ASTRONOMY 12

Problem Set 3 — Due Thursday, February 25

Longer Problems

1) Two stars in a face on visual binary are observed to orbit their center of mass with a period of 50 years. The two stars are separated by a total distance of 20 AU. Assume circular orbits. Star 1 is moving one-half as fast as Star 2. What are the masses, in solar masses, of the two stars?

2) Identify each and discuss with a few sentences.

a) Coronal gas

b) Molecular cloud

c) H II region

d) T-Tauri stars

3) a) What mass (in solar masses) would a cloud composed of pure hydrogen at a temperature 40 K and number density $n = 10,000\,\mathrm{cm}^{-3}$ need to have in order to collapse gravitationally? Assume the cloud is spherical and has constant density and temperature.

b) Would a more massive cloud having the same density and temperature be more tightly bound by gravity or unbound by thermal energy?

4) From computer models and neutrino observations the present sun has a central density of near 160 g cm^{-3} and temperature, 15 million K. Assuming a composition of 35% hydrogen and 65% helium at the sun's center (because about half the hydrogen has burned to helium there) and consulting the table in your notes for "F" in the ideal gas pressure equation. a) What is the ideal gas pressure for these conditions? b) What is the radiation pressure? c) If the sun's center were non-relativistically degenerate (it isn't), what would be the degeneracy pressure (use $Y_e = 0.68$)? d) What kind of pressure dominates in the sun's center?

5) The relation between the radius of a main sequence star and its mass is rather difficult to derive, but a reasonable overall fit is $R = R_\odot(M/M_\odot)^{0.65}$. Use this relation to answer the following questions.

a) Assuming uniform density, how does the average density of a star vary with mass on the main sequence? If the mass is halved, does the density go up or down and by what factor?

b) Assuming hydrostatic equilibrium continuing to assume constant density and the dominance of ideal gas pressure, how does the central temperature vary with mass? For a star of half the mass, how much does the central temperature change?

6) The fusion of hydrogen in the Sun's interior releases $6.4\times10^{18}\,\mathrm{erg\,g}^{-1}$ (see notes from Lecture 12). The combustion of gasoline with air releases $4.4\times10^{11}\,\mathrm{erg\,g}^{-1}$. [notice how many miles per gallon you could get if your car was fusion powered - just notice, don't compute]. Assuming the same fraction of the Sun's mass participates in the burning, that the luminosity of the Sun does not change, and that the lifetime of the Sun fusing hydrogen to helium is 10 billion years, what would be the Sun's lifetime if it were gasoline powered?

7) a) Considering the electrical repulsion between two protons, and assuming that each proton has only the kinetic energy available because the gas is hot, i.e., $KE = \frac{3}{2}kT$, estimate the temperature required for two protons to come within 10^{-13} cm of each other before being repelled apart by their mutual charge. [Hint: Use energy conservation. At large separation the total energy is just kinetic. At closest approach when the velocity goes to zero, the total energy is just electrical potential energy, $PE = e^2/r$, where e is the charge of the proton, 4.8×10^{-10} esu.]

b) Compare this temperature with what the virial theorem and other approaches give for the central temperature of the Sun - as discussed in class - about 10^7 K. How do you reconcile this difference?

8) Since the Sun generates its energy by the fusion of hydrogen into helium, two neutrinos are produced every time a helium nucleus is formed (two protons have somehow to turn into two neutrons along the way). Given how much energy is released each time a helium nucleus is produced (26.2 MeV $= 4.20 \times 10^{-5}$ erg) and the luminosity of the Sun, $L_\odot = 3.83 \times 10^{33}\,\mathrm{erg\,s^{-1}}$, estimate

a) the number of helium nuclei being produced per second in the Sun,

b) the number of neutrinos being produced per second in the Sun, and

c) the flux of neutrinos passing through the Earth's crust (and your body). Does your answer matter whether it is day or night?

[Hints: The answer to part b) is just $2\times$ the answer to part a) (Why?). For part c), once you have the number of neutrinos made per second, use the definition of flux ϕ in terms of luminosity to see how many arrive per cm^2 at Earth.]

Shorter Questions

1) In which phases of the interstellar medium would the following states of hydrogen likely be found a) H_2, b) H II, c) H I? In which phase does star formation occur?

2) Write down the name and series of nuclear reactions responsible for powering the Sun. DO NOT use shorthand notation. Above what stellar mass does the *CNO cycle* dominate in producing energy on the main sequence?

3) Why is there a minimum mass a star can have on the main sequence and what is the value of that mass? What happens to a contracting protostar below that mass?

4) The Sun rotates differentially, i.e., it rotates faster at its equator than at other latitudes. A point on its equator goes around once every 26.8 days, but a point at 75 degrees latitude (either north or south) takes 31.8 days to go around. In 10 years, how many more times has the Sun rotated at its equator than at 75 degrees latitude?

5) List the three kinds of gas pressure discussed in class. Indicate how each one depends on temperature and/or density (i.e., write down the proportionality, i.e., give x and y in $P \propto \rho^x T^y$). Give your answer for the two different kinds of degeneracy.

6) Give three ways that energy can be transported (moved from a smaller radius to a larger one) in main sequence stars. Which dominates in the center of the current sun? Near its surface? In the center of a 10 solar mass star? Near *its* surface?

7) Define each of the following with a sentence or two.

a) Little Ice Age

b) Umbra

c) Babcock model

8) Is the present sun on the main sequence growing more luminous with time or less? Larger in radius or smaller? Hotter in its center or cooler?

9) Sketch the location of the horizontal branch in the Hertzsprung Russel diagram. What fuel is burning in the centers of such stars?