Lecture 13

Presupernova Models, Core Collapse and Bounce

How a massive star dies is determined by its presupernova structure, especially that of its inner 2 solar mass core*, its composition, density and temperature (entropy) contours, and rotation rate. These in turn depend on the properties of the ZAMS star, especially its mass and angular momentum when it was born, and all that happened along the way.

* For He cores below about 40 $M_\odot$, ZAMS below about 100 $M_\odot$

Rotation – some limits.

The moment of inertia of a cold neutron star is approximately

$$I = 0.4 M R^2 = 0.4 (1.4) (2 \times 10^{32}) (1.0 \times 10^{26})^2 = 1.1 \times 10^{45} \text{ erg s}$$

The rotational energy is $\frac{1}{2} I \omega^2$, so the rotational frequency corresponding to a typical supernova kinetic energy, $10^{51}$ erg, is 1500 rad s$^{-1}$, or a period of 4 ms.

But during the time the explosion would develop, $t < 1$ s, the radius of the hot protoneutron star is larger, 20 - 50 km, so the requisite final period is even smaller.

Discussion of ms magnetars deferred.
In a calculation that included current approximations to all known mechanisms of angular momentum transport in the study, the final angular momentum in the iron core of a 10 solar mass star when it collapsed was $5 \times 10^{47}$ erg s.

This corresponds to a pulsar period of 17 ms, just a bit less than the Crab is believed to have been born with.

Spruit (2006) suggests modifications to original model that may result in still slower spins.

Therefore---
The explosion of the Crab
SN was probably not (initially) powered by rotation. The explosion was weak although historical accounts suggest that it was very bright. The observed explosion energy was $\sim 10^{50}$ erg.

Core compactness (a measure of preSN density structure):


Characterize possibility of an explosion based upon the compactness parameter, $\zeta$, of the preSN model

$$\zeta_M = \frac{2.5}{R(M_{bary} = 2.5M_\odot) / 1000 \text{ km}}$$

If $\zeta$ is big, R is small and the 2.5 solar mass point lies close in. The star is hard to explode. Based upon a series of 1D models they find stars with $\zeta$ over 0.45 are particularly difficult to explode.

$\zeta(\text{explosion}) < 0.45$

maybe too high – 0.25?

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O’Connor and Ott (2011) original plot

Results up here sensitive to poorly known mass loss rates

Black holes - (low metallicity, i.e., low mass loss)

Sukhbold, Woosley and Heger (2018)

\[ \mu_4 = \frac{d\ln(M_{\text{enc}})}{dr} \text{ (1000 km)} \]

Mu4M4 is the mass location of the point with entropy 4.0 (usually the edge of the Si core) and \( \mu_4 \) is the gradient of the enclosed mass there. When \( m_4 \) is large the density gradient is shallow. Small \( \mu_4 \) corresponds to small accretion rate and small \( \mu_4 M_4 \) to high accretion luminosity.
Specific Cases

Lower mass stars produce degenerate cores of carbon and oxygen or oxygen, magnesium and neon. All three models shown here are red supergiants with very low density extended hydrogenic envelopes.

If mass loss removes the envelope, one gets white dwarfs. If not, and the CO core grows to $1.27 \ M_\odot$, one gets edge lit carbon burning that converts the core to Ne, O and Mg. Further core growth to $1.38 \ M_\odot$ leads to collapse.
Presupernova density profiles of stars with ZAMS masses 8.75, 9.25, 9.5, 9.6, and 9.7 solar masses. These are essentially white dwarfs inside of loosely bound envelopes. They should explode easily. Al but the 8.75 M\(_\odot\) model are evaluated at iron core collapse. The 8.75 M\(_\odot\) model has a NeOMg of 1.345 M\(_\odot\) and a central density of 2 \times 10^9 g cm\(^{-3}\).

**ELECTRON-CAPTURE SUPERNOVAE**

\(e.g., M_\alpha = 2.2 M_\odot\) i.e., main sequence mass ="10 M\(_\odot\)" (Nomoto et al) (but dredge up reduces \(M_\alpha\) substantially in a calculation where the envelope is carried, so maybe main sequence mass \(\sim 8.5 M_\odot\))

O, Ne, Mg core develops - residual of carbon burning, but not hot enough to ignite Ne or O burning. Degenerate core (may) grow by thin helium shell burning. \(M \to 1.375 M_\odot\) if envelope not lost

\[^{24}Mg(e^-,\nu_e)^{24}Na, \quad ^{20}Ne(e^-,\nu_e)^{20}Na \quad \text{reduce } \rho \uparrow \text{ runaway collapse} \]

\[\Delta(^{24}Mg) = -13.933 \quad \Delta(^{24}Na) = -8.417 \quad Q_{ec} = -5.52 \text{ MeV} \]

\[\varepsilon_f = 11.1 \left(\rho_{10}/Y_e\right)^{1/3} \text{ MeV} \]

**ELECTRON-CAPTURE SUPERNOVAE**

At about \(1 - 2 \times 10^{10} \text{ g cm}^{-3}\), ignite oxygen burning, but matter is already falling in. Very degenerate runaway. Burn to iron group (NSE) but \(kT < \varepsilon_{\text{ Fermi}}\). No appreciable overpressure. Instead capture electrons on Fe group nuclei. Collapse accelerates.

Oxygen burning continues, but in a thin shell through which matter is falling supersonically. Collapse continues to nuclear density without ever having formed a large iron core.

A similar evolution may occur for accreting Ne-O white dwarfs (or very rapidly accreting CO-white dwarfs) in binary systems - an alternate outcome to Type Ia supernovae. This phenomena in a binary is generally referred to as “Accretion Induced Collapse (AIC)”.

Once the collapse is well underway, the outcome does not vary appreciably from what one would expect for a collapsing iron core of the same (zero temperature Chandrasekhar) mass.

The energy release from oxygen burning and silicon burning is small compared with the gravitational potential at which the burning occurs

\[ M_{\text{MS}} = 8.5 \ M_\odot \]
\[ M_{\text{He}} = 2.2 \ M_\odot \]


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Note: For models from 1.7 to 2.4 \( M_\odot \), conditions are given at the last model calculated and are not the terminal values. CO and NeO indicate the major constituents of the core at that time. Were the envelope not lost, continued growth of the core to the Chandrasekhar mass would lead to electron-capture supernovae in all cases from 1.8 to 2.5 \( M_\odot \) model.

Helium stars with mass loss

Explosion \( \sim 10^{30} \ erg \), basically the neutrino wind. Very little Ni or heavy elements ejected.

Faint supernova(?)

Star of \( \sim \) 10 solar masses suggested as progenitor of the Crab nebula by Nomoto et al. (1982, Nature, 299, 803)

Observed for Crab: KE = 0.6 to 1.5 \( x 10^{30} \ erg \) in 4.6+- 1.8 solar masses of ejecta (Davidson and Fesen 1985)
"FLAME" STARS (9.0 – 10.5 Solar Masses)

Due to plasma neutrino losses which increase rapidly with the density, a temperature inversion develops. Neon, oxygen and silicon burning ignite off center and burn inwards in "convectively bounded flames".

Convectively Bounded Flame (e.g., Timmes et al (1994))

Flame speed:
\[
\frac{\ell^2 k \rho}{c} \propto \frac{C_N T}{\epsilon_{\text{nuc}}} \Rightarrow \ell \sim \left( \frac{c C_N T}{k \rho \epsilon_{\text{nuc}}} \right)^{1/2}
\]

\[
V_{\text{flame}} \sim \frac{\ell}{\tau} \sim \left( \frac{c \epsilon_{\text{nuc}}}{k \rho C_M T} \right)^{1/2}
\]

\( T \) is fixed by convection.

Flame speed:

\[
\tau_{\text{cond}} \sim \tau_{\text{burn}}
\]

\[
\ell^2 k \rho \propto \frac{C_N T}{\epsilon_{\text{nuc}}} \Rightarrow \ell \sim \left( \frac{c C_N T}{k \rho \epsilon_{\text{nuc}}} \right)^{1/2}
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\]

\( \ell \) is cm to m s\(^{-1} \)

Single stars 6.5 - 12.0 M\(_{\odot}\) (Woosley and Heger 2015)

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Convectively bounded oxygen flame

Later in the same star silicon ignites with a powerful degenerate flash!

\[ T_9 = 3.2 \]
\[ \rho_9 = 0.47 \]
\[ \eta_9 = 9.30 \]

Afterwards silicon burns in as a second convectively bounded flame

Eventually an iron core forms and collapses to a neutron star
Single stars 10.5 solar masses and above ignite all post-helium burning stages in their centers without violent flashes (KEPLER)

E.g. Sukhbold and Woosley (2014)

Top: Carbon, neon, and oxygen burning

Bottom: Silicon burning. x-axis is log time until iron core collapse.

The convective burning shells occur in different places and times for different mass stars and “sculpt” the density structure around the final iron core.

Overview M > 10.5

- All stars up to very large values ($M_{\text{He,He}} \sim 65$) ignite all 6 fuels – H, He, C, Ne, O, Si – in their centers and burn to completion. The corresponding ZAMS mass depends on mass loss and rotation.

- The larger the star, the greater the mass of heavy elements, $Z > 2$, it produces. Stars lighter than 10.5 don’t contribute much nucleosynthesis (even though they explode easily).

- Generally, but not necessarily monotonically, and depending on mass loss, bigger stars are harder to blow up and will tend to leave black hole remnants.

- The iron core mass sets a lower bound on the baryonic mass of the compact remnant. It collapses as a unit, has steep density gradients at its edge and is composed of isotopes that the solar abundances tell us are rarely ejected.
Overview M > 10.5

- The explosion mechanism and nucleosynthesis are most sensitive to the mass of the helium core when the star dies. This will end up meaning that stars with the same initial mass have a different final fate in close binaries.

- Rotation may become a more important consideration to the explosion mechanism for the more massive stars. It probably (my opinion) is not very important for supernovae the mass of the Crab or even the most common supernovae. The importance of rotation depends on the rotation rate of the initial star, its mass loss history (and hence metallicity), and some uncertain physics previously discussed (e.g., magnetic torques).

- The following discussion should apply to most common core collapse supernovae — both Type II and Ibc. “Unusual” supernovae will be discussed separately. 15 M\(_\odot\) is often taken to be typical.

Stars of larger mass have thicker, more massive shells of heavy elements surrounding the iron core when it collapses.

Note that the final masses of the 15 and 25 solar mass main sequence stars are nearly the same — owing to mass loss.

Distribution of collapse velocity and \(Y_e\) (solid line) in the inner 2.5 solar masses of a 15 solar mass presupernova star. A collapse speed of 1000 km/s anywhere in the iron core is a working definition of "presupernova". The cusp at about 1.0 solar masses is the extent of the first convective core silicon burning.
**Core Collapse**

Once the collapse is fully underway, the time scale becomes very short. The velocity starts at $10^6$ cm s$^{-1}$ (definition of the "presupernova") and will build up to at least $c/10 = 30,000$ km s$^{-1}$ before we are through. Since the iron core only has an initial radius of 5,000 to 10,000 km, the next 0.2 seconds are going to be very interesting.

Neutrino Trapping

Trapping is chiefly by way of elastic neutral current scattering on heavy nuclei. Freedman, PRD, 9, 1389 (1974) and Janka (2017) give the cross section

$$
\sigma_{\text{coh}} = a_0^2 N^2 \left( \frac{\epsilon_v}{\text{MeV}} \right)^2 \sigma_0,
$$

and since $\kappa = n \sigma = \frac{\rho N A}{A} \sigma$

$$
\kappa_{\text{coh}} = a_0^2 N \sigma \left( \frac{\epsilon_v}{\text{MeV}} \right)^2 \times 1.76 \times 10^{-44} \text{ cm}^2 \text{ gm}^{-1}
$$

$$
= 5.3 \times 10^{-19} a_0^2 \left( \frac{N}{50} \right) \left( \frac{\epsilon_v}{\text{MeV}} \right)^2 \text{ cm}^2 \text{ gm}^{-1}
$$

$$
\kappa_{\text{coh}} = 2.7 \times 10^{-20} \left( \frac{N}{50} \right) \left( \frac{\epsilon_v}{\text{MeV}} \right)^2 \text{ cm}^2 \text{ gm}^{-1}
$$

if one takes $a_0^2 = \sin^2(\theta_w) = (0.229)^2 = 0.0524$

Janka (2017)

Neutrino Emission from Supernovae

PreSN

$\rho_c = 10^{12}$ is central density. Average and density at neutrinosphere is less

Generic description but could be 15 $M_\odot$ Janka article is on website

$\epsilon_v = 1.11(\rho Y) \times 10^{-13} \text{ MeV}$

$\sim 20 \text{ MeV}$ at $\rho = 10^3 \text{ g cm}^{-3}$

$\left( 3 \times 10^{-20} \right) \left( 100 \right) \rho \left( 10^3 \right) \sim 1 \Rightarrow \rho \sim 10^{11} \text{ g cm}^{-3}$

Therefore neutrino trapping will start when $\kappa \rho \sim 1$, $E_v = 10 \text{ MeV}$ $R \sim 10^7 \text{ cm}$

$\left( 3 \times 10^{-20} \right) \left( 100 \right) \rho \left( 10^7 \right) \sim 1 \Rightarrow \rho \sim 10^{11} \text{ g cm}^{-3}$

From this point on the neutrinos will not freely stream but, increasingly, will diffuse. Neutrino producing reactions will be inhibited by the filling of neutrino phase space. The total lepton number

$$
Y_L = Y_e + Y_\nu
$$

will be conserved, not necessarily the individual terms. At the point where trapping occurs $Y_L = Y_e \sim 0.37$. At bounce $Y_e \sim 0.29; Y_\nu \sim 0.08.$
Bounce

Up until approximately nuclear density the structural adiabatic index of the collapsing star is governed by the leptons – the electrons and neutrinos, both of which are highly relativistic. Hence it is nearly $\Gamma = 4/3$.

As nuclear density is approached however, the star first experiences the attractive nuclear force and $\Gamma$ goes briefly but dramatically below $4/3$.

At still higher densities, above $\rho_{\text{nuc}}$, the repulsive hard core nuclear force is encountered and abruptly $\Gamma >> 4/3$.

Throughout the collapse, nuclei stay, for the most part, bound, but above nuclear density it makes sense to talk of individual nucleons again.

$1$ MeV = $11.6$ billion K

$\rho = 1.66 \times 10^{15} n (\text{fm}^{-3}) \text{ g cm}^{-3}$

$= (N_a \times 10^{-39} \text{)}^{-1} n$
The portion of the core that collapses together is called the "homologous core". It collapses subsonically (e.g., Goldreich & Weber, ApJ, 238, 991 (1980); Yahil ApJ, 265, 1047 (1983)). This is also approximately equivalent to the "sonic core".

This part of the core is called homologous because it can be shown that within it, \( v_{\text{collapse}} \) is proportional to radius. Thus the homologous core collapses in a self-similar fashion. Were \( \Gamma = 4/3 \) for the entire iron core, the entire core would contract homologously, but because \( \Gamma \) becomes significantly less than 4/3, part of the inner core pulls away from the outer core.

As the center of this inner core approaches and exceeds \( \rho_{\text{nuc}} \), the resistance of the nuclear force is communicated throughout its volume by sound waves, but not beyond its edge. Thus the outer edge of the homologous core is where the shock is first born. Typically, \( M_{\text{HC}} = 0.6 \text{–} 0.8 \text{ solar masses.} \)

The larger \( M_{\text{HC}} \) and the smaller the mass of the iron core, the less dissipation the shock will experience on its way out.

Relevant Physics To Shock Survival

Photodisintegration:

As the shock moves through the outer core, the temperature rises to the point where nuclear statistical equilibrium favors neutrons and protons over bound nuclei or even \( \alpha \)-particles

\[
q_{\nu} (^{56}\text{Fe} \rightarrow 26p,30n) = 9.65 \times 10^{11} \left( \frac{492.26 \text{ MeV}}{56} \right) \\
= 8.5 \times 10^{38} \text{ erg cm}^{-2} \\
= 1.7 \times 10^{41} \text{ erg/0.1 M}_\odot
\]

Neutrino losses

Especially as the shock passes to densities below \( 10^{12} \text{ g cm}^{-3} \), neutrino losses from behind the shock can rob it of energy. Since neutrinos of low energy have long mean free paths and escape more easily, reactions that degrade the mean neutrino energy, especially neutrino-electron scattering are quite important. So too is the inclusion of \( \mu \)– and \( \tau \)–flavored neutrinos.
It is generally agreed that the so called “prompt shock mechanism” – worked on extensively by Bethe, and colleagues in the 1980’s – does not work. The shock fails and becomes in a short time (< 10 ms) an accretion shock. What happens next depends on the transport of energy by neutrinos.

Collapse and bounce in a 13 solar mass supernova. Radial velocity vs. enclosed mass at 0.5 ms, +0.2 ms, and 2.0 ms with respect to bounce. The blip at 1.5 solar masses is due to explosive nuclear burning of oxygen in the infall (Herant and Woosley 1996).