Lecture 1

Overview

Time Scales, Temperature-density Scalings, Critical Masses
I. Preliminaries

The life of any star is a continual struggle between the force of gravity, seeking to reduce the star to a point, and pressure, which holds it up. *Stars are gravitationally confined thermonuclear reactors.*

So long as they remain non-degenerate and have not encountered the pair instability, overheating leads to expansion and cooling. Cooling, on the other hand, leads to contraction and heating. Hence stars are generally stable. The Virial Theorem works.

But, since ideal gas pressure depends on temperature, stars must remain hot. By being hot, they are compelled to radiate. In order to replenish the energy lost to radiation, stars must either contract or obtain energy from nuclear reactions. Since nuclear reactions change their composition, stars must evolve.
The Virial Theorem implies that if a star is neither too degenerate nor too relativistic (radiation or pair dominated),

\[ \frac{GM^2}{R} \sim MN_A kT \]  
(for an ideal gas)

\[ T \sim \frac{GM}{N_A kR} \]

\[ R \sim \left( \frac{3M}{4\pi \rho} \right)^{1/3} \]  
for constant density

So

\[ T \sim \frac{GM^{2/3}}{N_A k} \rho^{1/3} \]

That is as stars of ideal gas contract, they get hotter and since a given fuel (H, He, C etc) burns at about the same temperature, more massive stars will burn their fuels at lower density, i.e., higher entropy.
at death the iron cores of massive stars are somewhat degenerate
Four important time scales for a (non-rotating) star to adjust its structure can be noted. The shortest by far is the time required to approach and maintain hydrostatic equilibrium. Stars not in a state of dynamical implosion or explosion maintain a balance between pressure and gravity on a few sound crossing times. The sound crossing time is typically comparable to the free fall time scale.

\[
\frac{dP}{dr} = -\frac{GM \rho}{r^2}
\]

\[
P_{\text{cent}} \sim \frac{GM \rho}{R}
\]

and

\[
c_s = \gamma \left( \frac{P}{\rho} \right)^{1/2} \sim \left( \frac{GM}{R} \right)^{1/2}
\]

\[
v_{\text{esc}} \sim \left( \frac{GM}{R} \right)^{1/2}
\]

So the escape speed and the sound speed in the deep interior are comparable.
The free fall time scale is \( \sim R/v_{esc} \) so

\[
\tau_{esc} \sim \frac{R}{v_{esc}} = \left( \frac{R^3}{2GM} \right)^{1/2}
\]

\[
\sim \left( \frac{3}{8\pi G \rho} \right)^{1/2}
\]

The number out front depends upon how the time scale is evaluated. The e-folding time for the density is 1/3 of this

\[
\tau_{HD} \sim \left( \frac{1}{24\pi G \rho} \right)^{1/2} \sim \frac{446}{\sqrt{\rho}} \text{ sec}
\]

which is often used by the nuclear astrophysicist
The second relevant adjustment time, moving up in scale, is the thermal time scale given either by radiative diffusion, appropriately modified for convection. This is the time it takes for a star to come into and maintain thermal steady state, e.g., for the energy generated in the interior to balance that emitted in the form of radiation at the surface.

\[ \tau_{\text{therm}} = \frac{R^2}{D_{\text{therm}}} \]

nb. These two time scales, \( \tau_{\text{HD}} \) and \( \tau_{\text{therm}} \) sometimes place limits on the accuracy with which jobs run in one code can be remapped into another (e.g., 1D -> 3D). \( \tau_{\text{KH}} \) can also be important.
The thermal diffusion coefficient (also called the thermal diffusivity) is defined as

\[ D = \left( \frac{\text{conductivity}}{\text{heat capacity}} \right) = \frac{K}{C_p \rho} \]

where \( K \) appears in Fourier's equation

\[ \text{heat flow} = -K \nabla T \]

For radiative diffusion the "conductivity", \( K \), is given by

\[ K = \frac{4acT^3}{3\kappa \rho} \quad \text{(see Clayton 3-12)} \]

where \( \kappa \) is the opacity (cm\(^2\) g\(^{-1}\)), thus

\[ D = \left( \frac{4a c T^3}{3\kappa C_p \rho^2} \right) \]

where \( C_p \) is the heat capacity (erg g\(^{-1}\) K\(^{-1}\))

\( D \) here thus has units cm\(^2\) s\(^{-1}\)

Note that

\[ D \approx \left( \frac{c}{\kappa \rho} \right) \left( \frac{a T^4}{\rho C_p T} \right) = \left( \frac{c}{\kappa \rho} \right) \left( \frac{\text{radiation energy content}}{\text{total heat content}} \right) \]
If radiation energy density is a substantial fraction of the internal energy (not true in the usual case), \( D \sim c/\kappa \rho \) with \( \kappa \) the opacity, and taking advantage of the fact that in massive stars electron scattering dominates so that \( \kappa = 0.2 \) to \( 0.4 \) cm\(^2\) g\(^{-1}\), the thermal time scale then scales like

\[
\tau_{rad} \approx \frac{R^2 \kappa \rho}{c} \sim \left( \frac{0.2 R^2}{c} \right) \left( \frac{3M}{4\pi R^3} \right) \propto \frac{M}{R}
\]

however, massive stars have convective cores so the thermal time is generally governed by the diffusion time in their outer layers. Since the dimensions are still (several) solar radii while the densities are less and the opacity about the same, the radiative time scales are somewhat less than the sun (\( \sim 10^5 \) yr; Mitalas and Sills, ApJ, 401, 759 (1992)).
Possible point of confusion. The radiative diffusion time vs the thermal time. The former is the time for a photon to diffuse out

$$\tau_{\text{rad}} \sim \frac{R^2}{\ell_{mfp} c} \sim \frac{R^2 \kappa \rho}{c}$$

The latter is the time for an appreciable fraction of the heat content to diffuse out. It is longer if most of the heat is in the ions, not the radiation

$$\tau_{\text{thermal}} \sim \tau_{\text{rad}} \left( \frac{\text{total thermal energy in star}}{\text{radiation content of star}} \right)$$

In the sun these differ by a factor of about 100 and the thermal time is of order the Kelvin Helmholtz time

$$\tau_{\text{thermal}} \sim \tau_{\text{rad}} \left( \frac{E_{\text{matter}}}{E_{\text{rad}}} \right) \sim \left( \frac{E_{\text{matter}}}{L_{\text{rad}}} \right) \sim \left( \frac{\alpha GM^2}{2RL_{\text{rad}}} \right) = \tau_{\text{KH}}$$

http://link.springer.com/article/10.1023/A%3A1022952621810
In general a diffusion coefficient is given by

\[ D \sim \frac{1}{3} \nu l \]

where \( \nu \) is a typical speed (light or a convective element) and \( l \) is a characteristic length scale (mean free path or pressure scale height).
A closely related time scale is the Kelvin Helmholtz time scale

\[ \tau_{KH} \approx \frac{GM^2}{RL} \propto \frac{M^{5/3} \rho^{1/3}}{L} \]

i.e. \( R \sim (M / \rho)^{1/3} \)

Except for very massive stars, \( L \) on the main sequence is proportional to \( M \) to roughly the power 2 to 4, and \( \rho \) decreases with \( M \) so the Kelvin Helmholtz time scale is shorter for more massive stars. Note that there are numerous Kelvin Helmholtz time scales for massive stars since they typically go through six stages of nuclear burning. During the stages after helium burning, \( L \) in the heavy element core is given by pair neutrino emission and the Kelvin Helmholtz time scale becomes quite short - e.g. a protoneutron star evolves in a few seconds.
Finally, there is the nuclear time,

\[ \tau_{\text{nuc}} = \left( \frac{1}{\frac{dX}{X \, dt}} \right)^{-1}, \]

where \( X \) is the mass fraction of the chief combustible fuel.

Usually, \( \tau_{\text{HD}} < \tau_{\text{thermal}} < \tau_{\text{KH}} < \tau_{\text{nuc}} \). During the late stages of massive stellar evolution, however, the inequality \( \tau_{\text{thermal}} < \tau_{\text{nuc}} \) actually begins to break down. During “explosive nucleosynthesis” in a supernova, there is near equality between \( \tau_{\text{nuc}} \) and \( \tau_{\text{HD}} \).

The life of a (non-degenerate) star is then typically a series of nuclear burning stages separated by periods of Kelvin-Helmholtz contraction. Hydrostatic equilibrium is maintained throughout the interior and thermal steady state is maintained if \( \tau_{\text{therm}} \) is short enough.

e.g., the sun is in thermal steady state. A presupernova star is not.
$\rho \propto T^3$
## Advanced Nuclear Burning Stages

(e.g., 20 solar masses)

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Main Product</th>
<th>Secondary Products</th>
<th>Temp ((10^9 \text{ K}))</th>
<th>Time ((\text{yr}))</th>
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<td>H</td>
<td>He</td>
<td>(^{14}\text{N})</td>
<td>0.02</td>
<td>10(^7)</td>
</tr>
<tr>
<td>He</td>
<td>C, O</td>
<td>(^{18}\text{O},^{22}\text{Ne})</td>
<td>0.2</td>
<td>10(^6)</td>
</tr>
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<td></td>
<td></td>
<td>s- process</td>
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<td></td>
</tr>
<tr>
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<td>Ne, Mg</td>
<td>Na</td>
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<td>Si</td>
<td>Fe</td>
<td>Ti, V, Cr</td>
<td>3.5</td>
<td>1 week</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mn, Co, Ni</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Final composition of a $25 M_0$ star:
25 $M_\odot$ Presupernova Star (typical for 9 - 130 $M_\odot$)

- 1400 $R_\odot$: H, He
- 0.5 $R_\odot$: He
- 0.1 $R_\odot$: C, O, O, Mg, Ne, He, C
- 0.003 $R_\odot$: Si, S, Ar, Ca, Fe

240,000 $L_\odot$
Examples of Time Scales

• In stellar explosions, the relevant time scale is the hydrodynamic one.

• Explosive nucleosynthesis happens when $\tau_{\text{HD}} \sim \tau_{\text{nuc}}$.

• During the first stages of stellar evolution $\tau_{\text{KH}} < \tau_{\text{nuc}}$. The evolution occurs on a Kelvin Helmholtz time.

• In a massive presupernova star in its outer layers, $\tau_{\text{nuc}} < \tau_{\text{thermal}}$. The outer layers are not in thermal equilibrium with the interior. It also happens in these same stars that $\tau_{\text{nuc}}(\text{core}) < \tau_{\text{KH}}(\text{envelope})$. The core evolves like a separate star.

• Rotation and accretion can add additional time scales. E.g. Eddington Sweet vs nuclear. Accretion vs nuclear. Convection also has its own time scale.
Central Conditions for Polytropes

So long as a star is in hydrostatic equilibrium, it satisfies
\[ \frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}. \]

If the density is assumed to be constant,
\[ \rho = \text{const} = \frac{3M}{4\pi R^3}, \]
direct integration implies
\[ P_c = \frac{1}{2} \frac{GM\rho_c}{R} \propto \frac{M^2}{R^4} \]
\[ \propto GR^2 \rho^2 \]
where here \( \rho_c = \bar{\rho} = \rho. \)

It follows that
\[ \frac{P_c}{\rho_c} = \frac{1}{2} GM \left( \frac{4\pi \rho_c}{3M} \right)^{1/3} \]
and
\[ \frac{P_c^3}{\rho_c^4} = \frac{4\pi}{24} G^3 M^2 \]
for constant density

and hence
\[ m(r) = \text{const}^* r^3 \]
More generally, for a polytrope of index $n$, $P \propto \rho^\gamma$; $\gamma = (n + 1)/n$, see e.g., Clayton, Eq. 2-313

$$P_c = \frac{4\pi R^2 G}{(n + 1)\zeta_1^2} \rho_c^2$$

$$\bar{\rho} = -\frac{3}{\zeta_1} \left( \frac{df}{d\zeta} \right)_{\zeta_1} \rho_c$$

$$= \frac{3M}{4\pi R^3}$$

where $\zeta_1$ is the Emden constant given, e.g., in Table 2.5 of Clayton.

From this it follows that

$$\frac{P_c^3}{\rho_c^4} = \frac{4\pi G^3}{\phi^2} \left( \frac{M}{\phi} \right)^2$$

where $\phi$ is a constant given by solution of the polytropic equation for index $n$,

$$\phi = (n + 1)^{3/2} \zeta_1^2 \left( \frac{df}{d\zeta} \right)_{\zeta_1}$$
\[
\begin{align*}
  n = 0 & \quad \phi = 4.8988 = \sqrt{24} \\
  n = 3 & \quad \phi = 16.145 \\
  n = 3/2 & \quad \phi = 10.73
\end{align*}
\]

Now, if \( P \) is \( P_{\text{ideal}} \) (NR, ND, ionized),

\[
P_{\text{ideal}} = \frac{N_A k}{\mu} \rho T
\]

where \( \mu \) is the mean molecular weight

**Aside: Abundance nomenclature**

In general the mass fraction of a species “\( i \)” is \( X_i \). The number density of \( i \) is then

\[
n_i = \rho N_A \frac{X_i}{A_i}
\]

with \( A_i \) the atomic mass number (integer) of isotope \( i \) and \( N_A \), Avogadro’s number, \( 6.02205 \times 10^{23} \) particles/mole, or approximately the reciprocal mass of the nucleon in grams.

most stars have \( 1.5 < n < 3 \)
In this class we will extensively use the notation

\[ Y_i = \frac{X_i}{A_i} \]

where \( Y_i \) is like a dimensionless number density

\[ Y_i = \frac{n_i}{\rho N_A} \]

Similarly we can define an electron abundance variable

\[ Y_e = \frac{n_e}{\rho N_A} \]

The total gas pressure for an ideal, non-relativistic, non-degenerate ionized gas is then

\[ P_{\text{ideal}} = \rho N_A k T [\Sigma Y_i + Y_e] \]

which implies

\[ \mu = [\Sigma Y_i + Y_e]^{-1} \]

Also the mean atomic weight, \( \bar{A} \), is given

\[ P = \Sigma n_i k T = \frac{N_A k}{\mu} \rho T \]
by

\[ \bar{A} = \frac{\Sigma n_i A_i}{\Sigma n_i} = \frac{\rho N_A \Sigma Y_i A_i}{\rho N_A \Sigma Y_i} = \frac{\Sigma X_i}{\Sigma Y_i} \]

\[ = (\Sigma Y_i)^{-1} \]

Similarly

\[ Y_e = \Sigma Z_i Y_i \]

and

\[ \mu = (\Sigma (1 + Z_i) Y_i)^{-1} \]

Some examples:

a) Pure hydrogen:

\[ Y_H = 1 \quad \bar{A} = 1 \quad Y_e = 1 \]

\[ \mu = (1 + 1)^{-1} = \frac{1}{2} \]

\[ P_{\text{ideal}} = 2\rho N_A kT \]

(The limit \( \mu = 2 \) is achieved as \( \bar{A} \) goes to infinity and \( Z = A/2 \), i.e. electrons dominate)
b) 75% H, 25% He:

\[ \bar{A} = \left( 0.75 + \frac{0.25}{4} \right)^{-1} \]

\[ = 1.23 \]

\[ Y_e = 0.75 + (2)\left(\frac{0.25}{4}\right) \]

\[ = 0.875 \]

\[ \mu = \left[ (1 + 1)(0.75) + (1 + 2)\left(\frac{0.25}{4}\right) \right]^{-1} \]

\[ = 0.5926 \]

\[ P_{\text{ideal}} = 1.69 \rho N_A kT \]

As an exercise to the reader, for pure helium, \( \bar{A} = 4 \), \( Y_e = 0.50 \), \( \mu = 4/3 \), and \( P_{\text{ideal}} = 0.75 \rho N_A kT \). For a mixture of 50% \(^{12}\text{C}\) and 50% \(^{16}\text{O}\), \( \bar{A} = 13.71 \), \( Y_e = 0.50 \) (as it always does for a gas of isotopes having neutron number = proton number), \( \mu \)
\[ P \propto \frac{\rho}{\mu} \]  

\[ T \propto \frac{\mu}{\rho} \]  

\[ \frac{P^2}{\rho^4} \propto M^2 \]

Thus implies for an ideal gas equation of state:

\[ P_c \propto \rho_c \frac{T_c}{\mu} \]

\[ \frac{T_c^3}{\rho_c} \propto M^2 \mu^3 \]

Therefore, for a given temperature, as might be necessary to burn a given fuel, for example, the central density will be lower for a star of higher mass. And, in fact, for a given constant mass and composition, so long as the star closely resembles a single polytrope, and the pressure remains ideal, the central density will scale as

\[ \rho_c \propto \left( \frac{T_c}{\mu} \right)^3 \]

\[ T_c \propto \mu \rho_c^{1/3} \]

For advanced stages of evolution where \( \bar{A} > 1 \), most of the pressure is due to the electrons (and radiation).
The decline of $T^3/\rho$ mostly reflects the fact that beyond H burning the star becomes a red giant and no longer is a single polytrope. $M_{\text{core}}$ is essentially reduced. This more than compensates for the increase in $\mu$. A decreasing $\beta$ also decreases $T^3/\rho$. In the final stages degeneracy becomes important.
Decrease in $\beta$ as star evolves acts to suppress $T^3/\rho$.

Since $\mu$ increases as the fuel burns to heavier ashes, the relation $\rho \propto T^3$ works pretty well at the stars center but tends to be an overestimate. The onset of degeneracy or near relativistic motion of the electrons at high temperature can also cause deviations.

Now, especially for massive stars, the radiation pressure will not be negligible. One traditionally defines a quantity

$$\beta = \frac{P_{\text{ideal}}}{P_{\text{ideal}} + P_{\text{rad}}} = \left( \frac{P_{\text{ideal}}}{P_{\text{total}}} \right)$$

$$P_{\text{tot}} = \frac{P_{\text{ideal}}}{\beta}$$

Then \( P_c^3/\rho_c^4 \propto M^2 \) implies

$$\frac{T_c^3}{\rho_c} \propto M^2 \beta^3 \mu^3,$$

that is, so long as beta doesn’t change much, one gets the same relation as before.
CRITICAL MASSES

Dropping, for now, the explicit dependence on $\mu$ and $\beta$, a contracting protostar of constant mass, or the contracting core of a massive star in between burning stages, so long as that core has an approximately constant polytropic index, will obey $T_c \propto \rho_c^{1/3} M^{2/3}$. Contraction leads to heating. The greater weight of the more compact configuration requires more pressure to hold it up and the pressure rises by increasing both $T$ and $\rho$. A plot of $\log T_c$ vs. $\log \rho_c$ gives a straight line with an upward slope of $1/3$. Lines for larger mass will lie above those for lower mass. As the density grows ever higher, three possibilities emerge: a) collapse to a black hole; b) a dynamical event of some sort (e.g., neutron star formation) or c) the onset of degeneracy. For now, we are most interested in c).
\[ \log T \]

\[ \log \rho \]

\[ \text{slope} = \frac{1}{3} \]

\[ \frac{d\log T}{d\log \rho} \]

\[ M_1 > M_2 > M_3 \]
A completely degenerate gas can be characterized by an equation of state of the form
\[ P_{\text{deg}} = K \gamma (\rho Y_e)^\gamma \] with \( \gamma \) between 4/3 and 5/3.

The case \( \gamma = 4/3 \) has a well known singularity. For an \( n = 3 \) polytrope, which is appropriate here,

\[
\frac{P_c^3}{\rho_c^4} = 4\pi G^3 \left( \frac{M}{16.14} \right)^2 = \frac{K_{4/3}^3 \rho_c^4 Y_e^4}{\rho_c^4}
\]

\[
M = \left( \frac{20.745 K_{4/3}^3}{G^3} \right)^{1/2} Y_e^2
\]

\[ K_{4/3} = 1.244 \times 10^{15} \text{ dyne cm}^{-2} \]

\[
\frac{M}{M_\odot} = 5.80 \ Y_e^2
\]

= 1.45 \( M_\odot \) if \( Y_e = 0.50 \)

neglecting Coulomb corrections and relativistic corrections. 1.39 \( M_\odot \) if they are included.
For lower densities and hence degenerate cores significantly less than the Chandrasekhar mass non-relativistic degeneracy pressure gives another solution ($\gamma = 5/3$)

\[
\frac{P^3_c}{\rho^4_c} = 4\pi G^3 \left( \frac{M}{10.73} \right)^2
\]

\[
\rho_c = \frac{4\pi G^3 M^2}{K^{3/3} Y_e^5 (10.73)^2}
\]

\[
= 4.05 \times 10^6 \left( \frac{0.5}{Y_e} \right)^5 \left( \frac{M}{M_\odot} \right)^2 \text{ g cm}^{-3}
\]

This implies, for each mass, a stable permanent configuration of fixed $\rho_c$ independent of $T$.

This is the well known central density mass relation for (non-relativistic) white dwarfs.
Specifying a core mass gives a maximum temperature achieved before degeneracy supports the star. If that maximum temperature exceeds a critical value, burning ignites

\[ M_1 > M_2 \]

\[ M_i < M_{Ch} \]
He core  
CO Core  
Ne-O Core  

100% He  
50% C - 50% O  
75% O - 25% Ne
### Critical Temperatures

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Main Product</th>
<th>Secondary Products</th>
<th>Temp ($10^9$ K)</th>
<th>Time (yr)</th>
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<td>$10^6$</td>
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<td>s-process</td>
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<tr>
<td>C</td>
<td>Ne, Mg</td>
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<tr>
<td>Ne</td>
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</tr>
<tr>
<td>Si</td>
<td>Fe</td>
<td>Ti, V, Cr, Mn, Co, Ni</td>
<td>3.5</td>
<td>1 week</td>
</tr>
</tbody>
</table>
All stars with main sequence mass above the Chandrasekhar mass could in principle go on to burn Si. In fact, that never happens below 9 solar masses. Stars develop a red giant structure with a low density surrounding a compact core. The convective envelope “dredges up” helium core material and causes it to shrink. Only for stars above about 7 or 8 solar masses does the He core stay greater than the Chandrasekhar mass after helium burning.

These calculations give critical masses:

<p>| | |</p>
<table>
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<tbody>
<tr>
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<td>O</td>
<td>1.39</td>
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<tr>
<td>Si</td>
<td>1.39</td>
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More detailed and physical calculations exist in the literature, see especially Nomoto and Hashimoto (1986). The following should be regarded as standard

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Min. Mass</th>
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<td>O</td>
<td>1.39</td>
</tr>
<tr>
<td>Si</td>
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</tr>
</tbody>
</table>

These are core masses. The corresponding main sequence masses are larger.
Main Sequence Critical Masses

0.08 \( M_\odot \) Lower limit for hydrogen ignition
0.45 \( M_\odot \) helium ignition
7.25 \( M_\odot \) carbon ignition
9.00 \( M_\odot \) neon, oxygen, silicon ignition (off center)
10.5 \( M_\odot \) ignite all stages at the stellar center
\(~ 80 \ M_\odot \) First encounter the pair instability (neglecting mass loss)
\(~ 35 \ M_\odot \) Lose envelope if solar metallicity star

These are for models calculated with the KELER code including semiconvection and convective overshoot mixing but ignoring rotation. With rotation the numbers may be shifted to slightly lower values. Low metallicity may raise the numbers slightly since less initial He means a smaller helium core. Other codes give different results typically to within 1 solar mass.
Between 7.25 and 10.5 solar masses the evolution can be quite complicated and code dependent owing to the combined effects of degeneracy and neutrino losses. Off-center ignition is the norm for the post-carbon burning stages.

Mass loss introduces additional uncertainty, especially with regard to final outcome. Does a 8.5 solar mass main sequence star produce a NeO white dwarf or an electron-capture supernova (TBD).

Above 9 solar masses an iron core eventually forms – on up to the pair instability limit – around 150 solar masses.
Off-Center Ignition

![Graph showing temperature (K) vs. mass (M_☉) with a peak at 1.25 M_☉ indicating off-center ignition.](image)
The death of a star and how it may potentially explode is also very sensitive to:

• **The density structure surrounding the iron core**

• **The rotation rate of the core and that material**

The density structure depends on the entropy of presupernova cores (TBD). Higher entropy cores occur for higher masses and are less degenerate and less centrally condensed.
Density Profiles of Supernova Progenitor Cores

These should be easy to explode

These make the heavy elements
Density Profiles of Supernova Progenitor Cores

These should be easy to explode

These make the heavy elements
Below 13 solar masses very easy to explode.
<table>
<thead>
<tr>
<th>Mass (solar masses)</th>
<th>End point</th>
<th>Remnant</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 7 to ~8</td>
<td>planetary nebula</td>
<td>CO white dwarf</td>
</tr>
<tr>
<td>~8 to ~11</td>
<td>degenerate core low energy SN</td>
<td>Ne-O WD below 9? neutron star above 9</td>
</tr>
<tr>
<td>~11 - ~20</td>
<td>neutrino powered supernova; SN Ibc in binary. Islands of explosion at higher mass</td>
<td>neutron stars and black holes</td>
</tr>
<tr>
<td>20 - 85</td>
<td>without mass loss probably no SN (unless rotationally powered); with mass loss SN Ibc</td>
<td>black hole if enough mass loss neutron star</td>
</tr>
<tr>
<td>85 – 150</td>
<td>pulsational pair SN if low mass loss</td>
<td>black hole</td>
</tr>
<tr>
<td>150 - 260</td>
<td>pair instability SN if low mass loss</td>
<td>none</td>
</tr>
<tr>
<td>&gt; 260</td>
<td>pair induced collapse if low mass loss</td>
<td>black hole</td>
</tr>
</tbody>
</table>