Lecture 1

Time Scales, Temperature-density Scalings, Critical Masses

I. Preliminaries

The life of any star is a continual struggle between gravity, seeking to reduce the star to a point, and pressure, which supports the star. Stars are gravitationally confined thermonuclear reactions.

So long as they remain non-degenerate, overheating leads to cooling and expansion, cooling to contraction and heating. Hence stars are generally stable.

But, since ideal gas pressure depends on temperature, stars must remain hot. By being hot, they are compelled to radiate.

In order to replenish the energy lost to radiation, stars must either contract or obtain energy from nuclear reactions. Since nuclear reactions change the composition, stars must evolve.

The Virial Theorm implies that if a star is neither too degenerate or too relativistic (radiation dominated)

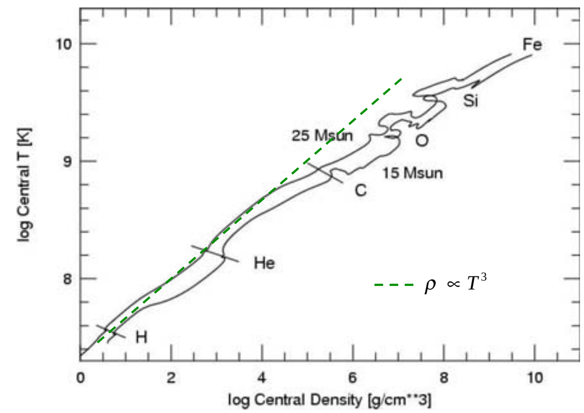
$$\frac{GM^2}{R} \sim MN_A kT \quad \text{(for an ideal gas)}$$

$$T \sim \frac{GM}{N_A kR}$$

$$\rho \sim \left(\frac{3M}{4\pi\rho}\right)^{1/3} \quad \text{for constant density}$$

So $$\boxed{T \sim \frac{GM^{2/3}}{N_A k}\rho^{1/3}}$$

That is as stars contract, they get hotter and since a given fuel (H, He, C etc) burns at about the same temperature, more massive stars will burn their fuels at lower density, i.e., higher entropy.



Four fundamental time scales for a star to adjust its structure can be noted. The shortest by far is the time required to approach and maintain hydrostatic equilibrium. Stars not in a state of dynamical implosion or explosion maintain a balance between pressure and gravity on a few sound crossing times. This is typically comparable to the free fall time scale.

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}$$

$$P \sim \frac{GM\rho}{R}$$

and

$$c_s = \gamma\left(\frac{P}{\rho}\right)^{1/2} \sim \left(\frac{GM}{R}\right)^{1/2}$$

$$v_{esc} \sim \left(\frac{GM}{R}\right)^{1/2}$$

So the escape speed and the sound speed are comparable

The free fall time scale is ~R/v_{esc} so

$$\tau_{esc} \sim \frac{R}{v_{esc}} = \left(\frac{R^3}{2GM}\right)^{1/2}$$

$$\sim \left(\frac{3}{8\pi G\rho}\right)^{1/2}$$

The number out front depends upon how the time scale is evaluated. The e-folding time for the density is 1/3 of this

$$\tau_{HD} = \left(\frac{1}{24\pi G\rho}\right)^{1/2} = \frac{446}{\sqrt{\rho}}\ \mathrm{sec}$$

The second relevant adjustment time is the thermal time scale given either by the radiative diffusion time or, where appropriate, the convective transport time. This is the time it takes for a star to come into and maintain thermal steady state, e.g., for the energy generated in the interior to balance that emitted in the form of radiation at the surface.

$$\tau_{therm} = \frac{R^2}{D_{therm}}$$

The diffusion coefficient is generally defined as

$$\mathrm{D} = \left(\frac{conductivity}{heat\ capacity}\right) = \frac{K}{C_p\rho}$$

where K appears in Fourier's equation

heat flow = $-K\ \nabla T$

For radiative diffusion the conductivity, K, is given by

$$K = \frac{4acT^3}{3\kappa\rho} \qquad \text{(see Clayton 3-12)}$$

where κ is the opacity (cm^2 g^{-1}), thus

$$\mathrm{D} = \left(\frac{4a\ cT^3}{3\kappa C_p\rho^2}\right)$$

where C_p is the heat capacity (erg g^{-1} K^{-1})

D here has units cm^2 s^{-1}

Note that

$$\mathrm{D} \approx \left(\frac{c}{\kappa\rho}\right)\left(\frac{aT^4}{\rho C_p T}\right)$$

Taking $D \sim c/\kappa\rho$ with κ the opacity, and taking advantage of the fact that in massive stars electron scattering dominates so that $\kappa = 0.2$ to $0.4\ \mathrm{cm^2\ g^{-1}}$, the thermal time scale then scales like

$$\tau_{\mathrm{Therm}} \approx \frac{R^2\kappa\rho}{c}$$

however, massive stars have convective cores so the thermal time is generally governed by the diffusion time in their outer layers. Since the dimensions are still (several) solar radii while the densities are less and the opacity about the same, thermal time scales are comparable to the sun ($\sim 10^5$ yr; Mitalas and Sills, ApJ, 401, 759 (1992)).

In general a diffusion coefficient is given by

$$D \sim \frac{1}{3} v\, l$$

where v is a typical speed (light or a convective element) and l is a characteristic length scale (mean free path or pressure scale height)

A third time scale of interest is the Kelvin Helmholtz time scale

$$\tau_{KH} \approx \frac{GM^2}{RL} \propto \frac{M^{5/3}\rho^{1/3}}{L}$$

Except for exceptionally massive stars, L on the main sequance is proportional to M to about the power 3, so the Kelvin Helmholtz time scale is faster for more massive stars. Note there are numerous Kelvin Helmholtz time scales for massive stars since they typically go through six stages of nuclear burning. During the stages after helium burning, L in the heavy element core is given by pair neutrino emission.

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Finally, there is the nuclear time,

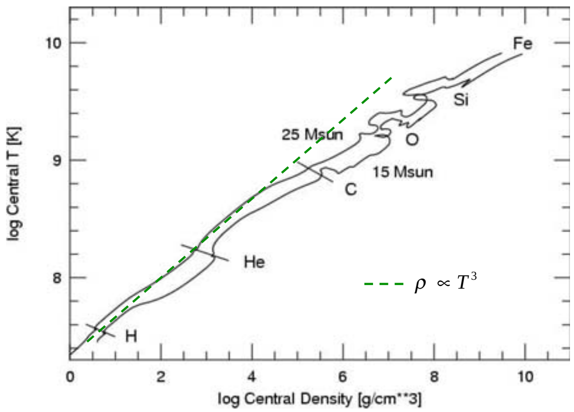
$$\tau_{\mathrm{nuc}} = \left(\frac{1}{X}\frac{dX}{dt}\right)^{-1},$$

where X is the mass fraction of the chief combustible fuel.

Usually, $\tau_{\mathrm{HD}} < \tau_{\mathrm{thermal}} < \tau_{\mathrm{KH}} < \tau_{\mathrm{nuc}}$. During the late stages of massive stellar evolution, however, the inequality $\tau_{\mathrm{thermal}} < \tau_{\mathrm{nuc}}$ actually begins to break down. During "explosive nucleosynthesis" in a supernova, there is near equality between τ_{nuc} and τ_{HD}.

The life of a (non-degenerate) star is then typically a series of nuclear burning stages separated by periods of Kelvin-Helmholtz contraction. Hydrostatic equilibrium is maintained throughout the interior and thermal steady state is maintained if τ_{therm} is short enough.

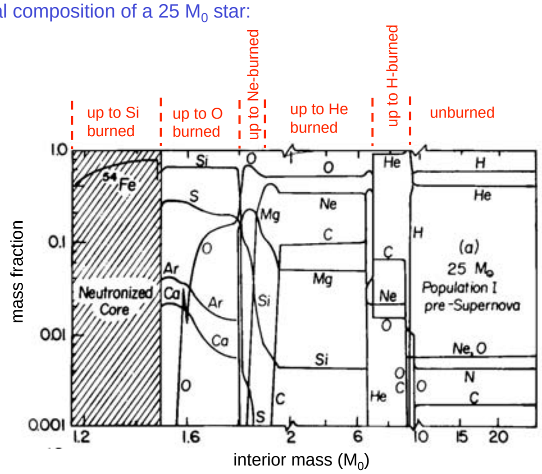
e.g., the sun is in thermal steady state. A presupernova star is not.



Advanced Nuclear Burning Stages
(e.g., 20 solar masses)

Fuel	Main Product	Secondary Products	Temp (10^9 K)	Time (yr)
H	He	^{14}N	0.02	10^7
He	C,O	^{18}O, ^{22}Ne s- process	0.2	10^6
C	Ne, Mg	Na	0.8	10^3
Ne	O, Mg	Al, P	1.5	3
O	Si, S	Cl, Ar K, Ca	2.0	0.8
Si	Fe	Ti, V, Cr Mn, Co, Ni	3.5	1 week

Final composition of a 25 M_0 star:



Examples of Time Scales

- In stellar explosions, the relevant time scale is the hydrodynamic one.
- Explosive nucleosynthesis happens when $\tau_{HD} \sim \tau_{nuc}$
- During the first stages of stellar evolution $\tau_{KH} < \tau_{nuc}$. The evolution occurs on a Kelvin Helmholtz time
- In a massive presupernova star in its outer layers, $\tau_{nuc} < \tau_{thermal}$. The outer layers are not in thermal equilibrium with the interior. It also happens in these same stars that τ_{nuc}(core) $< \tau_{KH}$ (envelope). The core evolves like a separate star.
- Rotation and accretion can add additional time scales.

Scaling Relations

So long as a star is in hydrostatic equilibrium, it satisfies

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}.$$

If the density is assumed to be constant,

$$\rho = \text{const} = \frac{3M}{4\pi R^3},$$

direct integration implies

$$P_c = \frac{1}{2}\frac{GM\rho_c}{R} \propto \frac{M^2}{R^4}$$

where here $\rho_c = \bar{\rho} = \rho$.

It follows that

$$\frac{P_c}{\rho_c} = \frac{1}{2}GM\left(\frac{4\pi\rho_c}{3M}\right)^{1/3}$$

and

$$\boxed{\frac{P_c^3}{\rho_c^4} = \frac{4\pi}{24}G^3M^2}$$ for constant density

More generally, for a polytrope of index n, $P \propto \rho^{\gamma}$; $\gamma = (n+1)/n$, see e.g., Clayton, Eq. 2-313

$$P_c = \frac{4\pi R^2 G}{(n+1)\zeta_1^2}\rho_c^2$$

$$\bar{\rho} = -\frac{3}{\zeta_1}\left(\frac{df}{d\zeta}\right)_{\zeta_1}\rho_c$$

$$= \frac{3M}{4\pi R^3}$$

where ζ_1 is the Emden constant given, e.g., in Table 2.5 of Clayton.

From this it follows that

$$\boxed{\frac{P_c^3}{\rho_c^4} = 4\pi G^3\left(\frac{M}{\phi}\right)^2}$$

where ϕ is a constant given by solution of the polytropic equation for index n,

$$\phi = (n+1)^{3/2}\zeta_1^2\left(\frac{df}{d\zeta}\right)_{\zeta_1}$$

$n = 0$ $\phi = 4.8988$ $= \sqrt{24}$

$n = 3$ $\phi = 16.145$

$n = 3/2$ $\phi = 10.73$

Now, if P is P_{ideal} (NR, ND, ionized),

$$P_{\text{ideal}} = \frac{N_A k}{\mu}\rho T$$

where μ is the mean molecular weight

Aside: Abundance nomenclature

In general the mass fraction of a species "i" is X_i. The number density of i is then

$$n_i = \rho N_A \frac{X_i}{A_i}$$

with A_i the atomic mass number (integer) of isotope i and N_A, Avogadro's number, 6.02205×10^{23} particles/mole, or approximately the reciprocal mass of the nucleon in grams.

In this class we will extensively use the notation

$$Y_i = \frac{X_i}{A_i}$$

where Y_i is like a dimensionless number density

$$Y_i = \frac{n_i}{\rho N_A}$$

Actually the dimensions of Y are Mole/gm and N_A has dimensions particles per Mole.

Similarly we can define an electron abundance variable

$$Y_e = \frac{n_e}{\rho N_A}$$

The total gas pressure for an ideal, non-relativistic, non-degenerate ionized gas is then

$$\boxed{P_{\text{ideal}} = \rho N_A kT\,[\Sigma Y_i + Y_e]}$$

$$P = \Sigma\, n_i\, kT = \frac{N_A k}{\mu}\rho T$$

which implies

$$\mu = [\Sigma Y_i + Y_e]^{-1}$$

Also the mean atomic weight, $\bar{A}$, is given by

$$\bar{A} = \frac{\Sigma\, n_i A_i}{\Sigma\, n_i} = \frac{\rho N_A \Sigma\, Y_i A_i}{\rho N_A \Sigma\, Y_i} = \frac{\Sigma\, X_i}{\Sigma\, Y_i} = (\Sigma\, Y_i)^{-1}$$

Similarly, it follows that

$$\boxed{Y_e = \Sigma\, Z_i Y_i}$$

and

$$\boxed{\mu = (\Sigma\,(1+Z_i)Y_i)^{-1}} \qquad 0.5 < \mu < 2$$

Some examples:

a) Pure hydrogen:

$$Y_H = 1 \qquad \bar{A} = 1 \qquad Y_e = 1$$

$$\mu = (1+1)^{-1} = \frac{1}{2}$$

$$P_{\rm ideal} = 2\rho N_A kT$$

(The limit μ=2 is achieved as A goes to infinity and Z = A/2)

b) 75% H, 25% He:

$$\bar{A} = \left(0.75 + \frac{0.25}{4}\right)^{-1} = 1.23$$

$$Y_e = 0.75 + (2)(\frac{0.25}{4}) = 0.875$$

$$\mu = \left[(1+1)(0.75) \ + \ (1+2)(\frac{0.25}{4})\right]^{-1} = 0.5926$$

$$P_{\rm ideal} = 1.69\,\rho N_A kT$$

As an exercise to the reader, for pure helium, $\bar{A} = 4$, $Y_e = 0.50$, $\mu = 4/3$, and $P_{\rm ideal} = 0.75\rho N_A kT$. For a mixture of 50% ^{12}C and 50% ^{16}O, $\bar{A} = 13.71$, $Y_e = 0.50$ (as it always does for a gas of isotopes having neutron number = proton number), μ

= 1.745, and $P_{\rm ideal} = 0.573\rho N_A kT$, (0.50 from e$^-$; 0.073 for ions).

For advanced stages of evolution where $\bar{A}$ > 1, most of the pressure is due to the electrons

Back to the main discussion:

$$\frac{P_c^3}{\rho_c^4} \propto M^2$$

thus implies for an ideal gas equation of state

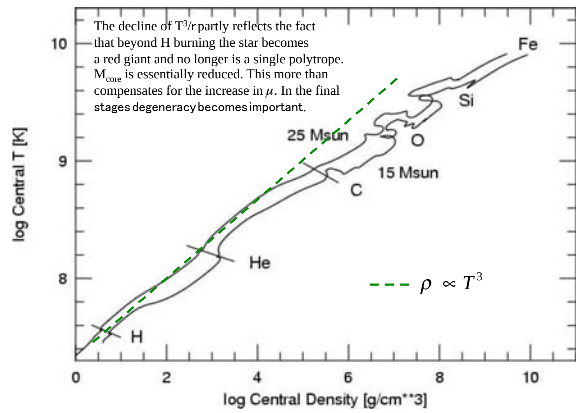
$$P_c \propto \rho_c \frac{T_c}{\mu}$$

$$\boxed{\frac{T_c^3}{\rho_c} \propto M^2 \mu^3}$$

This would suggest that the ratio would *increase* as the star evolved and m became greater.

Therefore, for a given temperature, as might be necessary to burn a given fuel, for example, the central density will be lower for a star of higher mass. And, in fact, for a given constant mass and composition, so long as the star closely resembles a single polytrope, and the pressure remains ideal, the central density will scale as

$$\boxed{\rho_c \propto \left(\frac{T_c}{\mu}\right)^3}$$



Since μ increases as the fuel burns to heavier ashes, the relation $\rho \propto T^3$ works pretty well at the stars center but tends to be an over-estimate. The onset of degeneracy or near relativistic motion of the electrons at high temperature can also cause deviations.

Now, especially for massive stars, the radiation pressure will not be negligible. One traditionally defines a quantity

$$\beta = \frac{P_{\rm ideal}}{P_{\rm ideal} + P_{\rm rad}}$$

$$P_{\rm tot} = P_{\rm ideal}/\beta$$

$$P_c \propto \frac{P_{ideal}}{\beta} \propto \rho_c \frac{T_c}{\mu\beta}$$

Then $P_c^3/\rho_c^4 \propto M^2$ implies

$$\boxed{\frac{T_c^3}{\rho_c} \propto M^2 \beta^3 \mu^3},$$

that is, so long as beta doesn't change much, one gets the same relation as before.

In the special case of an n=3 polytrope, some simple scaling relations are known. Of course technically, a star with a convective core like those over about about 2 $M_\odot$, is not

Decrease in b as star evolves acts to suppress T^3/r.

$$P_g = \frac{N_A k}{\mu} \rho T = \beta P \qquad P_{rad} = \frac{1}{3} a T^4 = (1-\beta) P$$

Equating P and solving for T gives

$$\frac{N_A k \rho T}{\mu\beta} = \frac{a T^4}{3(1-\beta)}$$

$$T = \left(\frac{N_A k}{\mu} \frac{3}{a} \frac{1-\beta}{\beta}\right)^{1/3} \rho^{1/3}$$

and putting this in the equation for P_{gas} gives

$$\mathrm{P} = \frac{N_A k \rho}{\mu\beta} \left(\frac{N_A k}{\mu} \frac{3}{a} \frac{1-\beta}{\beta}\right)^{1/3}$$

$$= \left[\left(\frac{N_A k}{\mu}\right)^4 \frac{3}{a} \frac{1-\beta}{\beta^4}\right]^{1/3} \rho^{4/3}$$

So a star with constant β is an n = 3 polytrope

an n=3 polytrope, still a star with an approximately constant β throughout is near n = 3 and most of the mass of even a quite massive stars is in radiative equilibrium, not convective. So the standard model can serve as a guide. Then for n = 3

$$\frac{M}{M_\odot} = 18.0 \frac{(1-\beta)^{1/2}}{\mu^2 \beta^2}$$

also known as "Eddington's quartic equation" (Clayton, eq. 2-307).

This shows that β will decrease monotonically with mass (see, e.g., Fig. 2-19, Clayton). Radiation pressure is more important for massive stars. But even for quite massive stars, β is still not very small. For 25 $M_\odot$ on the main sequence for example, $\mu = 0.84$ and $\beta = 0.75$. Using the boxed equation on the previous page, one also has for n = 3

Since for n = 3

$$\mathrm{T} = \left[\left(\frac{N_A K}{\mu}\right)\left(\frac{3}{a}\right)\left(\frac{1-\beta}{\beta}\right)\right]^{1/3} \rho^{1/3}$$

see previous page

(Aside) Polytropes

There exists stars that are close to polytropes:

1. Main sequences stars, M< 0.3 Msun fully convective, n = 3/2.

2. White dwarfs, M < 0.35 Msun, n = 3/2.

3. White dwarfs, M = 1.2 Msun n = 3; for masses 0.35 Msun < M < 1.2 Msun polytropes with 3/2 < n < 3.

4. MS, M > 1 Msun Sun, n = 3.

5. MS, M > 10 Msun inner convective cores, n=3/2.

(Clayton 2-318)

$$T_c = 17.4 \times 10^6 \ \mu\beta \left(\frac{M}{M_\odot}\right)^{2/3} \bar{\rho}^{1/3} \ \mathrm{K}$$
$$= 4.6 \times 10^6 \ \mu\beta \left(\frac{M}{M_\odot}\right)^{2/3} \rho_c^{1/3} \ \mathrm{K}$$

i.e., T goes as $\mu\beta \mathrm{M}^{2/3}\rho^{1/3}$ as discussed a few pages back

On the main sequence, with 50% H, 50% He and making the empirical approximation $\bar{\rho} \approx 1.4(M_\odot/M)$ g cm^{-3}, one has

$$T_c \approx 14 \times 10^6 K \left(\frac{M}{M_\odot}\right)^{1/3}$$

good only on the main sequence

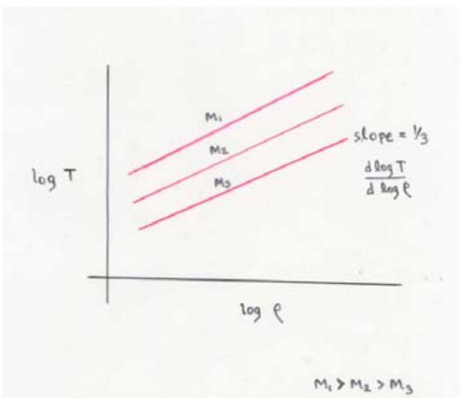
This tendency of the central temperature to increase with mass has many important implications. For massive stars, above about 2 $M_\odot$ (solar metallicity), the CNO cycle will dominate over the pp-chains. The opacity will become increasingly dominated by electron scattering. The central core will become convective (in part, responding to the high T-sensitivity of the CNO cycle). And as noted previously, radiation pressure will

become increasingly important.

1.2 Critical Masses

Dropping, for now, the explicit dependence on μ and β, a contracting protostar of constant mass, or the contracting core of a massive star in between burning stages, so long as that core has an approximately constant polytropic index, will obey $T_c \propto \rho_c^{1/3} M^{2/3}$. Contraction leads to heating. The greater weight of the more compact configuration requires more pressure to hold it up and the pressure rises by increasing both T and ρ. A plot of log T_c vs. log ρ_c gives a straight line with an upward slope of 1/3. Lines for larger mass will lie above those for lower mass. As the density grows ever higher, three possibilities emerge: a) collapse to a black hole; b) a dynamical event of some sort (e.g., neutron star formation) or c) the onset of degeneracy. For now, we are most interested in c).

Ideal gas



A completely degenerate gas can be characterized by an equation of state of the form $P_{\mathrm{deg}} = K_\gamma(\rho Y_e)^\gamma$ with γ between 4/3 and 5/3.

The case $\gamma = 4/3$ has a well known singularity. For an n = 3 polytrope, which is appropriate here,

The analog of this for a gas of radiation and ideal ions and electrons was Eddington's quartic eqn Relating M and β and μ.

$$\frac{P_c^3}{\rho_c^4} = 4\pi G^3 \left(\frac{M}{16.14}\right)^2 = \frac{K_{4/3}^3 \rho_c^4 Y_e^4}{\rho_c^4}$$

$\gamma = 4/3$

$$M = \left(\frac{20.745 K_{4/3}^3}{G^3}\right)^{1/2} Y_e^2$$

$$K_{4/3} = 1.244 \times 10^{15} \ \ \mathrm{dyne\ cm^{-2}}$$

$$\frac{M}{M_\odot} = 5.80 \ Y_e^2$$

$= 1.45 M_\odot$ if $\mathrm{Y}_e = 0.50$

The case $\gamma = 5/3$ (n = 3/2) has an analytic solution. For densities well below 10^7

g cm^{-3},

$$\frac{P_c^3}{\rho_c^4} = 4\pi G^3 \left(\frac{M}{10.73}\right)^2$$

$$\rho_c = \frac{4\pi G^3 M^2}{K_{5/3}^3 Y_e^5 (10.73)^2}$$

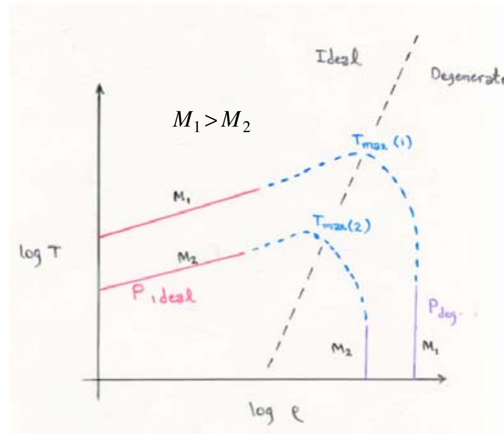
$$= 4.05 \times 10^6 \left(\frac{0.5}{Y_e}\right)^5 \left(\frac{M}{M_\odot}\right)^2 \text{ g cm}^{-3}$$

$\gamma = 5/3$, n = 1.5

This implies, for each mass, a stable permanent configuration of fixed ρ_c independent of T..

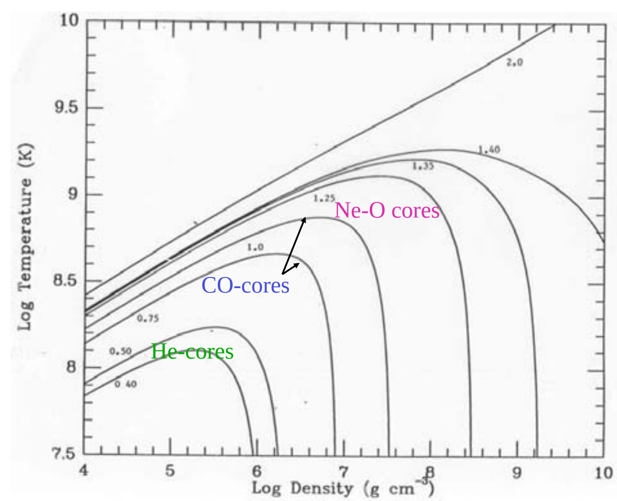
As the figure shows, putting this together with our previous discussion of the ideal gas case shows that contracting cores will first heat up, keeping $T \propto \rho^{1/3}$, and then, upon reaching a critical density and becoming degenerate, will actually cool down. At issue is the maximum temperature that is reached along the way. Will it surpass the ignition temperature for a given nuclear fuel?

Such arguments are well known to give a



critical mass for the ignition of hydrogen burning ($T_{ign} = 10^7$ K) of 0.08 $M_\odot$. But what about other fuels? To illustrate the solution, several stellar cores of constant composition were evolved using a full equation of state (arbitrary degeneracy, arbitrary relativity, Coulomb corrections, etc) using the Kepler code to be described later in the course. The cores consisted of 0.30, 0.40, 0.50 $M_\odot$ of He, 0.75 and 1.00 $M_\odot$ of carbon and oxygen, and 1.25, 1.35, 1.40, and 2.00 $M_\odot$ of 25% neon and 75% oxygen. Some of the results are shown on the following page.

The ignition temperature is not a precisely defined quantity and, especially for the more advanced burning stages, neutrino losses are important. But if the ignition temperature of helium burning is 1×10^8 K, the critical mass is about 0.30 $M_\odot$ of helium (note that the total stellar mass is larger). Adopting ignition temperatures for carbon, neon, oxygen, and silicon burning of 0.7, 1.3, 1.8, and



Advanced Nuclear Burning Stages
(e.g., 20 solar masses)

Fuel	Main Product	Secondary Products	Temp (10^9 K)	Time (yr)
H	He	^{14}N	0.02	10^7
He	C,O	^{18}O,^{22}Ne s- process	0.2	10^6
C	Ne, Mg	Na	0.8	10^3
Ne	O, Mg	Al, P	1.5	3
O	Si, S	Cl, Ar K, Ca	2.0	0.8
Si	Fe	Ti, V, Cr Mn, Co, Ni	3.5	1 week

3.0×10^9 K (these are known with some precision because of the large temperature exponent of the burning), one obtains critical masses of 1.0, 1.25, 1.39, and 1.39 $M_\odot$. Note the pile up of critical masses around the Chandrasekhar value (including Coulomb corrections). There can be no critical mass larger.

More detailed and physical calculations exist in the literature, see especially Nomoto and Hashimoto (1986). The following should be regarded as standard

Fuel	Min. Mass
He	0.25
C	1.06
Ne	1.37
O	1.39
Si	1.39

These correspond to masses on the main sequence of approximately (in $M_\odot$)

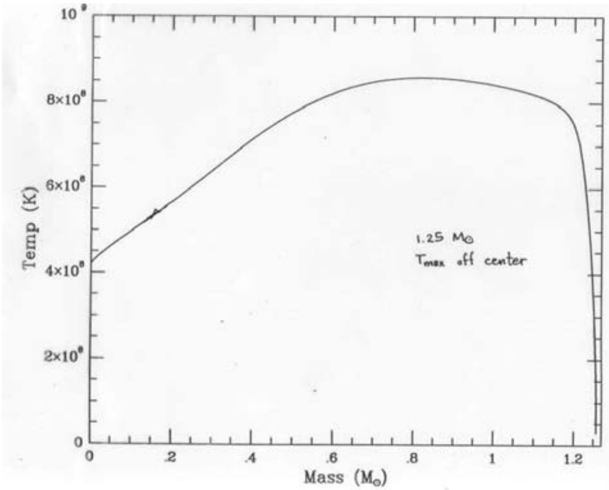
All stars above the Chandrasekhar mass could in principle go on to burn Si. In fact, that never happens. Stars develop a red giant structure with a low density surrounding a compact core. The convective envelope "dredges up" helium core material and causes it to shrink. Only for stars above about 7 or 8 solar masses does the He core stay greater than the Chandrasekhar mass after helium burning.

Critical Masses

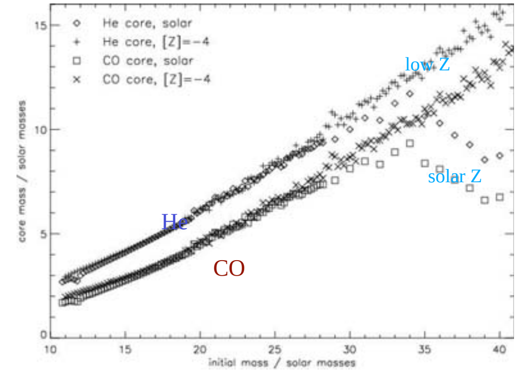
0.08 $M_\odot$	Lower limit for hydrogen ignition
0.45 $M_\odot$	helium ignition
7.25 $M_\odot$	carbon ignition
9.25 $M_\odot$	neon, oxygen, silicon ignition (off center)
~11 $M_\odot$	ignite all stages at the stellar center

These are for models that ignore rotation. With rotation the numbers may be shifted to lower values. Low metallicity may raise the numbers slightly since less initial He means a smaller helium core.

Between 8 and 11 solar masses the evolution can be quite complicated owing to the combined effects of degeneracy and neutrino losses. Off-center ignition is the norm for the post-carbon burning stages.



For non-rotating stars of a given metallicity and for given theories of convection and mass loss, there exists a well defined relation between main sequence mass and helium core mass.



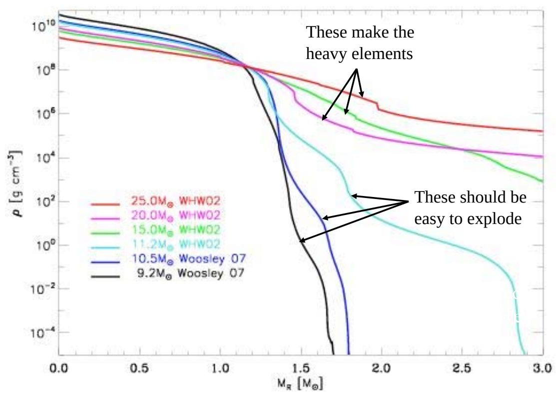
Later stages of evolution are more sensitive to this helium core mass than to the total mass of the star.

The death of a star and how it may potentially explode is also very sensitive to:

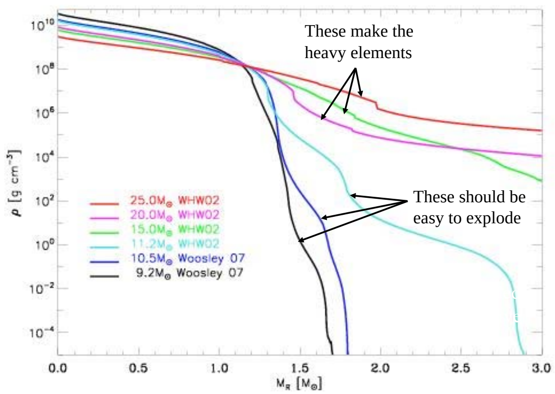
- *The density structure surrounding the iron core*
- *The rotation rate of the core and that material*

The density structure depends on the entropy of presupernova cores (TBD). Higher entropy cores occur for higher masses and are less degenerate and less centrally condensed.

Density Profiles of Supernova Progenitor Cores



Density Profiles of Supernova Progenitor Cores



$7 - 12\ M_\odot$ Stars

Poelarends, Herwig, Langer and Heger (ApJ, 675, 614, (2008))

Ignite carbon burning	$7.25\ M_\odot$	Super AGB stars
Heaviest to lose envelope by winds and thermal pulses	$9.0\ M_\odot$	
Ignite Ne and O burning	$9.25\ M_\odot$	

Range of e-capture NeO SNe $9.0 - 9.25\ M_\odot$

Expected number 4%; Maximum number 20%

Larger percentage at lower metallicity

$12\ M_\odot$ Model has binding 1×10^{50} erg external to $1.7\ M_\odot$ baryon; 1×10^{49} erg external to $2.6\ M_\odot$