

ASTRONOMY 12

Problem Set 1 – Due Thursday, January 26, 2012

Longer problems - 10 to 15 points each

Please work all of the following problems. Show your work and work individually. Considerable partial credit will be given for a correct approach even if you miss the actual numerical answer. If your answer sounds preposterous to you say so and suggest what might be wrong. Express your answers, where appropriate, to the accuracy implied by the statement of the problem (usually 2 figures) and use scientific notation. You will probably not be able to work some of the problems until we cover the appropriate material in class. You would be well advised, however, not to wait to the last minute. Work what you can early on. Each problem is individually fairly simple, but the total time required can add up.

1) Assume it is the summer solstice (June 21) for parts a), b), and c). a) What is the declination of the sun? Does this answer depend on your location on the earth? b) If the sun were located 60 degrees above the horizon at mid-day, at what latitude are you located? c) Would the sun rise due east, north of east, or south of east? d) What would your answers be to a), b), and c) on the vernal equinox, roughly March 21?

2) a) The latitude of Mauna Kea Hawaii (where the Keck telescopes are) is 19 degrees 50 minutes, north. The coordinates of the star α -Centauri are (approximately) RA = 14h 36m and $\delta = -60^\circ$. Is α -Centauri ever visible from Mauna Kea? Is it visible all the time or just some of the time? Show a calculation; don't just state the answer.

b) The right ascension of Sirius is 06 hr 45 m. If the sidereal time in Greenwich England is 11 hr 45 m when Sirius is on your celestial meridian, what is your **longitude**? Please express your answer in degrees east or west of Greenwich (this is the definition of longitude on the earth.)

3) If the luminosity of a main sequence star is proportional to its mass cubed (i.e., $L = \text{const } M^3$), and the mass of hydrogen the star has available to burn on the main sequence (and hence the total energy it can produce on the main sequence) is proportional to M , how does the lifetime of a star on the main sequence depend on its mass, i.e., $\tau \propto M^n$, what is n ? If the sun lives 10 billion years on the main sequence, how long would a 0.8 solar mass star live? If the most massive star remaining as a main sequence star in a globular cluster is 0.8 solar masses (all the rest having turned into something else, like a red giant or white dwarf), what is the age of that globular cluster?

4) Consider an astronaut descending feet first into a *ten* solar mass (one solar mass is 1.99×10^{33} gm) black hole. Assume the astronaut is tall (she soon will be) and has a height of 200 cm and has a mass of 60 kg (6×10^4 gm). a) What would be the tidal force between the bottom of her feet and the top of her head at a distance of 100 km from the hole? Give your answer in dynes and in pounds of force (1 dyne = 2.248×10^{-6} pound force). Falling freely, this is the stretching force her body would feel. This is a question about *tidal* forces. b) What is your answer if the astronaut were located twice as far away, i.e., 200 km away? Try to answer this question using ratios and without plugging in numbers again.

5) It is often hard to make direct measurements of the entire Milky Way galaxy (e.g., its optical luminosity), but we can study its near twin, the Andromeda galaxy. The Andromeda galaxy, like our own, has orbiting globular clusters and dwarf ellipticals. One of those globular clusters, G1, is located 150,000 light years from the center of Andromeda. G1 is observed to be orbiting Andromeda at a speed of 310 km s^{-1} . Assume the orbit of G1 is circular. What is the mass of the Andromeda galaxy in solar masses? [some unit conversions will be involved in this calculation]. The luminosity of the Andromeda galaxy is 8×10^9 solar luminosities. What is the mass-to-light ratio, measured in solar masses per solar luminosity, of Andromeda? The correct answer is much greater than 1. What do you infer about the nature of the matter that makes up Andromeda? Is it all solar mass stars, stars heavier than the sun, stars lighter than the sun, or what?

6) The total geothermal energy diffusing out of the Earth comes from a combination of radioactive decay and heat left over from the Earth's formation. Currently the total power is measured to be 45 terawatts = $4.5 \times 10^{20} \text{ erg s}^{-1}$

(http://en.wikipedia.org/wiki/Earth's_energy_budget).

a) Given the current radius and mass of the Earth (see constant sheet) and using expressions derived in class that assume constant density, what is the gravitational binding energy of the Earth?

b) Most of this binding energy was radiated away long ago, but to get an upper bound to what might still be available from the original formation of the Earth, divide this binding energy by the known age of the Earth. What would be the "luminosity" of the Earth in erg s^{-1} . Could this be the source of most of the geothermal power? Note that this is all independent of the much larger power the Earth receives from the sun.

7) Let's recreate the event that may have killed the dinosaurs 65 million years ago. Consider a small rocky asteroid of **radius** 5 kilometers and **density** 5 g cm^{-3} . You

may assume that it is spherical in shape.

a) Compute its total mass in grams.

Now presume this asteroid, starting at rest relative to the Earth, falls from a great distance (say infinity) and strikes the Earth's surface. Neglect, for the moment, the motion of the asteroid and the Earth around the sun. Just think of the Earth and asteroid both initially at rest and then accelerating toward one another due to their mutual gravitational attraction (actually the asteroid does all the moving).

b) With what velocity in km/s would the asteroid hit the earth?

c) What energy will be dissipated in the impact (as heat and kinetic energy of the ejecta)? Express your answer in ergs and in Megatons of high explosives (1 Megaton = 4.2×10^{22} erg).

Short questions - 3 to 6 points each

1) What is the highest angle the sun will reach above the horizon in Santa Cruz on the Vernal Equinox (March 21)? The latitude of Santa Cruz is 37 degrees north.

2) What are the right ascension and declination of the sun at the moment of the autumnal equinox?

3) The four fundamental forces were discussed in class. Which of the four forces is chiefly responsible for:

a) Holding the neutrons and protons together in a carbon nucleus?

b) Holding the electron to the proton in the hydrogen atom?

c) Holding oxygen and hydrogen together in the water molecule?

d) Keeping the earth in orbit around the sun.

e) Binding your fingers to your hand?

f) Holding your feet to the ground?

g) Allowing a free, unbound neutron to decay into a proton.

h) Keeping the stars bound together in a galaxy.

4) If a planet were discovered that had the mass of the earth but a radius half as large, how much larger would its average density be (i.e., by what factor would it go up) and by what factor would its escape velocity increase?

5) Write an equation that gives the kinetic energy of the earth around the sun in terms of the solar mass, the mass of the earth, and the AU (assume circular orbits). Write another equation, in the same quantities that gives the gravitational potential energy of the earth with respect to the sun, i.e., the energy that would be released if the Earth fell straight in from infinity to a distance 1 AU from the sun. Which is larger in magnitude (the binding energy will be a negative number) and what is the ratio of the two energies?

6) If there were a planet 3 AU from the sun, with a mass of 10 Earth masses, what would be its period around the sun measured in years? What if the planet were 20 earth masses instead?

7) The mass of the moon is 7.35×10^{25} gm and its radius is 1.74×10^8 cm. What is the moon's escape speed in km s^{-1} ? Compare this with the Earth's 11.2 km s^{-1} escape speed. Since the kinetic energy required to escape goes as v_{esc}^2 , the moon would make a nice "space station".

8) To two figure accuracy, what is the age of the earth in years? The age of the sun? Express your answer in scientific notation.