Lecture 6

$p+p$, Helium Burning and Energy Generation

Proton-proton reaction:

\[ p(p,e^+\nu_e)^2H \quad (+0.42\text{MeV}) \]

see Lecture 4 for energy yield

This cross section is far too small (~10^{-47} \text{cm}^2 \text{ at } 1 \text{ MeV})
to measure in the laboratory, but it does have a nearly constant,calucable S-factor.

The theory is straightforward, but complex(e.g., Clayton 366 - 368) because it includes a strong interaction and weak interaction happening in rapid succession.

Two stages:

- Temporarily form diproton (initial wave function can be
  probed experimentally with proton scattering). Initial diproton must
  have J = 0 because can’t have protons in identical states.

- Diproton experiences a weak interaction (with a
  spin flip) to make deuteron $^2\text{H}(J^p=1^+)$

We shall be terse in our discussion of this reaction, chiefly
because it involves a lot of concepts we have not discussed so far
(weak decays, axial/vector currents, etc), but also because it is
unimportant in massive stars. Read Adelberger et al, 1998, RMP
(Sec III) for background. This is given at the class website. See also
Kamionkowski and Bahcall (1994)

\[
S(0)=6\pi^2m_p\alpha\ln\frac{\Lambda^2}{\gamma^2}\left(\frac{G_F}{G_\gamma}\right)^2f_{pp}^R\left((\theta)_{\nu_\mu\to\nu_e}\right)(1+\delta)^2
\]

where $\alpha$ is the fine structure constant, $m_p$ is the mass of the proton,
c is the speed of light, $G_F$ and $G_\gamma$ are the Fermi and axial vector
weak-coupling constants, $\gamma=\left(2\mu E_d\right)= 0.23161 \text{ fm}^{-1}$ is the deuteron
binding wave number, $\mu$ is the proton-neutron reduced mass and $E_d$ is
the deuteron binding energy, $(\hbar=1)$, $f_{pp}^R$ is the phase space factor, $(\theta)_{\nu_\mu\to\nu_e}$
is the $(\theta)$ value for the superallowed $0^-\to0^+$ transitions, $A$ is
proportional to the overlap of the $pp$ and deuteron wave functions,
and $\delta$ is a small correction to the nuclear force for the exchange of
heavier mesons.
\( \Lambda^2 \) is given by the overlap integral between the initial \( pp \) wave function and the final state deuteron wave function. The wave functions are determined by integrating Schrödinger’s equation for the two nucleon system with an assumed nuclear potential. The potential for the \( pp \) wave function must fit the data on proton-proton scattering. Five different potentials* were explored by Kamionkowski and Bahcall (1994) and give results consistent with the quoted error bar. The deuteron wave function must be consistent with the deuteron binding energy and other experimental constraints. Seven different possibilities were explored. The overall error is in \( \Lambda^2 \) is about 0.2%.

\( f(t) \) and \( G_d/G_V \) are determined by measurements of weak decay in a variety of nuclei and especially the lifetime of the free neutron. The standard value for the latter is \( 881 \pm 2 \) seconds. But see Bumm, Science, 360, 605 (2018) 888 or 879? The weak decay here is of the Gamow-Teller type (\( \Delta J = 0,1 \)), not Fermi (\( \Delta J = 0 \)). GT is mediated by the axial current (A). Fermi is mediated by the vector current (V).

The other factors are either accurately measurable (deuteron BE), straightforward to calculate \( (f_{pp}) \), or complicated and not very important (\( \delta \)).

The overlap is insensitive to the form of nuclear potential assumed inside a few fm and is determined by the tail of the potential at the nuclear surface.

This is highly constrained by proton scattering experiments.


\[
S(0) = 4.01 \times 10^{-25} \text{ MeV barns} \\
\times \left( \frac{f(t)_{0^+ \rightarrow 0^+}}{3071.4 \text{ sec}} \right)^{-1} \left( \frac{\Lambda^2}{7.035} \right)
\]

\[
3.78 \pm 0.15 \times 10^{-6} \text{ in Bahcall (1968)}
\]

\[
= \left( \frac{G_d}{G_V} \right)^2 \left( \frac{f_{pp}}{1.2695} \right)^2 \left( \frac{R}{0.144} \right)^2 \left( \frac{1 + \delta}{1.01} \right)^2
\]

\[
= 4.01 \pm 0.04 \times 10^{-25} \text{ MeV barns}
\]

History:
Bethe and Critchfield (1938)
Salpeter (1952)
TABLE I. The Solar Fusion II recommended values for S(0), its derivatives, and related quantities, and for the resulting uncertainty on S(E) in the region of the solar Gamow peak – the most probable reaction energy – defined for a temperature of 1.55 \times 10^7 K characteristic of the Sun’s center. See the text for detailed discussions of the range of validity for each S(E). Also see Sec. VIII for recommended values of CNO electron capture rates, Sec. XI.B for other CNO S-factors, and Sec. X for the 3H neutrino spectral shape. Quoted uncertainties are 1σ.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Section</th>
<th>S(0) [(keV b)^{-1}]</th>
<th>S'(0) [(b/keV)^{-1}]</th>
<th>S''(0) [(b/keV)^{-2}]</th>
<th>Gamow peak uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(p,e^+\gamma)d</td>
<td>III</td>
<td>(4.01 \pm 0.04) \times 10^{-22}</td>
<td>(4.49 \pm 0.05) \times 10^{-24}</td>
<td>(5.10 \pm 0.06) \times 10^{-26}</td>
<td>± 0.7</td>
</tr>
<tr>
<td>d(p,\gamma)He</td>
<td>IV</td>
<td>(2.14 \pm 0.12) \times 10^{-4}</td>
<td>(5.65 \pm 0.33) \times 10^{-6}</td>
<td>(9.3 \pm 0.5) \times 10^{-8}</td>
<td>± 7.1</td>
</tr>
<tr>
<td>(^3)He(\alpha,\gamma)(^7)Be</td>
<td>V</td>
<td>(5.21 \pm 0.27) \times 10^{-1}</td>
<td>(-4.9 \pm 3.2)</td>
<td>(2.2 \pm 1.7) \times 10^{-2}</td>
<td>± 4.3</td>
</tr>
<tr>
<td>(^7)Be(\alpha,\gamma)(^9)Be</td>
<td>VI</td>
<td>(0.56 \pm 0.03)</td>
<td>(-3.6 \pm 0.2) \times 10^{-4}</td>
<td>(1.51 \pm 0.08) \times 10^{-4}</td>
<td>± 5.1</td>
</tr>
<tr>
<td>(^9)Be(p,e^+\gamma)(^7)Be</td>
<td>VII</td>
<td>(8.6 \pm 2.6) \times 10^{-20}</td>
<td>(-)</td>
<td>(-)</td>
<td>± 30</td>
</tr>
<tr>
<td>(^8)Be(p,e^-\gamma)(^7)Be</td>
<td>VIII</td>
<td>(8.6 \pm 2.6) \times 10^{-20}</td>
<td>(-)</td>
<td>(-)</td>
<td>± 30</td>
</tr>
<tr>
<td>p(p,e^-\gamma)d</td>
<td>IX</td>
<td>(2.08 \pm 0.16) \times 10^{-2}</td>
<td>(-3.1 \pm 0.3) \times 10^{-2}</td>
<td>(2.3 \pm 0.8) \times 10^{-2}</td>
<td>± 7.5</td>
</tr>
<tr>
<td>(\alpha) + (\alpha)</td>
<td>XIA</td>
<td>(6.65 \pm 0.12)</td>
<td>(-3.3 \pm 0.2) \times 10^{-1}</td>
<td>(4.4 \pm 0.1) \times 10^{-1}</td>
<td>± 7.2</td>
</tr>
</tbody>
</table>

*Error from phenomenological quadratic fit. See text.
*2(S(0)/S(0)) taken from theory; error is that due to S(0). See text.
*3(S(0)/S(0)) taken from theory; error is that due to S(0). See text.
*4Estimated error in the pp/pp rate ratio. See Eq. (46).
*5Error dominated by theory.

Helium Burning

Helium burning is a two-stage nuclear process in which two alpha-particles temporarily form the ground state of unstable \(^8\)Be\(^\ast\). Occasionally the \(^8\)Be\(^\ast\) captures a third alpha-particle before it flies apart. No weak interactions are involved.

The ground state of \(^8\)Be\(^\ast\) is unbound by 92 keV to \(\alpha\)-decay. It has a width \(\Gamma_\alpha = 6.8\) eV and a lifetime of \(\tau = \frac{\hbar}{\Gamma} = \frac{6.58 \times 10^{-22}\text{ MeV s}}{6.8 \times 10^{-6}\text{ MeV}} = 9.7 \times 10^{-17}\text{ sec}\).

\(\text{The 7.654 MeV excited state of } ^{12}\text{C plays a critical role in the } 3\alpha \text{ reaction. Its } \alpha \text{-width is much greater than its photon width, so it predominantly decays back to } ^{8}\text{Be}^\ast, \text{setting up an equilibrium abundance of } ^{12}\text{C}^\ast. \Gamma_\gamma \text{ is augmented by a small contribution from pair production.}\)
Recall the Saha equation: (e.g., Clayton p 29). For example, for ionized and neutral hydrogen:

\[
\frac{n(\text{H II})}{n(\text{H I})} = \frac{G(\text{H II})}{G(\text{H I})} \left( \frac{2\pi m_e kT}{h^3} \right)^{3/2} \exp(-\chi_e / kT)
\]

The same thermodynamic arguments (equilibrium, chemical potential, etc.) also give a nuclear Saha equation. In particular, the equilibrium concentration of an unbound transitory \(^8\text{Be}^+\) nucleus is given by \(^8\text{Be}^+ \rightleftharpoons 2 \ 4\text{He}^\text{+}\)

\[
\frac{n_0}{n(\text{Be})} = \left( \frac{G_0}{G(\text{Be}^+)} \right) \left( \frac{2\pi A\tilde{M}T^{3/2}}{N_A h^3} \right)^{3/2} \exp\left(-\frac{Q_{\alpha}(\text{Be})}{kT} \right) \quad G = 2J+1
\]

\[Q_{\alpha}(\text{He}) = BE(\text{Be}^+)-2BE(\alpha=56.4995-2(28.2957)) = -0.0919 \text{ MeV} \quad Q_{\alpha}(\text{He})/kT = -0.0919 \times 11.6045/T_y = -1.066/T_y
\]

The time scale for establishing this equilibrium is very short.

Now consider the excited state of \(^{12}\text{C}\) at 7.6542 MeV. Call it \(^{12}\text{C}^\ast\). It also has as its dominant width, \(\Gamma_{\alpha} > \Gamma_{\gamma}\). That is \(^{8}\text{Be}^+ + \alpha \rightleftharpoons ^{12}\text{C}^\ast\)

\[
\frac{n(8\text{Be}^+)}{n(12\text{C})} = 5.94 \times 10^{33} T_y^{-3/2} \left( \frac{4+8}{4+8} \right)^{3/2} e^{-Q_{\alpha}(8\text{Be}^+)/kT}
\]

\[n(12\text{C}) = (5.94 \times 10^{33})^{-1} T_y^{3/2} \left( \frac{12}{32} \right) n(8\text{Be}^+) n_{\alpha} \exp(-0.287 / kT)
\]

where \(Q_{\alpha}(8\text{Be}^+) = BE(12\text{C}) - BE(8\text{Be}^+) - BE(\alpha) - 7.6542 = 92.1617 - 56.4995 - 28.2957 - 7.6542 \text{ MeV} = -0.287 \text{ MeV} \) \((*1/k = 11.6045 \Rightarrow -3.330)\)

\[n(12\text{C}) = 3.87 \times 10^{-35} T_y^{-3/2} n(8\text{Be}^+) n_{\alpha} \exp(-3.330 / T_y) = 3.87 \times 10^{-35} (5.95 \times 10^{-35}) T_y^{-3} n_{\alpha} \exp(-3.330 / T_y) = 2.303 \times 10^{-60} T_y^{-3} n_{\alpha} \exp(-4.396 / T_y)
\]

\[n(8\text{Be}^+) = \left( 5.94 \times 10^{33} 2^{3/2} T_y^{3/2} \right)^{-1} n_{\alpha}^2 \exp(-1.066 / T_y)
\]

\[n(8\text{Be}^+) = n_{\alpha}^2 T_y^{-3/2} (5.95 \times 10^{-35}) \exp(-1.066/T_y) \text{ cm}^{-3}
\]

or, since \(n = \rho N_A Y \quad \text{and} \quad Y = \frac{X_A}{A}\)

\[X(8\text{Be}^+) = 1.79 \times 10^{-3} \frac{\rho X_A^2}{T_y^{3/2}} e^{-1.066/T_y}
\]

For example, at \(2 \times 10^8 \text{ K}, \rho = 10^3 \text{ g cm}^{-3}, X_A = 1\)

\[X(8\text{Be}^+) \approx 10^9 \quad n = 7.3 \times 10^{16} / \text{cm}^3
\]

This works because the dominant decay mode of \(^8\text{Be}^+\) is to the same products from which it is assembled, i.e., \(\alpha + \alpha \rightleftharpoons ^{8}\text{Be}^+\).
The number of $^{12}C$ formed permanently per second is

$$\Gamma_{3\alpha} = n(^{12}C^*) \frac{\Gamma_{\text{rad}}}{\hbar}$$

$\Gamma_{\text{rad}}$ is the one thing besides binding energies and excited state energy that has to be measured

$$\Gamma_{\text{rad}} = 3.41 \pm 1.12 \times 10^3 \text{ eV (1976)}$$

$$= 3.67 \pm 0.46 \text{ meV (1988)}$$

$$= 3.64 \pm 0.5 \text{ meV (1990)}$$

$$\Gamma_{e^+} = 60.5 \pm 3.9 \text{ meV}$$

see article by Hale (1997). Current error about 10% (Sam Austin 2013)

The current value is due to Caughlan and Fowler (1988) using measurements from Sam Austin

$$\lambda_{3\alpha} = 2.79 \times 10^{-8} \ T_y^3 \ \text{exp} \left(-4.396/ T_y \right)$$

This gives:

$$R_{3\alpha} = 1.28 \times 10^{-6} \ T_y^3 \ n_\gamma^3 \ \text{exp} \left(-4.396/ T_y \right) \ \text{cm}^{-3} \ \text{sec}^{-1}$$

$$\frac{dn_{12}}{dt} = R_{3\alpha} \ \frac{dn_{12}}{dt} = -3 R_{3\alpha}$$

converting to our standard, $Y_\gamma$ notation

$$n_a = \rho N_A Y_a \quad Y_a = \frac{X(^{4}\text{He})}{4}$$

$$n_{12} = \rho N_A Y_{12} \quad Y_{12} = \frac{X(^{12}\text{C})}{12}$$

$$\frac{dY_a}{dt} = \rho^2 Y_a (\lambda_{3\alpha} / 3!) \quad \frac{dY_a}{dt} = -3 \rho^2 Y_a (\lambda_{3\alpha} / 3!$$

where

$$\lambda_{3\alpha} = 3! \times N_A^2 R_{3\alpha} = 2.79 \times 10^{-8} \ T_y^3 \ \text{exp} \left(-4.396/ T_y \right) \ \text{cm}^6 \ \text{gm}^{-2} \ \text{Mole}^{-2} \ \text{sec}^{-1}$$

(the units are such that $\rho^2 Y_\gamma \lambda_{3\alpha}$ has units of Mole/s)

**Helium burning 2 – the $^{12}$C($\alpha$, $\gamma$) rate**

*Resonance in Gamow window - Oxygen is made!*

*No resonance in Gamow window – stop at oxygen*

Unlike most reactions in astrophysics, the temperature dependence here is not determined by barrier penetration but by the Saha equation. In fact, at high temperature ($T_y > 1.5$) the rate saturates and actually begins to decline slowly as the resonance slips out of the Gamow window.
Uncertainty in the $^{12}$C($\alpha,\gamma$) rate was, for an extended time, the single most important nuclear physics uncertainty in astrophysics.

Affects:

- C/O ratio $\rightarrow$ further stellar evolution (C-burning or O-burning ?)
- iron (and other) core sizes (outcome of SN explosion)

---

Sub-threshold resonances
(See Rolfs and Rodney, Cauldrons in the Cosmos, p. 185ff)

$$\sigma(E) = \pi \hbar^2 \omega \frac{\Gamma_1(E) \Gamma_3(E + Q)}{(E - E_\gamma)^2 + [\Gamma(E)/2]^2}$$

E.g., 1 is an $\alpha$-particle and 2 is a photon. $\Gamma_1$ is the probability that the $\alpha$ penetrates to the nuclear surface. $\Gamma_2$ is the photon width evaluated at $E + Q$. e.g., for dipole radiation

$$\Gamma_2 = \left(\frac{E + Q}{\epsilon_\gamma}\right)^3 \Gamma_\gamma(\epsilon_\gamma)$$

An excited state of a compound nucleus lies $E_r$ below the threshold of the reaction, $Q$. The excited state is known to decay by $\gamma$ emission and is characterized by a width $\Gamma_\gamma$. Because of this width the state extends energetically to both sides of $E_r$ on a rapidly decreasing scale.

---

**TABLE I** Astrophysical environments and burning stages where the $^{12}$C($\alpha,\gamma$)$^{16}$O reaction plays an important role. The temperatures of these environments dictate the energy ranges where the $^{12}$C($\alpha,\gamma$)$^{16}$O cross section must well known for an accurate calculation of the reaction rate.

<table>
<thead>
<tr>
<th>Burning Stages</th>
<th>Astros. Sites</th>
<th>Temp. Range (K)</th>
<th>Gamow Energy Range (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Helium Burning</td>
<td>AGB stars and Massive Stars</td>
<td>0.1-0.4</td>
<td>0.13-0.65</td>
</tr>
<tr>
<td>Core Carbon and Oxygen Burning</td>
<td>Massive Stars</td>
<td>0.6-2.7</td>
<td>0.44-2.5</td>
</tr>
<tr>
<td>Core Silicon Burning</td>
<td>Massive Stars</td>
<td>2.8-4.1</td>
<td>1.1-3.4</td>
</tr>
<tr>
<td>Explosive Helium Burning</td>
<td>Supernovae and X-Ray Bursts</td>
<td>$\geq 1$</td>
<td>0.6-1.25</td>
</tr>
<tr>
<td>Explosive Oxygen and Silicon Burning</td>
<td>Supernovae</td>
<td>$\geq 5$</td>
<td>$&gt; 1.45$</td>
</tr>
</tbody>
</table>
Woosley and Weaver, Physics Reports (2007)

Prediction:
\[ S(300 \text{ keV}) = 170 \text{ keV-barns} \]

See also Woosley & Weaver, Phys. Reports, 227, 65, (1993)


This corresponds to 1.2 times Buchman (1996) and is what has been used in KEPLER for many years.

deBoer et al 2017 (on class website)

TABLE IV Extrapolations of the S factor (MeV b)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dufour and Descouvemont (2008)</td>
<td>42</td>
<td>70 GCM</td>
<td>Dufour and Descouvemont (1993)</td>
<td>90</td>
</tr>
<tr>
<td>Descouvemont and Baye (1987)</td>
<td>160</td>
<td>70 GCM</td>
<td>Qualtieri et al. (1992)</td>
<td>1</td>
</tr>
<tr>
<td>Makii et al. (2009)</td>
<td>22</td>
<td>70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohen et al. (1997)</td>
<td>7.0</td>
<td>56</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Current best value
\[ S_\text{ast}(300 \text{ keV}) = 140+\ 21(\text{MC}) +18-11 (\text{model}) \text{ keV-barns} \]
FIG. 25 (Color online) The uncertainty in the $S$ factor as derived by combining the MC analysis (which includes the subthreshold state uncertainties) and the model uncertainties is shown in Fig. 25 a). The data from Schürmann et al. (2005) are shown for comparison. Fig. 25 b) shows the uncertainties relative to the best fit value for the Monte Carlo analysis (olive colored dashed line) and the uncertainties derived from the model (dot-dashed orange line). The total uncertainty, taken as the MC and model uncertainties summed in quadrature, is shown by the solid red line. The black vertical dashed line marks the region of typical astrophysical interest at $E_{\text{cm}} = 300$ keV.

Helium Burning Rate Equations

$$\frac{dY}{dt} = -3 \rho^2 Y^3 \lambda_{\alpha} \bar{\lambda}_{\alpha} / 6 - Y_\alpha \ Y(Y^{(12)C}) \rho \lambda_{\alpha} \left(1^{2}C\right)$$

$$\frac{dY(Y^{(12)C})}{dt} = \rho^2 Y^3 \lambda_{\alpha} \bar{\lambda}_{\alpha} / 6 - Y_\alpha \ Y(Y^{(12)C}) \rho \lambda_{\alpha} \left(1^{2}C\right)$$

$$\frac{dY(Y^{(16)O})}{dt} = Y_\alpha \ Y(Y^{(12)C}) \rho \lambda_{\alpha} \left(1^{2}C\right)$$

For binary reactions, $\lambda \equiv N_{\lambda} \langle \sigma v \rangle$

For $Y_{12}$ small or $\rho$ large

$\alpha \rightarrow 12^{C}$

For $Y_{12}$ large or $\rho$ small

$\alpha \rightarrow 16^{O}$
Because of the tendency of $\frac{\rho}{T}$ to decrease with increasing mass and the near constancy of helium burning temperatures, massive stars make a decreasing ratio of carbon to oxygen as M increases. Variation with Z reflects the different extent of convection during He burning resulting from e.g., mass loss in solar Z stars, red vs. blue supergiants.

**Nuclear Energy Yield**

When an arbitrary composition, $\{Y_i\}$, rearranges by nuclear reactions to a new composition, $\{Y_i'\}$, where $Y_i' = Y_i + \delta Y_i$, there is a change in internal energy that can be positive or negative

$$q_{\text{nucl}} = 1.602 \times 10^{-6} N_A \sum (\delta Y_i)(BE_i) - q_{\nu} \text{ erg/gm}$$

Here $1.602 \times 10^{-6}$ is the conversion factor from MeV (which are the units of BE) to erg and the $q_{\nu}$ corrects for any neutrinos that might be emitted by weak interactions or thermal processes (like pair annihilation). If there are no weak interactions and thermal neutrino losses are negligible, e.g., in helium burning, $q_{\nu} = 0$.

**Example:** Hydogen burning

a) $100\% \ ^1H \rightarrow \ ^4He \quad \delta Y(\ ^1H) = -1 \quad \text{BE}(\ ^1H) = 0$

$$\delta Y(\ ^4He) = \frac{1}{4} \quad \text{BE}(\ ^4He) = 28.296 \text{ MeV}$$

$$q = 9.65 \times 10^{17} \left( \frac{28.296}{4} \right) = 6.83 \times 10^{18} \text{ erg g}^{-1}$$

b) $70\% \ ^1H; \ 30\% \ ^4He \rightarrow \ ^4He \quad \delta Y(\ ^1H) = -0.7$

$$\delta Y(\ ^4He) = \frac{1}{4} \quad - \frac{0.3}{4}$$

$$q = 9.65 \times 10^{17} \left( \frac{1}{4} - \frac{0.3}{4} \right) 28.296 = 4.78 \times 10^{18} \text{ erg g}^{-1}$$

need to subtract off term for weak interactions and neutrino losses

**A related quantity, the energy generation rate is given by**

$$\dot{q}_{\text{nucl}} = 9.65 \times 10^{17} \sum \frac{dY_i}{dt} (BE_i) - q_{\nu,\text{weak}} - q_{\nu,\text{thermal}} \text{ erg g}^{-1} \text{ sec}^{-1}$$
Both these expressions are only good for strong interactions.
In a weak interaction one has to worry about n and p mass differences, electron masses created and destroyed, as well as the mean neutrino energy loss.

A correct expression uses the atomic mass excesses. To within a constant

\[ \hat{\epsilon}_{\text{nucl}} = -\sum_i \frac{dY_i}{dt} M_i (^4 Z) \text{ [neutrino losses]} \]

where

\[ M_i = \Lambda_i (931.49) + \Delta_i \text{ MeV} \]

and

\[ BE = Z \Delta_H + N \Delta_n - \Delta(^4 Z) \quad \text{and} \quad \sum_i \frac{dY_i}{dt} A_i = \sum_i \frac{dX_i}{dt} = 0 \text{ so that} \]

\[ \hat{\epsilon}_{\text{nucl}} = \sum_i \frac{dY_i}{dt} BE_i - \sum_i \frac{dY_i}{dt} (Z \Delta_H + N \Delta_n) \text{ [neutrino losses]} \]

In the absence of weak interactions the second and third term may be dropped. (this includes the energy that the positrons deposit when they annihilate in positron emission).