Lecture 10

Nucleosynthesis During Helium Burning and the s-Process

Woosley (2019)	helium stars
evolved at con	stant mass

Mass	Density	Temperature/10 ⁸
3.5	1430	1.75
4.0	1260	1.79
4.5	1140	1.82
6.0	910	1.89
7.0	814	1.92

Z = 0.02. Helium burning at central helium mass fraction of 50%.

At 1.9 x 10⁸ K, the temperature sensitivity of the 3α rate is approximately T^{20} .

A. Thermodynamic Conditions (Massive Stars)

Review: The helium core evolves like a separate star, mass M_{α} $\tau_{He} \sim 10^{6}$ years (+- factor of three); inside RSG, BSG, or as WR star n= 1.5 to 3; assume 3

$$M_{\alpha} = 18 \frac{\sqrt{1-\beta}}{\mu^{2}\beta^{4}} \qquad \mu = \frac{4}{3} \text{ (for pure helium)} \qquad \text{solve for } \beta$$
$$T_{c} = 4.6 \times 10^{6} \,\mu\beta \left(\frac{M_{\alpha}}{M_{\odot}}\right)^{2/3} \rho_{c}^{1/3} \quad \text{K}$$

E.g.,
$$M_{\alpha} = 6$$
 (a 20 M_o main sequence star)

$$\beta = 0.87$$

 $T_c = 1.8 \times 10^8 \left(\frac{\rho_c}{1000 \,\mathrm{g \ cm^{-3}}}\right)^{1/3} \mathrm{K}$

need temperatures > 10^8 to provide significant energy generation by 3α so this really just sets the density.

So, typical temperatures are $1.5 - 2 \ge 10^8$ K (higher in shell burning later). As the core evolves, the temperature and density go up significantly. Note non-degenerate.



Most mass loss occurs during helium core burning for single stars

The helium core itself can grow (single stars) or lose mass (close binary plus winds)

Helium burns in the star's center and later in a shell. Helium burning in the shell is incomplete and makes more carbon relative to oxygen.

During helium burning the star is a red or blue supergiant depending on semiconvection, or a WR star if it loses its envelope. See previous lecture.



Complications:

- If the helium core grows just a little bit towards the end of helium burning, the extra helium convected in greatly decreases the ¹²C synthesis.
- Mass loss from very massive WR stars can greatly increase the synthesis of both ¹²C and ¹⁶O for stars over 35 solar masses
- The uncertain rate for ${}^{12}C(\alpha,\gamma){}^{16}O$
- ¹²C/¹⁶O ratio may be affected by post-helium burning evolution and by black hole formation above some critical main sequence mass. ¹⁶O is made in the more massive (massive) stars.

Massive stars are definitely the site of oxygen nucleosynthesis and contribute appreciably to carbon.

B. Major Nucleosynthesis – C and O

In massive stars after helium burning in the stars center (calculations included semiconvection). Woosley et al (2007)

	M/M_{\odot}	$X(\underline{^{12}C})$	X(16O)	$X(\underline{^{20}Ne})$
In the sun,	12	0.229	0.752	1.48(-3)
$^{12}C/^{16}O = 0.32$	15	0.214	0.765	2.05(-3)
	19	0.202	0.776	3.15(-3)
	25	0.184	0.792	5.62(-3)
	35	0.169	0.801	1.03(-2)

Buchman ${}^{12}C(\alpha,\gamma){}^{16}O$ multiplied bv 1.2.

If the star contains appreciable metals there is, as we shall see also ²²Ne and ¹⁸O.

C. TRACE ELEMENT NUCLEOSYNTHESIS AND NEUTRONIZATION

Prior to the ignition of the 3α reaction the star goes through a brief stage of "nitrogen burning". All the initial CNO in the core has been converted to ¹⁴N which captures an α -particle and experiences a weak decay

> The number of CNO nuclei is $^{14}N(\alpha,\gamma)^{18}F(e^{+}v)^{18}O$ preserved $X(^{14}N) = 14. \cdot Y(^{14}N) = 14. \cdot (Y_i(^{14}N) + Y_i(^{12}C) + Y_i(^{16}O))$ $= 0.0095 (Z / Z_{\odot})$

This reaction is very important both as a source of ¹⁸O in nature and because it creates a neutron excess

$$\eta = 1 - 2Y_e = 1 - 2\sum_{i} Z_i Y_i = \sum_{i} (N_i + Z_i) Y_i - 2\sum_{i} Z_i Y_i$$

$$\eta = \sum_{i} (N_i - Z_i) Y_i - 1 < \eta < 1$$

Prior to this reation $Y_{a} = 0.50$ because both ⁴He and ¹⁴N have equal numbers of neutrons and protons and η =0. After this reaction $\eta > 0$ with a value that depends on the initial metallicity (i.e., initial mass fractions of C, N, and O) of the star.

Before ${}^{14}N(\alpha,\gamma){}^{18}F(e^+\nu){}^{18}O$ the composition was almost entirely ⁴He and ¹⁴N, hence $\eta \approx 0$ (actually a small posive value exists because of ⁵⁶Fe and the like).

After this reaction

 $\eta = 0.00136 \frac{Z}{Z_{\star}}$

During helium core burning, ¹⁸O is later mostly destroyed by ¹⁸O(α, γ)²²Ne, but the neutron excess is unchanged. η can only be changed by weak interactions.

This neutron excess is very important to the subsequent nucleosynthesis and one of the main reasons nucleosynthesis in low Z stars is different!

So one expects that, depending on mass and rates, some but not all of the ²²Ne will burn when the temperature goes up towards the end of helium burning.

The following table gives the temperature at the center of the given model and the mass fractions of ²²Ne, ²⁵Mg, and ²⁶Mg each multiplied by 1000, when the helium mass fraction is 1% and zero

	Μ	T _C	²² Ne	²⁵ Mg	²⁶ Mg
	12	2.42	13.4	0.51	0.61
Woosley,			12.3	1.17	1.05
(2007)	15	2.54	12.7	0.98	0.91
()			11.1	1.99	1.80
	19	2.64	11.5	1.73	1.54
			9.4	2.90	2.87
	25	2.75	9.8	2.67	2.59
			6.96	4.05	4.54
	35	2.86	7.37	3.87	4.22
			4.41	5.18	6.39

The remainder of the ²²Ne will burn early during carbon burning, but then there will be more abundant "neutron poisons", Na and Mg.

These numbers are quite sensitive to the uncertain reaction rate for $^{22}Ne(\alpha,n)^{25}Mg$ and may be *lower limits to the* ²²*Ne* consumption.

During helium shell burning, which does not go to completion, ¹⁸O remains undestroyed and this is the source of ¹⁸O in nature. Convection helps to preserve it

Lifetimes in years $X_{\alpha} = 0.5$, $\rho = 1000$ g cm⁻³

	T ₈ = 1.8	T ₈ = 2.0	
$^{14}N(\alpha,\gamma)$	15	1.1	destroyed
$^{18}\mathrm{O}(\alpha,\gamma)$	2700	98	destroyed in core
22 Ne(α , γ) + (α ,n)	4.1(6)	2.2(5)	partly burned
$^{16}\mathrm{O}(\alpha,\gamma)$	3.5(10)	2.3(9)	survives

Note that the reaction rate for ${}^{22}Ne(\alpha,n){}^{25}Mg$ is uncertain and this is an important source of error in s-process nucleosynthesis. Need all the ²²Ne to burn

D. The s-Process in Massive Stars

Late during helium burning, when the temperature rises to about 3.0 x 10⁸ K, ²²Ne is burned chiefly by the reaction ²²Ne(α ,n)²⁵Mg (with some competition from ²²Ne(α , γ)²⁶Mg).

Where do the neutrons go?

56

Some go on ⁵⁶Fe but that fraction is small:

$$f = \frac{\sigma_{56} Y_{56}}{\Sigma \sigma_i Y_i}$$
16O
$$Y_{16} \approx \frac{0.5}{16} = 3.1 \times 10^{-2}$$

$$Y_{16} \sigma_{16} \approx 1.2 \times 10^{-3}$$
But, ¹⁷O(α ,n)²⁰Ne destroys the ¹⁷O and restores the neutron
22Ne
$$Y_{22} \approx \frac{0.005}{22} = 2.3 \times 10^{-4}$$

$$Y_{22} \sigma_{22} \approx 1.3 \times 10^{-5}$$
For σ in mb
56Fe
$$Y_{56} \approx \frac{0.0013}{56} = 2.3 \times 10^{-5}$$

$$Y_{56} \sigma_{56} \approx 2.7 \times 10^{-4}$$

30 keV neutron capture cross sections

(mostly from Bao et al, ADNDT, 2000)

<u>Nucleus</u>	σ		<u>Nucleus</u>	<u>σ</u>	
	(mb)			(mb)	
^{12}C	0.0154	ŀ	⁵⁴ Fe	27.6	
*16O	0.038		⁵⁶ Fe	11.7	
²⁰ Ne	0.119		⁵⁷ Fe	40.0	
²² Ne	0.059		⁵⁸ Fe	12.1	
²⁴ Mg	3.3		⁵⁸ Ni	41.0	
²⁵ Mg	6.4		⁶⁴ Zn	59	
²⁶ Mg	0.126		⁶⁵ Zn	162	
²⁸ Si	2.9		⁶⁶ Zn	35	
			⁸⁸ Sr	6.2	(closed shell)

The large cross section of ²⁵*Mg is particularly significant since it is made by* ²²*Ne*(*α*,*n*) ²⁵*Mg*. * Igashira et al, *ApJL*, **441**, L89, (1995); factor of 200 upwards revision

Composition of a 25 solar mass star at the end of helium burning compared with solar abundances (Rauscher et al 2001).



and 0.0012 for ⁵⁶Fe and that all ²²Ne burns by (α,n) .

But only 10 - 20% of these actually add to iron. This is about 3 - 6 *neutrons per iron* and obviously not nearly enough to change e.g., Fe into Pb, but the neutron capture cross sections of the isotopes generally increase above the iron group and the solar abundances decrease. A significant s-process occurs that produces significant quantities of the isotopes with A < 90.







Here "s" stands for "slow" neutron capture

$$\tau_{\beta} \ll \tau_{n\gamma} \qquad \tau_{\beta} = \frac{1}{\lambda_{\beta}} = \frac{\tau_{1/2}}{\ln 2} \qquad \frac{dY_i}{dt} = -Y_i Y_n \rho \lambda_{n\gamma}(t) + \dots$$
$$\tau_{n\gamma} = \left(\rho Y_n \lambda_{n\gamma}\right)^{-1} \qquad \tau_{n\gamma} = \left(\frac{1}{Y_i} \frac{dY_i}{dt}\right)^{-1}$$

This means that the neutron densities are relatively small

E.g. for a ²²Ne neutron source

$$\frac{dY_n}{dt} \approx 0 \approx \rho Y_{\alpha} Y(^{22}Ne)\lambda_{\alpha n}(^{22}Ne)$$

$$- \rho Y_n Y(^{25}Mg)\lambda_{n\gamma}(^{25}Mg)$$

$$\rho \approx 1000, X_{\alpha} \approx 0.5, X(^{22}Ne) \approx 0.005, X(^{25}Mg) \approx 0.005$$



	$Y_n \approx \frac{Y_{\alpha} Y(^{22} Ne}{Y(^{25} Mg))}$	$\frac{\lambda_{\alpha n}(^{22}Ne)}{\lambda_{n\gamma}(^{25}Mg)} \sim$	$\frac{Y_{\alpha}\lambda_{\alpha n}(^{22}Ne)}{\lambda_{n\gamma}(^{25}Mg)}$	
	$Y_{\alpha} \sim 0.02 / 4; \ Y(^{22})$	Ne) $\sim 2 Y(^{25}\text{Mg})$; $\rho = 1000$	
T_8	$\lambda_{\alpha n}(^{22}Ne)$	$\lambda_{n\gamma}(^{25}Mg)$	$n_n = \rho N_A Y_n$	$\lambda_{n\gamma}({}^{56}Fe)$
2.0	9.1(-17)	1.2(6)	negligible	
2.5	1.5(-13)	1.1(6)	$\sim 10^{6}$	1.9(6)
3.0	2.6(-11)	1.0(6)	$\sim 10^8$	1.9(6)

Most of the s-process takes place around $T_8 = 2.5 - 3$, so the neutron density is about $10^6 - 10^8$ cm⁻³ (depends on uncertain rate for (α ,n) on ²²Ne and on how much ²²Ne has burned).

At these neutron densities the time between capture, even for heavy elements with bigger cross sections than iron, is days. For ⁵⁶Fe itself it is a few years λ_{re} (⁵⁶Fe) $\approx 2.6 \times 10^6$

Eg. at $T_8 = 2.5 (n_n \sim 10^6)$, the lifetime of ⁵⁶Fe is about

$$\tau({}^{56}Fe) = \left(\frac{1}{Y({}^{56}Fe)}\frac{dY({}^{56}Fe)}{dt}\right)^{-1} = \left(\rho Y_n \lambda_{n\gamma}({}^{56}Fe)\right)^{-1}$$

 \approx 7000 year (at 3 x 10⁸ K it is 7 years)

The s-process in massive stars only goes on during a brief period at the end of helium burning. The time scale is lengthened by convection.

<u>Reaction Rates (n,γ) :</u>

Either measured (Bao et al, ADNDT, 76, 70, 2000) or calculated using Hauser-Feshbach theory (Woosley et al., ADNDT, 22, 371, (1976) Holmes et al., ADNDT, 18, 305, (1976); Rauscher et al. ADNDT, 75, 1, (2000))

The calculations are usually good to a factor of two. For heavy nuclei within $kT\sim 30~keV$ of $Q_{n\gamma}$ there are very many resonances.

Occasionally, for light nuclei or near closed shells, direct capture is important: e.g., ¹²C, ^{20,22}Ne, ¹⁶O, ⁴⁸Ca







If there were locations where steady state is achieved then there

$$\frac{dn_A}{d\tau} \approx 0 = n_A \sigma_A - n_{A-1} \sigma_{A-1}$$

i.e., σn is locally a constant, and $n \propto \frac{1}{\sigma}$

Attaining steady state requires a time scale longer than a few times the destruction lifetime of the species in the steady state group. One has "local" steady state because any flux that would produce, e.g., lead in steady state would totally destroy all the lighter s-process species.

The flow stagnates at various "waiting points" along the *s*-process path, particularly at the closed shell nuclei.

Rate Equations: Their Solutions and Implications

Assume constant density, temperature, cross section, and neutron density and ignore branching (would never assume any of these in a modern calculation). Then for each A there is only one nucleus on the s-process path and

$$\frac{dY(^{A}Z)}{dt} \equiv \frac{dY_{A}}{dt} = -Y_{A}Y_{n}\rho\lambda_{n\gamma}(A) + Y_{A-1}Y_{n}\rho\lambda_{n\gamma}(A-1)$$

and since $n_n = \rho N_A Y_n$ and $\lambda_{n\gamma} = N_A \langle \sigma_{n\gamma} v \rangle \approx N_A \sigma_A v_{thermal}$ defining $\tau \equiv \int \rho N_A Y_n v_{thermal} dt = \int n_n v_{thermal} dt$, one has $\frac{dn_A}{d\tau} = -n_A \sigma_A + n_{A-1} \sigma_{A-1}$



Eg., $n_n \sim 10^8 \,\mathrm{cm}^{-3} \Rightarrow \rho Y_n \sim n_n / N_A \sim 10^{-16}$

 $\lambda_{n\gamma}$ experimentally at helium burning temperatures is $10^5 - 10^8$

$$\tau_{n\gamma} = \left(\rho Y_n \lambda_{n\gamma}\right)^{-1} = \left(\frac{d \ln Y_A}{dt}\right)^{-1} \sim 3 - 3000 \text{ years}$$

This can be greatly lengthened in a massive star by convection.

As a result nuclei with large cross sections will be in steady state while those with small ones are not. This is especially so in He shell flashes in AGB stars where the time scale for a flash may be only a few decades.

Implicit solution:

Assuming no flow downwards from A+1 and greater to A and below. (replace dn_A by $n_{new}(A)$ - $n_{old}(A)$ and n_A and n_{A-1} by $n_{new}(a)$ and $n_{new}(A-1)$)

$$\frac{dn_A}{d\tau} = -n_A \sigma_A + n_{A-1} \sigma_{A-1}$$
$$n_{new}(A) = \frac{n_{old}(A)/d\tau + n_{new}(A-1)\sigma_{A-1}}{1/d\tau + \sigma_A}$$

This is useful because in the do loop, $n_{new}(A-1)$ is updated to its new value before evaluating $n_{new}(A)$. Matrix inversion reduces to a recursion relation.



Sample output from toy model code micros2.f Start with n(30) = 1 and run for tau = 50



e.g., ¹¹⁷Sn, ¹¹⁸Sn, ¹¹⁹Sm, and ¹²⁰Sn are s,r isotopes. Sn is not a good place to look for σ n = const though because it is a closed shell.

Termination of the s-Process





 ^{147}Pm beta decays to ^{147}Sm in 2.6 years so the path is $^{146}\text{Nd}(n,\!\gamma)^{147}\text{Nd}(e\bar{\nu})^{147}\text{Pm}(e\bar{\bar{\nu}})^{147}\text{Sm}.$



5	imilarly	for Sm	or(mb)	n	ση
	1449m	9	9229	7.42(-3)	0.65
4	147 Sm	r, 5	1000 \$ 100	3.71 (-2)	37 ± 4
2=62	148 Sm	5	267 ±12	2.70(-2)	7.2 ± 0.3 ←
	149 Sm	r, s	14541 66	3.32(-2)	48.3 ± 2.2
	150 Sm	5	447 1 26	1,79(-2)	\$.0. ± 0.5 +
	152 Sm	5, r	375123	6.41(-2)	24.2 ± 1.5
	194 Sm	٣	293±19	5.45(-2)	16.0 ± 1.0

 $\tau_{1/2}(^{151}\text{Sm}) = 90 \text{ years}; \ \tau_{1/2}(^{153}\text{Sm}) = 46 \text{ hours}$

Based upon the abundances of s-only isotopes and the known neutron capture cross sections one can subtract the s-portion of s,r isotopes to obtain the r- and s-process yields separately.

The correlation between anomalies in isotopic abundances of ^{142}Nd and the Sm content, observed in meteorites, has provided evidence for extinct $^{146}Sm, \ t_{1/2} \ 6.8 \times 10^7$ yr decays by alpha emission to $^{142}Nd)$



http://www.nndc.bnl.gov/chart/

See

Producing the full s-process requires a distribution of neutron exposures. This precludes making the entire s-process in massive stars where there is only one (small) exposure.





The symbols are s-only nuclei. The solid lines are the model results for a standard (exponential) set of exposure strengths. Below A = 90 there is evidence for a separate additional s-process component.



mass number A. The solid curve is the calculated result of an exponential distribution of anutrus exposures. [P. A. Seeger, W. A. Fueler, and D. D. Cloyton, Astrophys. J. Suppl., 11:121 (1965). By permission of The University of Chicogo Press. Copyright 1965 by The University of Chicogo.]





Thermally populating 1st excited state of str -ray greatly accelerate its decay

Other T-sensitive branch points:

⁹³Zr, ¹¹³Cd, ¹³⁴Cs, ¹⁵¹Sm, ¹⁶³Dy, ¹⁷⁰Tm ¹⁷⁶Lu, ¹⁶³Ho, ⁷⁹Se, ⁸⁰Br, ¹⁵²Eu, ¹⁵⁴Eu, ¹⁸⁵W These can be analysed to estimate the temperature of the S-process.

12 0.514

3/2-

85 Rb

0.149

But where is the heavy s-process made?



Fig.2 s-process neutron density as a function of s-process temperature. Gensistant values for the main and weak component lie within the indicated areas [8]. The area of the weak component is hatched. The fact that there is no common range of values for the two components is interpreted as a strong avidence for their independence.



Figure 1 The variation with time of the location (in mass) of the base of the convective envelope and of the center of the helium-burning shell during the second dredge-up phase in a model of mass $S M_{\odot}$ and initial composition (Y, Z) = (0.28, 0.001).



Thin shell instability:

 $\frac{dP}{dm} = \frac{GM(R)}{4\pi R^4}$

If a shell is sufficiently thin, its pressure is set by the gravitational potential in which it rests. During a flash, this does not change. Burning raises the temperature but does not change the pressure so the density in the shell goes down. But the energy generation is very sensitive to the temperature and continues to go up. Finally sufficient burning occurs to cause enough overall expansion to reduce the pressure. This instability tends to happen at the edges of compact objects where the burning shells are quite thin compared with the radius of the core and the gravitational potential is thus constant.

Schwarzshild & Harm (1965) and Weigert (1966), Yoon, Langer and van der Sluys, *A&A*, **425**, 207 (2004)



The metallicity history of the s-process can be quite complicated.

In the simplest case in massive stars with a ²²Ne neutron source it is independent of metallicity because the neutron density is set by a balance between ²²Ne(a,n)²⁵Mg and ²⁵Mg(n,g)²⁶Mg. If protons are mixed into the helium burning core then a strong s-process can result.

At very low Z things can become complicated because of the effect of neutron poisons, ¹²C and ¹⁶O. In AGB stars the mixing between H and He shells is Z dependent. Some metal poor stars are actually very s-process rich. Cristallo et al (ApJ, 696, 797 (2009)). Important modification:

s-process giants derived from AGB stars in the solar neighborhood do not show the large ²⁶Mg excesses one would expect if the neutron source were ²²Ne(α ,n)²⁵Mg [as it surely is in massive stars]. Moreover these stars are too low in mass for ²²Ne(α ,n)²⁵Mg to function efficiently. A different way of making neutrons is required. Probably

 ${}^{4}He(2\alpha,\gamma){}^{12}C(p,\gamma){}^{13}N(e^{+}\nu){}^{13}C$ ${}^{13}C(\alpha,n){}^{16}O$

with the protons coming from mixing between the helium burning shell and the hydrogen envelope. Each p mixed in becomes an n.

McWilliam and Lambert, *MNRAS*, **230**, 573 (1988) and Malaney and Boothroyd, *ApJ*, **320**, 866 (1987) Hollowell and Iben, *ApJL*, **333**, L25 (1988); *ApJ*, **340**, 966, (1989) many more since then