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Mass-dependent mass loss rates of Wolf-Rayet stars

N. Langer

Universitäts-Sternwarte Göttingen, Geismarlandstrasse 11, D-3400 Göttingen, Federal Republic of Germany

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Summary. According to the theory of the structure of Wolf-Rayet (WR) stars it is suggested that the mass loss rate of WNE, WC, and WO type stars may be expressed as a function of the mass of the WR star, e.g. $\dot{M}_{WR} \sim M^{\alpha}_{WR}$. We constructed a simple computational scheme in order to simulate the evolution of WR stars. Thereby, we computed several hundred WR evolution sequences and elaborated the consequences of mass dependent mass loss rates. Furthermore, we computed IMF-averages for many observable quantities, which allows a straightforward comparison of our results with observations of WR stars. We find that several contradictions, which arise from standard WR evolution theory (i.e. assuming $\dot{M}_{WR}$=const.), are avoided by mass dependent WR mass loss rates. Main effects of mass dependent mass loss rates are: 1. The mean mass of WNE and WC/WO stars is very small, despite initially large WR mass or ZAMS mass. 2. The mean luminosity of WNE/WC/WO stars is low ($5.5 \gtrsim \log(L_{WR}/L_{\odot}) \gtrsim 4.8$) 3. The predicted scatter of WNE and WC/WO masses is rather small, the main part of them having masses in the range $20\,M_{\odot} \gtrsim M_{WR} \gtrsim 5\,M_{\odot}$, regardless of the initial mass. 4. The predicted scatter of observed WR mass loss rates is rather small. 5. The mean WR lifetime is increased. 6. The ratio of the number of WC over WNE stars is increased. 7. The ZAMS mass of WNE progenitors is much smaller than that of WC progenitors. 8. The mean final mass of WR stars is small ($5-10\,M_{\odot}$), yielding agreement with recent studies of type Ib supernovae. According to a comparison of our results with observations we propose a mass loss rate for WNE/WC/WO stars as $\dot{M}_{WR} = (0.6-1.0)\,10^{-7}\,(M_{WR}/M_{\odot})^{2.5}\,[M_{\odot}\,\mathrm{yr}^{-1}]$.

Key words: Wolf-Rayet stars – massive stars – mass loss – stellar evolution – supernovae

1. Introduction

Wolf Rayet (WR) stars are generally regarded as central helium burning massive stars, which have lost the main part or all of their hydrogen rich envelope, consequently showing products of hydrogen burning (WN stars) or helium burning (WC/WO stars) at their surface (cf. Chiosi and Maeder, 1986). Especially, the recent measurements of very high carbon abundances in WC stars by several groups (Torres, 1988; Smith and Hummer, 1988; de Freitas Pacheco and Machado, 1988) confirm this scenario. The WN stars are separated into early (WNE) and late (WNL) types, where the latter ones still show a considerable concentration of hydrogen, while in WNE stars hydrogen seems to be very rare or absent (cf. Willis, 1982). In the following we will therefore use the expression "WNE star" for WN stars with vanishing surface hydrogen abundance. Also the WC stars are divided into early and late types according to spectroscopic criteria. However, in this case no relation to surface abundance differences of both types is evident (cf. e.g. Torres, 1988), so we have no possibility to distinguish the early and late WC phase in our theoretical models (cf. however, Langer and Kiriakidis, 1988). The same applies to the WO stars, which may be characterized as early WC stars with very strong O VI lines in their spectra (cf. Barlow and Hummer, 1982), and which are classified within the WC spectral sequence in the catalogue of van der Hucht et al. (1981). Therefore, we regard the WO stars as a member of the WC sequence in the following.

The dominance of emission lines in spectra of WR stars is provoked by their intense stellar winds, which cause very high mass loss rates of the order of $3\,10^{-5}\,M_{\odot}\,\mathrm{yr}^{-1}$. Due to these high mass loss rates together with a WR lifetime of the order of several 10^5 yr it is clear that the mass loss considerably affects the evolution of WR stars, i.e. that they loose a large fraction of their initial mass during the WR phase. For this reason it is very probable, that the mass loss rate itself also varies during the evolution of WR stars. A proper knowledge of the dependence of the mass loss rate on the stellar parameters is of great importance in order to correctly predict e.g. WR lifetimes, final WR masses, or ratios of WN over WC lifetimes from stellar evolution calculations. It seems not yet possible to compute mass loss rates of WR stars from first principles for given WR stellar models. Even the physical mechanism which causes the mass loss is still unclear, radiation pressure in absorption lines (Abbott, 1982) or stellar vibrational instability according to the ε-mechanism (Maeder, 1985) being possible candidates.

Observationally determined WR mass loss rates, on the other hand, are rather uncertain. Though there has been a big progress on the accuracy of the mass loss rates due to application of new techniques in recent years, determinations of other fundamental quantities of WR stars, e.g. masses, luminosities, radii, or surface temperatures are still very uncertain. Therefore, also a conclusive deduction of the dependence of the WR mass loss rate on stellar parameters from observations seems hardly possible (cf. Sect. 2).

In this situation, many (if not all) groups which performed stellar evolution calculations for massive stars leading through the WR phase used a constant rate of mass loss for WR stars (cf. Maeder, 1983; Prantzos et al., 1986; Langer and El Eid, 1986; Maeder and Meynet, 1987). This simplification seemed to be justified since the mass loss rates of the different WR subtypes were found to be all of the same order of magnitude of $\sim 3\, 10^{-5}\, M_{\odot}\, \mathrm{yr}^{-1}$ (cf. Barlow et al., 1981).

In Sect. 2 we intend to demonstrate, that according to theoretical arguments the mass loss rates of WR stars are not at all constant with time, rather that a strong dependence on the mass of WR stars may exist, which is shown to be not inconsistent with observations. Adopting mass loss rates of the form $\dot{M}_{\mathrm{WR}} \sim M_{\mathrm{WR}}^{\alpha}$ in simplified WR evolutionary calculations we elaborate their observable consequences in Sect. 3. Those are confronted with observations in Sect. 4, where we show that several contradictions of the standard WR evolution with observations can be removed. Conclusions are given in Sect. 5.

2. Arguments in favour of mass-dependent WR mass loss rates

First, we want to point out, that the following considerations apply only for WR stars of spectral types WNE, WC, and WO (according to our definitions, cf. Sect. 1), but not for WNL type stars. Actually, WNL stars should be considered as a completely different kind of object compared to stars of other WR types for the following reasons:

1. In contrast to all other WR stars, WNL stars contain a considerable amount of hydrogen in their surface layers. Therefore, the microphysics (e.g. opacity, mean molecular weight) in WNL envelopes is quite different compared to other types of WR stars.

2. The internal structure of WNL stars is much more complicated than that of other WR stars: in contrast to those they contain a hydrogen burning shell, which may e.g. considerably contribute to the stellar luminosity.

3. The intrinsic radii (i.e. the radii of the hydrostatic bulks of WR stars) of WNL stars are ~ 10 times larger than those of other WR types (cf. e.g. Maeder and Meynet, 1987).

4. Observations indicate, that WNL stars on average are much more luminous (cf. e.g. Smith and Willis, 1983) and much more massive (Niemela, 1983) than other types of WR stars.

For these reasons we do not expect a common mass loss law for WNL stars and other WR types. In particular, the fact that the order of magnitude of the mean mass loss rates for WNL stars and WNE/WC/WO stars is similar ($\sim 3\, 10^{-5}\, M_{\odot}\, \mathrm{yr}^{-1}$) may be purely accidental.

2.1. Theoretical aspects

A very simple argument in favour of a mass dependent mass loss rate, which, however, should not be overlooked, is the necessary condition, that the mass loss rate has to tend to zero when the WR mass tends to zero. From WR stars in binary systems we know, that several of them have masses below or of the order of $10\, M_{\odot}$. The helium burning lifetime of these stars, which may be a good approximation for the duration of the WR phase, is of the order of $6\, 10^{5}$ yr. Therefore, a constant mass loss rate of $3\, 10^{-5}\, M_{\odot}\, \mathrm{yr}^{-1}$ cannot be adopted for these objects, since it would imply a total mass loss of $20\, M_{\odot}$ for the whole WR phase, which is impossible for a $10\, M_{\odot}$ WR star.

The basic argument for the mass being the main parameter in determining the mass loss rate of WNE/WC/WO stars comes from Langer (1989): he could show that within a good approximation the whole structure of a WR star of the considered spectral types is fixed when the mass is given, similar to the case of main sequence stars. The surface chemical composition may also have some influence (Langer et al., 1988; Langer, 1989), which is, however, of minor importance. For radiation pressure driven winds Kudritzki et al. (1986) estimate the metal dependence of the radiation pressure driven mass loss to be relatively weak ($\dot{M} \sim Z^{0.5}$) for small metallicities. In the case of large metallicities as e.g. for WC/WO stars one may expect an even weaker dependence because of saturation effects in the relevant absorption lines. In Sect. 3 we will also investigate the influence of an increased mass loss rate in the WC/WO phase.

We note that in this situation where all properties of WR stars may be expressed as a function of only one parameter, one may of course select another variable as the WR mass as independent parameter. E.g. the mass loss rate may well be expressed as a function of the luminosity of WR stars. Then, such relation could be easily converted into a mass dependent mass loss rate via the mass-luminosity relation for WR stars.

Accepting that the mass loss rate of WNE/WC/WO stars may be expressed as a function of the WR mass, we now consider the question how large the coefficient α in the mass loss law of the form $\dot{M}_{\mathrm{WR}} = k \cdot M_{\mathrm{WR}}^{\alpha}$ may be. On the basis of a comprehensive set of WR stellar models, Langer (1989) has compiled the respective values of α for the dependence of quantities which might be relevant to the mass loss rate, as a function of the WR mass, e.g. LR/GM ($\alpha \in [1, 2.5]$), or L/v_{esc}^{2} ($\alpha \in [1, 2.5]$). For a mass loss rate, which Abbott (1982) proposed for the mass loss rate of WR stars on the basis of the radiation driven wind theory of Castor et al. (1975), Langer (1989) finds $\alpha \in [2, 5]$. For the case of mass loss caused by vibrational instability, Maeder (1985) proposed $\dot{M}_{\mathrm{WR}} \sim L_{\mathrm{WR}}^{0.7}$, leading to $\dot{M}_{\mathrm{WR}} \sim M$, i.e. $\alpha \simeq 1$.

In summary, theoretical considerations lead to values of α in the range of 1 to 5. Surprisingly, we shall see in Sect. 3, that the exact value of α is not particularly important; in reality, α itself may be a function of the WR mass. For any value of α in the considered range we find the same important differences for the evolution of WR stars compared to the standard WR evolution, i.e. $\alpha = 0$.

2.2. Observational aspects

As mentioned in Sect. 1, observational determinations of masses and mass loss rates for WR stars are very difficult, and the results correspondingly uncertain. However, recently several authors investigated the possibility of $\dot{M}_{\mathrm{WR}}$ vs. M_{WR} relations from observational data. We have repeated the analyses of these authors, but we omitted the WNL stars. In Table 1 we have compiled the coefficients resulting from least square fits. In Refs. (1) to (3) we also omitted the WC7 star WR 79 for the reasons discussed by St.-Louis et al. (1988).

In this context we have to note, that standard least square fit methods – i.e. such where the sum of the squares of the vertical distances of the data points from the straight line is minimized – may involve some arbitrariness in the case of a relatively low correlation coefficient of the data. For example, the exchange of X- and Y-axis – corresponding to a minimization of the horizontal distance squares – could lead to a quite different linear relation. The only unequivocal way (at least in a log-log- relation, where the axes have no units) is a minimization of the squares of the rectangular distances of the points to the straight line, which we designate as rectangular least square fit. Note that the standard least square fit method tends to favour flat linear relations. This is clearly visible in Table 1, where the results of the rectangular fits are shown together with those from standard least square fits. Because of the above argument we tend to favour the results of the rectangular fits.

The data of Table 1 show, that observations cannot definitely verify the possibility of a $\dot{M}_{WR}$ vs. M_{WR} relation, but they show also, that such relationship may well exist. The correlation coefficient of the data used by Doom (1988) is very small, but at least these data are not inconsistent with the other results. Also in the case of Schmutz et al. (1989) the correlation coefficient is very small. The reason is that though all data points lie rather well on a straight line, this line is very steep.

In summary it is remarkable, that all 5 cases are not inconsistent with a mass loss law exponent in the range $2 \lesssim \alpha \lesssim 3$, and that this range again is not inconsistent with the theoretical expectations described above.

3. Evolution of WR stars with mass-dependent mass loss rates

3.1. The computational scheme

In order to simulate the evolution of WNE/WC/WO stars, we constructed a simple numerical scheme, which works as follows. For a WR star of a given mass, we can derive its luminosity according to the M–L relation of Langer (1989). Also on the basis of the WR models of Langer (1989), we can construct a $M_{WR}-M_{cc}$ relation, i.e. a relation of the convective core mass as a function of the stellar mass. Since Langer (1989) investigated also WR models including convective overshooting, we could include this effect also in our $M_{WR}-M_{cc}$ relation, introducing a parameter α_{over} (which measures the overshooting distance in units of the pressure scale height), i.e. we got $M_{cc}=M_{cc}(M_{WR}, \alpha_{over})$. Note that both relations, $L(M_{WR})$ and $M_{cc}(M_{WR}, \alpha_{over})$ are in very good agreement with WR models computed in stellar evolution sequences (cf. discussion in Langer, 1989).

Now, for a WR model at time t_0 of given mass and given convective core chemical composition (assuming within a very good approximation, that the core is composed of helium, carbon, and oxygen only), we can compute the change of internal chemical composition within a small timestep Δt from the consideration of global energy conservation: the amount of energy $L(M_{WR})\cdot\Delta t$ has to be supplied by nuclear transformations within the convective core, i.e.

$$L\cdot\Delta t=\left(\frac{B_Y\Delta Y}{A_Y}+\frac{B_C\Delta C}{A_C}+\frac{B_O\Delta O}{A_O}\right)\cdot N_A\cdot M_{cc}, \quad (1)$$

with ΔY, ΔC, ΔO being the changes of the mass fractions of helium, carbon, and oxygen in the convective core from time t_0 to $t=t_0+\Delta t$, A_Y, A_C, and A_O the atomic weights of the respective nuclei, B_Y, B_C, B_O their binding energies, and N_A the Avogadro number. In Eq. (1) ΔO can be expressed by ΔY and ΔC through the condition of mass conservation (i.e. $\Delta Y+\Delta C+\Delta O=0$), and ΔC can be computed from ΔY according to the $C(Y)$-fit to helium burning nucleosynthesis in massive stars of Langer (1989) (cf. his Eq. (1) and Fig. 1), which yields C as a function of Y with a very good precision. Now, ΔY is the only unknown in Eq. (1) and can thus be computed. For a complete helium burning sequence we used about 10000 timesteps which ensures high numerical precision and yields a very good agreement to sophisticated stellar evolution calculations.

Now we can adopt a specific mass loss law of the form $\dot{M}=-k\cdot M^{\alpha}$, which can be easily integrated and yields

$$M(t)=(M_0^{1-\alpha}+(\alpha-1)kt)^{1/1-\alpha}\ (\alpha\neq 1). \quad (2)$$

We express the parameter k through the mass M_{cal} at which the mass loss rate achieves the typical vaue of $3\ 10^{-5}\ M_\odot\ yr^{-1}$, i.e. $k=3\ 10^{-5}\ M_\odot\ yr^{-1}/M_{cal}^{\alpha}$.

We start our computations at the beginning of the WNE phase, which implies that the radiative envelope of our initial model is composed of pure helium. Since in general the WNE phase is not entered at the beginning of core helium burning but at a certain time during helium burning, we have to introduce the initial helium mass fraction within the convective core Y_0 as a further parameter.

Our computations yield $M_{WR}(t)$, $L_{WR}(t)$, $\dot{M}_{WR}(t)$, $M_{cc}(t)$. Furthermore, the surface helium concentration Y_s is determined via the relation $Y_s(t)=Y_{cc}(t_1)$, where $t_1<t$ is given by the condition $M_{WR}(t)=M_{cc}(t_1)$. (For $M_{WR}(t)>M_{cc}(t_0)$ we have $Y_s(t)=1$.) From these quantities we can then derive the final WR mass M_f, the duration of the WNE phase τ_{WNE} and of the WC phase τ_{WC}, and mean values $\overline{M_{WR}}$, $\overline{\dot{M}_{WR}}$, and $\overline{L_{WR}}$, where $\bar{X}:=\int X\,dt/\int dt$. From the initial central helium mass fraction Y_0 we can estimate the time t_0 from central helium ignition ($t=0$) to the beginning of our sequence according to

$$t_0=(C_{cc}(t_0)Q_{He\to C}+O_{cc}(t_0)Q_{He\to O})\frac{M_{cc}(t_0)}{L(t_0)}, \quad (3)$$

with $Q_{He\to C}=5.8\ 10^{17}\ erg\ g^{-1}$ representing the energy gain of converting 1 g helium into carbon, and $Q_{He\to O}=8.6\ 10^{17}\ erg\ g^{-1}$ that of converting 1 g helium into oxygen. Then we can compute the helium burning timescale τ_{He} as $\tau_{He}=t_0+\tau_{WNE}+\tau_{WC}$.

In summary, our computations involve the 5 parameter $M_{WR}(t_0)$, Y_0, α_{over}, α, and M_{cal}, and the solution of Eqs. (1) and (2) yields all global quantities characterising WR stellar evolution from the beginning of the WNE phase up to the end of central helium burning, which – concerning observable properties of WR stars – may be considered as the end of WR evolution. As an example, we present the results of a $M_i=25\ M_\odot$ sequence in Table 2a.

Table 1. Coefficients of $\dot{M}_{WR}$ vs. M_{WR} relations of the form $\log\dot{M}=k+\alpha\log M_{WR}$ as fitted to observational data of the respective references (column 1) according to the least square fit method (columns 3 and 4) and according to the rectangular least square fit (columns 5 and 6; see text). N is the number of data points involved, and R is the correlation coefficient of the data

Ref.	N	k	α	$k_\perp$	$\alpha_\perp$	R
(1)	7	-6.1 ± 0.8	1.0 ± 0.6	-7.6 ± 4.8	2.3 ± 1.5	0.60
(2)	4	-7.3 ± 1.4	2.7 ± 1.3	-8.6 ± 4.4	3.8 ± 1.9	0.82
(3)	4	-7.4 ± 0.5	2.5 ± 0.5	-7.6 ± 1.4	2.6 ± 0.5	0.97
(4)	4	-5.1 ± 2.4	0.9 ± 2.8	-19.7 ± 67.4	18.2 ± 60.7	0.21
(5)	22	-5.1 ± 0.6	0.6 ± 0.6	-15.8 ± 17.8	11.2 ± 12.3	0.21

References: (1) St.-Louis et al. (1988), WR 79 and WNL stars omitted; (2) Cassinelli and van der Hucht (1987), WR 79 omitted; (3) Abbott et al. (1986), WR 79 omitted; (4) Doom (1988), WNL stars omitted, WR masses are derived from luminosities with the M-L relation of Langer (1989); (5) Schmutz et al. (1989), WNL stars omitted, WR masses are derived from luminosities with the M-L relation of Langer (1989).

Table 2a. Evolutionary sequence of a WR star which enters its WNE phase with a mass of $M_i = 25\ M_\odot$ at a central helium content of $Y_0 = 0.54$ (corresponding to $M_{ZAMS} \simeq 51.6\ M_\odot$; cf. Table 2b) about 1.36 10^5 yr after central helium ignition, computed with $\alpha = 2.5$, $M_{cal} = 12\ M_\odot$, and $\alpha_{over} = 0.25$. N gives the timestep number of the computations, t is the time, M_{WR} the actual WR mass, L the corresponding luminosity, M_{cc} the convective core mass, and $\dot{M}$ the actual mass loss rate. Y, C, and O are the mass fractions of helium, carbon, and oxygen within the convective core, and Y_s and C_s designate the surface helium and carbon mass fractions. At the bottom we present several global quantities which characterise the sequence, and which enter in Table 2b

N	t (10^5 yr)	M_{WR} ($M_\odot$)	$\log L/L_\odot$	M_{cc} ($M_\odot$)	$\log \dot{M}$ ($M_\odot$ yr^{-1})	Y	C	O	Y_s	C_s
1	1.36	25.0	5.86	20.4	−3.7	0.54	0.34	0.12	1.00	0.00
1000	1.66	20.6	5.73	16.3	−3.9	0.45	0.36	0.16	1.00	0.00
2000	1.96	17.7	5.63	13.7	−4.1	0.38	0.41	0.22	0.49	0.37
3000	2.26	15.7	5.54	11.8	−4.2	0.31	0.41	0.28	0.44	0.39
4000	2.56	14.1	5.46	10.5	−4.4	0.26	0.40	0.35	0.39	0.40
5000	2.86	12.9	5.39	9.4	−4.4	0.21	0.37	0.42	0.35	0.41
6000	3.16	11.9	5.33	8.5	−4.5	0.17	0.34	0.50	0.32	0.41
7000	3.46	11.1	5.27	7.8	−4.6	0.13	0.30	0.58	0.29	0.41
8000	3.76	10.4	5.22	7.3	−4.7	0.09	0.25	0.66	0.27	0.40
9000	4.07	9.8	5.17	6.7	−4.7	0.06	0.20	0.74	0.23	0.39
10000	4.43	9.2	5.11	6.2	−4.8	0.03	0.13	0.84	0.20	0.37
10923	4.77	8.7	5.07	5.8	−4.9	0.00	0.07	0.93	0.17	0.35

⇒: $\overline{M_{WR}} = 13.5\ M_\odot$, $\overline{M_{WNE}} = 22.5\ M_\odot$, $\overline{M_{WC}} = 12.6\ M_\odot$, $\log \bar{L}/L_\odot = 5.44$, $\tau_{WNE} = 3.2\ 10^4$ yr, $\tau_{WC} = 3.1\ 10^5$ yr, $\log \overline{\dot{M}_{WR}} = -4.3$

Table 2b. Results of a set of evolution sequences computed with $\alpha = 2.5$, $M_{cal} = 12\ M_\odot$, and $\alpha_{over} = 0.25$. M_i gives the WR mass at the beginning of the WNE phase, Y_i the corresponding central helium mass fraction, and M_{ZAMS} the corresponding ZAMS mass. M_f designates the WR mass at central helium exhaustion, and τ_{WNE} and τ_{WC} the duration of the WNE and WC phase, respectively. $\bar{\dot{M}}$ is the mean WR mass loss rate, and $\overline{M_{WNE}}$ and $\overline{M_{WC}}$ are the mean masses of the WR star in the WNE and WC phases, respectively (cf. Sect. 3.1). The arrow indicates the entries from Table 2a. At the bottom we present several IMF mean quantities, which enter into Table 2c

M_{ZAMS} ($M_\odot$)	Y_i	M_i ($M_\odot$)	M_f ($M_\odot$)	τ_{WNE} (10^5 yr)	τ_{WC} (10^5 yr)	$\log \bar{\dot{M}}$ ($M_\odot$ yr^{-1})	$\overline{M_{WNE}}$ ($M_\odot$)	$\overline{M_{WC}}$ ($M_\odot$)
30.0	0.001	10.0	9.98	0.01	0.00	−4.7	10.0	—
31.4	0.040	11.0	10.07	0.43	0.00	−4.7	10.5	—
32.9	0.082	12.0	10.03	0.82	0.00	−4.6	11.0	—
35.0	0.15	13.5	9.81	1.32	0.05	−4.6	10.5	9.8
37.2	0.22	15.0	9.53	1.03	0.83	−4.5	12.9	10.3
40.1	0.31	17.0	9.18	0.78	1.62	−4.5	14.8	10.8
44.4	0.40	20.0	9.07	0.53	2.29	−4.4	17.7	11.7
→51.6	0.54	25.0	8.72	0.31	3.10	−4.3	22.5	12.6
61.7	0.74	32.0	7.99	0.18	4.11	−4.3	29.4	13.0
73.3	0.72	40.0	8.54	0.11	3.89	−4.1	37.2	14.7
83.4	0.69	47.0	9.01	0.08	3.67	−4.0	44.0	16.0
102.1	0.67	60.0	9.51	0.05	3.48	−3.8	56.1	17.7

⇒: $\langle \tau_{WNE} \rangle = 0.48\ 10^5$ yr, $\langle \tau_{WC} \rangle = 2.00\ 10^5$ yr, $\langle M_{ZAMS}(\mathrm{WNE}) \rangle = 40.7\ M_\odot$, $\langle M_{ZAMS}(\mathrm{WC}) \rangle = 62.9\ M_\odot$, $\langle M_f \rangle = 9.2\ M_\odot$, $\log \langle \dot{M}_{WR} \rangle = -4.2\ M_\odot\ \mathrm{yr}^{-1}$, $\langle \overline{M_{WNE}} \rangle = 15.5\ M_\odot$, $\langle \overline{M_{WC}} \rangle = 13.5\ M_\odot$

Table 2c. IMF mean quantities for sets of WR sequences computed with different mass loss parameters α and M_{cal}: mean mass loss rate $\langle \dot{M}_{WR} \rangle$, mean duration of WNE and WC phase ($\langle \tau_{WNE} \rangle$ and $\langle \tau_{WC} \rangle$, respectively), mean WNE and WC mass ($\langle M_{WNE} \rangle$ and $\langle M_{WC} \rangle$), mean final WR mass $\langle M_f \rangle$, and mean WNE and WC progenitor ZAMS masses $\langle M_{ZAMS}^{WNE} \rangle$ and $\langle M_{ZAMS}^{WC} \rangle$). The arrow indicates the entries from Table 2b

α	M_{cal}	$\langle \dot{M}_{WR} \rangle$ ($M_\odot$ yr^{-1})	$\langle \tau_{WNE} \rangle$ (10^5 yr)	$\langle \tau_{WC} \rangle$ (10^5 yr)	$\langle M_{WNE} \rangle$ ($M_\odot$)	$\langle M_{WC} \rangle$ ($M_\odot$)	$\langle M_f \rangle$ ($M_\odot$)	$\langle M_{ZAMS}^{WNE} \rangle$ ($M_\odot$)	$\langle M_{ZAMS}^{WC} \rangle$ ($M_\odot$)	$\langle \tau_{WC} \rangle / \langle \tau_{WNE} \rangle$
0	—	3.0 –	1.31	0.66	26.7±14.2	20.5±8.6	18.1±12.0	57±21	54±13	0.50
0	—	5.0 –	0.81	1.37	26.2±14.4	17.6±10.9	13.1±11.7	57±22	54±17	1.69
1	10	5.7±2.6	0.65	1.52	20.4±11.4	18.6±8.2	11.6±5.0	48±17	60±18	2.34
1	12	5.1±2.3	0.76	1.33	20.6±11.4	20.6±8.8	13.3±6.0	48±17	61±18	1.75
1	14	4.6±2.1	0.87	1.18	20.9±11.4	22.2±9.2	14.6±6.7	49±17	62±18	1.36
2	10	6.1±3.0	0.41	2.27	16.2±8.1	12.1±2.4	7.6±1.3	42±12	61±18	5.54
2	12	6.1±3.5	0.54	1.80	16.7±8.2	15.3±3.8	9.8±0.8	43±12	61±18	3.33
2	14	5.7±3.6	0.67	1.51	17.3±8.3	18.1±4.7	11.6±1.7	43±13	64±18	2.25
2.5	10	5.8±2.5	0.34	2.57	14.9±6.7	10.4±1.1	7.1±1.5	40±10	62±19	7.56
→2.5	12	6.0±3.2	0.48	2.00	15.5±6.8	13.5±2.1	9.2±0.6	41±10	63±19	4.17
2.5	14	5.8±3.6	0.62	1.65	16.2±7.0	16.4±3.1	11.0±0.7	42±11	64±18	2.66
3	10	5.5±2.2	0.30	2.79	13.9±5.5	9.5±0.5	7.2±1.5	38±8	62±19	9.30
3	12	5.8±3.0	0.44	2.16	14.6±5.7	12.4±1.1	9.1±0.8	39±9	63±19	4.91
3	14	5.7±3.5	0.58	1.76	15.4±5.9	15.2±2.0	10.7±0.4	41±9	65±18	3.03
4	10	5.2±2.0	0.24	3.01	12.6±3.9	8.7±0.3	7.2±1.4	37±6	62±19	12.5
4	12	5.5±2.7	0.38	2.35	13.4±4.1	11.2±0.4	9.0±0.9	38±6	63±19	6.18
4	14	5.5±3.2	0.53	1.90	14.4±4.3	13.8±0.8	10.6±0.4	39±7	65±18	3.58
1–4	10	5.5±2.7	0.31	2.69	17.5±12.3	10.0±1.8	7.4±1.3	44±18	60±18	8.68
1–4	12	5.8±3.4	0.45	2.06	17.4±11.3	13.2±3.0	9.3±0.8	44±17	61±18	4.58
1–4	14	5.7±3.7	0.63	1.65	17.9±10.7	16.2±4.4	11.2±1.2	44±16	63±17	2.62
2.5*	10	5.8±2.7	0.56	2.21	15.9±6.9	10.9±0.9	8.0±1.8	41±11	65±18	3.95
2.5*	12	5.9±3.5	0.74	1.62	16.9±7.2	14.4±1.8	10.1±1.0	43±11	66±18	2.19
2.5*	14	5.5±3.9	0.91	1.25	17.9±7.4	17.8±2.6	12.1±1.0	44±12	69±17	1.37

3.2. Choice of input parameters

In order to get results which can be easily compared with observations, we have computed WR sequences for a whole spectrum of initial masses $M_i(j), j=1, 12$. For a convolution with an initial mass function we have to define a $M_{ZAMS}(M_i)$-relation. On the basis of the stellar sequences of Maeder and Meynet (1987) we got a linear relation of the form $M_{ZAMS}=1.442\,M_i+15.576$ (units: $M_\odot$), which, for initial WNE masses in the range $10\,M_\odot \le M_i \le 60\,M_\odot$ leads to $30\,M_\odot \le M_{ZAMS} \le 102\,M_\odot$ (cf. Table 2b). Note that M_i is not the helium core mass for given ZAMS mass at core helium ignition, but rather the stellar mass at the beginning of the WNE phase. Also the corresponding values of Y_i have been taken from the sequences of Maeder and Meynet (1987) (for $M_{ZAMS}=40\,M_\odot, 60\,M_\odot, 85\,M_\odot, 120\,M_\odot$), and interpolated otherwise), leading to the values presented in Table 2b. For $30\,M_\odot \le M_{ZAMS} \lesssim 60\,M_\odot$, Y_i is increasing approximately linear with M_{ZAMS}, the relevant WR formation scenario being the post-RSG scenario, while for $M_{ZAMS} \gtrsim 60\,M_\odot$ $Y_i \simeq$const. corresponds to the LBV scenario of WR formation (cf. Maeder and Meynet, 1987; Langer, 1987). Particularly, towards small masses $Y_i=0$ is reached at $M_i=10\,M_\odot$, corresponding to $M_{ZAMS}=30\,M_\odot$. This means that $30\,M_\odot$ corresponds to the lower mass limit for the formation of WNE stars in our computations. At high masses we have a cut-off at $M_{ZAMS} \simeq 100\,M_\odot$.

The overshooting parameter has been adopted as $\alpha_{over}=0.25$, in agreement with Maeder and Meynet (1987), who argue that this value is most consistent with observations.

The mass loss parameters α and M_{cal} have been varied over a wide range in order to elaborate their effects on the observable quantities (see below).

3.3. Results

In Table 2c we have compiled the basic IMF-convolved quantities describing the WR evolution for different values of the mass loss parameters α and M_{cal}. As IMF we have used a Salpeter distribution of the form $\Phi(M_{ZAMS}) \sim M_{ZAMS}^{-2.35}$, $\Phi(M_{ZAMS})\,dM_{ZAMS}$ being proportional to the number of new born stars in the mass interval dM_{ZAMS}. The mean timescales $\langle \tau_{WNE} \rangle$ and $\langle \tau_{WC} \rangle$ are thus computed according to

$$\langle \tau_x \rangle = \frac{\int_{30}^{100} \tau_x(M_{ZAMS})\Phi(M_{ZAMS})\,dM_{ZAMS}}{\int_{30}^{100} \Phi(M_{ZAMS})\,dM_{ZAMS}}; \quad x=\text{WNE, WC} \quad (4)$$

and the same procedure has been used to compute the mean final mass $\langle M_f \rangle$. The mean WNE and WC masses $\langle M_{WNE} \rangle$ and $\langle M_{WC} \rangle$ are computed from the respective mean masses per stellar sequence $\overline{M_{WNE}}(M_{ZAMS})$ and $\overline{M_{WC}}(M_{ZAMS})$ according to

$$\langle M_x \rangle = \frac{\int_{30}^{100} \overline{M_x}(M_{ZAMS})\tau_x(M_{ZAMS})\Phi(M_{ZAMS})\,dM_{ZAMS}}{\int_{30}^{100} \tau_x(M_{ZAMS})\Phi(M_{ZAMS})\,dM_{ZAMS}}; \quad x = \text{WNE, WC.} \tag{5}$$

This means, $\langle M_x \rangle$ is computed with τ_x as weightening coefficient, i.e. it corresponds to the mean mass of observed WR stars of subtype x. Correspondingly, $\langle \dot{M} \rangle$ was computed with $\tau_{WNE} + \tau_{WC}$ as weightening coefficient. Finally, $\langle M_{ZAMS}(\text{WNE}) \rangle$ and $\langle M_{ZAMS}(\text{WC}) \rangle$ are computed as

$$\langle M_{ZAMS}(x) \rangle = \frac{\int_{30}^{100} M_{ZAMS}\tau_x(M_{ZAMS})\Phi(M_{ZAMS})dM_{ZAMS}}{\int_{30}^{100} \tau_x(M_{ZAMS})\Phi(M_{ZAMS})\,dM_{ZAMS}}; \quad x = \text{WNE, WC.} \tag{6}$$

Therefore, if e.g. a WR star of spectral type WNE is observed, $\langle M_{ZAMS}(\text{WNE}) \rangle$ represents its most probable ZAMS mass.

In Table 2c we also give the scatter Δ of the values (designated as $\pm$ – values), computed as

$$(\Delta(x))^2 = \frac{\int_{30}^{100} (x(M_{ZAMS}) - \langle x \rangle)^2 w_x(M_{ZAMS})\Phi(M_{ZAMS})dM_{ZAMS}}{\int_{30}^{100} w_x(M_{ZAMS})\Phi(M_{ZAMS})dM_{ZAMS}}, \tag{7}$$

with w_x as weightening coefficient for the variable x.

The data of Table 2c show clearly, that WR evolution proceeds quite different when $\alpha > 0$ is used compared to the standard case of $\alpha = 0$. Surprisingly, we note that the mean mass loss rate $\langle \dot{M} \rangle$ is almost independent of the mass loss parameters α and M_{cal} for $\alpha > 1$, rather it is $\langle \dot{M} \rangle \simeq 5.5$–$6\ 10^{-5}\ M_\odot\ \text{yr}^{-1}$, and also the predicted scatter is relatively small. α has also no effect on the mean final WR mass $\langle M_f \rangle$ for $\alpha > 1$. Here, however, $\langle M_f \rangle$ is slightly increasing with increasing M_{cal}. Note that for small α $\langle M_f \rangle$ becomes large, and also $\Delta \langle M_f \rangle$ becomes large while it is almost zero for larger α.

Both parameters, α and M_{cal}, have a clear influence on the mean timescales $\langle \tau_{WNE} \rangle$ and $\langle \tau_{WC} \rangle$: larger α or smaller M_{cal} tend to increase $\langle \tau_{WC} \rangle$ and to decrease $\langle \tau_{WNE} \rangle$, which drastically increases the ratio $\langle \tau_{WC} \rangle / \langle \tau_{WNE} \rangle$. Note that the increase of $\langle \tau_{WC} \rangle$ is larger than the decrease of $\langle \tau_{WNE} \rangle$, which means that the total WR lifetime is increased for larger α or smaller M_{cal} values.

Concerning the mean WR masses, we see that M_{cal} has no influence on $\langle M_{WNE} \rangle$, but a decreased M_{cal} leads to smaller mean WC masses $\langle M_{WC} \rangle$. An increase of α reduces both, $\langle M_{WNE} \rangle$ and $\langle W_{WC} \rangle$, and also the expected scatter of these quantities.

Finally, we may consider the original ZAMS masses of the considered WR types. Here again, the case $\alpha = 0$ is clearly different from the cases $\alpha \neq 0$: For $\alpha = 0$ both WR types, WNE and WC, are predicted to originate on average from the same ZAMS mass of $M_{ZAMS} \simeq 55\ M_\odot$, most WNE stars being formed from a larger ZAMS mass interval ($35\ M_\odot \lesssim M_{ZAMS}(\text{WNE}) \lesssim 80\ M_\odot$) compared to most WC stars ($40\ M_\odot \lesssim M_{ZAMS}(WC) \lesssim 70\ M_\odot$). In contrary, for $\alpha > 0$ it is clearly predicted, that WC stars evolve from initially more massive stars compared to WNE stars. For $\alpha > 1$ the results are very uniform: most WNE stars are predicted to originate from stars with $30\ M_\odot \lesssim M_{ZAMS}(\text{WNE}) \lesssim 50\ M_\odot$ while most WC stars are formed from stars with $44\ M_\odot \lesssim M_{ZAMS}(\text{WC}) \lesssim 80\ M_\odot$. The parameter M_{cal} is found to have almost no influence on these numbers.

We also performed computations with a mass dependent mass loss coefficient $\alpha(M_{WR})$, varying α linearly from 1 to 4 for M_{WR} between 60 and $5\ M_\odot$. (Note that Langer, 1989, finds $d\alpha/dM_{WR} < 0$ for the mass dependence of all quantities x which may influence the WR mass loss, of the form $x \sim M_{WR}^{\alpha}$.) The results show no big differences compared to computations with constant α for $1 < \alpha < 4$ concerning mean mass loss rate, timescales, and mean ZAMS and final masses. Just the mean WNE mass is somewhat increased ($\langle M_{WNE} \rangle \simeq 17.5\ M_\odot$, independent of M_{cal}), and the scatter of WNE masses as well as that of ZAMS masses of WNE precursers becomes somewhat larger.

Finally, for reasons discussed in the next Section, we performed computations with $\alpha = 2.5$ for three different M_{cal} values, including the condition that the mass loss rate in the WNE phase is divided by 2. This has, of course, the effect to increase the mean WNE lifetime on the expences of the mean duration of the WC phase, therefore strongly reducing the ratio $\langle \tau_{WC} \rangle / \langle \tau_{WNE} \rangle$. Also the mean WNE mass is slightly increased (by $\sim 1\ M_\odot$). All other quantities, however, remain almost unaffected.

4. Comparison to observations

Due to the simplicity of our numerical scheme we have been able to compute a large number of WR evolution sequences as the example presented in Table 2a. Therefore we could compute, for a given set of mass loss parameters, a set of WR sequences covering the whole realistic range of initial masses (cf. the example in Table 2b). This enabled us to convolute our results with an IMF, leading to average values which can be easily compared with observations. These values are compiled in Table 2c for a wide range of mass loss parameters α and M_{cal}; in total 276 stellar evolution sequences were necessary in order to derive these data.

4.1. Timescales

From the mean timescales $\langle \tau_{WNE} \rangle$ and $\langle \tau_{WC} \rangle$ we computed the ratio $\langle \tau_{WNE} \rangle / \langle \tau_{WNC} \rangle$ (cf. Table 2c), which corresponds to the expected ratio of the observed number of WC to WNE stars in a complete sample of WR stars. According to the data compiled by van der Hucht et al. (1988) on WR stars in the solar neighbourhood ($d < 2.5$ kpc), the WC/WN number ratio is 2, i.e. there are twice as many WC stars as WN stars in the solar neighbourhood. This implies $N_{WC}/N_{WNE} > 2$. (Note that the WC/WN ratio will be smaller for an incomplete sample, since the WNL stars are by far the visually brightes WR stars.) Actually, according to van der Hucht et al. (1988) we find $N_{WC}/N_{WNE} = 2.5 - 7.5$, the exact value depending on the number of WN6 stars counted as WNE or WNL: out of 15 WN stars (compared to 30 WC's), 8 are of spectral type WN6. Smith (1988) quotes values of 2.4 and 3.7 according to different data sets.

The quoted numbers rule out the computed WR sequences with either small α and large M_{cal} or large α and small M_{cal}. In the first case the WC/WNE number ratio is too small, in the second case it is too large. Note, however, that a too large ratio may be prevented by assuming, that the mass loss rate of WNE stars is smaller than that of WC stars of equal mass (cf. Sect. 4.2).

We should also mention that larger α or smaller M_{cal} values lead to an increase in $\langle \tau_{WNE} \rangle + \langle \tau_{WC} \rangle$, i.e. to an increase in the total WR lifetime. This may help to achieve a better agreement with the observed number ratio of O stars to WR stars (cf. Conti et al., 1983), which seems hard to match with constant WR mass loss rates (cf. Maeder and Meynet, 1987).

4.2. *Mass loss rates*

According to the data of Table 2c, the predicted mean mass loss rate for WNE and WC stars is approximately 5–6 $10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$ for all computations with mass dependent mass loss rates, independent of the mass loss parameters. This seems to be too high as compared to observational data presented by Barlow et al. (1981) or Abbott et al. (1986), who quote values of 2–3 $10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$. However, Schmutz and Hamann (1986) argued, that since not He^{++} but He^{+} is the dominant ion in the radio emitting region of the WR winds, the mass loss rates quoted above should be enhanced by about a factor of 2. Additionally, van der Hucht et al. (1986) showed, that especially the mass loss rates of WC stars have been yet considerably underestimated due to the neglect of the high abundances of carbon and oxygen in WC atmospheres. They estimate the WC mass loss rates to be 2–3 times larger than previously predicted. Therefore, the mean observed mass loss rate of WNE and WC stars may well be in the range 5–6 $10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$. Note that in all our computations with $\alpha \neq 0$, the mean WC lifetime is much larger than the mean WNE lifetime, which means that the value of $\langle \dot{M} \rangle$ is dominated by the mass loss rates in the WC stage.

Recently, Schmutz et al. (1989) determined mass loss rates for 30 galactic WR stars and found $-5.3 \leq \log \dot{M}_{\mathrm{WNE}} \leq -3.9$ and $-4.6 \leq \log \dot{M}_{\mathrm{WC}} \leq -4.4$, with $\langle \dot{M}_{\mathrm{WNE}} \rangle = 5.4\ 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$ and $\langle \dot{M}_{\mathrm{WC}} \rangle = 3.5\ 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$. These values may not be significant, since Schmutz et al. did not use a complete sample of WR stars. It is striking, however, that the scatter is much larger for the WNE stars compared to the WC stars. This compares well with our sequences with $\alpha > 1$, where the scatter in the WNE masses is much larger than that for the WC masses (cf. Sect. 4.3), which should of course be reflected in the scatter of the corresponding mass loss rates. Also a smaller mean WC mass loss rate compared to that for WNE stars is not in contradiction with the above hypothesis, that the mass loss rates for WC stars is larger than that of WNE stars of equal mass; it may simply indicate a smaller mean WC mass compared to the mean WNE mass, which is in fact indicated by the low luminosities for WC stars derived by Schmutz et al., and which fits well to the results of our computations with $\alpha > 1$ and $M_{\mathrm{cal}} \lesssim 12\ M_\odot$ (see below).

Note that recently de Freitas Pacheco and Machado (1988) determined the mass loss rates for WC stars to be in the range $4.3\ 10^{-5}\ M_\odot\ \mathrm{yr}^{-1} \lesssim \dot{M}_{\mathrm{WC}} \lesssim 1.5\ 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$, with $\langle \dot{M}_{\mathrm{WC}} \rangle \lesssim 7.2 \pm 2.4\ 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$, also pointing towards very high mass loss rates for WC stars.

In order to investigate the effect of a larger mass loss rate in the WC stage compared to that of the WNE phase as indicated by the work of van der Hucht et al. (1986), and which would also be a consequence if the WR mass loss is driven by radiation pressure in metal absorption lines (see e.g. Abbott, 1982), we performed computations with a moderate value of $\alpha = 2.5$, adopting $\dot{M}_{\mathrm{WR}} \sim f \cdot M_{\mathrm{WR}}^{\alpha}$, with $f = 1$ for the WC phase, but $f = 0.5$ for the WNE phase (designated as $\alpha = 2.5^*$ in Table 2c). We see from Table 2c, that the effect on the mean mass loss rate is small, and also all other quantities remain almost unaffected. Only the timescale ratio $\langle \tau_{\mathrm{WC}} \rangle / \langle \tau_{\mathrm{WNE}} \rangle$ is decreased by approximately a factor of 0.5. Consequently, the sequences with $\alpha = 2.5^*$ and $M_{\mathrm{cal}} = 10\ M_\odot$ predict a WC/WNE ratio of 4, which is now in agreement with the observations mentioned in the previous subsection.

We should mention that for constant mass loss rates (i.e. $\alpha = 0$), $\dot{M}_{\mathrm{WR}} = 5\ 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$ is the maximum possible value. A slightly higher mass loss rate would lead to vanishing WR masses before central helium exhaustion for intermediate initial WNE masses. Therefore, if the high mean WNE + WC mass loss rates will be confirmed by observations, a constant WR mass loss law is completely impossible.

4.3. *WR masses*

Basically two methods exist to derive masses for observed WR stars. The first, applicable only for WR stars in binary systems, is to deduce the mass ratio of the binary, and then to estimate the mass of the companion star (which is often an O star), or to estimate the inclination angle. For mean masses of WR stars in binary systems we must be aware, that these numbers may involve relatively many WR stars, which formed through the binary scenario. The second way is to derive the bolometric luminosity of a WR star and to compute its mass from theoretical mass luminosity relations (cf. Maeder and Meynet, 1987; Langer, 1989). To derive bolometric luminosities for WR stars, however, is also very difficult; particularly, no set of luminosities for a complete sample of WR stars exists yet. Therefore, a comparison of our theoretically computed mean masses for WNE and WC stars can show up only trends, which, however, turn out to be quite distinct.

Niemela (1983) made a statistic of masses of WR stars in binary systems, and she found $\langle M_{\mathrm{WNE}} \rangle_{\mathrm{bin}} = 7.5\ M_\odot$, $\langle M_{\mathrm{WC}} \rangle_{\mathrm{bin}} = 15\ M_\odot$, and $\langle M_{\mathrm{WNL}} \rangle_{\mathrm{bin}} = 63\ M_\odot$. Comparing these values to the corresponding mean masses of Table 2c we see: 1) The sequences with small α (i.e. $\alpha = 0$, $\alpha = 1$) predict too high mean masses, while those with larger α match the order of magnitude quite well; and 2) a mean WNE mass, which is smaller than the mean WC mass is incompatible with all computed sequences.

Also, observationally derived luminosities for WNE and WC stars are found to be very low, indicating that the corresponding masses are also too small in order to be matched by standard WR evolutionary calculations (i.e. $\alpha = 0$). Nussbaumer et al. (1982) and Smith and Willis (1983) derived luminosities for WNE and WC stars in the range $5.1 \gtrsim \log(L/L_\odot) \gtrsim 4.6$, indicating a mass interval of $10\ M_\odot \gtrsim M_{\mathrm{WR}} \gtrsim 4.5\ M_\odot$, while according to Schmutz et al. (1989) we find $5.5 \gtrsim \log(L_{\mathrm{WNE}}/L_\odot) \gtrsim 5.0$, i.e. $17\ M_\odot \gtrsim M_{\mathrm{WNE}} \gtrsim 9\ M_\odot$, and $5.1 \gtrsim \log(L_{\mathrm{WC}}/L_\odot) \gtrsim 4.6$, i.e. $10\ M_\odot \gtrsim M_{\mathrm{WC}} \gtrsim 4.5\ M_\odot$.

A further important point is the large gap between observed mean masses of WNE and WNL stars, mentioned above. This can only be consistent with an evolutionary conection of the kind WNL→WNE, if the massive WNE state lasts very short. This is exactly the result for the case $\alpha > 0$, but standard evolution ($\alpha = 0$) cannot account for this phenomenon.

In summary, though the data on observationally determined WR masses is rather inhomogeneous, the very small values found seem to rule out the standard WR evolution, i.e. $\dot{M}_{\mathrm{WR}} = \mathrm{const.}$ Note that, of course, a larger constant mass loss rate leads to smaller mean masses. However, as mentioned above, $\dot{M}_{\mathrm{WR}} = 5\ 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$ is the maximum possible value, which still results in a mean WNE mass as high as 26 $M_\odot$. On the contrary, the results of our sequences computed with large

$\alpha(\alpha \gtrsim 2)$ match the trend of small WNE and WC masses very well. They fit particularly well to the recent data of Schmutz et al. (1989), which indicate a smaller mean WC mass compared to the mean WNE mass.

4.4. *WR progenitor ZAMS masses*

The WR sequences computed with a constant mass loss rate (i.e. $\alpha=0$) infer that both, WNE and WC stars, form from stars with ZAMS masses in intervals peaked at $\sim 55\,M_\odot$, while all sequences with $\alpha \neq 0$ indicate that WNE stars form from stars with smaller ZAMS masses compared to WC progenitors, i.e. $\langle M_{\rm ZAMS}({\rm WNE})\rangle \simeq 38-48\,M_\odot$, and $\langle M_{\rm ZAMS}({\rm WC})\rangle \simeq 60-69\,M_\odot$.

Schild and Maeder (1984) determined ZAMS masses for different WR subtypes by a comparison of cluster turnoffs and stellar evolutionary tracks for open star clusters which contain one or more WR stars. They find that WNE stars form from $60\,M_\odot \gtrsim M_{\rm ZAMS} \gtrsim 18\,M_\odot$ and WC stars from stars with $M_{\rm ZAMS} \gtrsim 35\,M_\odot$, and that the bulk of WR stars originates from stars with $M_{\rm ZAMS} \gtrsim 40\,M_\odot$. Here, the preference of WC progenitor ZAMS masses towards higher values as compared to the WNE's again is a clear indication in favour of $\alpha \neq 0$. The very small value of the lower ZAMS mass limit for WNE star formation of $18\,M_\odot$ is not necessarily a contradiction of the corresponding limit of $30\,M_\odot$ adopted by us (according to the stellar evolution calculations of Maeder and Meynet, 1987): van der Hucht et al. (1988) found that (in the solar neighbourhood) binary WR stars form preferentially from less massive O type stars compared to single WR stars.

van der Hucht et al. (1988) and Humphreys et al. (1985) also determined ZAMS masses of WR progenitors. These authors do, however, not distinguish between WNE and WNL stars, and so we can derive no information enabling us to discriminate between the cases $\alpha=0$ and $\alpha \neq 0$ from these papers. They determined lower mass limits for WR formation of $\sim 25\,M_\odot$ (van der Hucht et al.) and $30\,M_\odot$ (Humphreys et al.), which are consistent with the value adopted here.

4.5. *Final WR masses and SN Ib progenitors*

In recent years, a subclass of type I supernovae (SNe) has been identified (cf. Branch, 1986), and WR stars have been proposed as possible stellar progenitors of these events (besides other propositions; see e.g. references in Ensman and Woosley, 1988). A stringent constraint on WR stars as type Ib progenitors comes from Ensman and Woosley (1988): their lightcurves coincide only with WR masses of $4-7\,M_\odot$.

Note that these masses are consistent with the small value of WC masses derived from their luminosities ($4.5\,M_\odot \lesssim M_{\rm WC} \lesssim 10.2\,M_\odot$; cf. Sect. 4.3). The existence of very low mass WC stars has also been demonstrated recently by St.-Louis et al. (1987): they derived a mass of the WC component of the binary HD 152270 of $5\pm2\,M_\odot$.

Such small final WR masses are clearly in contradiction with standard WR evolution (i.e. $\dot M_{\rm WR}=$const.), but note that they fit rather well to our sequences with $\alpha \geq 2$ and $M_{\rm cal} \simeq 10\,M_\odot$. Note also that the predicted scatter in the final WR masses is very small in these cases (cf. also Table 2b), which means that almost all WR stars loose sufficient mass during their evolution in order to appear as what people observe as type Ib SN when they explode, regardless of the initial WR or ZAMS mass; i.e. one may well expect quite a number of type Ib events. Therefore, if most type Ib's really turn out to originate from WR stars, this will clearly favour values of $\alpha \geq 2$ and $M_{\rm cal} \lesssim 12\,M_\odot$.

On the other hand, in the previous Sections we collected quite some evidence in favour of mass dependent mass loss rates of WR stars independently of this subsection. Therefore, the present investigation supports the hypothesis that type Ib SNe may originate from WR stars.

5. Discussion and conclusions

Theoretical models for WR stars of spectral types WNE/WC/WO indicate, that mainly the mass of the WR star determines all its global properties, while the surface chemical composition may have an additional effect for WC/WO stars (cf. Langer, 1989). Therefore it seems reasonable to adopt the WR mass as the basic quantity which determines the WR mass loss rate, i.e. to assume $\dot M_{\rm WR} \sim M_{\rm WR}^\alpha$, α being a free parameter with $1 \lesssim \alpha \lesssim 5$ (cf. Sect. 2.1). Also observational results are not inconsistent with such a mass loss relation; though the correlation of the data is quite weak, all observations are consistent with $2 \lesssim \alpha \lesssim 3$ (cf. Sect. 2.2).

On the basis of simplified but accurate numerical simulations of WR evolution we investigated the consequences of mass-dependent WR mass loss rates for a wide parameter range. A basic result, which has already been qualitatively discussed by Abbott et al. (1986) is, that for values of α in the relevant range, shortly after the WNE phase is entered the actual stellar mass falls within a rather narrow range ($25\,M_\odot \gtrsim M_{\rm WNE} \gtrsim 5\,M_\odot$) regardless of the initial WNE mass (cf. Fig. 9 of Abbott et al.). Consequently, the observed scatter in the WR mass loss rates will be relatively small. For this reason, a mass loss rate of the form $\dot M_{\rm WR} \sim M_{\rm WR}^\alpha$, which implies that $\dot M_{\rm WR}$ varies over orders of magnitude during the life of an e.g. initially massive WNE star, is not in contradiction with a narrow range of observed WR mass loss rates, since the duration of the very high mass loss is very short (cf. scatter of $\dot M_{\rm WR}$ in Table 2c). However, this implies further, that when observations indicate a small scatter of WR mass loss rates, one may *not* conclude that $\dot M_{\rm WR}$ depends only weakly on the WR properties.

In summary, our computations indicate, that a mass loss law of the form

$$\dot M_{\rm WR} = (0.6-1.0)\,10^{-7}\left(\frac{M_{\rm WR}}{M_\odot}\right)^{2.5}\ [M_\odot\,{\rm yr}^{-1}] \tag{8}$$

(the factor $(0.6-1.0)\,10^{-7}$ corresponding to $M_{\rm cal}=10-12\,M_\odot$, where smaller values might apply to WNE stars and higher ones to WC stars according to the discussion of Sect. 4.2) leads to good agreement with observations, while standard WR evolution (i.e. $\dot M_{\rm WR}=$const.) does not, the main points being the following:

—The ratio of the number of WC to WNE stars of $2.5-7.5$ is matched by WR sequences computed with Eq. (8), while it is too small in the standard case ($\alpha=0$, i.e. $\dot M_{\rm WR}=$const.). Furthermore, the total duration of the WR phase is increased for larger α, increasing therefore the ratio of the number of WR to O stars.

—Recently determined high values of WR mass loss rates are in good agreement with mean WR mass loss rates obtained with $\alpha>0$, but are hardly to recover with $\alpha=0$.

—$\alpha>0$ leads to low mean WR masses with a low scatter (e.g. $\langle M_{\rm WNE}\rangle = 16\pm7\,M_\odot$, and $\langle M_{\rm WC}\rangle = 11\pm1\,M_\odot$ for Eq. 8), which agrees much better with observations than results of standard

WR sequences (e.g. $\langle M_{WNE} \rangle = 27 \pm 14\, M_\odot$, and $\langle M_{WC} \rangle = 21 \pm 9\, M_\odot$ for $\dot{M}_{WR} \equiv 3\; 10^{-5}\, M_\odot\, yr^{-1}$).

—Computations with $\alpha > 0$ predict a higher mean ZAMS mass for WC progenitors than for WNE progenitors, which is in agreement with observations, while $\alpha = 0$ leads to the same mean ZAMS mass for both cases.

—$\alpha > 0$ leads to low final WR masses within a narrow range (e.g. $\langle M_f \rangle = 8 \pm 2\, M_\odot$ for Eq. 8), which makes WR stars to be very good candidates for SN Ib progenitors. Contrary, final WR masses according to standard WR evolution are too large and cover a too wide range (e.g. $\langle M_f \rangle = 18 \pm 12\, M_\odot$ for $\dot{M}_{WR} \equiv 3\; 10^{-5}\, M_\odot\, yr^{-1}$) in order to account for the observation type Ib SN frequency.

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Note added in proof: Very recently, L.F. Smith and A. Maeder (1989, *Astron. Astrophys.* **211**, 71) discussed the observational evidence of an $\dot{M}_{WR}$ vs L_{WR} relation, and especially the exponent β of the proportionality $\dot{M}_{WR} \sim L_{WR}^{\beta}$. As stated above and discussed by Smith and Maeder, such equation can be easily converted into $\dot{M}_{WR} \sim M_{WR}^{\alpha}$, with $\alpha \simeq \beta$, since approximately it is $L_{WR} \sim M_{WR}^{\gamma}$ with γ of the order of 1. For this reason, the paper of Smith and Maeder is of direct relevance to the present work.

The paper of Smith and Maeder strongly supports our concept of mass-dependent mass loss rates for WR stars in general. Their Fig. 2 clearly indicates a correlation $\dot{M}_{WR} \sim L_{WR}^{\beta}$ with $\beta > 0$, and we conclude that therefore $\alpha > 0$. Since our results for $\alpha > 0$ do not depend sensitively on α – though they are quite different from those obtained with $\alpha = 0$, i.e. the standard case – their paper is a support of our results.

The exact value of α may be a matter of debate, however. From their Fig. 2, Smith and Maeder that conclude $\beta \simeq 0.7$, which leads to $\alpha \simeq 1.0$, while our results fit best to observations for $\alpha \simeq 2.5$. In this context we have to mention that in our present work we discuss all WR subtypes with the exception of late WN (i.e. WNL) stars. In the beginning of Sect. 2 we compiled several arguments in favour of the supposition that WNL stars should not obey the same mass loss law as the other WR subtypes. By inspection of Fig. 2 of Smith and Maeder under this aspect, i.e. omitting the WNL stars, we find that then a value of $\beta \simeq 2$ would fit the remaining data, leading to $\alpha \simeq 2.6$. This is in agreement with our results.