Lecture 7

Evolution of Massive Stars on the Main Sequence and During Helium Burning - Basics

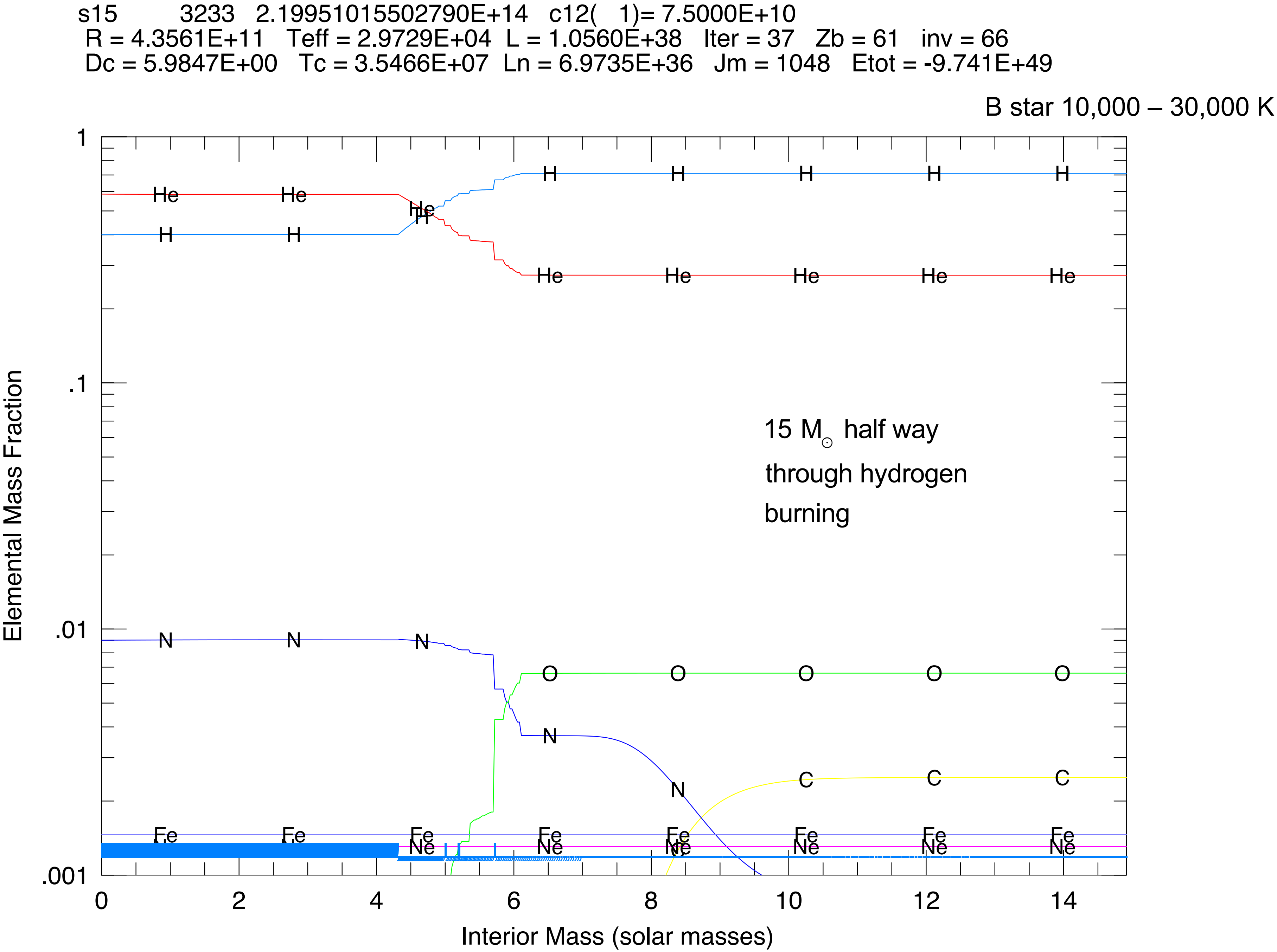
Massive Stars

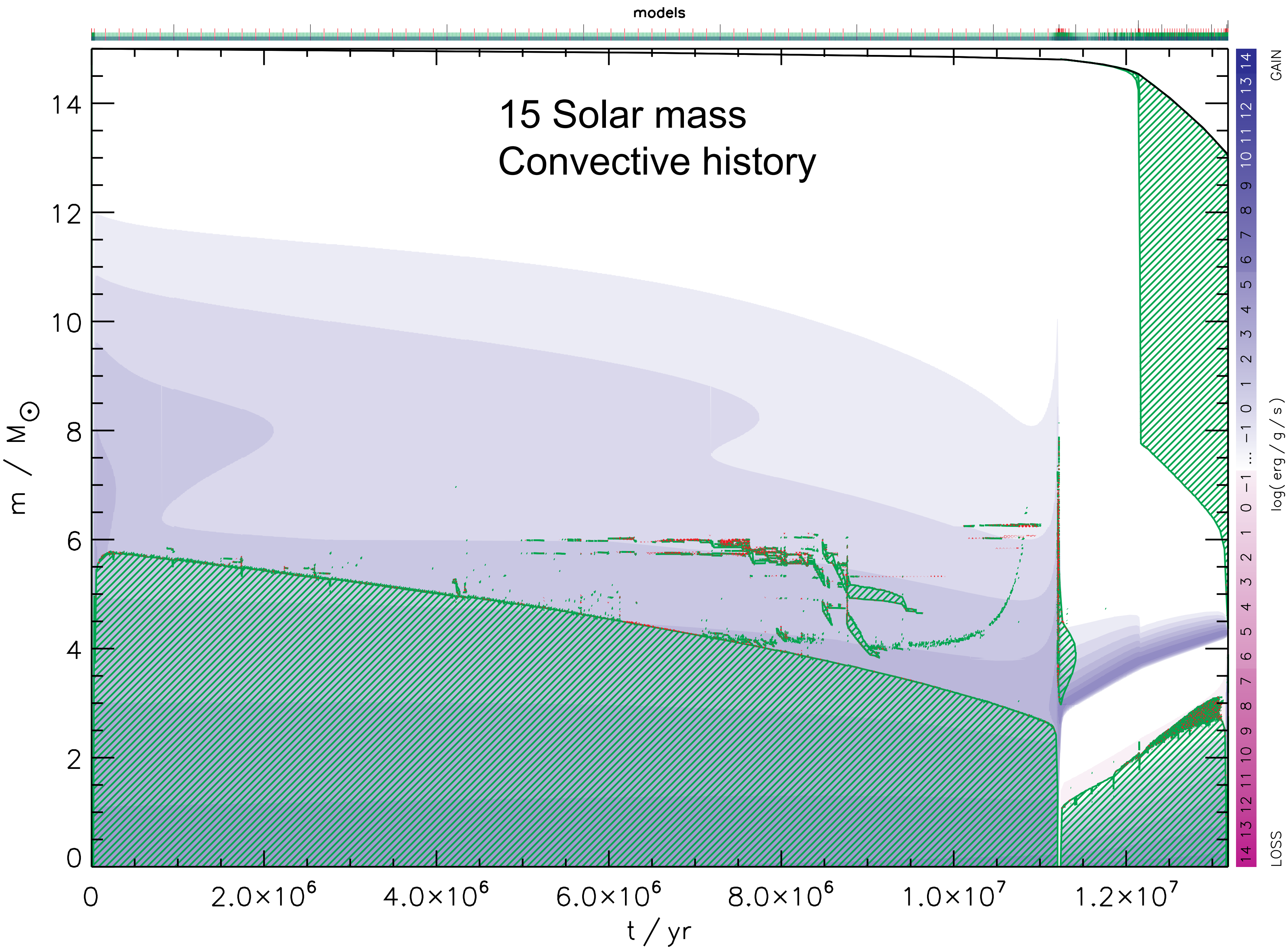
Generalities:

Because of the general tendency of the interior temperature of main sequence stars to increase with mass*, stars of over two solar mass are chiefly powered by the CNO cycle(s) rather than the pp cycle(s). The high temperature sensitivity of the CNO cycle (n = 17 instead of 4 for pp-cycle) makes the energy generation very centrally concentrated. This, plus the increasing fraction of pressure due to radiation, makes their cores convective. Because of the greater temperature, the opacity in their interiors is dominantly due to electron scattering.

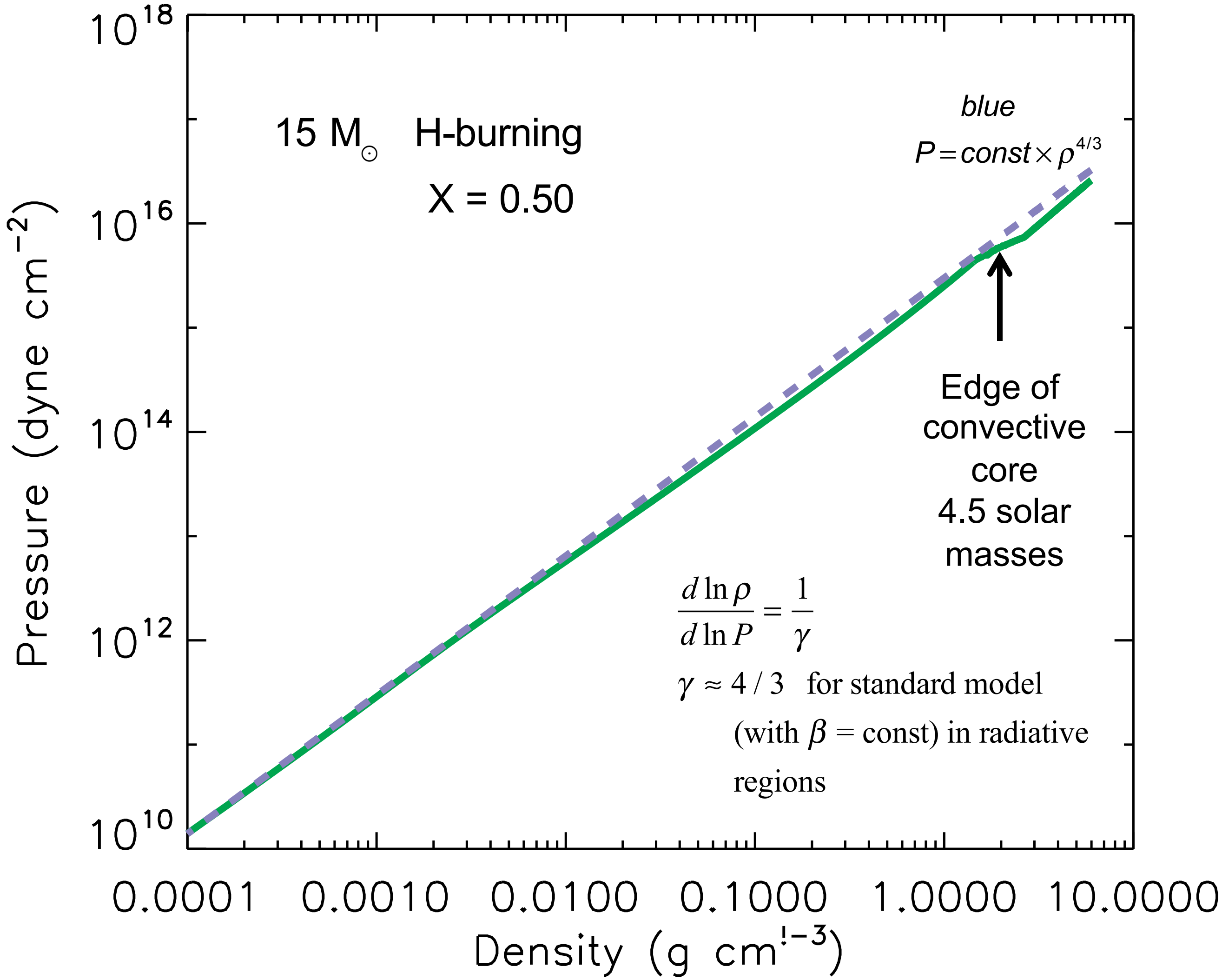
Despite their convective cores, the overall main sequence structure can be crudely represented as an n = 3 polytrope. This is especially true of the outer radiative part of the star that typically includes the majority of the mass.

* To provide a luminosity that increases as M^3


s15 3233 2.19951015502790E+14 c12(1)= 7.5000E+10
R = 4.3561E+11 Teff = 2.9729E+04 L = 1.0560E+38 Iter = 37 Zb = 61 inv = 66
Dc = 5.9847E+00 Tc = 3.5466E+07 Ln = 6.9735E+36 Jm = 1048 Etot = -9.741E+49
B star 10,000 – 30,000 K
15 $M_\odot$ half way
through hydrogen
burning
Elemental Mass Fraction
Interior Mass (solar masses)
H
He
N
O
C
Fe
Ne
1
.1
.01
.001
0
2
4
6
8
10
12
14

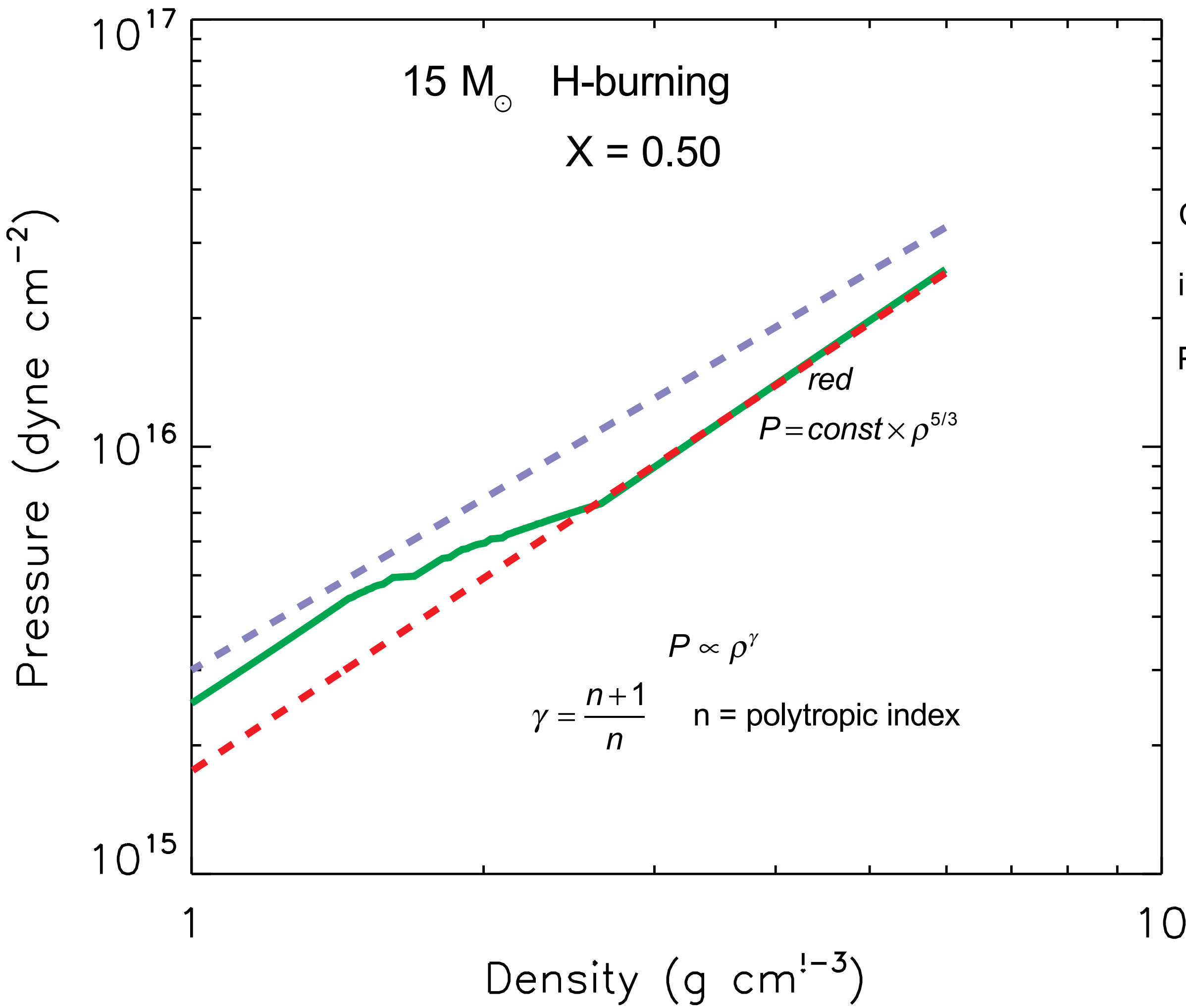


models
15 Solar mass
Convective history
m / $M_\odot$
t / yr
0
2.0×10^6
4.0×10^6
6.0×10^6
8.0×10^6
1.0×10^7
1.2×10^7
0
2
4
6
8
10
12
14
GAIN
LOSS
log(erg / g / s)



15 M☉ H-burning
X = 0.50
blue
$P = const \times \rho^{4/3}$
Edge of convective core 4.5 solar masses
$\frac{d \ln \rho}{d \ln P} = \frac{1}{\gamma}$
$\gamma \approx 4/3$ for standard model (with β = const) in radiative regions
Pressure (dyne cm^{-2})
Density (g cm^{-3})
10^{10}
10^{12}
10^{14}
10^{16}
10^{18}
0.0001
0.0010
0.0100
0.1000
1.0000
10.0000

The convective core (30% of the mass) resembles more an n=1.5 polytrope as expected for constant entropy for an ideal gas with P proportional to $\rho^{5/3}$ (see appendix). Overall though γ = 4/3 is not bad



Constant entropy and ideal gas implies $\frac{T^{3/2}}{\rho}$ is a constant hence

$P \propto \rho^{5/3}$

If μ and β were constant throughout the star, this would imply that the star was an n=3 polytrope

$$\beta \equiv \frac{P_{gas}}{P} = \frac{P_{gas}}{P_{rad} + P_{gas}} \Rightarrow \beta P = P - P_{rad}$$

$$P = \frac{P_{rad}}{(1-\beta)} = \frac{aT^4}{3(1-\beta)} \Rightarrow T = \left[\frac{3P(1-\beta)}{a}\right]^{1/4}$$

$$P = \frac{P_{gas}}{\beta} = \frac{N_A k}{\mu\beta}\rho T \Rightarrow P = \frac{N_A k}{\mu\beta}\rho\left[\frac{3P(1-\beta)}{a}\right]^{1/4}$$

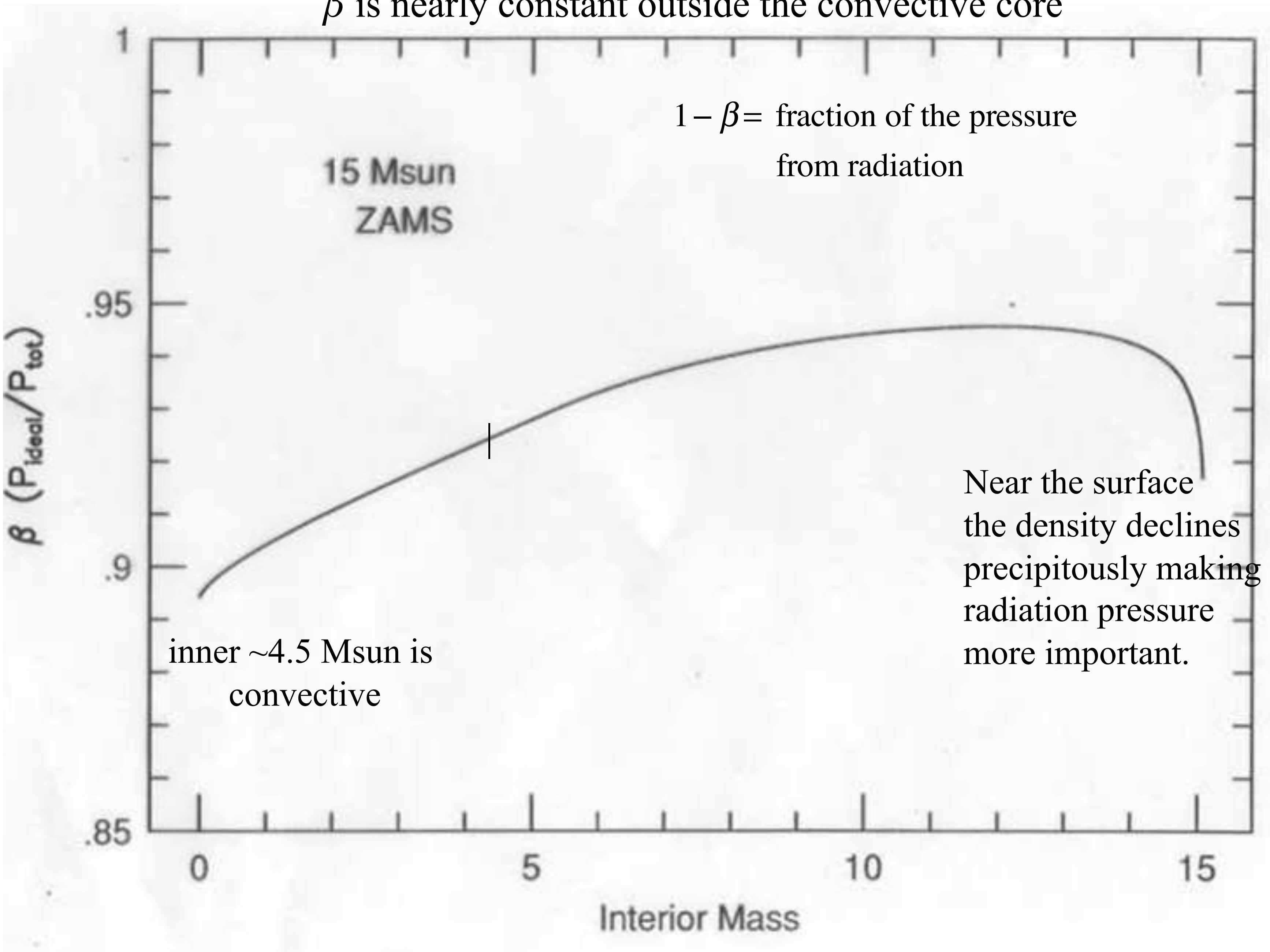
hence $P^{3/4} = \frac{N_A k}{\mu\beta}\rho\left[\frac{3(1-\beta)}{a}\right]^{1/4}$ and

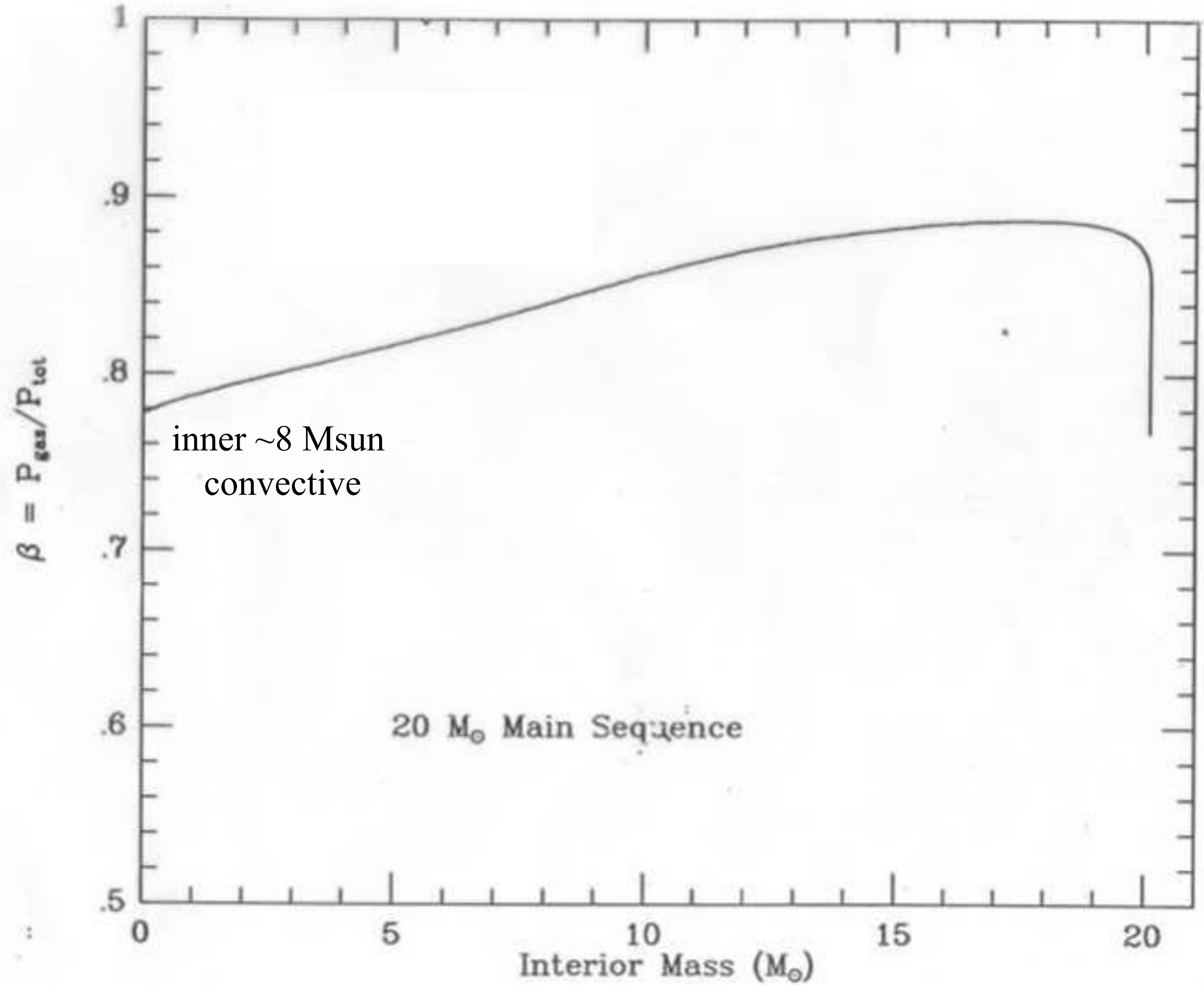
$$\boxed{P = \left[\frac{3(N_A k)^4(1-\beta)}{a(\mu\beta)^4}\right]^{1/3} \rho^{4/3}}$$

If μ and β are constants throughout the star this is an n = 3 polytrope

See also Clayton eq. 6-7
β will be constant if
κ L(r) /M(r) is everywhere
constant in radiative equilibrium.

β is nearly constant outside the convective core





1
.9
.8
.7
.6
.5
0
5
10
15
20
$\beta = P_{gas}/P_{tot}$
Interior Mass ($M_\odot$)
inner ~8 Msun
convective
20 $M_\odot$ Main Sequence

Eddington's standard model (n=3)

Consider a star in which radiation pressure is important (though not necessarily dominant) and energy transport is by radiative diffusion

$$\frac{dP_{rad}}{dr} = \frac{d}{dr}\left(\frac{1}{3}aT^4\right) = \frac{4}{3}aT^3\frac{dT}{dr}$$

But for radiative diffusion, $\frac{dT}{dr} = \frac{3\kappa\rho}{16\pi acT^3}\frac{L(r)}{r^2}$ so

$$\frac{dP_{rad}}{dr} = -\frac{\kappa\rho}{4\pi c}\frac{L(r)}{r^2}$$

but hydrostatic equilibrium requires

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}$$

Divide the 2 eqns

$$\frac{dP_{rad}}{dP} = \frac{\kappa L(r)}{4\pi Gmc} = \frac{L(r)}{L_{Edd}}$$

where $\boxed{L_{Ed} = \frac{4\pi GMc}{\kappa}}$

Since $P_{rad} = P - P_{gas} = (1-\beta)\,P$ and

$$\frac{dP_{rad}}{dP} = (1-\beta) = \frac{\kappa L(r)}{4\pi Gmc} = \frac{L(r)}{L_{Edd}}$$

If, and it is a big IF, β (or 1-β) were a constant throughout the star, then one could write everywhere, including the surface

$$\boxed{L(r) = \left(1-\beta\right)\, L_{Ed}}$$

EDDINGTON'S QUARTIC EQUATION

Fom polytropes $$M = -\frac{(n+1)^{3/2}}{\sqrt{4\pi}} \left(\xi_1^2 \frac{d\theta}{d\xi}\right)_{\xi_1} \left(\frac{K}{G}\right)^{3/2} \rho_c^{\frac{3-n}{2n}}$$

(Clayton 155- 165)

$$K = \left[\frac{3(N_A k)^4(1-\beta)}{a(\mu\beta)^4}\right]^{1/3} \qquad \beta = \frac{P_{gas}}{P_{total}} \qquad \mu = \left[\sum (Z_i + 1) X_i / A_i\right]^{-1}$$

For n = 3 (β= constant), ρ_c drops out and this becomes

2.01824

$$M = -\frac{4}{\sqrt{\pi}} \left(\xi_1^2 \frac{d\theta}{d\xi}\right)_{\xi_1} \left(\frac{K}{G}\right)^{3/2} = 4.56 \left(\frac{K}{G}\right)^{3/2}$$

$$M = 4.56 \left[\frac{3(N_A k)^4(1-\beta)}{a(\mu\beta)^4 G^3}\right]^{1/2}$$

$$\boxed{M = \frac{18.1\, M_\odot}{\mu^2} \left(\frac{1-\beta}{\beta^4}\right)^{1/2}}$$

$$\lim_{\beta \to 1} M \to 0$$

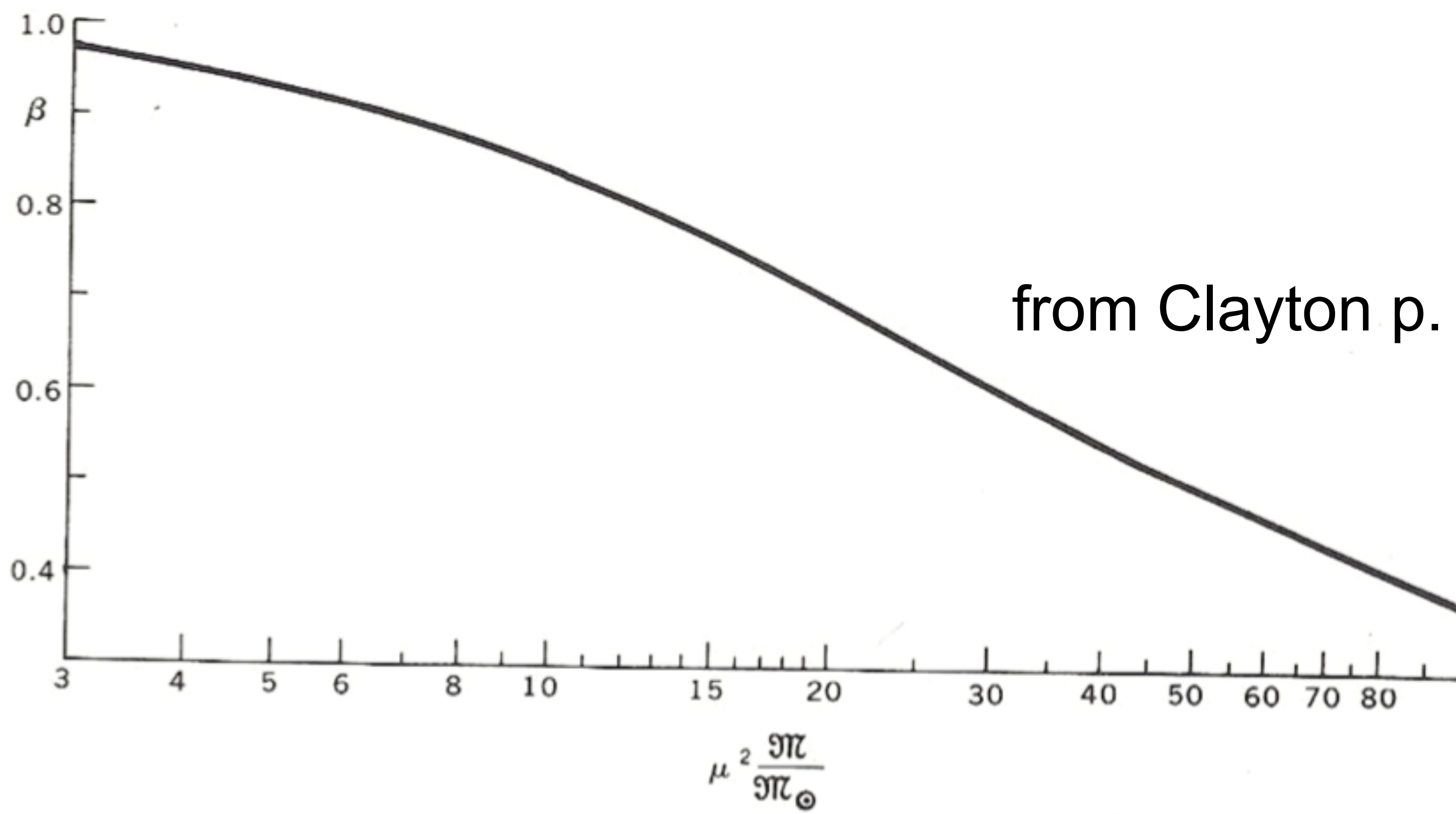
$$\lim_{\beta \to 0} M \to \infty$$

Eddington's quartic equation

$$\mu = \left[\sum (1+Z_i)\, Y_i \right]^{-1} = 0.73 \text{ for } 50\% \text{ H}, 50\% \text{ He}$$

$$0.64 \text{ for } 75\% \text{ H}, 25\% \text{ He}$$

For 20 $M_\odot$ $\beta \approx 0.85$ $\mu^2 M \approx 11$



from Clayton p. 163

$$M = \frac{18.1\,M_\odot}{\mu^2}\left(\frac{1-\beta}{\beta^4}\right)^{1/2}$$

$$1-\beta = 5.12\times 10^{-4}\left(\frac{M}{M_\odot}\right)^2\left(\frac{\mu}{0.64}\right)^4\beta^4 \quad \text{and since}$$

$$L(r) = (1-\beta)\,L_{Edd} = \left(1-\beta\right)\frac{4\pi G M c}{\kappa}$$

For low M, $\left(1-\beta\right)\propto M^2$, and $L\propto M^3$

For high M, $\beta \ll 1$ and $L \rightarrow L_{Ed} \propto M$

This was obtained with no mention of nuclear reactions.

$$M = \frac{18.1\, M_\odot}{\mu^2}\left(\frac{1-\beta}{\beta^4}\right)^{1/2}$$

For M not too far from $M_\odot$ β is close to 1 and $L \propto M^3$.

At higher masses however the mass dependence of β becomes important. Eventually $\beta^4 \propto M^{-2}$ so that $L \propto M$. In fact, the luminosity of very massive stars approaches the Eddington limit as $\beta \to 0$ ($L(r) = \left(1-\beta\right) L_{Edd}$)

$$L_{Edd} = \frac{4\pi G M c}{\kappa} = 1.47 \times 10^{38}\ \mathrm{erg\ s^{-1}} \left(\frac{M}{M_\odot}\right)\left(\frac{0.34}{\kappa}\right)$$

For n = 3 one can also derive useful equations for the central temperature based upon the original polytropic equation for mass

$$M = -4\pi\alpha^3\rho_c\,\xi_1^2 \left.\frac{d\theta}{d\xi}\right)_{\xi_1} = 2.01824\,(4\pi\alpha^3\rho_c)$$

and the definitions $\alpha = \left[\frac{P_c(n+1)}{4\pi G\rho_c^2}\right]^{1/2} = \left[\frac{P_c}{\pi G\rho_c^2}\right]^{1/2}$

and $P_c = \frac{P_{ideal}}{\beta} = \frac{\rho_c N_A k T_c}{\mu\beta}$ and $\frac{\rho_c}{\bar{\rho}} = \frac{4\pi R^3\rho_c}{3M} = 54.18$

$$P_c = 1.242\times10^{17}\left(\frac{\left(M/M_\odot\right)^2}{\left(R/R_\odot\right)^4}\right)$$

$$T_c = 19.57\times10^6\,\beta\mu\left(\frac{\left(M/M_\odot\right)}{\left(R/R_\odot\right)}\right)K$$

$$T_c = 4.62\times10^6\,\beta\mu\left(M/M_\odot\right)^{2/3}\rho_c^{1/3}\;K$$

For the n=3 polytrope

$$T_c = 4.6 \times 10^6 \ \text{K} \ \mu\beta \left(\frac{M}{M_\odot}\right)^{2/3} \rho_c^{1/3} \qquad \text{(in general for n = 3)}$$

For massive stars on the main sequence and half way through hydrogen burning, $\mu \approx$ 0.84 and, unless the star is very massive, $\beta \approx$ 0.8 - 0.9. Better values are given in Fig 2-19 of Clayton replicated on the next page.

The density is not predicted from first principles since the actual radius depends upon nuclear burning, but detailed main sequence models (following page) give $\rho_c \approx 10\left(\frac{10 \ M_\odot}{M}\right)$ gm cm^{-3},So

$$\boxed{T_c \approx 3.9 \times 10^7 \beta \left(\frac{M}{10 M_\odot}\right)^{1/3} \ \text{K}} \qquad \text{(main sequence only)}$$

$\beta \sim 0.8 - 0.9$

All evaluated in actual models at a core H mass fraction of 0.30 for stars of solar metallicity (but outer layers still unburned).

M	$T_c/10^7$	ρ_C	$L/10^{37}$	L/L_{Ed}
9	3.27	9.16	2.8	.02
12	3.45	6.84	7.0	.04
15	3.58	5.58	13	.06
20	3.74	4.40	29	.10
25	3.85	3.73	50	.14
40	4.07	2.72	140	.24
60(57)	4.24	2.17	290	.35
85(78)	4.35	1.85	510	.44
120(99)	4.45	1.61	810	.56

$L \propto M^{2.5}$ (for M = 9 to 40)

$$L \sim (1-\beta)\, L_{Ed}$$

ρ_c decreases with mass as a general consequence of the fact that $\dfrac{T_c^3}{\rho_c} \propto M^2\beta^3\mu^3$ and H burning happens at a relatively constant temperature. Until about 40 $M_\odot$, the density decreases roughly as M^{-1}. After that it decreases more slowly. Recall $\beta \propto M^{-1/2}$ for *very* large masses

Complications

- Beta not really constant, star not really an n=3 polytrope to begin with

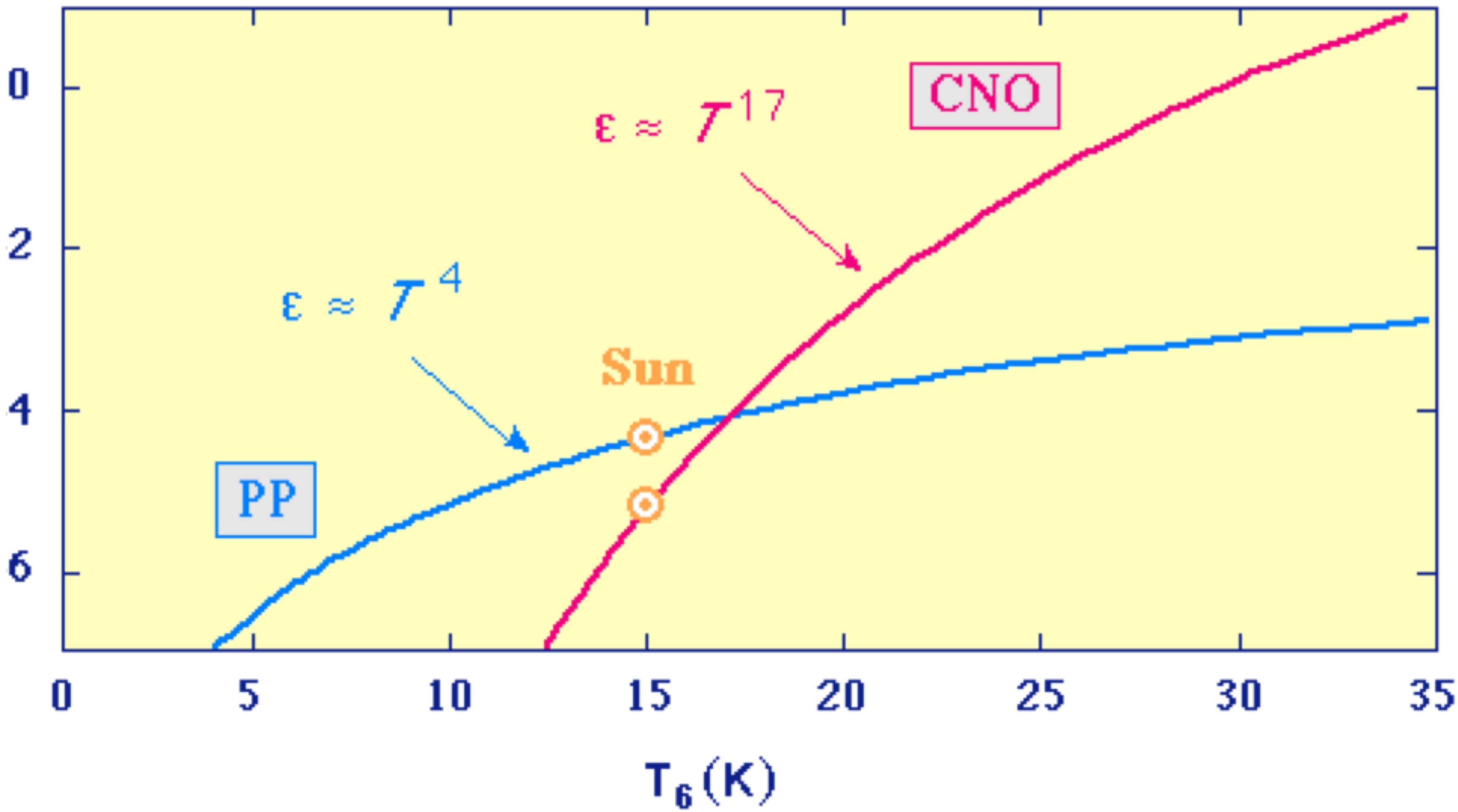
- Star evolves and develops variations in β and μ from center to surface

- L varies almost a factor of two from beginning to end of main sequence

- Opacity not constant and not all due to electron scattering.

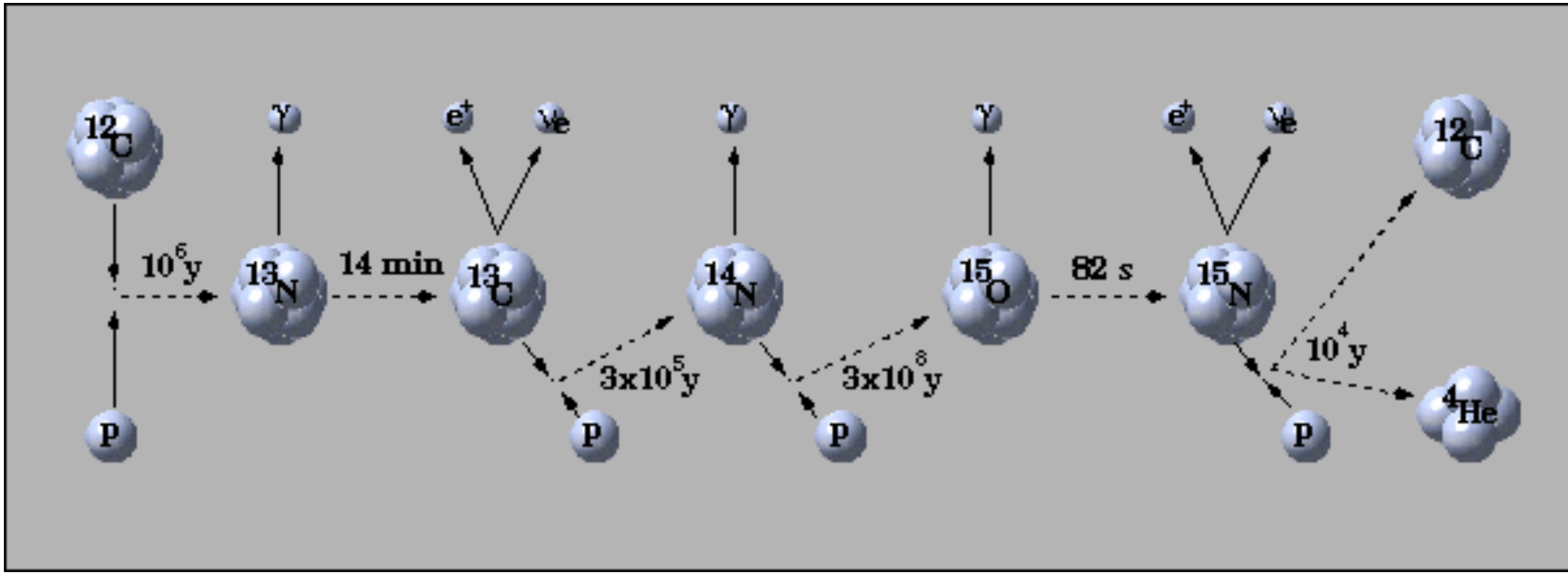
- L good to about a factor of two for conditions given (i.e., current μ, current M). Temperature to better than 10%.

Competition between the p-p chain and the CNO Cycle



The temperature dependence of the CNO cycle is given by the sensitivity of the proton capture rate of ^{14}N. See previous lectures

The Primary CNO Cycle



In a low mass star

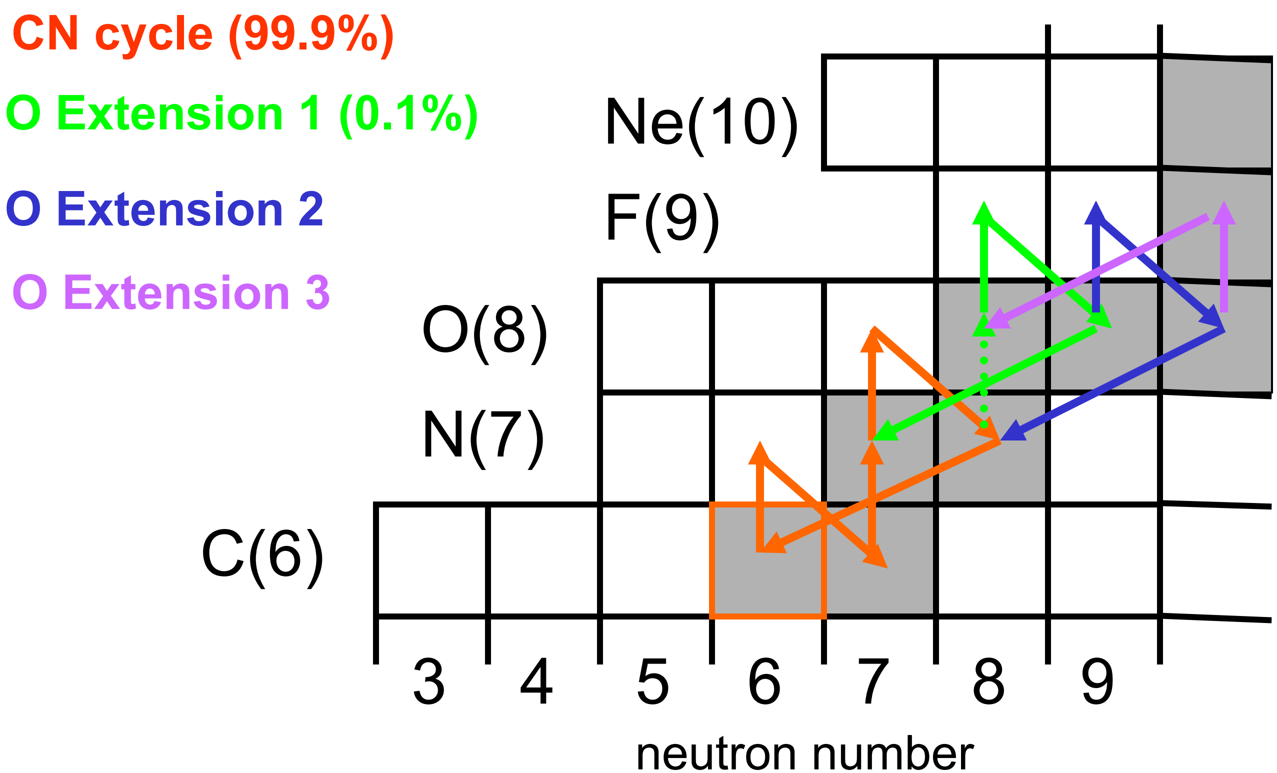
The slowest reaction is $^{14}N(p,\gamma)^{15}O$. For temperatures near 2.5 x 10^7 K.

$$\varepsilon_{nuc} \propto T^n \qquad n = \frac{\tau - 2}{3} \qquad \tau = 4.248 \left(\frac{7^2 1^2 \frac{14 \cdot 1}{14+1}}{0.025} \right)^{1/3} = 51.9$$

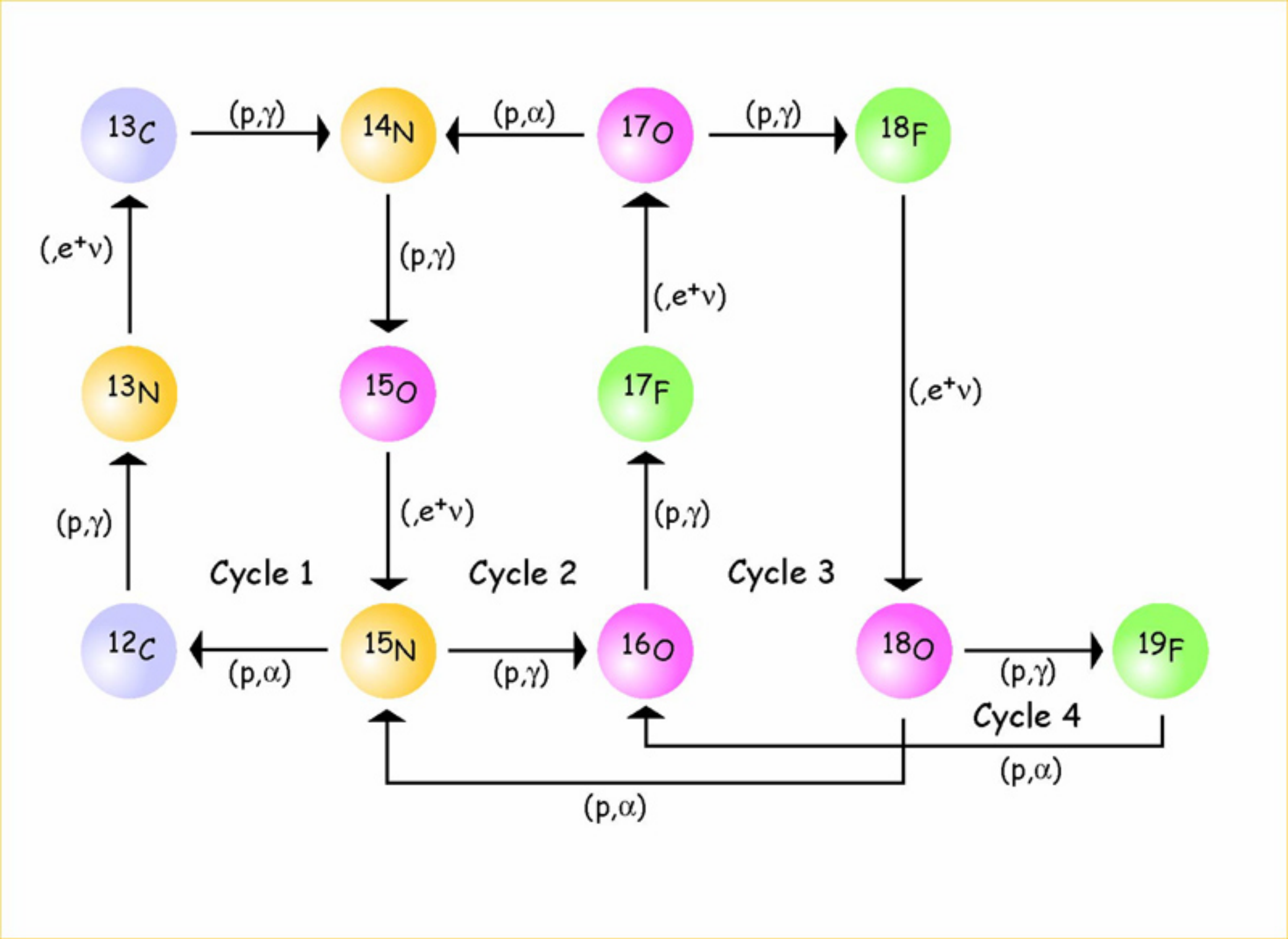
$n = 17$ (less at higher T)

(More on nucleosynthesis later)

CNO tri-cycle



All initial abundances within a cycle serve as catalysts and accumulate at largest τ, i.e., where the (p,γ) reaction rate is smallest



The extra loops are mainly of interest for nucleosynthesis and for bringing ^{16}O into the cycle

In general, the rates for these reactions proceed through known resonances whose properties are all reasonably well known.

There was a major revision of the rate for $^{14}N(p,\gamma)^{15}O$ in 2001 by Bertone et al., Phys. Rev. Lettr., 87, 152501. The new rate was about half as large as the old one, so the main sequence lifetime of massive stars is longer (but definitely not linear in the reciprocal rate). Mainly affected globular cluster ages (0.7 to 1 Gy increase in lifetime due to the importance of the CNO cycle at the end of the MS life and during thick H shell burning).

Equation of state

Well defined if tedious to calculate up to the point of iron core collapse.

Ions - ideal gas - $P = \frac{\rho}{\mu} N_A k T$

Radiation $P = \frac{1}{3} a T^4$

Electrons - the hard part - can have arbitrary relativity and degeneracy

(solve Fermi integrals or use fits or tables).

At high T must include electron - positron pairs.

Beyond 10^{11} g cm^{-3} - neutrino trapping, nuclear force, nuclear excited states, complex composition, etc.

Opacity

In the interior on the main sequence and within the helium core for later burning stages, electron scattering dominates.

In its simplest form:

$$\kappa_e = \frac{n_e \sigma_{\mathrm{Th}}}{\rho} = \frac{\rho \mathrm{N_A} Y_e \sigma_{\mathrm{Th}}}{\rho} = Y_e (\mathrm{N_A} \sigma_{\mathrm{Th}})$$

$$\sigma_{\mathrm{Th}} = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2$$

$$\kappa_e = 0.40\ Y_e\ \mathrm{cm^2\ gm^{-1}}$$

Recall that for 75% H, 25% He, $Y_e = 0.875$, so $\kappa_\mathrm{e} = 0.35$

For He and heavier elements $\kappa_e \approx 0.20$.

There are correction terms that must be applied to κ_{es} especially at high temperature and density

1) The *electron-scattering cross section* and Thomson cross section differ at high energy. The actual cross section is smaller.

Klein-Nishina

$$\sigma_{KN} = \sigma_{Th}\left[1 - \left(\frac{2h\nu}{m_e c^2}\right) + \frac{26}{5}\left(\frac{h\nu}{m_e c^2}\right)^2 + \ldots\right]$$

$$h\nu << 10^{20}\ Hz$$

2) *Degeneracy* – at high density the phase space for the scattered electron is less. This decreases the scattering cross section. Opacities small in white dwarfs and evolved stellar cores

3) *Incomplete ionization* – especially as the star explodes as a supernova. Use the Saha equation.

4) *Electron positron pairs* may increase κ at high temperature.

Effects 1) and 2) are discussed by

Chin, *ApJ*, **142**, 1481 (1965)
Flowers & Itoh, *ApJ*, **206**, 218, (1976)
Buchler and Yueh, *ApJ*, **210**, 440, (1976)
Itoh et al, *ApJ*, **382**, 636 (1991) and references therein

Electron conduction is not very important in massive stars but is important in white dwarfs and therefore the precursors to Type Ia supernovae

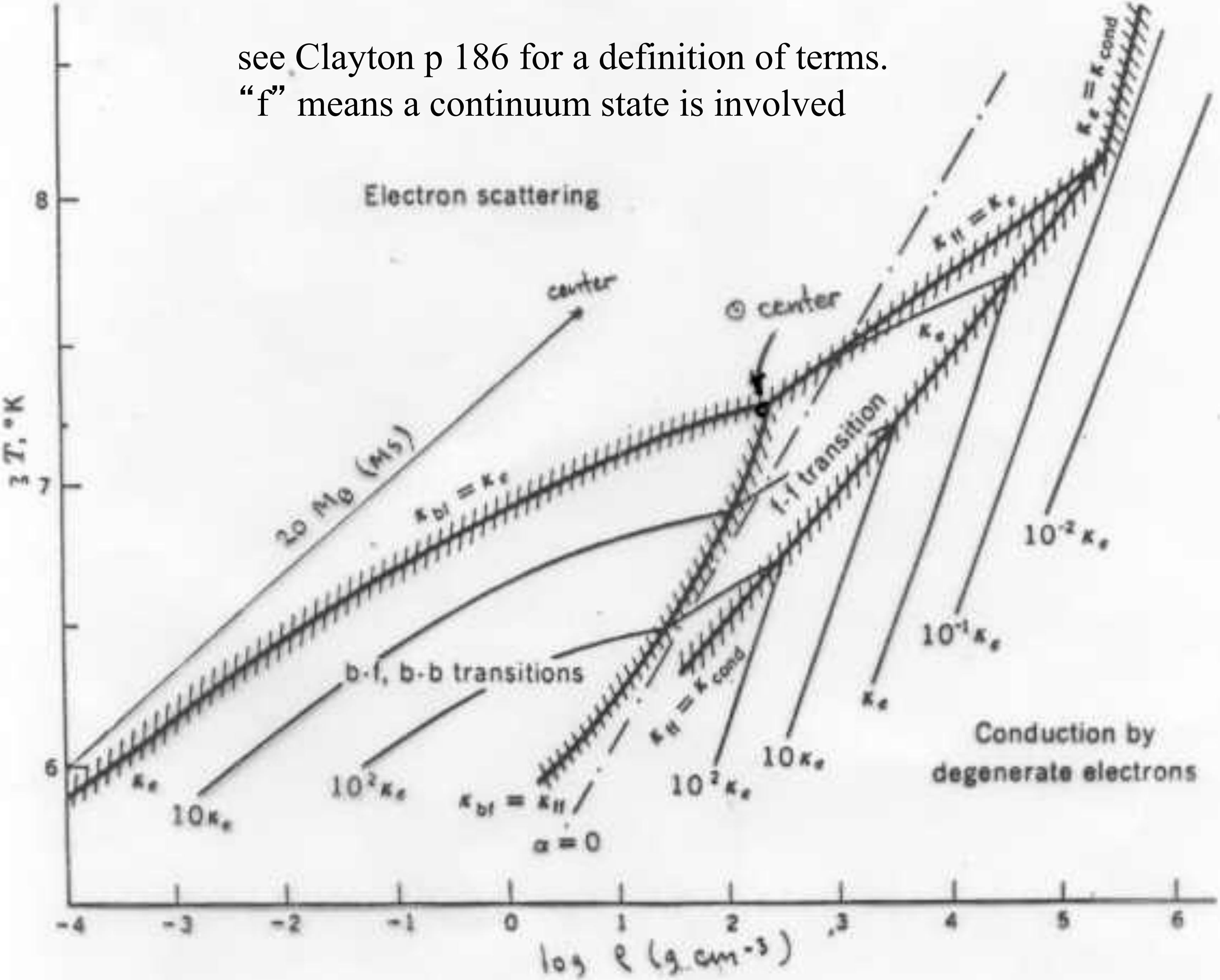
Itoh et al, *ApJ*, **285**, 758, (1984) and references therein

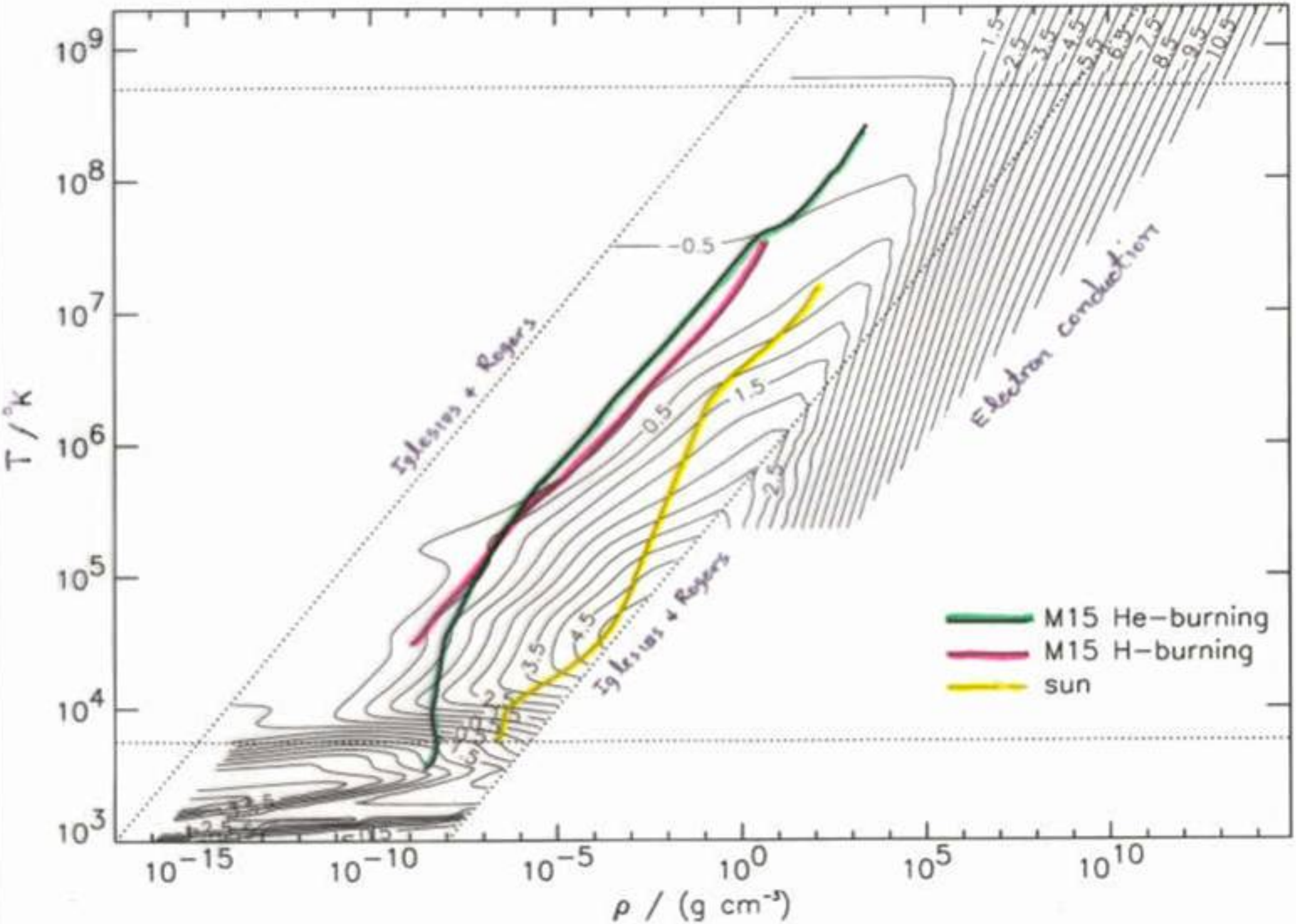
For radiative opacities other than κ_{es}, in particular κ_{bf} and κ_{bb},

Iglesias and Rogers, *ApJ*, **464**, 943 (1996)

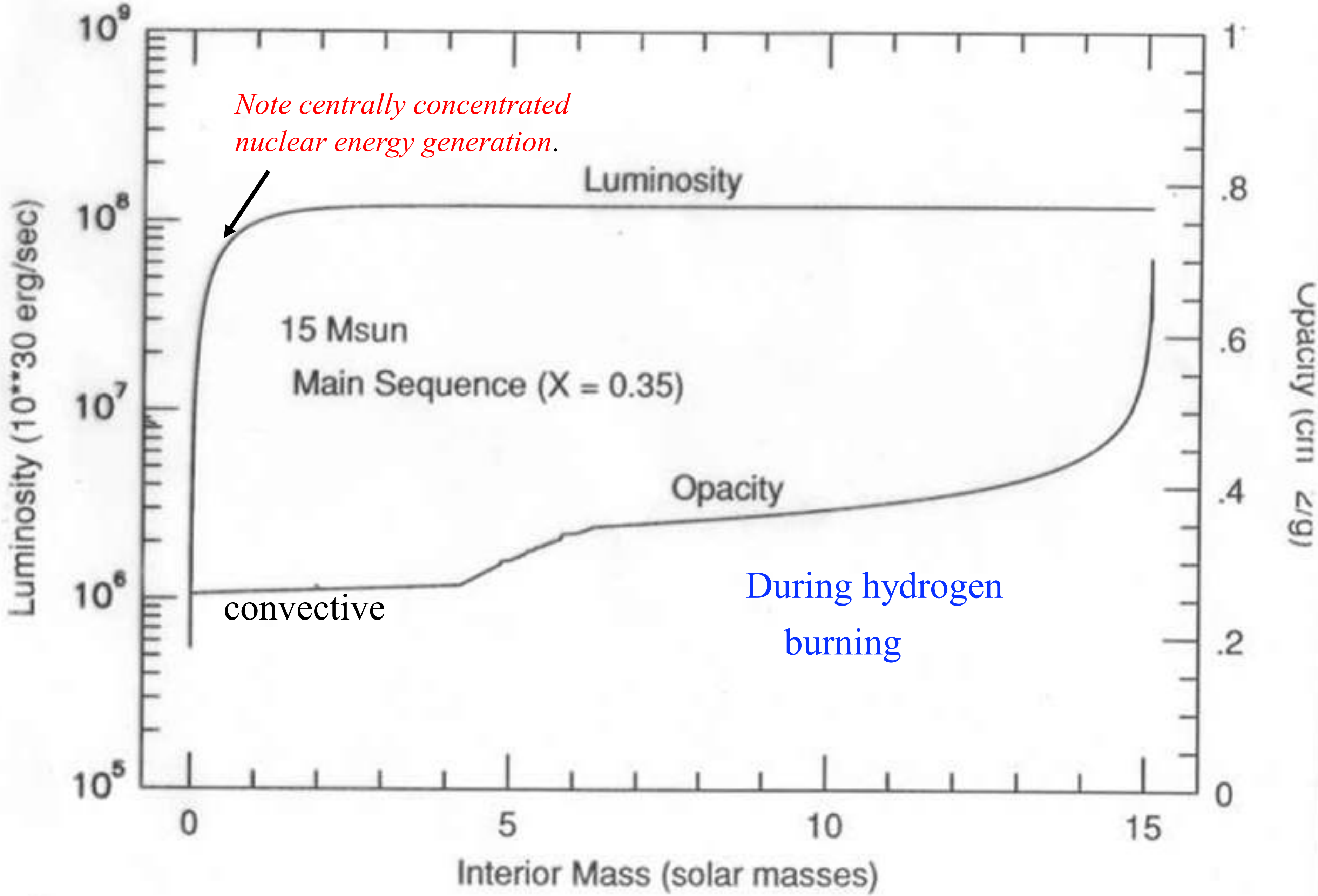
Rogers, Swenson, and Iglesias, *ApJ,* **456**, 902 (1996)

see Clayton p 186 for a definition of terms.
"f" means a continuum state is involved





T / °K
ρ / (g cm⁻³)
M15 He-burning
M15 H-burning
sun
Iglesias & Rogers
Iglesias & Rogers
Electron conduction



Note centrally concentrated nuclear energy generation.
Luminosity
15 Msun
Main Sequence (X = 0.35)
Opacity
convective
During hydrogen burning
Luminosity (10**30 erg/sec)
Opacity (cm²/g)
Interior Mass (solar masses)
10⁹
10⁸
10⁷
10⁶
10⁵
0
5
10
15
1
.8
.6
.4
.2
0

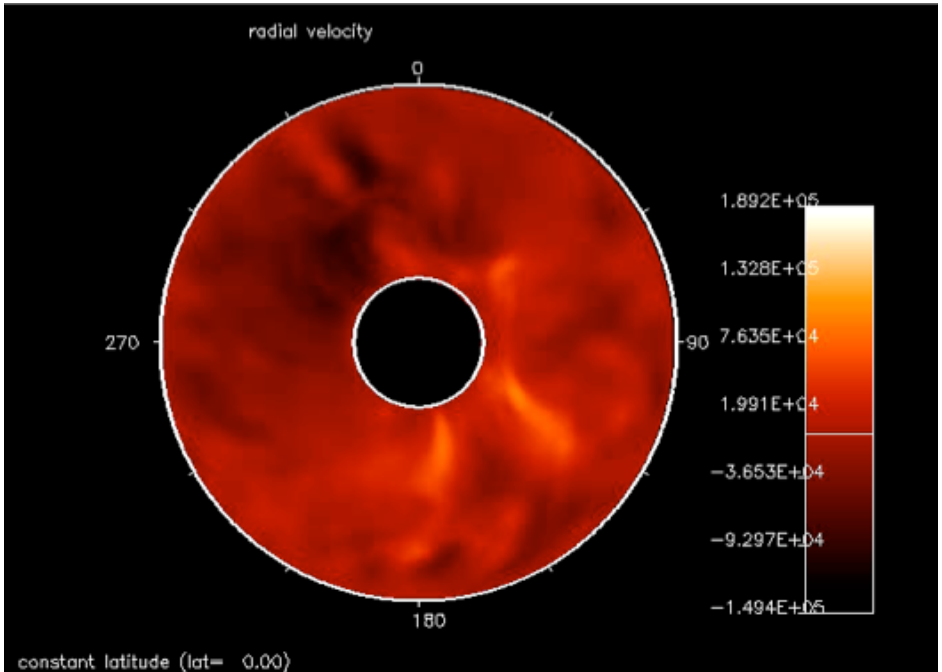
Convection

All stellar evolution calculations to date, except for brief snapshots, have been done in one-dimensional codes.

In these convection is universally represented using some variation of mixing length theory.

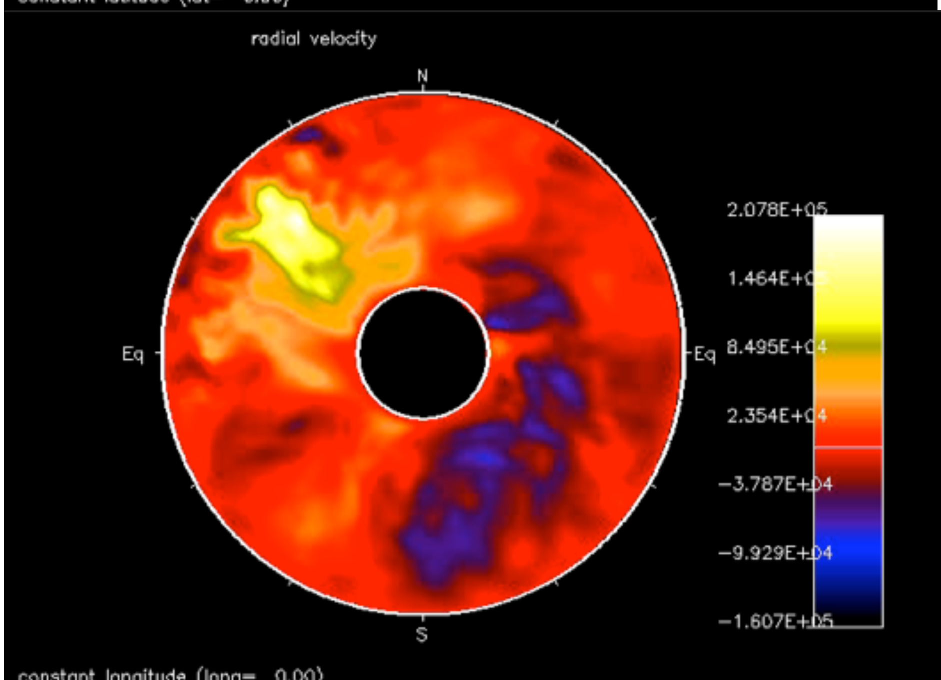
Caveats and concerns:

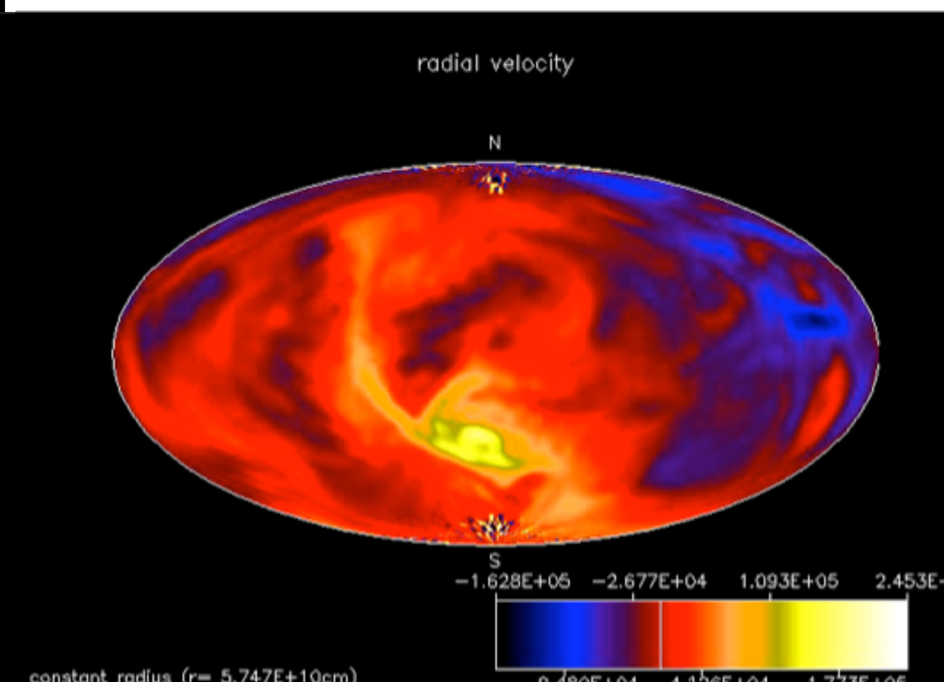
- The treatment must be time dependent
- Convective overshoot and undershoot (next lecture)
- Semiconvection (next lecture)
- Convection in parallel with other mixing processes, especially rotation (next lecture)
- Convection in situations where evolutionary time scales are not very much longer than the convective turnover time.

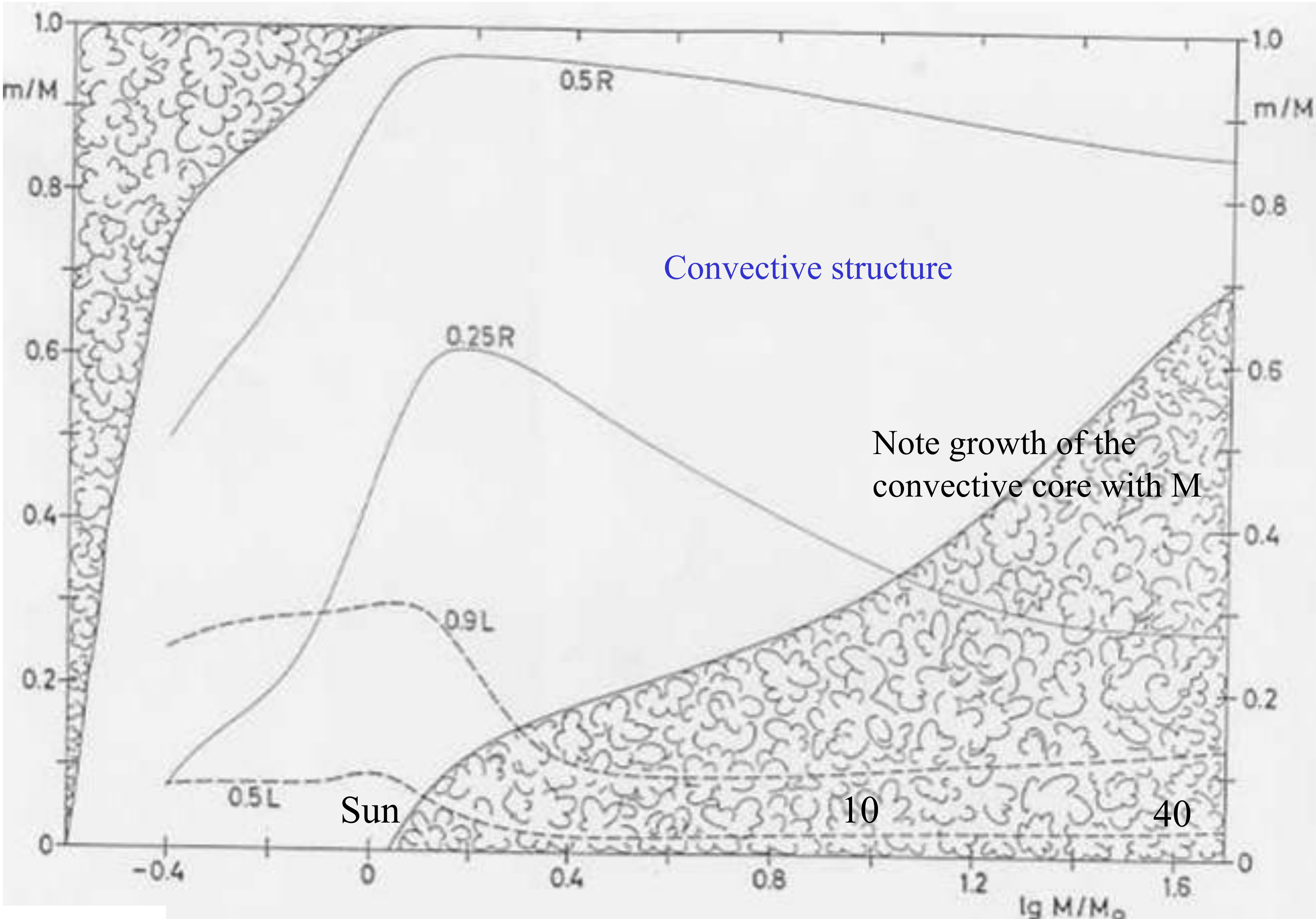


Kuhlen, Woosley, and Glatzmaier explored the physics of stellar convection using 3D anelastic hydrodynamics.
See also Meakin and Arnett (2007)
Gilet et al (ApJ, 2013)

The model shown is a 15 solar mass star half way through hydrogen burning. For now the models are not rotating. Mixing length theory is not a bad description of the overall behavior except at boundaries.







from Kippenhahn and Wiegert

The (Swartzschild) adiabatic condition can be written in terms of the temperature as

$$\frac{dP}{P}+\frac{\Gamma_2}{1-\Gamma_2}\frac{dT}{T}=0$$

This defines Γ_2 (see Clayton p 118)

For an ideal gas $\Gamma_2 = 5/3$, but if radiation is included the expression is more complicated

Convective instability is favored by a large fraction of radiation pressure, i.e., a small value of β (and of course by large L).

$$\left(\frac{dT}{dr}\right)_{\text{star}} > \left(1-\frac{1}{\Gamma_2}\right)\frac{T}{P}\left(\frac{dP}{dr}\right) \Rightarrow \text{convection}$$

$$\Gamma_2 = \frac{32-24\beta-3\beta^2}{24-18\beta-3\beta^2} \qquad \frac{4}{3} < \Gamma_2 < \frac{5}{3} \qquad \text{(Clayton 2-129)}$$

$$\text{For } \beta=1, \left(1-\frac{1}{\Gamma_2}\right)=0.4; \quad \text{for } \beta = 0, \left(1-\frac{1}{\Gamma_2}\right) = 0.25$$

$$\text{for } \beta = 0.8 \quad \left(1-\frac{1}{\Gamma_2}\right) = 0.294$$

So even a 20% decrease in β causes a substantial decrease in the critical temperature gradient necessary for convection. Since β decreases with increasing mass, convection becomes more extensive.

Also more massive stars are generating a lot more energy in a star whose physical dimension is not much larger than the sun.

M	$T_c/10^7$	ρ_C	$L/10^{37}$	$Q_{conv\ core}$
9	3.27	9.16	2.8	0.26
12	3.45	6.84	7.0	0.30
15	3.58	5.58	13	0.34
20	3.74	4.40	29	0.39
25	3.85	3.73	50	0.43
40	4.07	2.72	140	0.53
60(57)	4.24	2.17	290	0.60
85(78)	4.35	1.85	510	0.66
120(99)	4.45	1.61	810	0.75

All evaluated at a core H mass fraction of 0.30 for stars of solar metallicity.

The convective core shrinks during hydrogen burning

During hydrogen burning the mean atomic weight is increasing from near 1 to about 4. The ideal gas entropy is thus decreasing.

Also convection is taking entropy out of the central regions and depositing it farther out in the star.

As the central entropy decreases compared with the outer layers of the star it becomes increasingly difficult to convect through most of the star's mass.

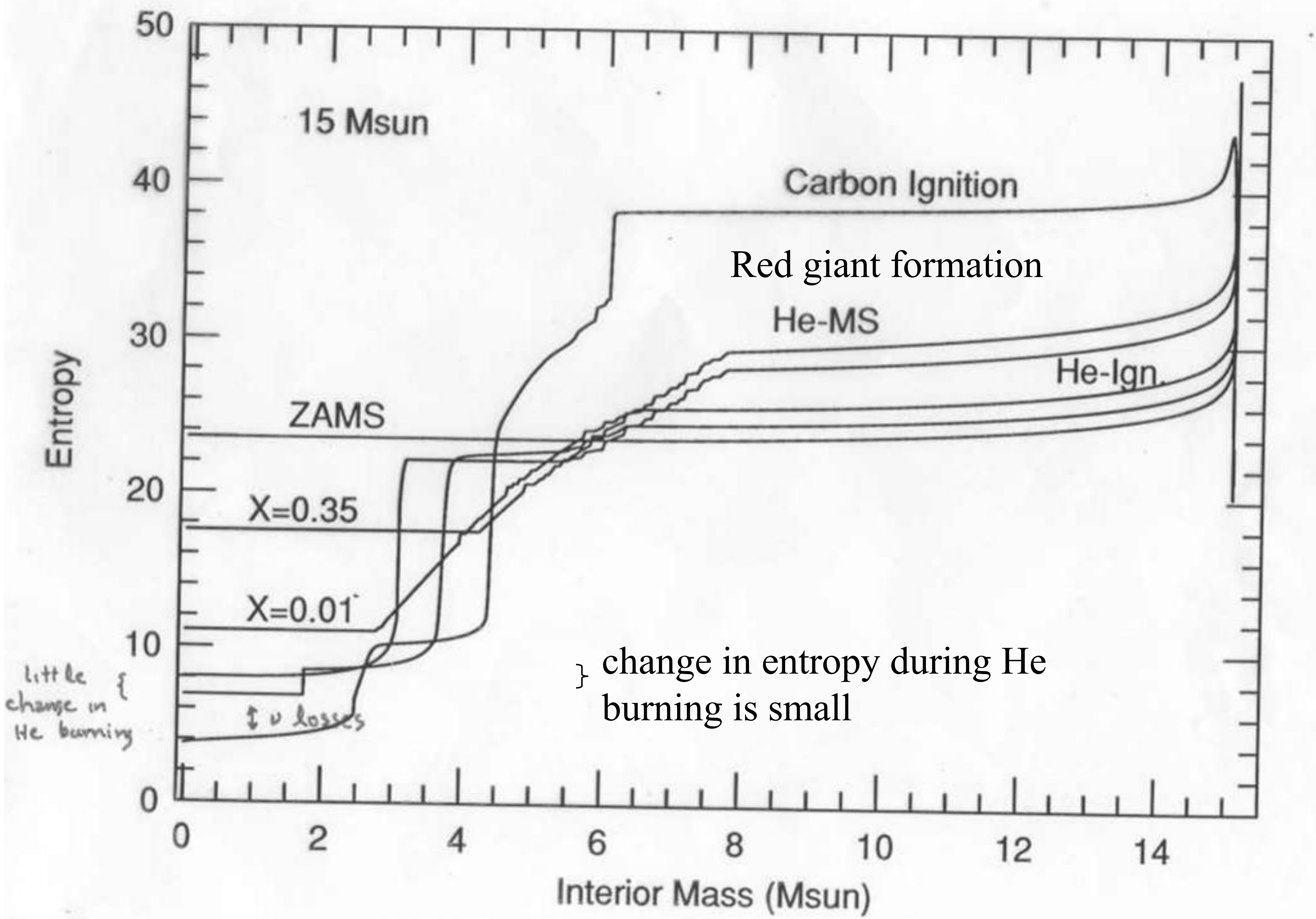
For an ideal gas plus radiation:

(see Clayton p. 123)

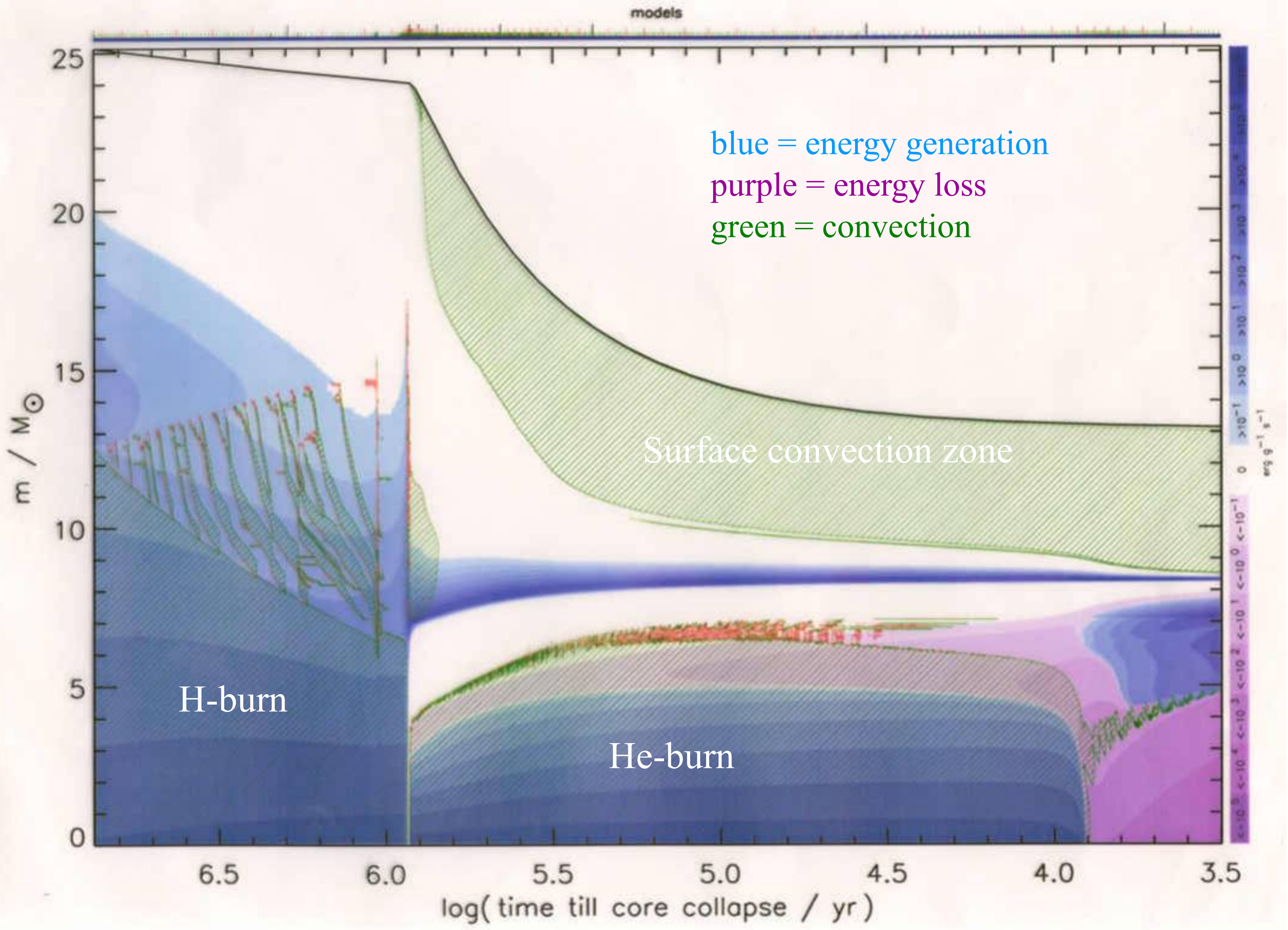
$$S = const + \frac{N_A k}{\mu} ln(\frac{T^{3/2}}{\rho}) + \frac{4a}{3}\frac{T^3}{\rho}.$$

$\mu = \frac{1}{2}$ for pure hydrogen; $\frac{4}{3}$ for pure helium

also $\frac{T^{3/2}}{\rho}$ decreases with time



15 Msun
Carbon Ignition
Red giant formation
He-MS
He-Ign.
ZAMS
X=0.35
X=0.01
change in entropy during He burning is small
little change in He burning
ν losses
Entropy
Interior Mass (Msun)
0
10
20
30
40
50
0
2
4
6
8
10
12
14


models
blue = energy generation
purple = energy loss
green = convection
Surface convection zone
H-burn
He-burn
m / M⊙
log(time till core collapse / yr)
25
20
15
10
5
0
6.5
6.0
5.5
5.0
4.5
4.0
3.5

The convective core grows during helium burning.

During helium burning, the convective core grows, largely because the mass of the helium core itself grows. This has two effects:

a) As the mass of the core grows so does its luminosity, while the radius of the convective core stays nearly the same (density goes up). For a 15 solar mass star:

	He mass fraction	Radius conv core	Lum conv core	Lum star
3.0 $M_\odot$	1	0.87×10^{10} cm	3.2×10^{37} erg s^{-1}	2.16×10^{38} erg s^{-1}
3.6 $M_\odot$	0.5	1.04×10^{10} cm	6.8×10^{37} erg s^{-1}	2.44×10^{38} erg s^{-1}

The rest of the luminosity is coming from the H shell..

b) As the mass of the helium core rises its β decreases.

$$\frac{T^3}{\rho} \sim M^2 \beta^3 \qquad \frac{1/3aT^4}{(\rho/\mu)N_A kT} \sim \frac{1-\beta}{\beta} \sim M^2\beta^3$$

Lecture 1

This decrease in β favors convection.

The entropy during helium burning also continues to decrease, and this would have a tendency to diminish convection, but the β and L effects dominate and the helium burning convective core grows until near the end when it shrinks due both to the decreasing central energy generation and central entropy.

This growth of the helium core can have several interesting consequences:

- Addition of helium to the helium convection zone at late time increases the O/C ratio made by helium burning
- If a star loses its envelope to a companion the helium core will not grow but will shrink due to mass loss. Presupernova helium cores may be smaller in mass exchanging binaries
- In very massive stars with low metallicity the helium convective core can grow so much that it encroaches on the hydrogen shell with major consequences for stellar structure and nucleosynthesis.

METALLICITY

Metallicity affects the evolution in four distinct ways:

- Mass loss
- Energy generation
- Opacity
- Initial H/He abundance

lower main sequence: homology – see appendix

κ decreases if Z decreases $\quad L \sim \kappa_o^{-16/13}\,\varepsilon_o^{1/13} \uparrow$

$$T_c \sim \left(\kappa_o \varepsilon_o\right)^{-2/15} \uparrow$$

For example, 1 $M_\odot$ at half hydrogen depletion

	Z = 0.02	Z = 0.001
$\log T_c$	7.202	7.238
L	$L_\odot$	$2.0\,L_\odot$

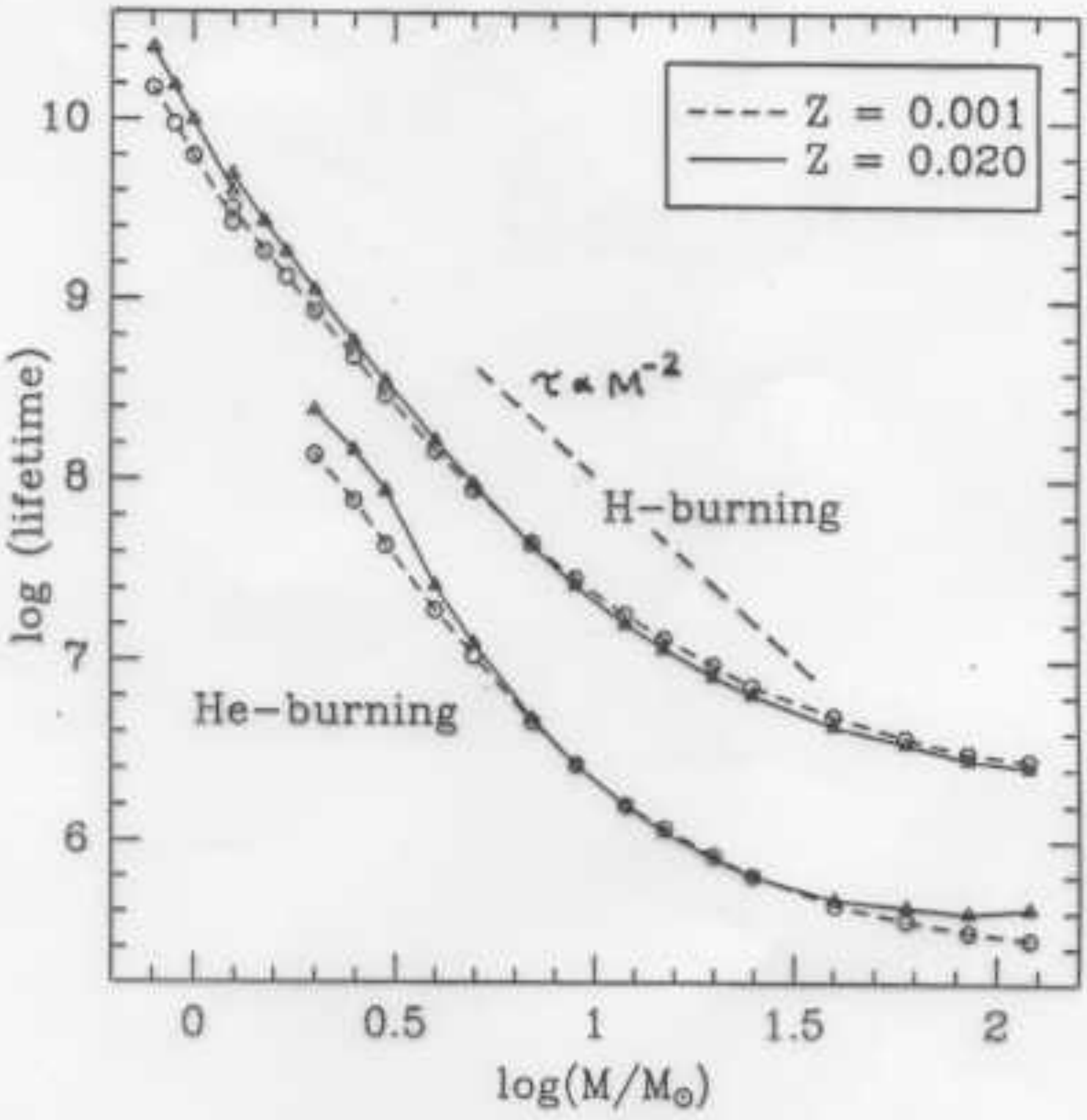
Because of the higher luminosity, the lifetime of the lower metallicity star is shorter (it burns about the same fraction of its mass). But this is the sun, it’s opacity is not due to electron scattering and so depends on Z

Upper main sequence:

The luminosities and lifetimes are very nearly the same because the opacity is, to first order, independent of the metallicity. The central temperature is a little higher at low metallicity because of the decreased abundance of ^{14}N to catalyze the CNO cycle.

For example in a 20 solar mass star at X_H = 0.3

	$Z = 0.02$	$Z = 0.001$
$\log T_c$	7.573	7.647
$\log L / L_\odot$	4.867	4.872
Q_{cc}	0.390	0.373
$M / M_\odot$	19.60	19.92 (mass loss)



Z = 0.001
Z = 0.020
τ ∝ M^-2
H-burning
He-burning
log (lifetime)
log(M/M⊙)
10
9
8
7
6
0
0.5
1
1.5
2

Zero and low metallicity stars may end their lives as compact blue giants – depending upon semiconvection and rotationally induced mixing

For example, $Z = 0$, presupernova, full semiconvection

a) 20 solar masses

$R = 7.8 \times 10^{11}$ cm $T_{eff} = 41{,}000$ K

b) 25 solar masses

$R = 1.07 \times 10^{12}$ cm $T_{eff} = 35{,}000$ K

$Z = 0.0001\ Z_{\odot}$

a) 25 solar masses, little semiconvection

$R = 2.9 \times 10^{12}$ cm $T_{eff} = 20{,}000$ K

b) 25 solar masses, full semiconvection

$R = 5.2 \times 10^{13}$ cm $T_{eff} = 4800$ K

Solar metallicity $R = 9.7 \times 10^{13}$ cm $T_{eff} = 3500$ K

Quite massive stars, M ~ 100 M_o

As radiation pressure becomes an increasingly dominant part of the pressure, β decreases in massive stars. See quartic equation.

This implies that the luminosity approaches the Eddington limit. But even in a 100 solar mass main sequence star, β is still 0.55.

Recall for n = 3

$$L(r) = (1-\beta)\frac{4\pi GMc}{\kappa}. = (1-\beta)\ L_{Ed}$$

So L is about ½ Eddington

Except for a thin region near their surfaces, such stars will be entirely convective and will have a total binding energy that approaches zero as β approaches zero. But the calculation applies to those surface layers which must stay bound.

Completely convective stars with a luminosity proportional to mass have a constant lifetime, which is in fact the shortest lifetime a (main sequence) star can have.

$$L_{Edd} = \frac{4\pi GMc}{\kappa} = 1.47 \times 10^{38} \text{ erg s}^{-1} \left(\frac{M}{M_\odot}\right)\left(\frac{0.34}{\kappa}\right)$$

$$q_{nuc} = 4.8 \times 10^{18} \text{ erg/g}$$

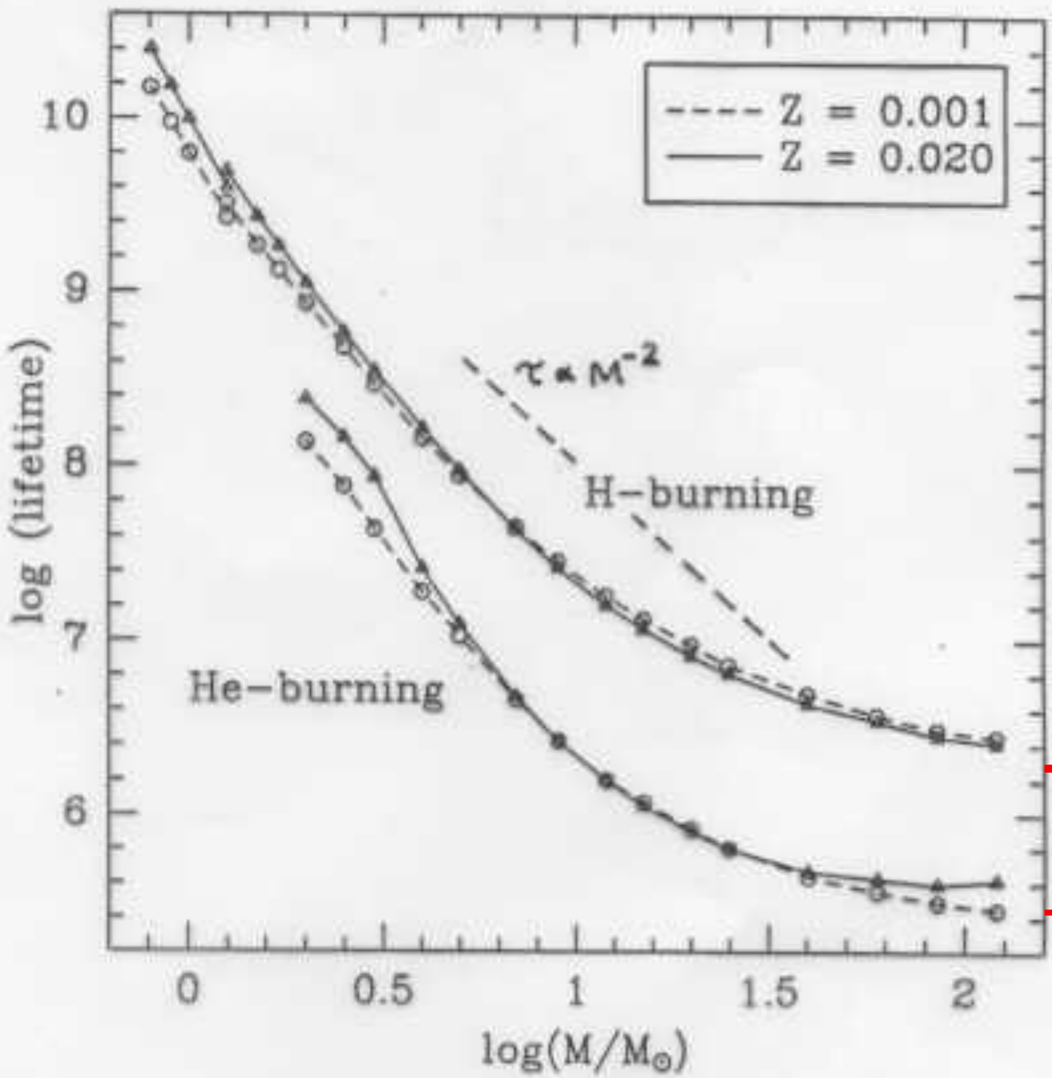
$$\tau_{\mathrm{MS}} = q_{nuc} M / L_{Edd} = 2.1 \text{ million years}$$

(exception *super*massive stars over 10^5 solar masses – post-Newtonian gravity renders unstable on the main sequence)

Similarly there is a lower bound for helium burning. The argument is the same except one uses the Q-value for helium burning to carbon and oxygen.

One gets 7.3×10^{17} erg g^{-1} from burning 100% He to 50% each C and O.

Thus the minimum (Eddington) lifetime for helium burning is about 300,000 years.



Limit

Limit

Note that homology holds pretty well for the helium cores too.

Since $\Gamma \sim 4/3$, very massive stars are loosely bound (total energy much less than gravitational or internal energy) and are subject to large amplitude pulsations. These can be driven by either opacity instabilities (the κ mechanism) or nuclear burning instabilities (the ε mechanism). β is less than 0.5 for such stars on the main sequence, but ideal gas entropy still dominates.

For solar metallicity it has long been recognized that such stars (well over 100 solar masses) would pulse violently on the main sequence and probably lose much of their mass before dying.

Ledoux, *ApJ,* **94**, 537, (1941)
Schwarzschild & Harm, *ApJ*, 129, 637, (1959)
Appenzeller, *A&A*, **5**, 355, (1970)
Appenzeller, A&A, **9**, 216, (1970)
Talbot, *ApJ*, **163**, 17, (1971)
Talbot, *ApJ*, **165**, 121, (1971)
Papaloizou, *MNRAS*, **162**, 143, (1973)
Papaloizou, *MNRAS*, **162**, 169, (1973)

Upper mass limit: theoretical predictions

Ledoux (1941)	radial pulsation, e- opacity, H	100 $M_\odot$
Schwarzchild & Härm (1959)	radial pulsation, e- opacity, H and He, evolution	65-95 $M_\odot$
Stothers & Simon (1970)	radial pulsation, e- and atomic	80-120 $M_\odot$
Larson & Starrfield (1971)	pressure in HII region	50-60 $M_\odot$
Cox & Tabor (1976)	e- and atomic opacity Los Alamos	80-100 $M_\odot$
Klapp et al. (1987)	e- and atomic opacity Los Alamos	440 $M_\odot$
Stothers (1992)	e- and atomic opacity Rogers-Iglesias	120-150 $M_\odot$

Calculations suggested that strong non-linear pulsations would grow, steepening into shock in the outer layers and driving copious mass loss until the star became low enough in mass that the instability would be relieved.

But what about at low metallicity?

Ezer and Cameron, *Ap&SS*, **14**, 399 (1971) pointed out that $Z = 0$ stars would not burn by the pp-cycle but by a high temperature CNO cycle using catalysts produced in the star itself, $Z \sim 10^{-9}$ to 10^{-7}. High temperature suppresses T sensitivity

Maeder, *A&A*, **92**, 101, (1980) suggested that low metallicity might raise M_{upper} to 200 solar masses.

Baraffe, Heger, and Woosley, *ApJ*, **550**, 890, (2001) found that zero metallicity stars (Pop III) are stable on the main sequence up to at least several hundred solar masses. This only concerns the main sequence though. Mass may be lost later as a giant, especially if nitrogen is produced by dredge up of carbon from the helium burning core.

WHAT IS SEEN?

Based on luminosities and stellar models

Table 4 Initial mass estimates for the R136a stars. $M_{C10}^{initial}$ and $M_{C10}^{current}$ are initial and current masses obtained from evolutionary tracks by Crowther et al. (2010). M_{G11} are upper limits from the homogeneous M-L relations of Gräfener et al. (2011) using a hydrogen mass fraction of 0.7 and the luminosities based on K-band, WFPC2 and WFPC3 photometry (see Table 3), assuming the distance and extinction of Crowther et al. (2010). All values are in $M_{\odot}$.

Star	$M_{C10}^{initial}$	$M_{C10}^{current}$	M_{G11}^{K}	M_{G11}^{WFPC2}	M_{G11}^{WFPC3}
R136a1	320^{+100}_{-40}	265^{+80}_{-35}	372	230	336
R136a2	240^{+45}_{-45}	195^{+35}_{-35}	285	234	347
R136a3	165^{+30}_{-30}	135^{+25}_{-20}	206	200	203
R136c	220^{+55}_{-45}	175^{+40}_{-35}	271	223	231

R136 is in S-Doradus in the LMC. Once thought to have a mass of 1000 solar masses

Binaries:

- NGC 3603-A1 – 120+26-17 (Crower et al 2010)
- WR20a – 85+-5, 82 +- 5 (Bonanos et al 2005)
- R145 –116 +- 33 (Schnurr et al (2009))

Read F. Martins (2015) on the class website.

Hoyle and Fowler (1963)
Feynman (1963)
Iben (1963)
Chandrasekhar (1964)
Fuller, Woosley and Weaver (1984)

General Relativistic Stars

The first order general relativistic correction to gravity leads to its strengthening because all kinds of energy contribute to gravity, not just rest mass. The Tolman – Oppenheimer-Volkov (TOV) equation for hydrostatic equilibrium is

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}$$

Chandrasekhar (1939) gives the local adiabatic index for very high entropy (radiation dominated stars)

$$\Gamma_1 = 4/3 + \frac{\beta}{6} + O(\beta^2)$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{\partial P}{\partial m_r} - \frac{G_{rel} m_r}{r^2}$$

$$G_{rel} = G\left(1 + \frac{P}{\rho c^2} + \frac{2Gm_r}{rc^2} + \frac{4\pi r P^3}{m_r c^2}\right)$$

Instability when (Chandrasekhar (1964)

$$R_{crit} \approx \frac{6.8}{\beta}\left(\frac{2GM}{c^2}\right) \qquad \rho_{crit} = 1.99\times 10^{18}\left(\frac{0.5}{\mu}\right)^3\left(\frac{M_\odot}{M}\right)^{7/2} \text{ g cm}^{-3}$$

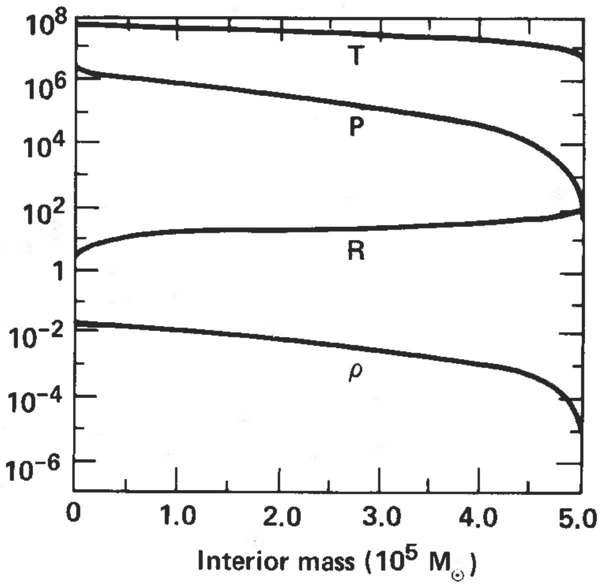


FIG. 1.—Run of temperature T (K), density ρ (g cm^{-3}), radius R (10^{12} cm), and pressure P (10^{10} ergs cm^{-3}) with interior mass coordinate for the 5×10^5 $M_\odot$ star of metallicity $Z = 5 \times 10^{-3}$ at the general relativistic instability point. At this point the run of the radius and the thermodynamic variables is very closely that for an index $n = 3$ polytrope. There are 145 mass zones in this calculation.

Unstable at H-ignition - 3×10^5 M_O

Unstable at He ignition - 4×10^4 M_O

Fuller, Woosley, and Weaver (1984)

TABLE 1

RESULTS

Initial Mass $M/10^5\ M_\odot$ (1)	Initial Metallicity $Z_{\rm init}$ (2)	Fate (3)	Cumulative Time for $L > 10^{45}$ ergs $\rm s^{-1}$ [a] (4)
1	0	Stable	...
5	0	Black hole	...
5	2×10^{-3}	Black hole	...
5	5×10^{-3}	2.1×10^{56} ergs He: $0.249 \rightarrow 0.282$	$> 3 \times 10^7$ s
5	1×10^{-2}	2×10^{56} ergs He: $0.247 \rightarrow 0.275$	$> 2.6 \times 10^8$ s
2.5	0	Black hole	...
10	0	Black hole	...
10	6×10^{-3}	Black hole	...
10	1×10^{-2}	2.5×10^{57} ergs He: $0.25 \rightarrow 0.42$	$> 10^8$ s

[a] The quantity L is the photon luminosity.

From a current calculation

$3 \times 10^4\ M_0$ helium star at helium burning (50% burned)

Central density 24.5 g cm^{-3}
Central temperature 3.18×10^8 K
Luminosity 7.8×10^{42} erg s^{-1}
Effective temperature 1.1×10^5 K
Radius = 8.6×10^{12}
μ = 1.5
$\beta = 0.016$

$\Gamma_1 = 1.336$ throughout most of the mass

Hovering on collapse

$5 \times 10^4\ M_0$ helium star collapses at helium ignition

Appendix

Homology and Entropy

Homology

$$\frac{dP}{dr} = -\frac{m(r)G}{r^2}\rho; \qquad \frac{P}{R} \sim \frac{MG}{R^2}\cdot\frac{M}{R^3}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{dL}{dr} = 4\pi r^2 \rho\epsilon; \qquad \frac{L}{R} \sim R^2 \frac{M}{R^3}\epsilon \sim R^2 \frac{M}{R^3}\epsilon_0 \frac{MT^n}{R^3}$$

$$L = \frac{4\pi r^2}{\rho}\frac{c}{\kappa}\frac{d}{dr}\left(\frac{1}{3} a T^4\right); \qquad L \sim R^2 \frac{1}{\rho\kappa}\frac{T^4}{R} \sim R^2 \left(\frac{R^3}{M}\right)^2 \frac{T^{7/2}}{\kappa_0}\frac{T^4}{R}$$

$$P = \frac{\rho k T}{\mu m_H}; \qquad P \sim \frac{M}{R^3}\frac{T}{\mu} \quad \text{(gas pressure)}$$

$$P = \frac{1}{3} a T^4; \qquad P \sim T^4 \quad \text{(radiation pressure)}$$

Energy generation rates:

$\epsilon = \epsilon_0 \rho T^n$ $\quad$ n ~ 18

Opacity: $\quad$ Kramer's opacity law (bound-free)

$\kappa = \kappa_0 \rho T^{-7/2}$ $\quad$ κ = *const* if eleectron scattering

For a non-degenerate gas, the entropy is given by (Clayton 2-136)

integrate the 1st law of thermo-dynamics
$T\,dS = dU + P\,dV$

$$\boxed{S = const + \frac{N_A k}{\mu} ln(\frac{T^{3/2}}{\rho}) + \frac{4a}{3}\frac{T^3}{\rho}.}$$

For an ideal gas

$$S_0 = \frac{3}{2}\ln\left(\frac{2\pi mk}{h^2}\right) + \frac{5}{2}$$

(Reif - Statistical Physics - 7.3.6)

The electrons are included in μ and in S_0

Ideal gas (convective with negligible radiation entropy):

$$P = \text{const} \times \rho \times T \propto \rho \times \rho^{2/3} = \rho^{5/3} = \rho^{\gamma}$$

$$\text{since } \frac{T^{3/2}}{\rho} = \text{constant}$$

$$\gamma = \frac{n+1}{n} \Rightarrow n = \frac{3}{2}$$

For non relativistic, but possibly partly degenerate electrons, the electrons are given as a separate term see Clayton 2-145.

Radiation dominated gas or a gas with constant β:

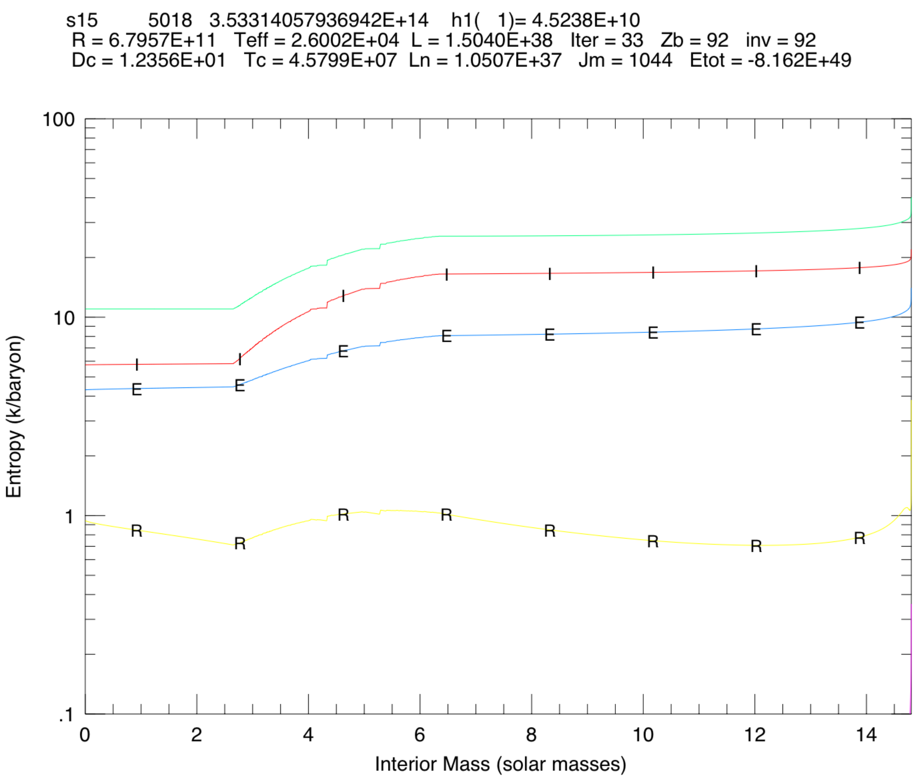
$$P = \frac{1}{3}aT^4 \propto \rho^{4/3}$$

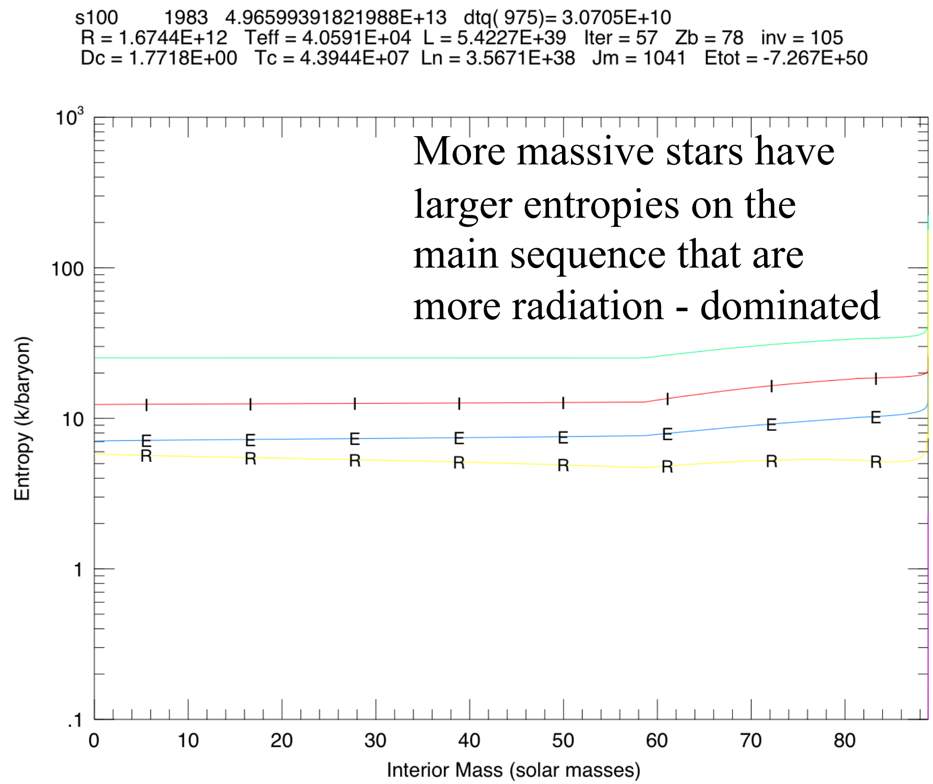
$$\text{if } \frac{T^3}{\rho} \propto \frac{P_{rad}}{P_{ideal}} = \frac{1-\beta}{\beta} = \text{constant}$$

$$\beta = \frac{P_{gas}}{P_{tot}} = \frac{P_{gas}}{P_{gas} + P_{rad}}$$

$$\gamma = \frac{n+1}{n} \Rightarrow n = 3$$

$(S/N_A k)$ half way through hydrogen burning 15 and 100 solar masses





For normal massive stars, the ionic entropy always dominates on the main sequence, but for very massive stars S_{elec}, S_{rad} and S_{ionic} can become comparable.

Not surprisingly then, it turns out that massive stars are typically hybrid polytropes with their convective cores having $3 > n > 1.5$ and radiative envelopes with n approximately 3.

Overall $n = 3$ is not bad.

Aside:

For an ideal, non-degenerate gas our (and Clayton's) equations suggest that the electronic entropy is proportional to Y_e (i.e., the number of electrons) and the ionic entropy to $1/\bar{A}$ (the number of ions). For hydrogen burning composition (75% H, 25% He) $Y_e = .875$ and $1/\bar{A} = 0.81$ (Lecture 1)

This suggests that the entropy of the electrons and ions should be about equal in the envelopes on the previous page. Our equation for the entropy is too simple and contains only the T and rho dependent terms for an ideal gas plus radiation. There are additive constants that depend on the mass of the particle

For an ideal gas

$$S_0 = \frac{3}{2}\ln\left(\frac{2\pi mk}{h^2}\right) + \frac{5}{2}$$

(Reif - Statistical Physics - 7.3.6)

$\frac{3}{2}\ln(1836) = 11.3$

so in H envelope

$S_i \approx S_e + 10$

Different equation for electrons when they are degenerate – Fermi integrals