

## ASTRONOMY 12

### *Problem Set 1 – Due Thursday, January 21, 2016*

*Please work all of the following problems. Show your work. You may discuss problems with others, but your work should reflect your own thought and not duplicate what is submitted by someone else. Considerable partial credit will be given for a correct approach even if you miss the actual numerical answer. If your answer sounds preposterous to you say so and suggest what might be wrong. Express your answers, where appropriate, to the accuracy implied by the statement of the problem (usually 2 figures) and use scientific notation. You will probably not be able to work some of the problems until we cover the appropriate material in class. You would be well advised, however, not to wait to the last minute. Work what you can early on. Each problem is individually fairly simple, but the total time required can add up.*

*The TA additionally asks that you please follow these additional guidelines so that points are not lost accidentally or through lack of clarity:*

- 1) Staple all pages together. Don't fold corners or use paper clips.*
- 2) Put a box around all answers and include the correct units where appropriate.*
- 3) Insure that your homework is legible to a human being other than yourself.*
- 4) Submit your solutions in numerical order: 1, 2, 3, 4, ... not 2, 7, 1, 3, ...*

*Longer problems - 10 to 15 points each*

- 1)** Assume it is the vernal equinox (approximately March 21) for parts a), b), and c).
  - a) What is the declination of the Sun? Does this answer depend on your location on the Earth?
  - b) If the Sun were located 55 degrees above the horizon at mid-day that day, at what latitude would you be located?
  - c) Would the Sun rise due east, north of east, or south of east?
  - d) To what maximum altitude above the horizon would the sun rise, at the same location, on the summer solstice, roughly June 21?
- 2)** a) Suppose an astronomer wants to study the star Alpha Centauri in detail. Its coordinates are (approximately)  $RA = 14\text{h } 36\text{m}$ ,  $dec = -60^\circ$ . Should she use a telescope at Lick Observatory (located east of San Jose, at latitude  $37^\circ$  N) or at Keck Observatory (located in Hawaii, at latitude  $19^\circ 50'$  N)? Show your calculations; don't just state the answer.
  - b) The right ascension of Sirius is 06 h 45 m. If the sidereal time in Greenwich, England

is 11 h 45 m when Sirius is on your celestial meridian, what is your **longitude**? Please express your answer in degrees east or west of Greenwich (this is the definition of longitude on the Earth.)

**3)** If the luminosity of a main sequence star is proportional to its mass cubed (i.e.,  $L = \text{const } M^3$ ), and the mass of hydrogen the star has available to burn on the main sequence (and hence the total energy it can produce on the main sequence) is proportional to  $M$ , how does the lifetime of a star on the main sequence depend on its mass, i.e., if  $\tau \propto M^n$ , what is  $n$ ? If the Sun lives 10 billion years on the main sequence, how long would a 0.8 solar mass star live? If the most massive star remaining as a main sequence star in a globular cluster is 0.8 solar masses (all the rest having turned into something else, like a red giant or white dwarf), what is the age of that globular cluster?

**4)** Consider an astronaut descending feet first into a *ten* solar mass (one solar mass is  $1.99 \times 10^{33}$  gm) black hole. Assume the astronaut is tall (she soon will be) and has a height of 200 cm and has a mass of 60 kg ( $6 \times 10^4$  gm). a) What would be the tidal force between the bottom of her feet and the top of her head at a distance of 100 km from the hole? Give your answer in dynes and in pounds of force (1 dyne =  $2.248 \times 10^{-6}$  pound force). Falling freely, this is the stretching force her body would feel. This is a question about *tidal* forces. b) What is your answer if the astronaut were located twice as far away, i.e., 200 km away? Try to answer this question using ratios and without plugging in numbers again.

**5)** a) The Hubble Space Telescope is located about 600 km *above the Earth's surface*. How many minutes does it take for the telescope to make one complete orbit around the Earth? How fast is it moving, in km/s? Please calculate; don't try to look it up.

b) Communication and broadcast satellites are often placed in "geosynchronous orbits" around the Earth. An object in such an orbit has an orbital period equal to one Earth day. How high *above the Earth's surface* would a satellite be located in order to be in a geosynchronous orbit? Express your answer in kilometers and in Earth radii.

**6)** An early (and incorrect) theory attempting to identify the Sun's source of energy claimed that gravitational contraction could release sufficient energy to account for the Sun's luminosity. Let's examine this theory here and see why it fails.

a) Calculate the gravitational binding energy of the Sun, given its current mass and radius and assuming a constant density.

b) The gravitational binding energy represents the amount of energy lost as the Sun collapsed from an extended cloud of gas into its present size and shape. According to the Virial theorem, what fraction of this energy was lost as radiation? (The Virial theorem explains why light energy is generated from gravitational collapse.)

c) Given the current luminosity of the Sun, and assuming it has remained constant over its lifetime, how long could the Sun radiate away the energy released from gravitational contraction?

d) Why is gravitational contraction NOT a likely explanation for the Sun's energy source today?

7) Let's recreate the event that may have killed the dinosaurs 65 million years ago. Consider a small rocky asteroid of **radius** 3 kilometers and **density**  $5 \text{ g cm}^{-3}$ . You may assume that it is spherical in shape.

a) Compute its total mass in grams.

Now presume this asteroid, starting at rest relative to the Earth, falls from a great distance (say infinity) and strikes the Earth's surface. Neglect, for the moment, the motion of the asteroid and the Earth around the Sun. Just think of the Earth and asteroid both initially at rest and then accelerating toward one another due to their mutual gravitational attraction (actually the asteroid does all the moving).

b) With what velocity, in km/s, would the asteroid hit the Earth? Be sure you know which object (Earth or asteroid) each variable refers to in your formula.

c) What energy will be dissipated in the impact (as heat and kinetic energy of the ejecta)? Express your answer in ergs and in Megatons of high explosives (1 Megaton =  $4.2 \times 10^{22}$  erg).

8) Write an equation that gives the kinetic energy of the Earth around the Sun in terms of the solar mass,  $M_{\odot}$ , the mass of the Earth,  $M_E$ , and the AU (assume circular orbits). Write another equation, in the same quantities, that gives the gravitational potential energy of the Earth with respect to the Sun, i.e., the energy that would be released if the Earth fell straight in from infinity to a distance 1 AU from the Sun. Which is larger in magnitude (the potential energy will be a negative number) and what is the ratio of the two energies? Your answers should involve variables only, not numerical values.

*Short questions - 3 to 6 points each*

1) Associate each of the following properties with a low mass star (less massive than the sun) or a high mass star, say of several solar masses: a) rare or common, b) luminous or less luminous; c) long lived or short lived; d) red or blue. A table might be an easy way to short the answer.

2) What is the age of the earth? The age of the sun? The two most abundant elements in the sun? The element that constitutes most of the mass of your body?

The approximate diameter of the bright disk of the Milky Way Galaxy measured in light years?

3) What are the right ascension and declination of the Sun at the moment of the autumnal equinox?

4) Where on the Earth would the Sun be directly overhead at its highest point on the equinoxes? *Explain your answer (perhaps with a diagram).*

5) The four fundamental forces were discussed in class. Which of the four forces is chiefly responsible for:

a) Holding the neutrons and protons together in a carbon nucleus?

b) Holding the electron to the proton in the hydrogen atom?

c) Holding oxygen and hydrogen together in the water molecule?

d) Keeping the Earth in orbit around the Sun?

e) Binding your fingers to your hand?

f) Holding your feet to the ground?

g) Allowing a free, unbound neutron to decay into a proton?

h) Keeping the stars bound together in a galaxy?

6) If a planet were discovered that had the mass of the Earth but a radius twice as large, how much smaller would its a) average density and b) escape velocity be compared to the Earth's (i.e., by what factor would it go down in each case)?

7) If there were a planet 2 AU from the Sun, with a mass of 10 Earth masses, what would be its period around the Sun measured in years? What if the planet were 5 Earth masses instead?

8) The mass of the Moon is  $7.35 \times 10^{25}$  gm and its radius is  $1.74 \times 10^8$  cm. What is the Moon's escape speed in  $\text{km s}^{-1}$ ? Compare this with the Earth's  $11.2 \text{ km s}^{-1}$  escape speed. Since the kinetic energy required to escape goes as  $v_{esc}^2$ , the Moon would make a nice "space station".