Neutrino Losses and Advanced Stages of Stellar Evolution - I

The late stages (> helium burning) of evolution in massive stars are characterized by large luminosities, carried away predominantly by neutrinos, and consequently, by short evolutionary time scales.

The nuclear physics can become quite complicated because of the presence of many species and occurrence of many reactions at high temperature.

Woosley, Heger, and Weaver (2002)
Rev. Mod. Phys., 74, 1016.

Thermal Neutrino Emission

Fowler and Hoyle, ApJS, 2, 201, (1964)
Chiu in Stellar Physics p 259-282

In nature, both of the following weak interactions follow all the necessary conservation laws:

\[ \nu_e + e^- \rightarrow \nu_e + e^- \]
\[ e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e \]

In about 1970 it was realized that neutral currents could lead to additional reactions and modifications of the rates of old ones. Where one had \( \nu_e \), one could now have \( \nu_e, \nu_\mu, \) or \( \nu_\tau \).

(Dicus, Phys. Rev D, 6, 941, (1972)).

Thus, with different coupling constants

\[ e^+ + e^- \rightarrow \nu_\mu + \bar{\nu}_\mu \] or \( \nu_\tau + \bar{\nu}_\tau \)

also

\[ \nu + \bar{\nu} \rightarrow \nu + \bar{\nu} \] for \( \nu_e, \nu_\mu, \nu_\tau \)

a coherent process with large cross section

Stellar Neutrino Energy Losses

(see Clayton p. 259ff, especially 272ff; Fowler and Hoyle 1964, eq. 3)

1) Pair annihilation - dominant in massive stars

\[ kT \geq 10\% \ m_e c^2 \ (T_e > 0.5) \quad \text{i.e., post helium burning} \]

\[ \sigma_\nu = \frac{G^2}{3\pi} \left( \frac{\hbar}{m_e c} \right)^2 \left[ \left( \frac{E}{m_e c^2} \right)^2 - 1 \right] \]

\( E \) includes electron rest mass

\[ \frac{\hbar}{m_e c} = \text{Compton wavelength}/2\pi \text{ of } e^- = 3.86 \times 10^{-11} \text{ cm} \]

\[ E^2 = m_e^2 c^4 + p^2 c^2 = \gamma^2 m_e^2 c^4 \quad p = m_e v; \ v \text{ is the relative velocity} \]

\[ G^2 = 3.00 \times 10^{-12} \text{ (dimensionless)} \quad \text{of the pair} \quad c \]

\[ \sigma_\nu = 1.42 \left( \frac{c}{v} \right) \left[ \left( \frac{E}{m_e c} \right)^2 - 1 \right] \times 10^{48} \text{ cm}^2 \]
Aside:

\[
\sigma_{\pm} = \frac{G_n^2 c}{3\pi \nu} \left( \frac{\hbar}{m_e c} \right)^2 \left( \frac{E}{m_e c^2} \right)^2 - 1
\]

\[
= \left( \frac{G_n \hbar c}{e^2} \right)^2 \left( \frac{e^2}{m_e c^2} \right) \left( \frac{E}{m_e c^2} \right)^2 - 1
\]

Thomson cross section for pair annihilation

\[
\left( \frac{G_n \hbar c}{e^2} \right)^2 = (137 \times G_n)^2 \sim 10^{-19}
\]

In 1941 Gamow and Schonberg proposed this neutrino loss mechanism as the cause for core collapse in massive stars. Hoyle 46 said photodisintegration

Clayton (Chap 4) and Lang in Astrophysical Formulae give some approximations (not corrected for neutral currents)

**NDNR**

\[
P_\pm = 4.9 \times 10^{15} T_9^3 \exp(-11.86/T_9) \text{ erg cm}^{-3} \text{ s}^{-1}
\]

\[T_9 < 2\]

\[
2m_e c^2 / kT
\]

**NDR**

\[
P_\pm = 4.6 \times 10^{15} T_9^3 \text{ erg cm}^{-3} \text{ s}^{-1}
\]

(better is \(3.2 \times 10^{15}\))

\[T_9 > 3, \text{ but not too bad at } T_9 > 2 \text{ (fac 2)}\]

Note origin of \(T^9\):

If \(n_\pm\) is relativistic, \(n_\pm \propto T^3\) (like radiation)

\[
\sigma \propto \frac{E^2}{v} \propto (kT)^2 \frac{1}{v} \quad (3 \text{ pages back})
\]

energy carried per reaction \(\sim kT\)

\[
P_\pm = \left( T^8 \right) \left( T^2 \right) \left( T \right) = T^9
\]

\[
n_\pm n_\pm \sigma v E
\]

\[v \text{ cancels } 1/v \text{ in } \sigma\]

These formulae are very crude; factor of 2 at best. For more accurate results use subroutine neut01.f on the class website.

Want energy loss per cm\(^3\) per second. Integrate over thermal distribution of \(e^+\) and \(e^-\) velocities. These have, in general, a Fermi-Dirac distribution.

\[
P_\pm = n_\pm n_e < \sigma v E >
\]

\[E = \text{total energy including rest mass}\]

\[
n_\pm = \frac{1}{\pi^2} \left( \frac{m_e c}{\hbar} \right)^3 \int_{W}^{W} W \left( W^2 - 1 \right) dW \exp(\theta W \pm \phi)^{-1}
\]

\[\theta = \frac{m_e c^2}{kT} = 5.93 / T_9 \quad W = \frac{E}{m_e c^2} \quad \text{c/m energy}\]

\[\phi = \text{Chemical potential/kT}\]

(determined by the condition that \(n_+ - n_- = n_e \) (matter) = \(\rho N_A Y_e\))

More frequently we use the energy loss rate per gram per second

\[
\varepsilon_v = \frac{P}{\rho} \text{ erg gm}^{-3} \text{ s}^{-1}
\]

In the non-degenerate limit \(\varepsilon_v\) from pair annihilation declines as \(\rho^{-1}\).

In degenerate situations, the filling of phase space suppresses the creation of electron-positron pairs and the loss rate plummets. Usually pair annihilation neutrino emission dominates other processes when the matter is non-degenerate. This includes most of the advanced stages of stellar evolution (especially when electron capture on nuclei is negligible).
2) Photoneutrino process: (Clayton p. 280)

\[ e^- + \gamma \rightarrow e^- + \nu + \bar{\nu} \]

Analogue of Compton scattering with the outgoing photon replaced by a neutrino pair. The electron absorbs the extra momentum. This process can be marginally significant during helium and carbon burning.

*When non-degenerate and non-relativistic*

\( P_{\text{photo}} \) is proportional to the density (because it depends on the electron abundance) and \( \dot{\epsilon}_{\text{photo}} \) is independent of the density. At high density, degeneracy blocks the phase space for the outgoing electron. Also for relativistic electrons the density dependence is weaker.

3) Plasma Neutrino Process: (Clayton 275ff)

This process is important at high densities where the plasma frequency is high and \( \hbar \omega_{\text{plasma}} \) can be comparable to \( kT \). This limits its applicability to essentially white dwarfs, and to a lesser extent, the evolved cores of massive stars. It is favored in degenerate environments.

A "plasmon" is a quantized collective charge oscillation in an ionized gas. For our purposes it behaves like a photon with rest mass.

note that units here are erg cm\(^{-3}\) s\(^{-1}\) not erg g\(^{-1}\) s\(^{-1}\)
Per unit volume pair rate is roughly independent of density until degeneracy cuts it off.
A photon of any energy in a vacuum cannot decay into $e^+$ and $e^-$ because such a decay would not simultaneously satisfy the conservation of energy and momentum (e.g., a photon that had energy just equal to 2 electron masses, $h\nu = 2m_ee^2$, would also have momentum $h\nu/c = 2m_ec$, but the electron and positron that are created, at threshold, would have no kinetic energy, hence no momentum. Such a decay is only allowed when the photon couples to matter that can absorb the excess momentum.

The common case is a $\gamma$-ray of over 1.02 MeV passing near a nucleus, but the photon can also acquire an effective mass by propagating through a plasma.

$$\gamma_{\text{plasmon}} \rightarrow e^+ + e^-$$

Separate cloud of $N_e$ electrons into two pieces separated by $r$

$$F = m_e a = \frac{N_e \sqrt{2} e^2}{r} = \frac{4\pi^3 n_e e^2}{3}$$

$$\tau = \left(\frac{2r}{a}\right) = \left(\frac{2r m_6}{4\pi n_e e^2}\right) \left(\frac{3m}{4\pi n_e e^2}\right) = \frac{2\pi}{\omega}$$

$$\omega \propto \left(\frac{n_e e^2}{m}\right)^{\frac{1}{2}}$$

Consider a neutral plasma, consisting of a gas of positively charged ions and negatively charged electrons. If one displaces by a tiny amount all of the electrons with respect to the ions, the Coulomb force pulls back, acting as a restoring force.

If the electrons are cold it is possible to show that the plasma oscillates at the plasma frequency.

$$ND \quad \omega_p = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} = 5.6 \times 10^4 \frac{1}{n_e^{1/2}}$$

$$D \quad \omega_p = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} \left[1 + \frac{h}{m_e c} \left(\frac{3\pi^2 n_e}{2}\right)^{2/3} \frac{2e_r}{m_e c^2}\right]^{1/2}$$

For moderate values of temperature and density, raising the density implies more energy in the oscillations and raising the temperature excites more oscillations. Hence the loss rate increases with temperature and density.

However, once the density becomes so high that, for a given temperature $h\omega_p > kT$, raising the density still further freezes out the oscillations. The thermal plasma no longer has enough energy to excite them. The loss rate plummets exponentially.

$$a) \quad h\omega_p \leq kT$$

$$P_{\text{plasma}} = 7.4 \times 10^{32} \left(\frac{h\omega_p}{m_e c}\right)^{1/3} \left(\frac{kT}{m_e c^2}\right) \exp(\omega / T) \text{ erg cm}^{-3} \text{ s}^{-1}$$

$$b) \quad h\omega_p \gg kT$$

$$P_{\text{plasma}} = 3.3 \times 10^{32} \left(\frac{h\omega_p}{m_e c}\right)^{1/3} \left(\frac{kT}{m_e c^2}\right)^{3/2} \exp(-h\omega_p / kT) \text{ erg cm}^{-3} \text{ s}^{-1}$$
This is a relevant temperature for Type Ia supernovae and the red line a relevant density.

The late stages of stellar evolution are accelerated by (pair) neutrino losses.

4) Ordinary weak interactions – neutrinos from the decay of unstable nuclei

- Beta-decay
- Electron capture
- Positron emission

Electron capture – and to a lesser extent beta-decay can be very important in the final stages of stellar evolution – especially during silicon burning and core collapse.

Typically these are included by studying each nucleus individually, its excited state distribution, distribution of weak strength, etc. The results are then published as fitting functions at f(T,ρ).


**Carbon Burning**
Approximate initial conditions:

As we shall see, the temperature at which carbon burns in a massive star is determined by a state of balanced power between neutrino losses by the pair process and nuclear energy generation. This gives $8 \times 10^8$ K for carbon core burning. Burning in a shell is usually a little hotter at each step, about $1.0 \times 10^9$ K for carbon burning.

Assuming that $T^3/\rho$ scaling persists at the center, and that helium burned at $2 \times 10^8$ K and 1000 gm cm$^{-3}$, this implies a carbon burning density around a few $\times \ 10^5$ gm cm$^{-3}$.

Initial composition:

The initial composition is the ashes of helium burning, chiefly C and O in an approximate 1 : 4 ratio (less carbon in more massive stars).

There are also many other elements present in trace amounts:

- $^{22}$Ne, $^{25,26}$Mg from the processing of CNO elements in He-burning
- The light s-process
- Traces of other heavy elements present in the star since birth
- Up to $\sim 1\%$ $^{20}$Ne from $^{16}$O($a$,g)$^{20}$Ne during He-burning

Principal nuclear reaction

\[
^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg}^* \rightarrow ^{23}\text{Mg} + n - 2.62 \text{ MeV} \\
\rightarrow ^{20}\text{Ne} + \alpha + 4.62 \text{ MeV} \\
\rightarrow ^{23}\text{Na} + p + 2.24 \text{ MeV}
\]

Measured to about 2.5 MeV and S-factor is overall smooth but shows poorly understood broad “structures” at the factor of 2 level. See Rolfs and Rodney, p 419 ff - alpha cluster? Not seen in $^{16}$O + $^{16}$O
Many important secondary reactions:

- $^{20}\text{Ne}(\alpha,\gamma)^{24}\text{Mg}$
- $^{23}\text{Na}(\alpha,p)^{26}\text{Mg}$
- $^{23}\text{Na}(p,\alpha)^{20}\text{Ne}$
- $^{26}\text{Mg}(p,\gamma)^{27}\text{Al}$
- $^{23}\text{Mg}(\alpha,n)^{25}\text{Mg}$
- $^{25}\text{Mg}(n,\gamma)^{26}\text{Mg}$
- $^{20}\text{Ne}(p,g)^{21}\text{Na}$
- $^{21}\text{Na}(p,\alpha)^{20}\text{Ne}$
- $^{21}\text{Na}(e^+n)^{21}\text{Ne}$
- $^{22}\text{Na}(e^+n)^{22}\text{Ne}$

and dozens (hundreds?) more

There are also some important weak interactions that can change the neutron excess $\eta$.

- The neutron branch of $^{12}\text{C} + ^{12}\text{C}$ itself makes $^{23}\text{Mg}$. At lower temperature this decays by $^{23}\text{Mg}(e^+n)^{23}\text{Na}$. At higher temperature it is destroyed by $^{23}\text{Mg}(n,p)^{23}\text{Na}$. The former changes $\eta$; the latter does not, so there is some temperature, hence mass dependence of the result.

- $^{20}\text{Ne}(\alpha,\gamma)^{21}\text{Na}(e^+n)^{21}\text{Ne}$

- $^{21}\text{Na}(p,\gamma)^{22}\text{Na}(e^+n)^{22}\text{Ne}$

Together these reactions can add - a little - to the neutron excess that was created in helium burning by $^{14}\text{N}(\alpha,\gamma)^{18}\text{F}(e^+\nu)^{18}\text{O}$ or, in stars of low metallicity they can create a neutron excess where none existed before.
Principal Nucleosynthesis in carbon burning:

$^{20,21}\text{Ne}$, $^{23}\text{Na}$, $^{24,25,26}\text{Mg}$, $^{26,27}\text{Al}$, and to a lesser extent, $^{29,30}\text{Si}$, $^{31}\text{P}$

The $^{16}\text{O}$ initially present at carbon ignition essentially survives unscathed. There are also residual products from helium burning – the s-process, and further out in the star H- and He-burning continue.

A typical composition going into neon burning – major abundances only would be

- 70% $^{16}\text{O}$
- 25% $^{20}\text{Ne}$
- 5% $^{24}\text{Mg}$
- and traces of heavier elements

D. Energy Generation

Suppose we make $^{20}\text{Ne}$ and $^{24}\text{Mg}$ in a 3:1 ratio (approximately solar)

$$ 7\left(^{12}\text{C}\right) \rightarrow 3\left(^{20}\text{Ne}\right) + ^{24}\text{Mg} $$

$$ \epsilon_{\text{nuc}} = 9.65 \times 10^{17} \sum \left(\frac{dY_i}{dt}\right) \text{BE}_i \text{ erg g}^{-1} \text{ s}^{-1} $$

$$ \frac{dY(20\text{Ne})}{dt} = \frac{3}{7} \frac{dY(^{12}\text{C})}{dt} $$

$$ \frac{dY(24\text{Mg})}{dt} = -\frac{1}{7} \frac{dY(^{12}\text{C})}{dt} $$

$$ \frac{dY(^{12}\text{C})}{dt} = -2 \rho Y^2(^{12}\text{C}) \lambda_{12,12}/2 $$

$$ \epsilon_{\text{nuc}} = 9.65 \times 10^{17} \left[ -\frac{3}{7} (160.646) - \frac{1}{7} (198.258) + 1(92.160) \right] \frac{dY(^{12}\text{C})}{dt} $$

$$ \epsilon_{\text{nuc}} = 4.84 \times 10^{18} \rho Y^2(^{12}\text{C}) \lambda_{12,12} \text{ erg g}^{-1} \text{ s}^{-1} $$

where $\lambda_{12,12}$ was given a few pages back.
The total energy released during carbon burning is

\[ q_{\text{mc}} = 9.65 \times 10^{17} \sum \Delta Y_i (BE_i) \]

\[ \Delta Y_{12} = \frac{1}{12} \Delta X_{12} \]

\[ \Delta Y_{20} = -\frac{3}{7} \Delta Y_{12} \]

\[ \Delta Y_{24} = -\frac{1}{7} \Delta Y_{12} \]

\[ q_{\text{mc}} = 4.03 \times 10^{17} \Delta X_{12} \text{ erg g}^{-1} \]

Since \( \Delta X_{12} \ll 1 \), this is significantly less than helium burning (\( 10^{18} \text{ erg g}^{-1} \))

For carbon burning \( u = 2 \quad s = 30 \)

neutrino losses \( u = 0 \quad s \sim 16 \)

\[ \langle \epsilon_{\nu} \rangle / \langle \epsilon_{\text{mc}} \rangle = 1 = \left( \frac{\epsilon_{\nu,0}}{16^{3/2}} \right) / \left( \frac{\epsilon_{\text{mc},0}}{36^{3/2}} \right) \]

\[ \Rightarrow \epsilon_{\text{mc},0} = 3.4 \epsilon_{\nu,0} \]

---

**E. Balanced Power**

Averaged over the burning region, which is highly centrally concentrated

\[ \langle \epsilon_{\text{mc}} \rangle = \langle \epsilon_{\nu} \rangle \quad \text{since} \quad L_{\nu} = \int \epsilon_{\nu} \, dM \gg L_{\gamma} \]

Neutrino losses in carbon burning are due to pair annihilation.

Near \( T = 1 \) the non-relativistic, non-degenerate formula applies and \( \epsilon_{\nu} \) is approximately proportional to \( T^{10} \) (at \( \rho \sim 10^5 \text{ gm cm}^{-3} \))

Fowler and Hoyle (1964) showed that averaged over an \( n = 3 \) polytrope a density and temperature sensitive function has an average:

\[ \langle \epsilon \rangle = \frac{\int \epsilon \, dM}{\int dM} = \epsilon_{\nu} \frac{3.2}{(3u + s)^{1/2}} \]

where \( \epsilon_{\nu} \) is the central value of \( \epsilon \), and \( \epsilon \propto \rho^{-1} T^{4/3} \)

Energy is also provided by the Kelvin-Helmholz contraction of the core and this decreases the ignition temperature just a little. In more massive stars where \( X(^{12}\text{C}) \) is less than about 10%, carbon burning and neon burning at the middle generate so little energy that the core never becomes convective. The carbon and neon just melt away without greatly exceeding the neutrino losses.

Further out in a shell, the burning temperature is higher (set by the gravitational potential at the bottom of the shell - similar energy generation has to come from less fuel set by the pressure scale height).
assuming the burning density scales as $T^3$

Carbon core burning not centrally convective in more massive stars.

Convection

F. Approximate lifetime

$$\tau = \left( \frac{q\Delta X(C)}{\epsilon_{\text{nuc}}} \right) = \left( \frac{5 \times 10^{17} \cdot 0.2}{2 \times 10^7} \right) = 5 \times 10^7 \text{ s} = 1500 \text{ years}$$

But

- Hotter in more massive stars
- Gets shorter as temperature rises during C burning
- Lengthened by convection

See problem set 3
G. How long does it take the envelope to adjust its structure? Its Kelvin Helmholtz time.

\[ \tau_{KH} = \frac{GM_{\text{core}}M_{\text{env}}}{(R/2)(L/2)} \]

For 15 M_\odot \cdot M_{\text{core}} = 4 M_\odot \cdot 10 M_\odot \cdot R = 3 \times 10^{13} \text{ cm}

\[ L = 3 \times 10^{38} \]

\[ \tau_{KH} = \frac{\left(6.7 \times 10^{-6}\right)\left(8 \times 10^{33}\right)\left(2 \times 10^{34}\right)}{\left(1.5 \times 10^{13}\right)\left(1.5 \times 10^{38}\right)} = 150 \text{ years} \]

The actual time is more like several thousand years to go from e.g., a red supergiant to a blue one.
The atmosphere does not contract uniformly. The inner part contracts to 10^{11} - 10^{12} cm while the surface stays near its original value. Should use that smaller radius in the above estimate.

### Burning Stages in the Life of a Massive Star

<table>
<thead>
<tr>
<th>Stage</th>
<th>T_9</th>
<th>Radius</th>
<th>L_\gamma</th>
<th>L_\nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-burn</td>
<td>0.03</td>
<td>4.36(11)</td>
<td>1.06(38)</td>
<td>7.0(36)</td>
</tr>
<tr>
<td>He-burn</td>
<td>0.18</td>
<td>3.21(13)</td>
<td>1.73(38)</td>
<td>7.4(36)</td>
</tr>
<tr>
<td>C-ign</td>
<td>0.50</td>
<td>4.76(13)</td>
<td>2.78(38)</td>
<td>7.1(37)</td>
</tr>
<tr>
<td>C-dep</td>
<td>1.2</td>
<td>5.64(13)</td>
<td>3.50(38)</td>
<td>3.5(41)</td>
</tr>
<tr>
<td>O-dep</td>
<td>2.2</td>
<td>5.65(13)</td>
<td>3.53(38)</td>
<td>3.8(43)</td>
</tr>
<tr>
<td>Si-dep</td>
<td>3.7</td>
<td>5.65(13)</td>
<td>3.53(38)</td>
<td>2.3(45)</td>
</tr>
<tr>
<td>PreSN</td>
<td>7.6</td>
<td>5.65(13)</td>
<td>3.53(38)</td>
<td>1.9(49)</td>
</tr>
</tbody>
</table>

### Neon Burning

Following carbon burning, at a temperature of about 1.5 x 10^9 K, neon is the next abundant nucleus to burn. It does so in a novel “photodisintegration rearrangement” reaction which basically leads to oxygen and magnesium (nb. not ^{20}\text{Ne} + ^{20}\text{Ne} burns to ^{40}\text{Ca})

\[ 2^{(20}\text{Ne}) \rightarrow ^{16}\text{O} + ^{24}\text{Mg} + \text{energy} \]

The energy yield is not large, but is generally sufficient to power a brief period of convection. It was overlooked early on as a separate burning stage, but nowadays is acknowledged as such.

The nucleosynthetic products resemble those of carbon burning but lack ^{23}\text{Na} and have more of the heavier nuclei, ^{26},^{27}\text{Al}, ^{28,30}\text{Si}, and ^{31}\text{P}. 

### Table of Nuclear Burning Stages

<table>
<thead>
<tr>
<th>Burning Stage</th>
<th>T_9 (K)</th>
<th>M (M_\odot)</th>
<th>L (L_\odot)</th>
<th>R (R_\odot)</th>
<th>\rho (g cm^{-3})</th>
<th>t (Myr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td></td>
<td>1.15</td>
<td>1.39</td>
<td>0.11</td>
<td>1.37</td>
<td>0.09</td>
</tr>
<tr>
<td>Helium</td>
<td></td>
<td>0.25</td>
<td>0.71</td>
<td>3.1</td>
<td>6.7</td>
<td>0.01</td>
</tr>
<tr>
<td>Oxygen</td>
<td></td>
<td>0.35</td>
<td>1.41</td>
<td>5.7</td>
<td>7.8</td>
<td>0.01</td>
</tr>
<tr>
<td>Silicon</td>
<td></td>
<td>0.43</td>
<td>1.7</td>
<td>8.1</td>
<td>3.3</td>
<td>0.01</td>
</tr>
<tr>
<td>Carbon</td>
<td></td>
<td>0.50</td>
<td>1.89</td>
<td>11.4</td>
<td>1.39</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The table above shows the mass, luminosity, radius, and density of the burning stages in the life of a massive star.
**A. Basics:** The composition following carbon burning is chiefly $^{16}\text{O}$, $^{20}\text{Ne}$, $^{24}\text{Mg}$ but $^{16}\text{O}$ is not the next to burn (influence of $Z = N = 8 = \text{magic}$)

<table>
<thead>
<tr>
<th>Species</th>
<th>$S_o$(MeV)</th>
<th>energy required to remove an $\alpha$-particle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}\text{O}$</td>
<td>7.16</td>
<td></td>
</tr>
<tr>
<td>$^{20}\text{Ne}$</td>
<td>4.73</td>
<td></td>
</tr>
<tr>
<td>$^{24}\text{Mg}$</td>
<td>9.32</td>
<td></td>
</tr>
</tbody>
</table>

Before the temperature becomes hot enough for oxygen to fuse ($T_o = 1.8$ as we shall see), photons on the high energy tail of the Bose-Einstein distribution function begin to induce a new kind of reaction -

$^{20}\text{Ne}(\gamma,\alpha)^{16}\text{O}$

The $\alpha$-particle “photo-disintegrated” out of $^{20}\text{Ne}$ usually just adds back onto $^{16}\text{O}$ creating an “equilibrated link” between $^{16}\text{O}$ and $^{20}\text{Ne}$. Sometimes though an $\alpha$ captures on $^{20}\text{Ne}$ to make $^{24}\text{Mg}$. When this happens the equilibrium between $^{16}\text{O}$ and $^{20}\text{Ne}$ quickly restores the $\alpha$ that was lost.

**B. Photodisintegration Reaction Rates**

At high temperatures, the inverse reaction to radiative capture, $[\gamma(n,\gamma),(p,\gamma),(\alpha,\gamma)]$ becomes important as there exists an appreciable abundance of $\gamma$-rays out on the tail of the Bose-Einstein distribution that have energy in excess of several MeV. The reactions these energetic photons induce are called photodisintegration reactions – the major examples being $(\gamma,n),(\gamma,p)$, and $(\gamma,\alpha)$

Consider

$I + j \rightarrow L + \gamma$

and

$L + \gamma \rightarrow I + j$

The net result is that $2\left(^{20}\text{Ne}\right) \rightarrow ^{16}\text{O} + ^{24}\text{Mg}$ at a rate that is determined by how fast $^{20}\text{Ne}$ captures alpha particles from the equilibrium concentration set up by $^{16}\text{O}$ and $^{20}\text{Ne}$.

**Other secondary reactions:**

$^{24}\text{Mg}(\alpha,\gamma)^{28}\text{Si}$

$^{25}\text{Mg}(\alpha,n)^{28}\text{Si}$

$^{26}\text{Mg}(\alpha,n)^{30}\text{Si}$

$^{30}\text{Si}(p,\gamma)^{31}\text{P}$

etc.

**Products:**

some more $^{16}\text{O}$ and $^{24}\text{Mg}$, $^{29,30}\text{Si}$, $^{31}\text{P}$, $^{26}\text{Al}$ and a small amount of s-process.

In equilibrium, the abundances must obey the Saha equation

For the reaction $I + j \rightleftharpoons L + \gamma$

$$
\frac{n_jn_I}{n_L} = \left( \frac{g_Ig_L}{g_j} \right) \left( \frac{A_IA_J}{A_L} \right)^{3/2} \frac{2\pi kT}{h^2 N_A} \exp(-Q_{ij} / kT)
$$

(derivable from considerations of entropy and the chemical potential and the fact that the chemical potential of the photon is zero). Thus, in equilibrium (a more stringent condition than "steady state")

$$
\left( \frac{n_jn_I}{n_L} \right) = 5.942 \times 10^{33} \ T_o^{3/2} \left( \frac{g_I g_L}{g_j} \right) \left( \frac{A_IA_J}{A_L} \right)^{3/2} \ exp(-11.60485 Q_{ij} / T_o)
$$

for $Q_{ij}$ measured in MeV
Equilibrium in the reaction \( I + j \rightleftharpoons L + \gamma \) also implies
\[
Y_i Y_j \rho \lambda_{ij}(I) = Y_L \lambda_{ij}(L)
\]
and since \( Y_i = \frac{n_i}{\rho N_A} \)
\[
\frac{n_i n_j}{n_L} = \frac{\rho N_A Y_i Y_j}{Y_L} = \frac{\lambda_{ij}(L) N^4}{\lambda_{ij}(I)} = 5.942 \times 10^{13} T_9^{3/2} \left( \frac{g_i g_j}{g_L} \right) \left( \frac{A_i A_j}{A_L} \right)^{3/2} e^{-11.6048 Q_{ij}/T_9}
\]
\[
\lambda_{ij}(L) = \lambda_{ij}(I) \cdot 5.942 \times 10^9 T_9^{3/2} \left( \frac{g_i g_j}{g_L} \right) \left( \frac{A_i A_j}{A_L} \right)^{3/2} \exp(-11.6048 Q_{ij}/T_9)
\]

Where 9.686 \times 10^9 = \frac{5.942 \times 10^{13}}{N_A}

And since
\[
\frac{dY^{(20\text{Ne})}}{dt} = -2 Y^{(20\text{Ne})} \rho \lambda_{\alpha Y}^{(20\text{Ne})} \equiv -2f
\]
and
\[
\frac{dY^{(24\text{Mg})}}{dt} = +f \quad \frac{dY^{(16\text{O})}}{dt} = +f
\]

\[
\lambda_{\alpha Y}^{(20\text{Ne})} = 9.87 \times 10^9 T_9^{3/2} \left( \frac{16 \cdot 4}{20} \right)^{3/2} \lambda_{\alpha Y}^{(16\text{O})} e^{-11.6048 - 4.73/T_9}
\]
\[
Y_\alpha = 5.65 \times 10^{10} T_9^{3/2} \left( \frac{Y^{(20\text{Ne})}}{\rho Y^{(16\text{O})}} \right) e^{-54.89/T_9}
\]
\[
\epsilon_{\text{nuc}} = 9.65 \times 10^{17} \left[ \frac{1}{2} (198.258) + \frac{1}{2} (127.62) - 160.646 \right] \cdot 2f
\]

\[
\epsilon_{\text{nuc}} = \left( 9.65 \times 10^{17} \right) (2.29) 2f \text{ erg g}^{-1} \text{ s}^{-1}
\]

\[
\epsilon_{\alpha Y} = 2.49 \times 10^{29} T_9^{3/2} \left( \frac{Y^{(20\text{Ne})}}{Y^{(16\text{O})}} \right) \lambda_{\alpha Y}^{(20\text{Ne})} e^{-54.89/T_9} \text{ erg g}^{-1} \text{ s}^{-1}
\]

\[\text{Independent of density}\]

\[
q_{\text{nuc}} = 9.65 \times 10^{17} \frac{1.29}{20} \Delta X^{(20\text{Ne})}
\]

\[q_{\text{nuc}} = 1.1 \times 10^{17} \Delta X^{(20\text{Ne})} \text{ erg g}^{-1}\]

\[\text{Relatively small}\]
Example

\[ X(^{16}\text{O}) = 0.7 \quad X(^{20}\text{Ne}) = 0.2 \quad \rho \sim 10^6 - 10^7 \]

Near \( T_9 = 1.5 \) \( \lambda_{\text{v} \gamma} (^{20}\text{Ne}) \approx 3.43 \times 10^{-3} T_9^{10.5} \)

\[ \varepsilon_{\text{nuc}} \propto T_9^{1.5} T_9^{10.5} \exp(-54.89/T_9) \sim T_9^{49} \quad \text{very temperature sensitive} \]

Above approximation for \( \lambda_{\text{v} \gamma} \Rightarrow \)

\[ \varepsilon_{\text{nuc}} \approx 3.3 \times 10^{10} \left( \frac{T_9}{1.5} \right)^{49} \text{ erg g}^{-1} \text{ s}^{-1} \]

---

**Balanced Power During Neon Burning**

<table>
<thead>
<tr>
<th>( T_9 )</th>
<th>( \varepsilon_{\text{nuc}} )</th>
<th>( \varepsilon_{v} (\rho=10^9) )</th>
<th>( \varepsilon_{v} (\rho=10^7) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>1.1(9)</td>
<td>8.9(9)</td>
<td>2.9(8)</td>
</tr>
<tr>
<td>1.5</td>
<td>3.3(10)</td>
<td>2.2(10)</td>
<td>7.8(8)</td>
</tr>
<tr>
<td>1.6</td>
<td>7.8(11)</td>
<td>4.9(10)</td>
<td>2.0(9)</td>
</tr>
</tbody>
</table>

\[ \varepsilon_{\text{nuc}} = \varepsilon_{v} \left[ \frac{3u + s}{3u + s} \right]_{\text{nuc}}^{3/2} \varepsilon \propto \rho^{\nu-1} T^\eta \]

\[ = \varepsilon_{v} \left( \frac{3 + 49}{13} \right)^{3/2} = 8.5 \varepsilon_{v} \]

The density is closer to \( 10^7 \) than \( 10^6 \) at least in a \( 15 \text{ M}_\odot \) star so neon burns at about \( 1.4 - 1.5 \text{ GK} \) (somewhat higher in a higher mass star or a shell where the density is less)

---

**Lifetime Burning Neon**

At a temperature between \( T_9 = 1.5 \)

\[ \tau \sim q_{\text{nuc}} \Delta X(^{20}\text{Ne}) / \varepsilon_{\text{nuc}} = \frac{1.1 \times 10^{17} \cdot 0.2}{10^9} = 2 \times 10^6 \text{ s} \]

This is lengthened by convection – if it occurs – because of high \( T \) sensitivity. Typically \( \sim \text{ years} \).
Nucleosynthesis from neon burning

The principal nuclei with major abundances at the end of neon burning are $^{16}$O and $^{24}$Mg. Most of the neutron excess resides in $^{25,26}$Mg. Most of the $^{16}$O has in fact survived even since helium burning.

In terms of major production of solar material, important contributions are made to

$^{16}$O, $^{24,25,26}$Mg, $^{26}$Al, $^{29,30}$Si, and $^{31}$P

Oxygen Burning:

After neon burning the lightest nucleus remaining with appreciable abundance is $^{16}$O. This not only has the lowest Coulomb barrier but because of its double magic nature, has a high $\alpha$-particle separation energy. It is the next to burn.

Because of its large abundance and the fact that it is a true fusion reaction, not just a rearrangement of light nuclei, oxygen burning releases a lot of energy and is a very important part of the late stages of stellar evolution in several contexts (e.g., pair-instability supernovae).

It is also very productive nucleosynthetically. It’s chief products being most of the isotopes from $^{28}$Si to $^{40}$Ca as well as (part of) the $p$-process.

Initial composition:

$^{16}$O, $^{24}$Mg, $^{28}$Si

Nuclear reactions:

$^{16}$O + $^{16}$O $\rightarrow$ (32$^S$)* $\rightarrow$ $^{31}$S + n + 1.45 MeV 5%

$\rightarrow$ $^{30}$P + d − 2.41 MeV ≤ 5%

$\rightarrow$ $^{31}$P + p + 7.68 MeV 56%

$\rightarrow$ $^{28}$Si + $\alpha$ + 9.59 MeV 34%

The deuteron, $d$, is quickly photodisintegrated into a free neutron and proton.
3) Onset of "quasi-equilibrium" clusters

e.g. \( ^{28}\text{Si} + n \leftrightarrow ^{29}\text{Si} + \gamma \quad ^{29}\text{Si} + p \leftrightarrow ^{30}\text{P} + \gamma \) etc.


4) Weak interactions increase \( \eta \) markedly during oxygen core burning (much less so during oxygen shell burning where the density is less and the time scale shorter).

\[ ^{33}\text{S}(e^-,\nu_e)\rightarrow^{33}\text{P} 
^{37}\text{Ar}(e^-,\nu_e)\rightarrow^{37}\text{Cl} \]

\[ ^{35}\text{Cl}(e^-,\nu_e)\rightarrow^{35}\text{S} \]

---

Nuclear energy generation
Approximation \( 2(^{16}\text{O}) \rightarrow ^{32}\text{S} + 16.54 \text{ MeV} \)
(More correctly \( ^{28}\text{Si}, ^{32}\text{S}, ^{36}\text{Ar}, ^{40}\text{Ca} \) in approximate proportions 10:5:1:1)

\[
\frac{dY_{16}}{dt} = -2 \rho \gamma_{16}^2 \lambda_{16,16} / 2 \\
\frac{dY_{32}}{dt} = -2 \frac{dY_{16}}{dt} \\
\frac{dY_{40}}{dt} = -2 \frac{dY_{32}}{dt} \\
\Delta X_{16} = 9.65 \times 10^{17} \rho \gamma_{16}^2 \lambda_{16,16} \left[ \frac{1}{2} \frac{1}{8.271 \text{ MeV}} \right] \\
 \approx 6 \times 10^4 \rho \left( \frac{T_9}{2} \right)^{35} \frac{X_{16}}{0.5} \text{ erg g}^{-1} \text{ s}^{-1} \\
\rho \approx 4 \times 10^6 \text{ g cm}^{-3}
\]

\[ q_{\text{nuc}} = 9.65 \times 10^{17} \left( \frac{8.271 \text{ MeV}}{16} \right) \Delta X_{16} = 5.0 \times 10^{17} \Delta X_{16} \text{ erg g}^{-1} \]

\( \Delta X_{16} \) can be large

---

**Diagram**: Oxygen burns at about 2.0 GK
Lifetime is approximately \( q/\epsilon_{\text{nuc}} \sim 3 \times 10^8 \text{ s} \)
(lengthened by convection)
Inner 1 M\_\odot of a 25 M\_\odot model near the time of central oxygen depletion (X(\^{16}\text{O}) = 0.04). Neutron excess \(\eta = 0.0073\). Too large to make solar abundances. This matter must stay behind in the neutron star.

Nucleosynthesis in Si shell where neutronization has been less.

\(^{28}\text{Si}, ^{32,33}\text{S}, ^{35,37}\text{Cl}, ^{36,38}\text{Ar}, ^{39,41}\text{K}, ^{40,42}\text{Ca}, ^{40,42}\text{Ti}, ^{50}\text{Cr}\) some p-process

Element-wise: Si, S, Ar, Ca in roughly solar proportions.

Destruction of the s-process

Si-shell 25 M\_\odot presupernova star (region just outside the iron core, \(\eta = 0.0018\) to \(0.0028\). Much more solar-like pattern. This will be altered by explosive nucleosynthesis.

They [the atoms] move in the void and catching each other up and jostle together, and some recoil in any direction that may chance, and others become entangled with one another in various degrees according to the symmetry of their shapes and sizes and positions and order, and they remain together and thus the coming into being of composite things is effected.

Solving the wave equation in a plasma

\[ m_e \ddot{\vec{r}} = e \vec{E} \quad J \equiv n_e e \dot{\vec{r}} \]

\[ \frac{\partial J}{\partial t} = \frac{\partial}{\partial t} (n_e e \vec{r}) = n_e e \dot{\vec{r}} = n_e e \left( \frac{e \vec{E}}{m_e} \right) \]

combine a plane wave \( \vec{E} = E_0 \exp(i(kx - \omega t))\hat{x} \)

which satisfies the wave equation

\[ \nabla^2 \vec{E} = -k^2 E_0 \exp(i(kx - \omega t))\hat{x} \quad \text{with} \]

\[ \nabla^2 \vec{E} = -\frac{\partial^2 \vec{E}}{c^2 \frac{\partial^2 \vec{E}}{\partial t^2}} = \frac{4\pi}{c^2} \frac{\partial J}{\partial t} \]

(e.g., http://scienceworld.wolfram.com/physics/PlasmaFrequency.html)

gives

\[ \nu_{\text{phase}}^2 = \frac{\omega^2}{k^2} = c^2 \left( 1 - \frac{4\pi n_e e^2}{m_e \omega_p^2} \right)^{-1} \]

Undefined if \( \omega < \omega_p \)