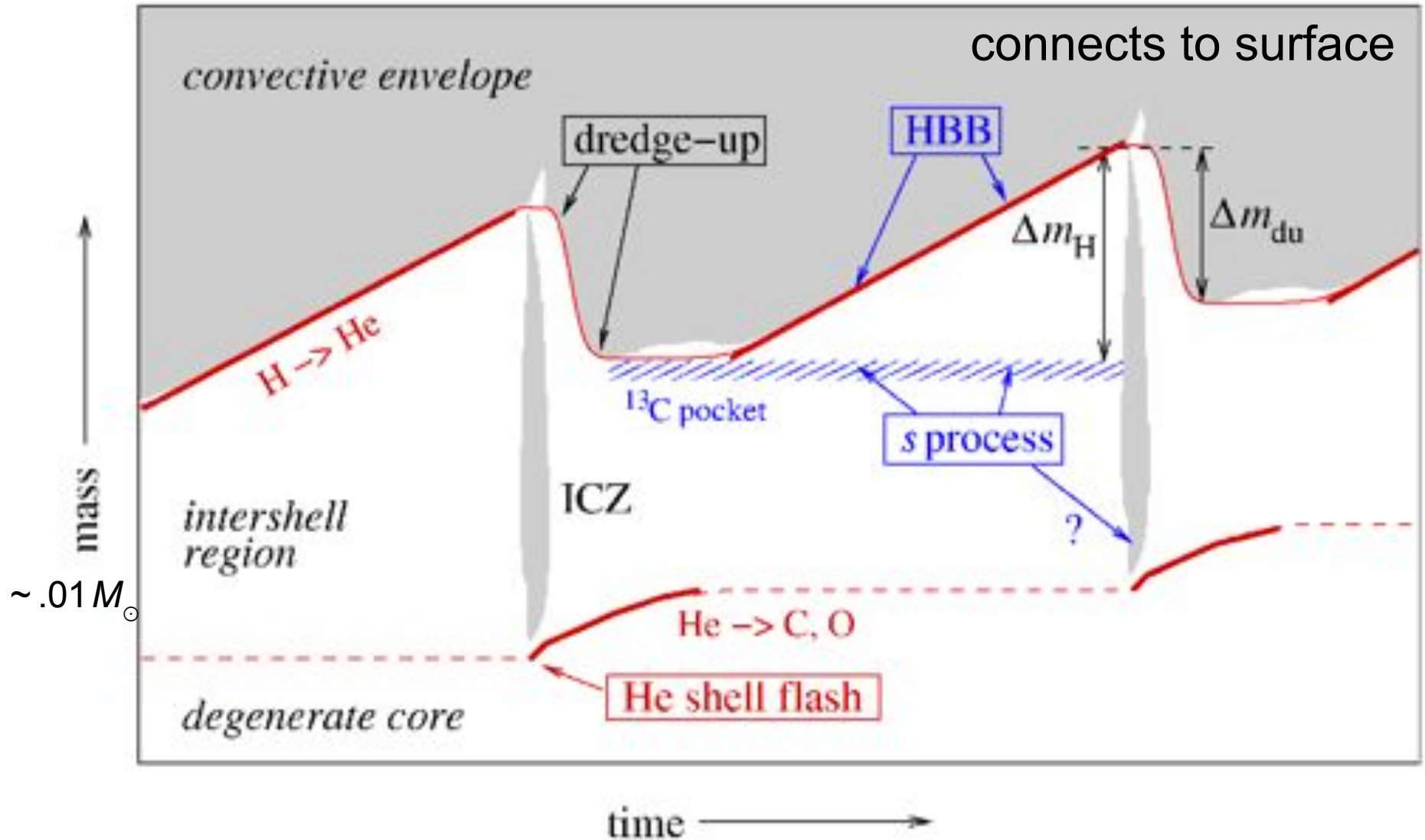


Figure 11.1. Evolution of luminosities (upper panel) and internal structure (lower panel) with time in a $5 M_{\odot}$ star (with composition $X = 0.70$, $Z = 0.02$) during the last stages of helium burning and on the AGB. Compare with Fig. 10.3 for the same star. The early AGB starts at point H, when He burning shifts quite suddenly from the centre to a shell around the former convective core. The H-burning shell extinguishes and at point K second dredge-up occurs. The H-burning shell is re-ignited some time later at point J. This is the start of the double shell-burning phase, which soon afterward leads to thermal pulses of the He-burning shell (and break-down of this particular model). The first thermal pulses can be seen in the inset of the upper panel which shows the last 20 000 yr of this model calculation. Strong mass loss is then expected to remove the stellar envelope within $\lesssim 10^6$ yr, leaving the degenerate CO core as a cooling white dwarf.

Pols 11.3



The time axis is non-linear. Each He flash lasts about 100 years. The period between flashes is about 10,000 – 100,000 years.

helium luminosity

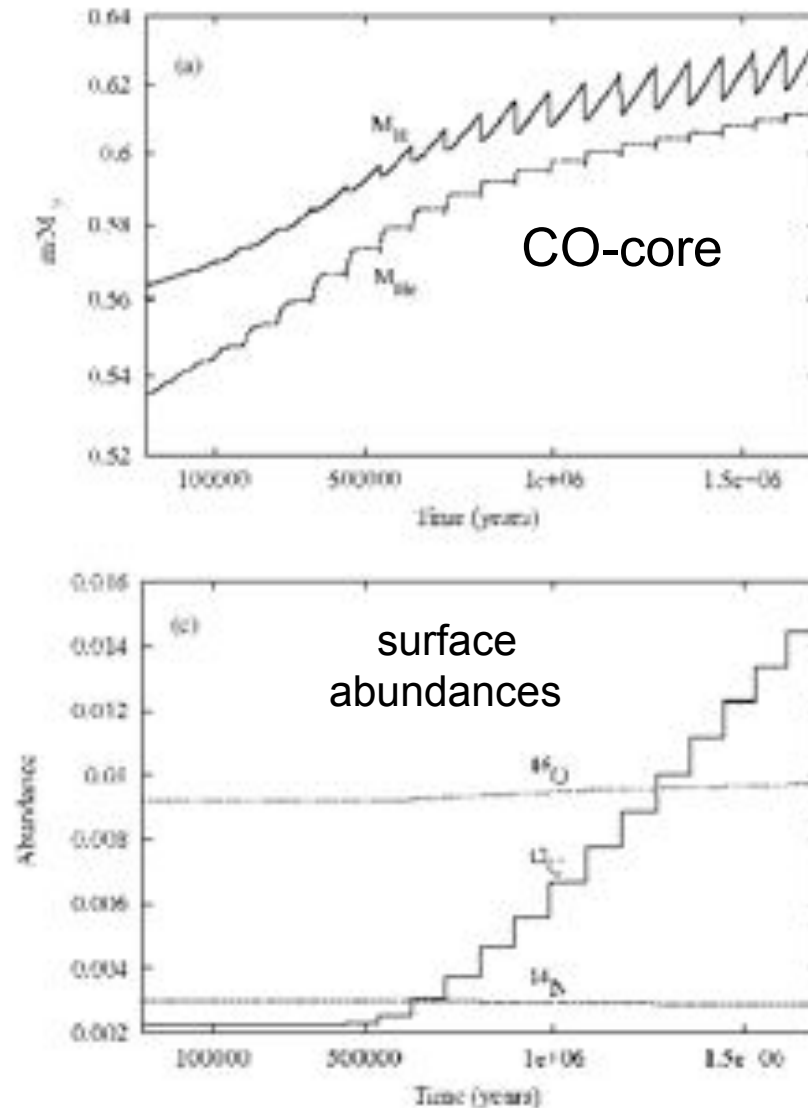


Figure 11.4. Evolution of a $3 M_{\odot}$ star with $X = 0.7$, $Z = 0.02$ during the TP-AGB phase. Time is counted since the first thermal pulse. The three panels show (a) the growth of the hydrogen-exhausted core mass and helium-exhausted core mass, (b) the He-burning luminosity and (c) the changes in surface abundances by mass fraction of ^{12}C , ^{14}N and ^{16}O . Except for the first few pulses, each thermal pulse is followed by a dredge-up episode (sudden drop in core mass) and a sudden increase in ^{12}C abundance. Figure adapted from Stancliffe et al. (2004, MNRAS 352, 984).

Note the consequences for nucleosynthesis here.

The outer layers of the star contain hydrogen and helium to be sure, but also **nitrogen from CNO processing and C** and O from helium burning. It is thought that stars in this mass range are responsible for producing most of the **nitrogen** and maybe 40 – 80% of the **carbon** in the universe.

The rest of carbon and most other elements come from massive stars.

Additional Nucleosynthesis – The s-Process.

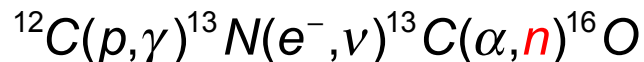
During a helium shell flash the temperature goes up to values well above what helium ordinarily attains during core burning. Helium burning also operates in very close proximity to hydrogen and some small amount of hydrogen may find its way into the convective shell flash. These circumstances give rise to two side reactions capable of producing a substantial concentration ($\sim 10^8 \text{ cm}^{-3}$) of free neutrons.

1) Prior to the helium flash, during the CNO cycle that deposits helium in the shell, $\text{CNO} \rightarrow {}^{14}\text{N}$. During helium burning



(high mass AGB stars; high T, weak exposure)

2) OR for a few protons mixed in



(low mass AGB stars; low T, strong exposure)

Where do these neutrons go?

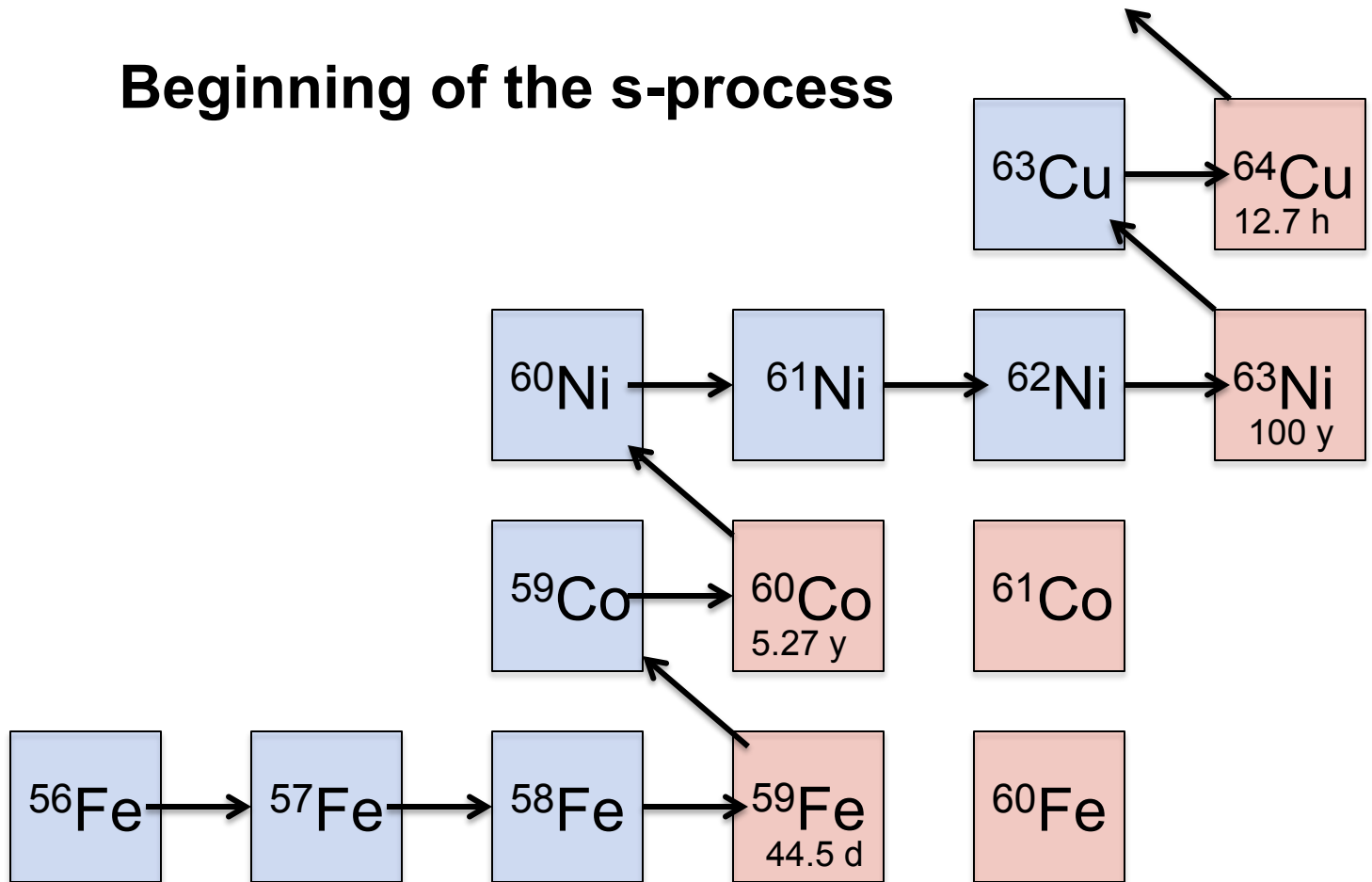
Helium, carbon, nitrogen, oxygen, and neon have very small cross sections for capturing neutrons.

In sequence 1), ^{25}Mg captures a substantial fraction of the neutrons produced, but not all.

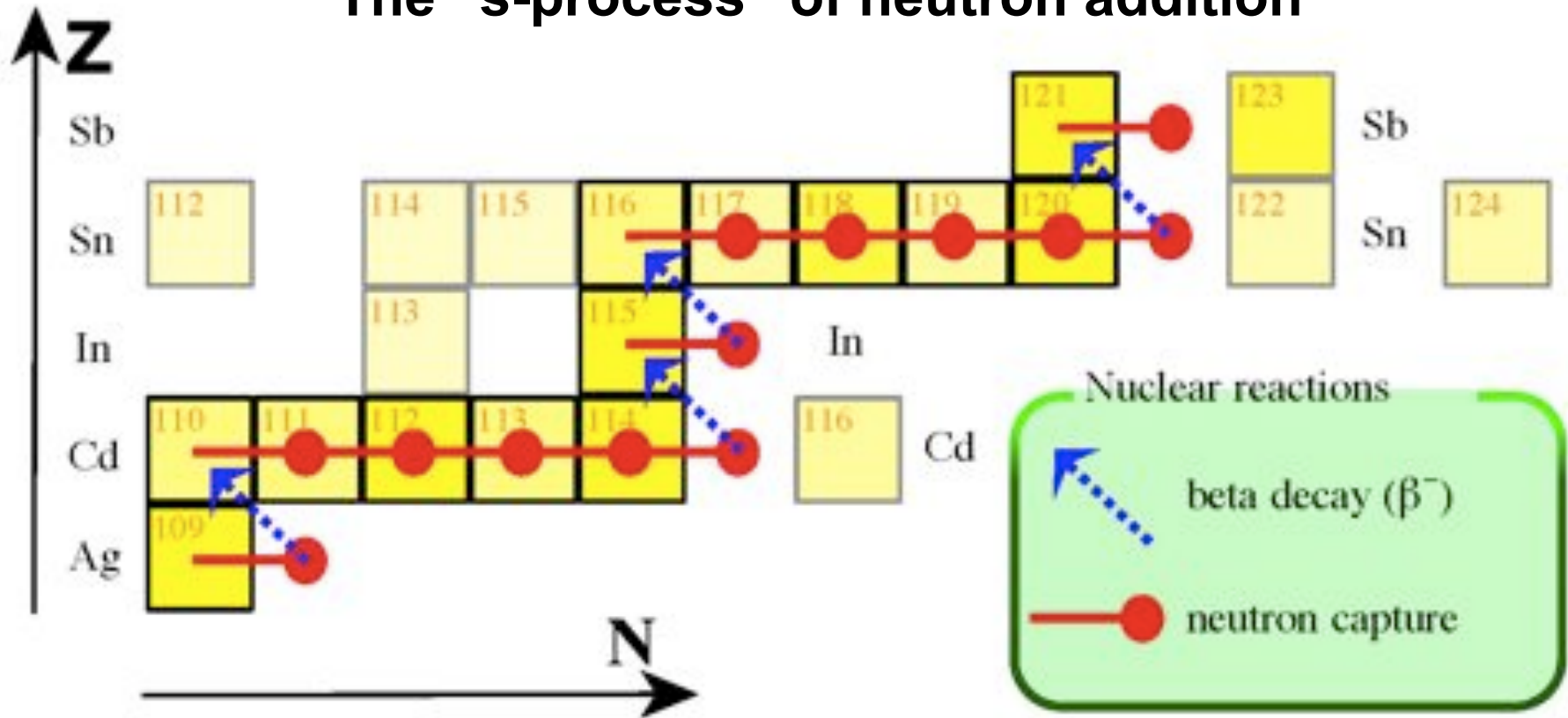
The element that is present with significant abundance and large neutron capture cross section is iron.

Beginning of the s-process

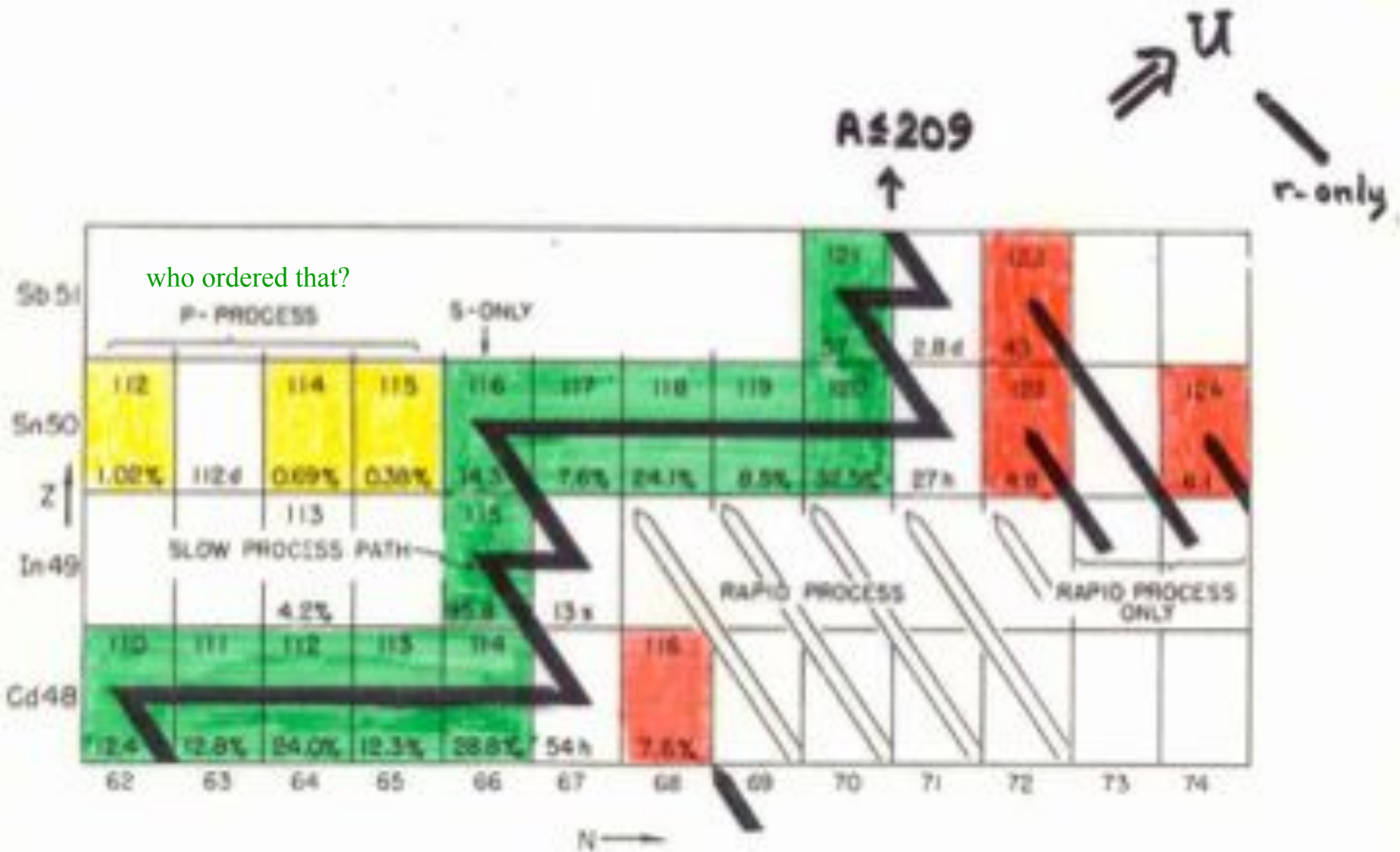
Z
↑



The “s-process” of neutron addition

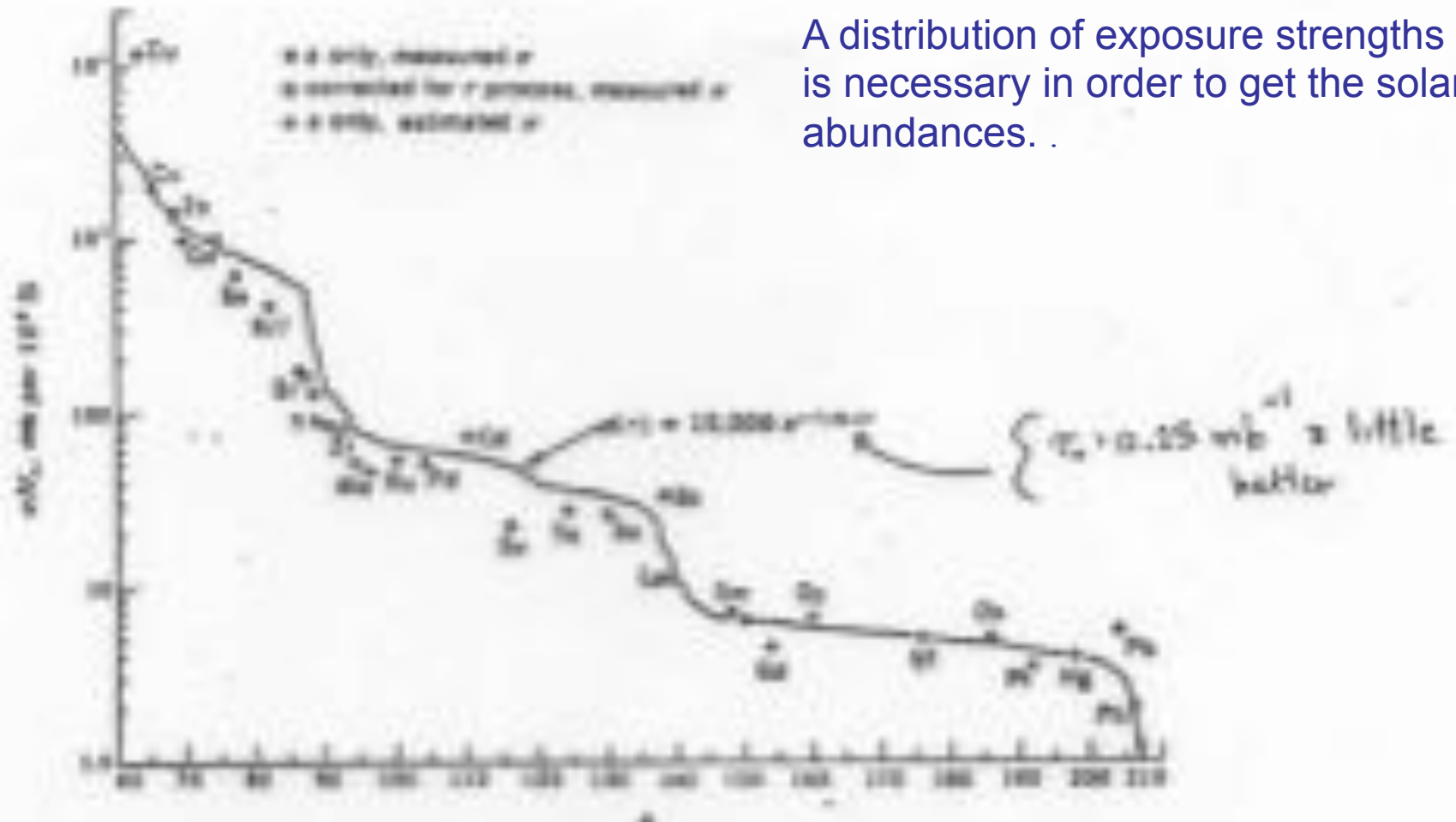


Each neutron capture takes you one step to the right in this diagram. Each decay of a neutron to a proton inside the nucleus moves you up a left diagonal. This goes all the way up to bismuth – element number 83 – where alpha decay finally terminates the s-process

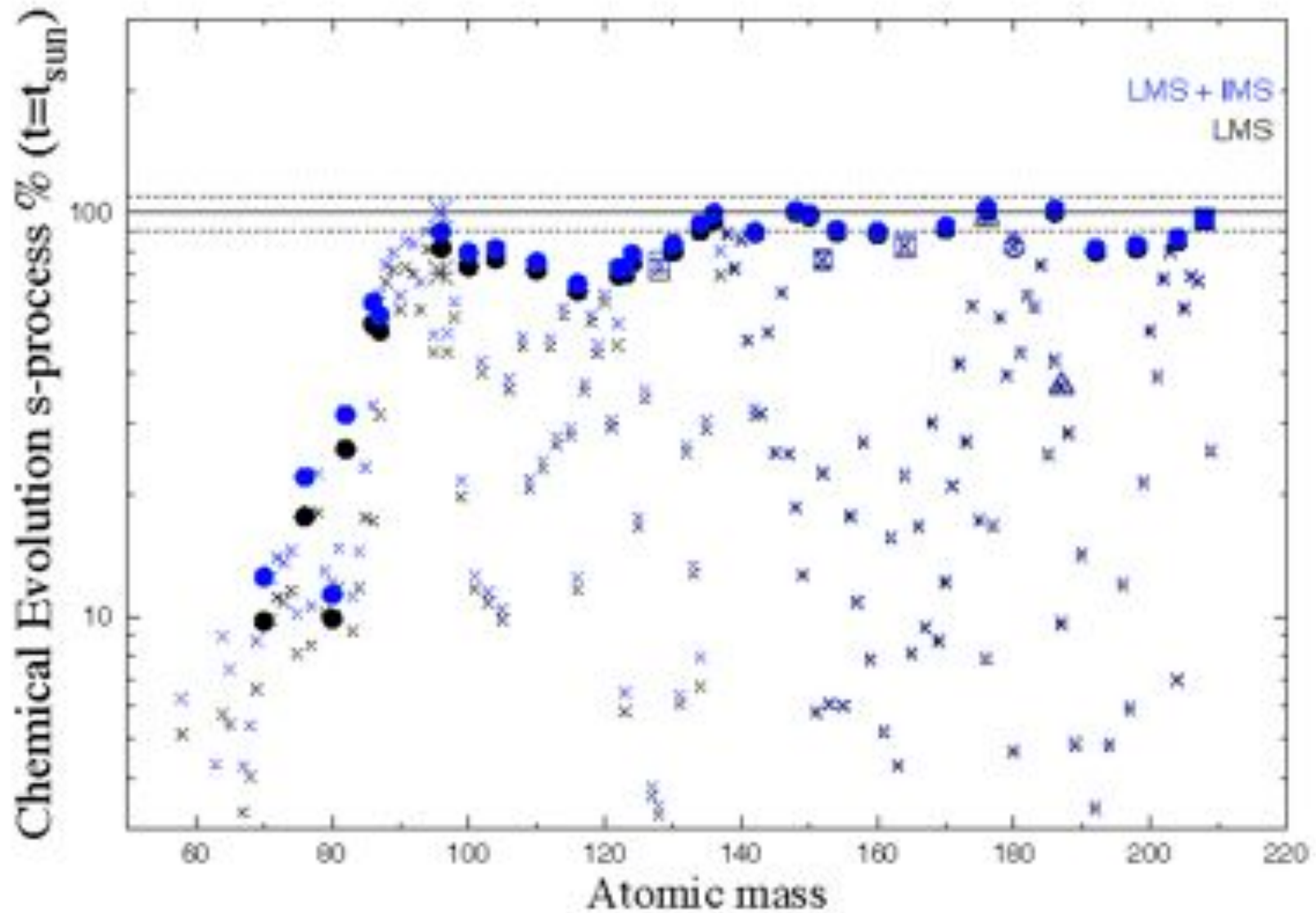


e.g., ^{117}Sn , ^{118}Sn , ^{119}Sn , and ^{120}Sn are s,r isotopes. Sn is not a good place to look for s n = const though because it is a closed shell.

A distribution of exposure strengths is necessary in order to get the solar abundances. .



Clayton

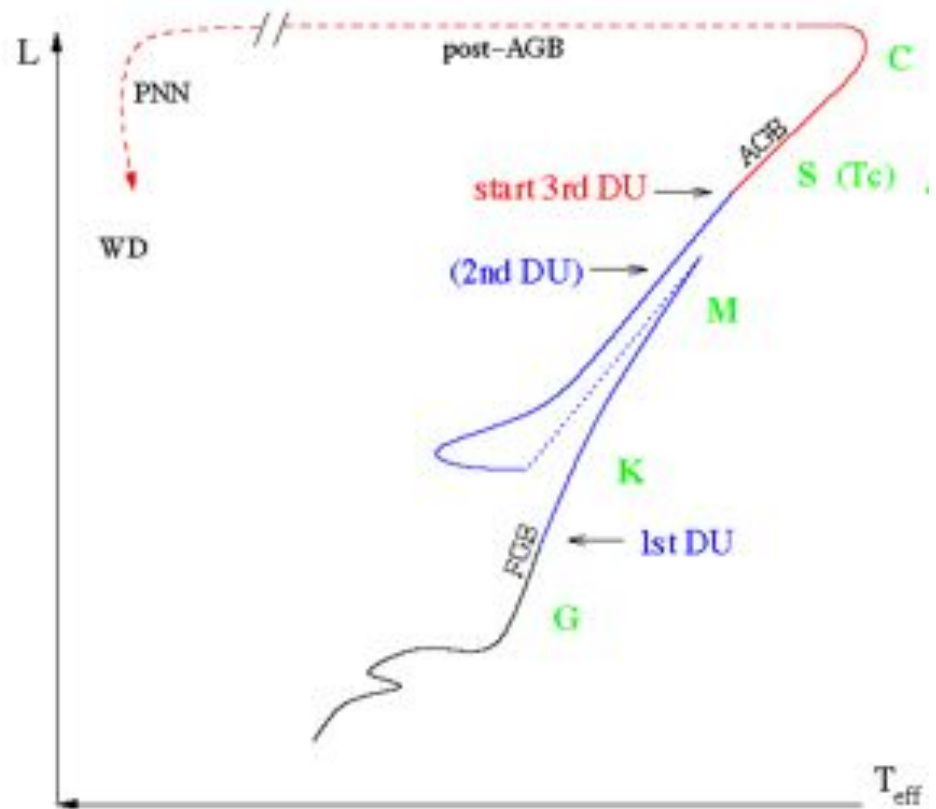


The blue circles are “s-only” isotopes. AGB stars, integrated over mass, can make the solar r-process above $A = 90$

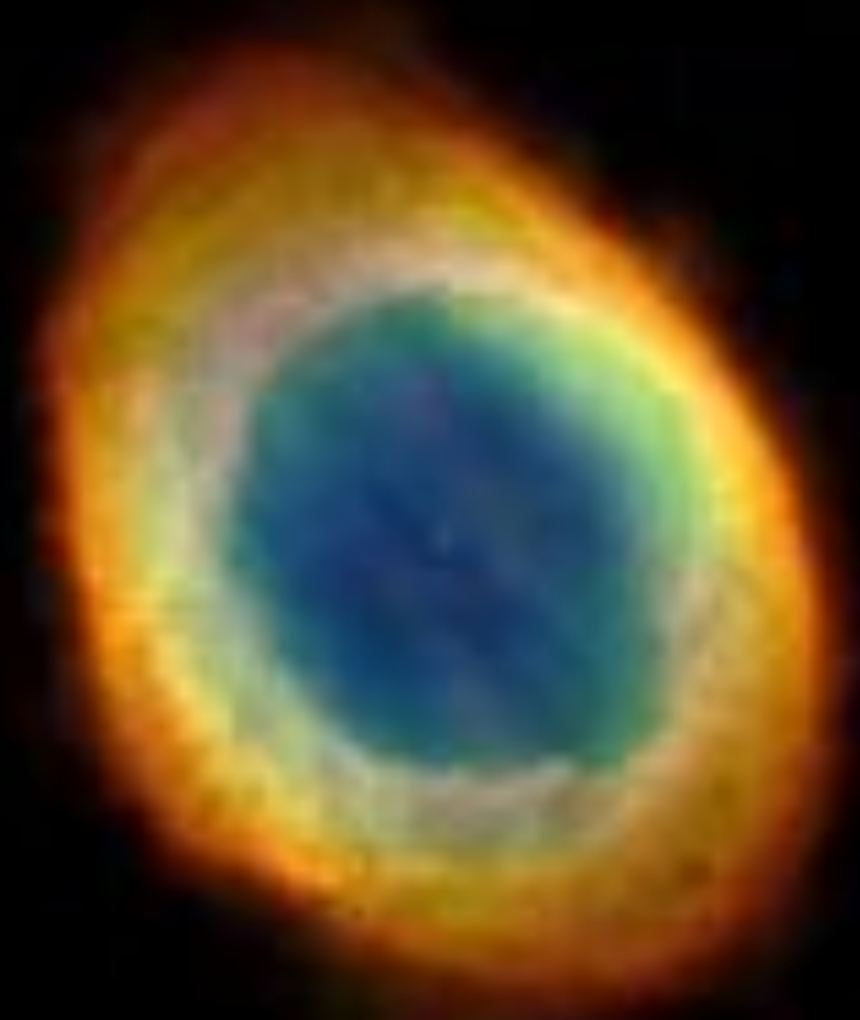
AGB stars are known to lose mass at a prodigious rate during their final stages, around 10^{-5} - 10^{-4} solar masses per year. This obviously cannot persist for much over 100,000 years.

The mass loss is driven in part by the pulsational instability of the thin helium shell. These pulses grow more violent with time. Also, and probably more importantly, the outer layers of the star get so large and cool owing to the high luminosity, that they form dust. The dust increases the opacity and material is blown away at speeds $\sim 10 - 30 \text{ km s}^{-1}$

The evolution is terminated as the outer layers of the star are blown away.



http://en.wikipedia.org/wiki/Ring_Nebula



The Ring Nebula in Lyra (M57)
700 pc; magnitude 8.8

PLANETARY NEBULAE

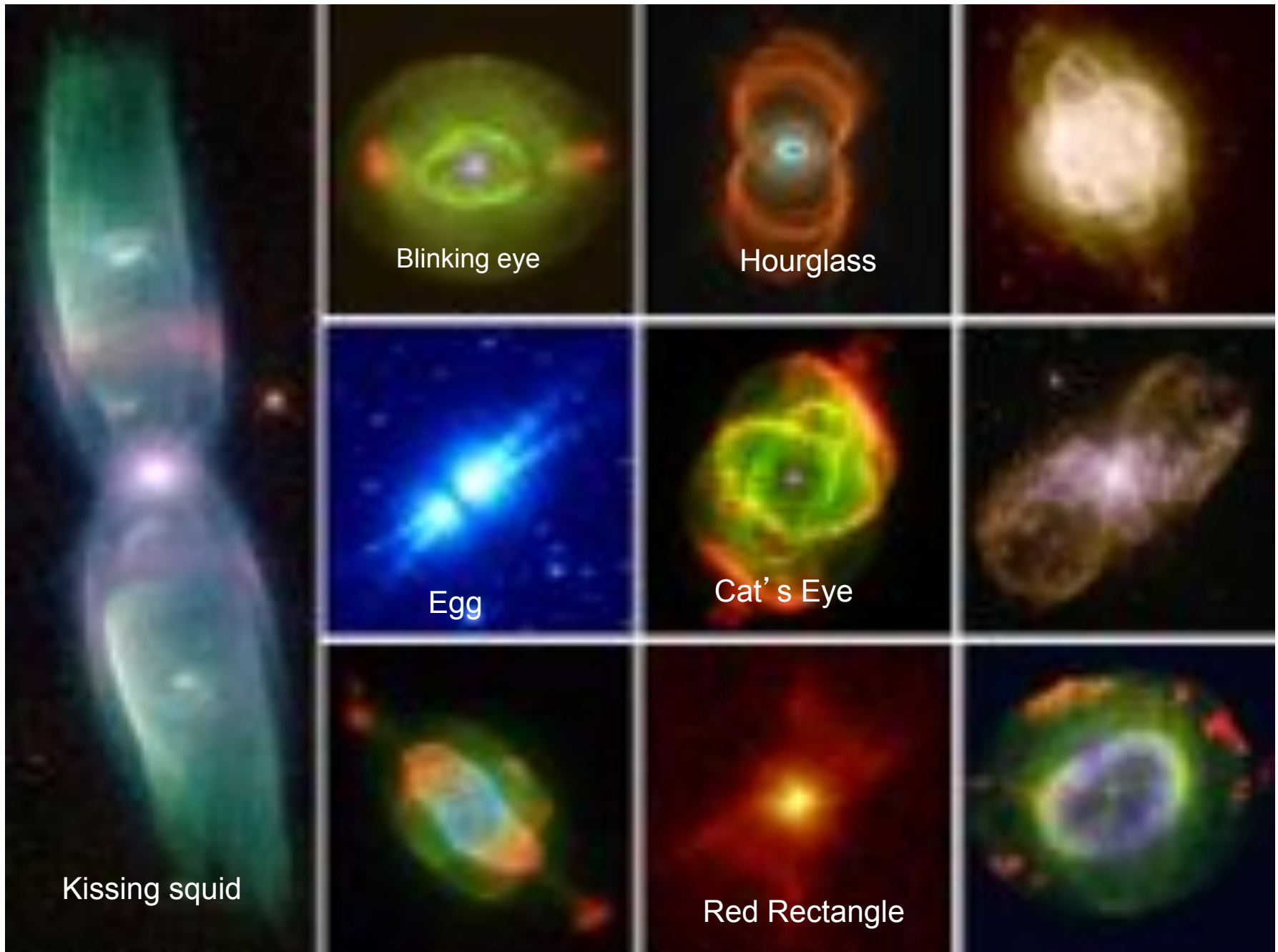
- Transition phase from a star to a white dwarf. Occurs for all stars under 8 solar masses
- About 3000 known in our galaxy though most stars go through this phase
- Short lived – about 10,000 years
- Densities $\sim 100 - 10,000$ particles per cm^3 ; roughly one light year across. Velocities $20 - 50 \text{ km s}^{-1}$
- Masses $0.1 - 1$ solar masses, a substantial fraction of the star's mass

PLANETARY NEBULAE

- UV radiation from central star heats to about 10,000 K
- Rich in carbon, nitrogen and oxygen. May be the source of most of the carbon and nitrogen in the universe
- Only about 20% are spherically symmetric. Rest are deformed by asymmetric outflow, binary companions, magnetic fields, etc.
- Emission lines e.g., of O III, O II, and N II
- Central stars 20,000 – 100,000 K



NGC 2440 – White dwarf ejecting envelope. One of the hottest white dwarfs known is in the center of the picture About 200,000 K and 250 times the sun's luminosity

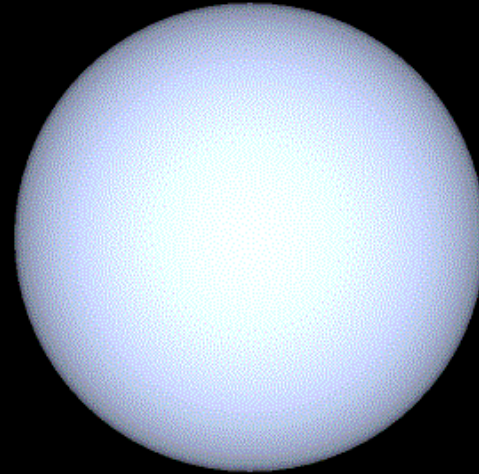


White Dwarfs

- Low mass stars are unable to reach high enough temperatures to ignite elements heavier than carbon in their core become white dwarfs.
- Hot exposed core of an evolved low mass star.
- Supported by **electron degeneracy pressure**. This is the tendency of atoms to resist compression.
- The more massive a white dwarf, the **smaller** it is. A solar mass white dwarf is about the size of the Earth.
- As white dwarfs radiate energy, they become cooler and less luminous gradually fading into oblivion, but it can take a long time....



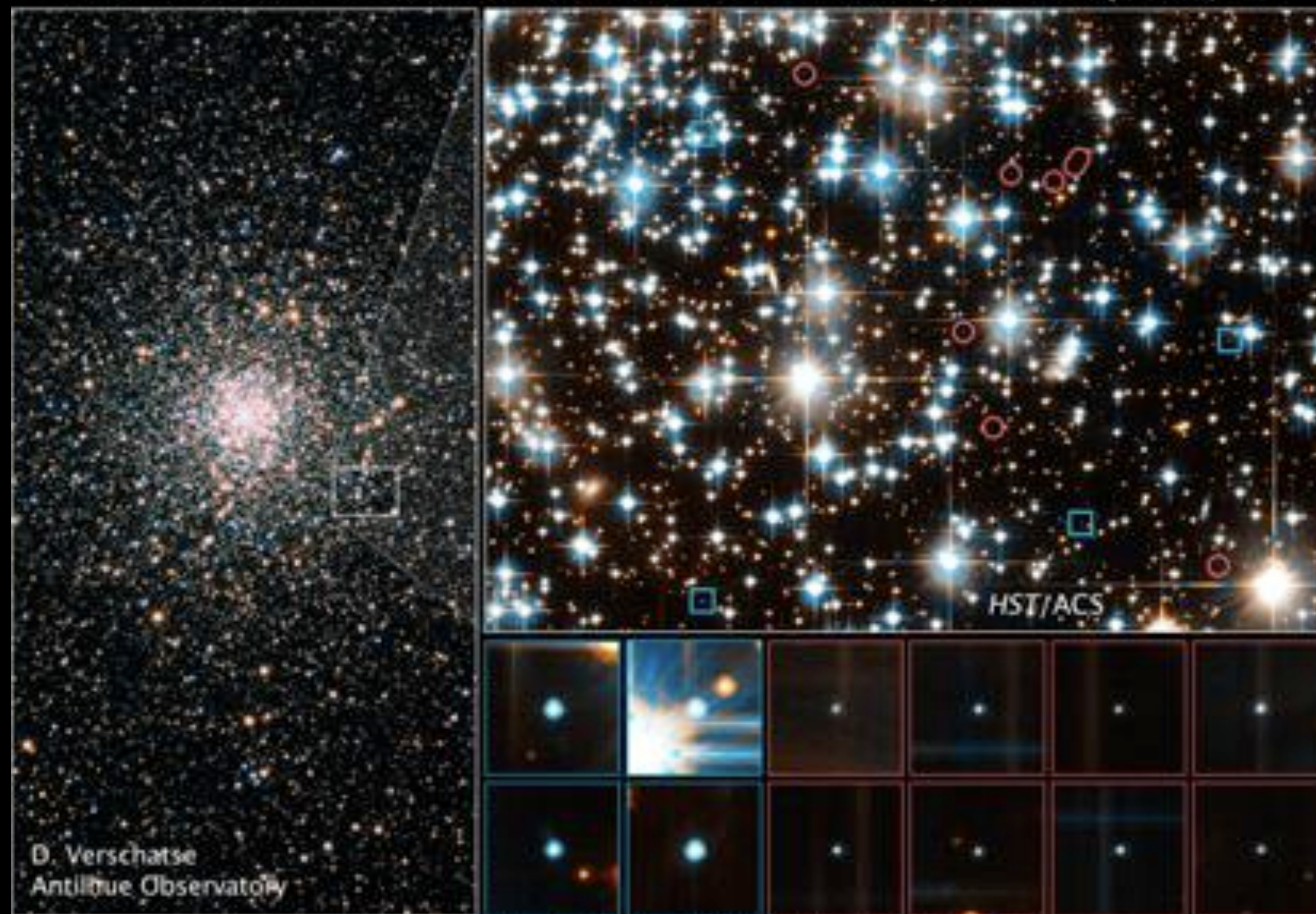
White Dwarf



Sirius B
 $M \approx 1.0 M_{\text{sun}}$
 $R \approx 5800 \text{ km}$

A white dwarf is the remnant of stellar evolution for stars between 0.08 and 8 solar masses (below 0.08 one can have brown dwarfs). They can be made out of helium, or more commonly carbon and oxygen (rarely NeOMg).

White Dwarf Stars in Globular Cluster NGC 6397 • Hubble Space Telescope ACS/WFC



MASS RADIUS RELATION FOR WHITE DWARFS

Previously we derived for non-relativistically degenerate white dwarfs

$$R = 8800 \text{ km} \left(\frac{Y_e}{0.5} \right)^{5/3} \left(\frac{M_\odot}{M} \right)^{1/3} = 0.0127 R_\odot \left(\frac{Y_e}{0.5} \right)^{5/3} \left(\frac{M_\odot}{M} \right)^{1/3}$$

This relation loses accuracy as the white dwarf mass approaches 1 solar mass and the electrons become partially relativistic. A more accurate relation that combines both relativistic and non-relativistic limits is given by

$$R = 0.0127 R_\odot \left(\frac{Y_e}{0.5} \right)^{5/3} \left(\frac{M_\odot}{M} \right)^{1/3} \left[1 - \left(\frac{M}{M_{Ch}} \right)^{4/3} \right]^{1/2} \quad M_{Ch} \approx 1.456 \left(\frac{Y_e}{0.5} \right)^2 M_\odot$$

Approximate Appearance in HR diagram

$$L \approx 0.01 L_{\odot} = 3.84 \times 10^{31} \text{ erg s}^{-1}$$

$$= 4\pi R^2 \sigma T_e^4 \quad R \approx 5000 \text{ km}$$

$$T_e = \left[\frac{L}{4\pi R^2 \sigma} \right]^{1/4} = \left[\frac{3.84 \times 10^{31}}{4\pi (5 \times 10^8)^2 (5.67 \times 10^{-5})} \right]^{1/4}$$
$$= 21,500 \text{ K}$$

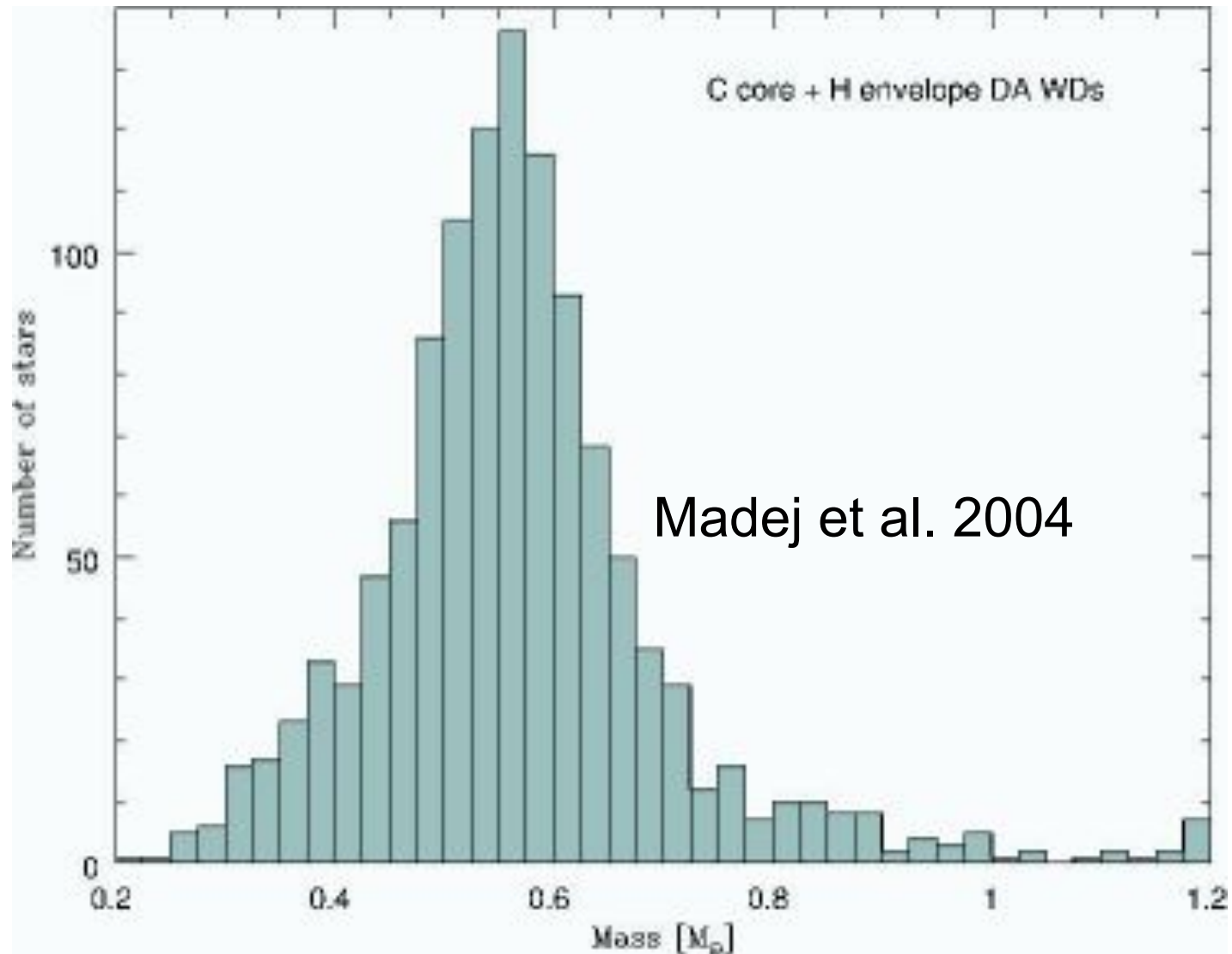
$$\lambda_{\text{max}} = \frac{2.89 \times 10^7 \text{ A}}{21,500} = 1340 \text{ A}$$

White dwarfs are
known with
temperatures
ranging from 4000 K
to 200,000 K

Mass distribution

Most WDs cluster around $0.6 M_{\odot}$.

Narrow mass distribution



Evolution of White Dwarfs

- White dwarfs shine solely because of the heat retained from their earlier evolution as stars. They have no source of nuclear energy
- The energy is stored in the heat capacity of the ions. This energy can power the low luminosity of a white dwarf for a long time. The ions remain an ideal gas though the electrons are degenerate and provide most of the pressure.
- The heat is transported by electron conduction to a thin layer near the surface where it then diffuses out through non-degenerate surface layers. Most of the mass, which is degenerate, is nearly isothermal.

Following GK 16-10

For the thin radiative layer, which will be taken to be an ideal gas, the mass is small and we can treat the total mass inside radius r as a constant, M

$$\frac{dP}{dr} = -\frac{GM\rho}{R}$$

The luminosity flowing through this layer is a constant

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa\rho}{T^3} \frac{L}{4\pi r^2}$$

Assume a Kramers opacity $\kappa = \kappa_0 \rho T^{-7/2} = \frac{\kappa_0 \mu P T^{-9/2}}{N_A k}$

$$\frac{dT}{dr} = -\frac{3}{16\pi ac} \frac{\kappa_0 \mu P}{T^{15/2} N_A k} \rho \frac{L}{r^2}$$

Following GK 16-10

Divide $\frac{dT}{dr} = -\frac{3}{16\pi ac} \frac{\kappa_0 \mu P}{T^{15/2} N_A k} \rho \frac{L}{r^2}$ by $\frac{dP}{dr} = -\frac{GM\rho}{R}$

to get $\frac{dT}{dP} = \frac{3\kappa_0 \mu}{16\pi ac N_A k G} \frac{P}{T^{15/2}} \frac{L}{M}$

$$P dP = \frac{16\pi ac N_A k G}{3\kappa_0 \mu} \frac{M}{L} T^{15/2} dT$$

Integrate from r to the surface at R. At the surface

$$P = T = 0$$

$$\int_0^P P dP = \frac{P^2}{2} = \frac{16\pi ac N_A k G}{3\kappa_0 \mu} \frac{M}{L} \int_0^T T^{15/2} dT$$

$$P = \left[\frac{64\pi ac N_A k G}{51\kappa_0 \mu} \frac{M}{L} \right]^{1/2} T^{17/4}$$

$$P = \left[\frac{64\pi a c N_A k G}{51\kappa_0 \mu} \frac{M}{L} \right]^{1/2} T^{17/4} \quad \rho = \frac{P \mu}{N_A k T}$$

$$\rho = \left[\frac{64\pi a c \mu G}{51\kappa_0 N_A k} \frac{M}{L} \right]^{1/2} T^{13/4}$$

This holds everywhere in the envelope and in particular is approximately true at the boundary with the degenerate

$$\text{core where } K(\rho Y_e)^{5/3} \approx \frac{N_A k \rho T}{\mu} \text{ or } T \approx \frac{\mu K Y_e^{5/3} \rho^{2/3}}{N_A k} \approx \frac{K Y_e^{2/3} \rho^{2/3}}{N_A k}$$

where we used $\mu^{-1} \approx Y_e$ for heavy ions like C and O. Then

$$T = \frac{K Y_e^{2/3}}{N_A k} \left[\left[\frac{64\pi a c \mu G}{51\kappa_0 N_A k} \frac{M}{L} \right]^{1/2} T^{13/4} \right]^{2/3}$$

$$L / M = \frac{64\pi a c \mu G Y_e^2}{51\kappa_0 N_A^4 k^4} T_{\text{core}}^{7/2} \approx 6.8 \times 10^{-3} \frac{L_{\odot}}{M_{\odot}} \left(\frac{T_{\text{core}}}{10^7 \text{ K}} \right)^{7/2}$$

$$L \sim 0.01 L_{\odot} \left(\frac{T_{\text{core}}}{10^7 \text{ K}} \right)^{7/2}$$

Given the luminosity and the heat capacity of the ions

$$U_{ion} = \frac{3}{2} \frac{N_A k}{\mu_{ion}} M T_{core}$$

one can estimate the cooling time

$$L = \frac{dU}{dt} = \frac{3}{2} \frac{N_A k}{\mu_{ion}} M \frac{dT_{core}}{dt} \text{ and since } T_{core} \propto L^{2/7} \quad \frac{dT_{core}}{dt} \propto \frac{2}{7} L^{-5/7}$$

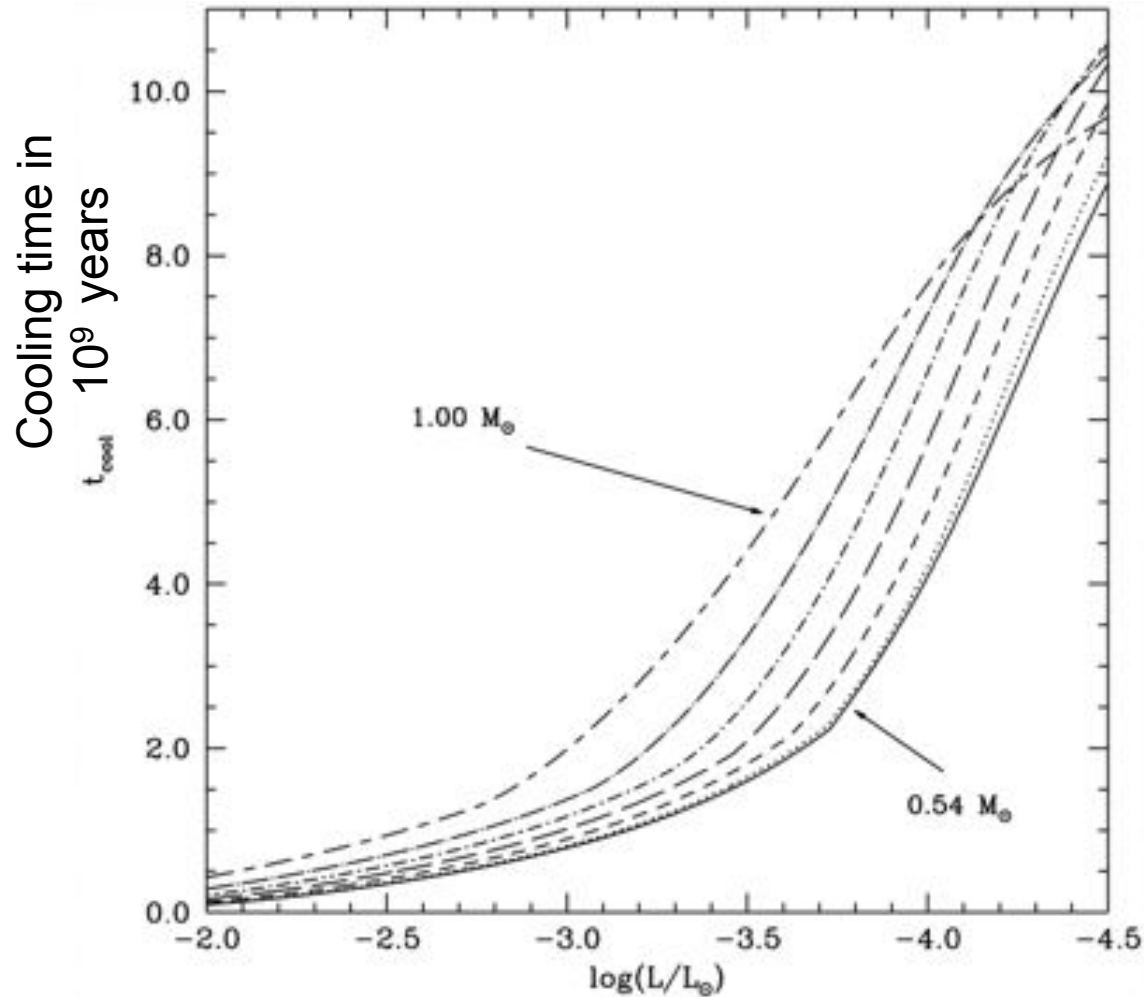
$$L \propto \frac{2}{7} L^{-5/7} \frac{dL}{dt} \text{ and the cooling time}$$

$$\tau_{cool} = \left(\frac{1}{L} \frac{dL}{dt} \right)^{-1} \propto L^{-5/7}$$

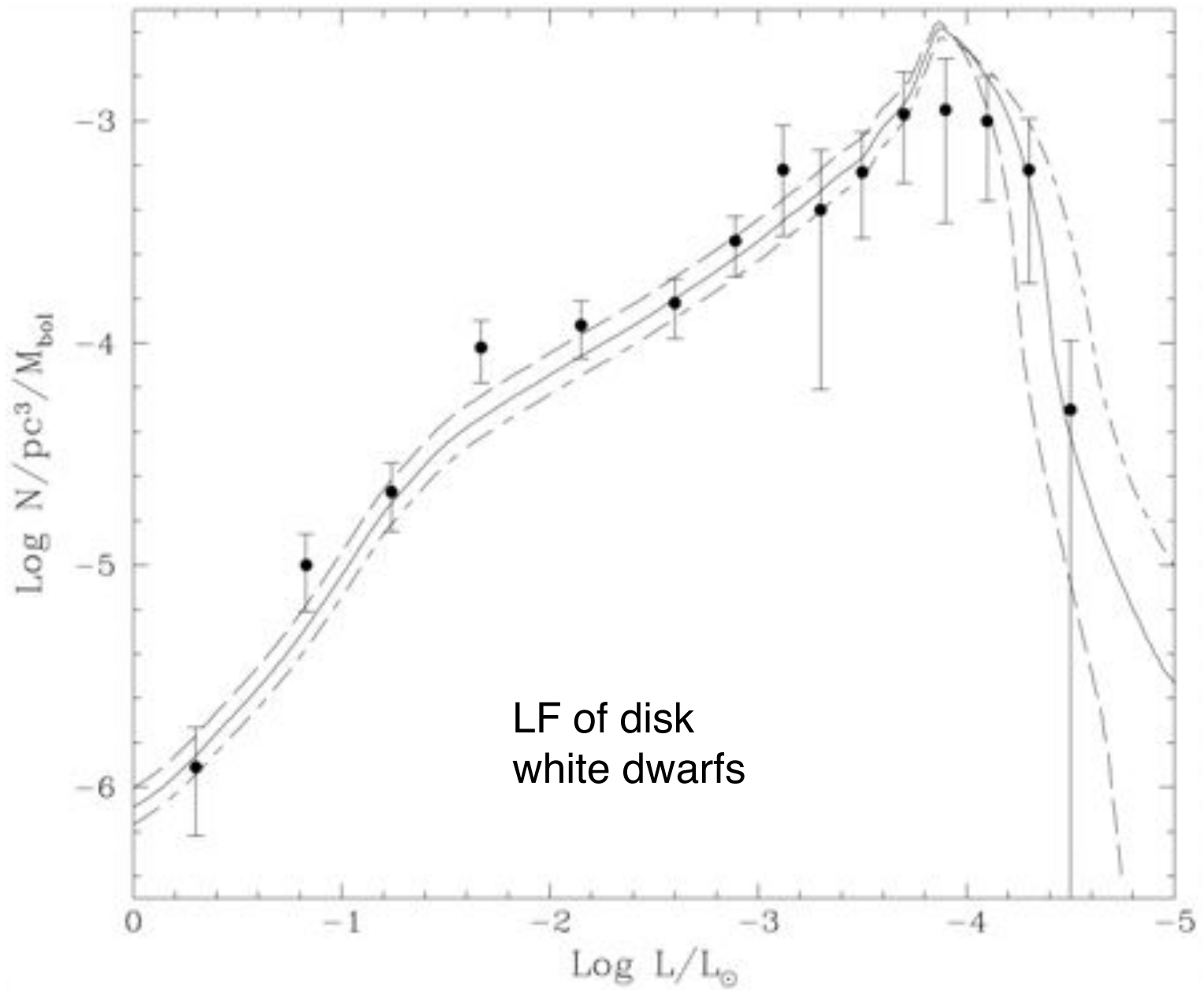
The fainter a white dwarf becomes, the longer it takes to cool down further. The ratio of the number of white dwarfs with luminosity L_1 to the number with luminosity L_2 should be $(L_2 / L_1)^{5/7}$ i.e. many more faint WDs than bright ones.

The coolest, faintest white dwarfs still have a surface temperature of ~ 4000 K. The universe is not old enough for “black dwarfs” to have formed yet.

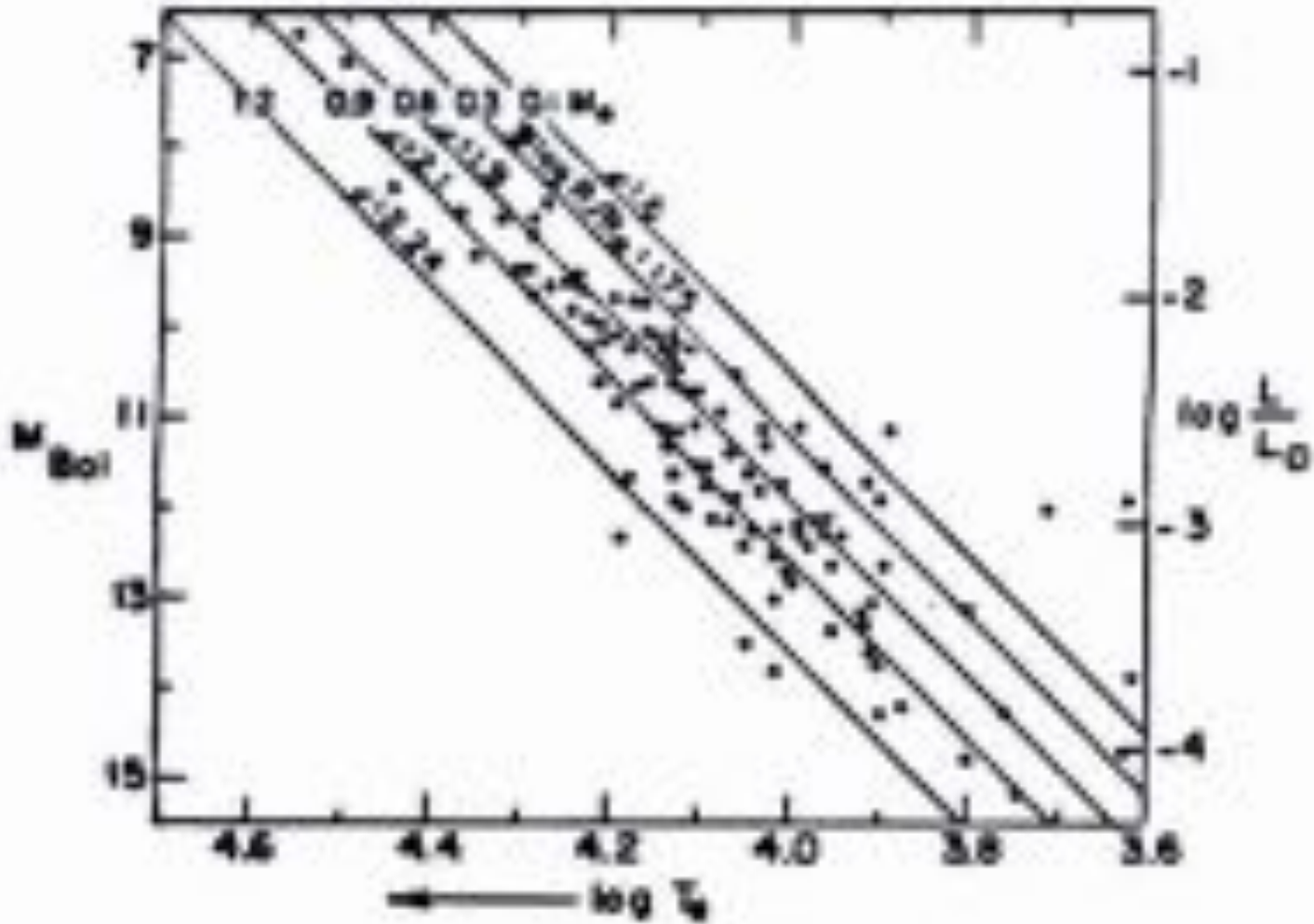
Salaris et al
(1997)



Luminosity function



EVOLUTION OF WHITE DWARF STARS



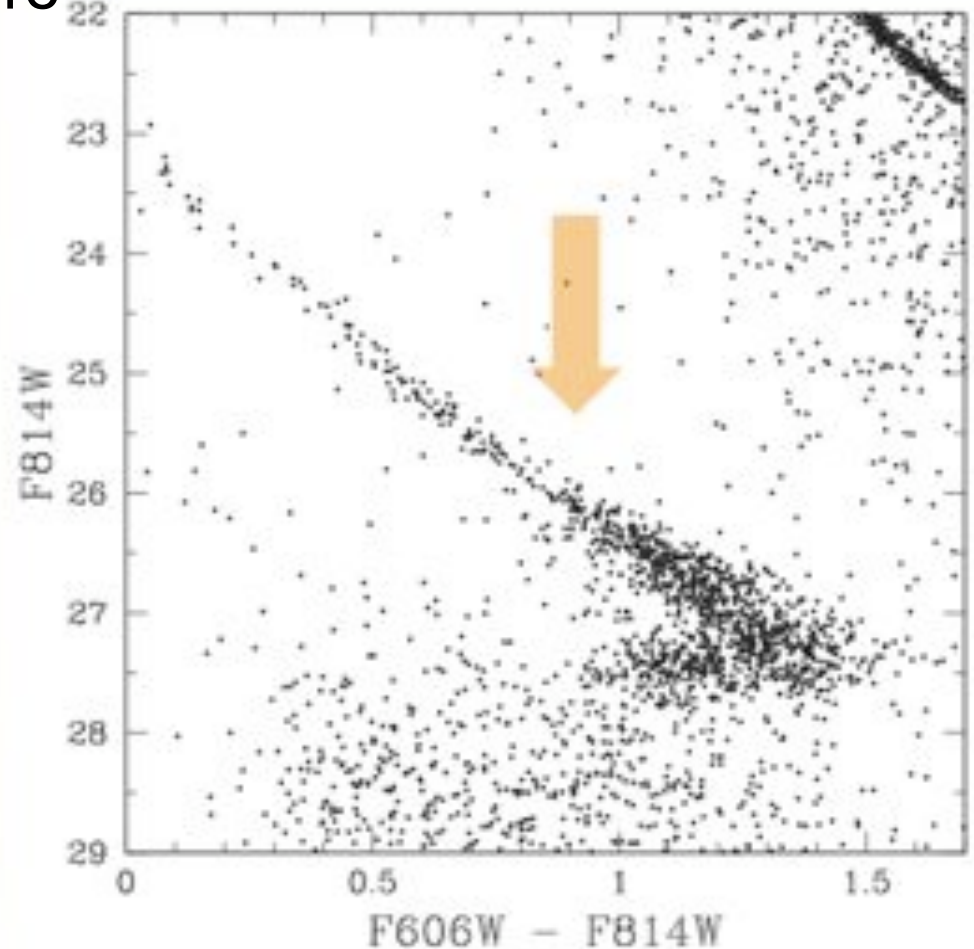
For a WD of constant mass, $R = \text{constant}$

Crystallization in white dwarfs

Hansen et al (2007)
NGC 6397 - globular cluster

When the interior temperature declines to ~ 5000 K, the carbon and oxygen start to crystallize into a lattice. This crystallization releases energy and provides a source of luminosity that slows the cooling.

The number counts pile up.



What happens to a star more massive than 1.4 solar masses?

1. There aren't any
2. They do not shrink to zero size right away
3. They explode - or
4. They become something else (a neutron star)