ASTRONOMY 12

Problem Set 4 – Due March 10, 2016

1) After helium burning in a low mass star like the sun, a degenerate core composed of carbon and oxygen is formed that has a temperature of roughly 4×10^8K, i.e., a white dwarf star. The ions (i.e., the nuclei) are not degenerate, even though the electrons are. The total *thermal* energy in the core (which is all that can be radiated away) is then $E \sim N_{ion} kT$ where N_{ion} is the number of nuclei in the core. a) Calculate the total number of carbon nuclei (which will be N_{ion}) in a (pure) carbon core of 0.7 $M_\odot$. (hint: 1 gram of carbon contains $N_A/12$ carbon ions where $N_A = 6.02 \times 10^{23}$). b) What is the total thermal energy in this 0.7 $M_\odot$ of carbon nuclei if the temperature is 3×10^8 K? c) How long could this energy provide a luminosity of 0.01 $L_\odot$? d) of 0.001 $L_\odot$? This is where white dwarfs get their energy. They are the cooling "embers" of helium burning.

2) Big and bright as a supernova is, it still radiates like a blackbody with an effective temperature not much hotter than the solar photosphere. A value of 5500 K is typical for a Type IIp supernova. But the supernova has an enormous radius, $\sim 3 \times 10^{15}$ cm (i.e., 200 AU). Assuming these values for the temperature (as measured e.g., by Wien's Law) and the radius, calculate the luminosity of the supernova in erg s^{-1} and in solar luminosities. In what part of the HR diagram would supernovae be found (upper left, upper right, lower left, or lower right?). Incidentally, this makes Type IIp supernovae useful standard candles. We don't know their radius directly but can calculate it from the age of the supernova and its expansion rate (as determined by spectral line widths) and then calculate their luminosity, making them a standard candle.

3) Assume that a typical neutron star has a radius of 10 km and a mass of 1.4 $M_\odot$. a) If the neutron star can be approximated as a sphere of constant density, what is its gravitational binding energy in erg (see lecture 4 if you don't remember how)? b) This much energy is radiated away as neutrinos during the three second Kelvin-Helmholtz evolution of the neutron star. What is the approximate neutrino luminosity of the neutron star in erg/s during those 3 seconds? c) Compare the binding energy with the rest mass energy Mc^2 of the neutron star. d) Compute the escape speed of the neutron star and compare it with the speed of light.

NB. The neutron star actually weighs significantly less than the iron core that collapsed to it. The difference is radiated away as neutrinos. This is another example of binding energy, in this case gravity, reducing the effective mass of the system. Other examples are nuclei (mass reduced by nuclear binding) and atoms (mass reduced by electrical binding).

4) A neutron star has a radius of 10 km and a mass of 1.4 $M_\odot$. It has a strong magnetic field and is rotating so it may be a pulsar. What is the shortest period that pulsar

can have? Stated another way, what is the fastest the neutron star can rotate without flying apart at its equator? Consider a gram of matter at the equator and the balance of centrifugal and gravitational forces there. A real neutron star would deform, but for this calculation assume that it remains a sphere.

5) If the net gravitational binding energy of a white dwarf just before it explodes as a Type Ia supernova is -3×10^{50} erg and if the nuclear fusion reactions release 10^{51} erg and the mass of the dwarf is 1.4 $M_{\odot}$, what will be the average velocity - in km/s - of the ejecta in the explosion? [Hint: Use energy conservation. What is the total energy of the white dwarf before the explosion? Afterwards, the energy of the ejecta is all kinetic.]

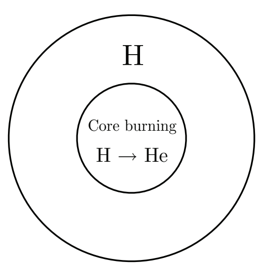
6) What are the chief differences with respect both to mechanism and observations between a Type Ia and a Type II supernova? Give 5 characteristics of each - "what kind of stars give each, how often do they occur, what does the spectrum look like, how bright are they, how long does each last, what velocities are seen, what elements do they make, what do they leave behind, how does each explode", etc. are questions you might consider.

Shorter Questions and lists

1) Define and discuss each of the following with two or three sentences.

a) s-process b) Helium flash c) Carbon burning d) Chandrasekhar mass
e) Planetary nebula

2) Sketch the interior structure of a low-mass star during each of the following stages: red giant branch star, horizontal branch star, asymptotic giant branch star. In each diagram, indicate a) where fusion is occurring (core and/or shell) and the fusion reaction taking place there (e.g., H $\rightarrow$ He) and b) the dominant element present in each non-fusing region. See the example diagram below for a main sequence star. Do not worry about drawing your diagrams to scale.



Main sequence star

3) In a few sentences describe the currently favored astrophysical model for each of

the following:

a) Classical nova

b) Pulsar

c) Binary (stellar massed) x-ray source

4) What is the relevance of each of the following masses in stellar evolution? That is, what happens above or below this mass that is different?

a) 0.08 $M_\odot$ b) 0.5 $M_\odot$ c) 1.44 $M_\odot$ d) 2 $M_\odot$ (two things) e) 8 $M_\odot$ f) $\sim$150 $M_\odot$ (what happens and why?).

5) Suppose you could watch the evolution of a cluster of stars over time. Which do you expect to occur first in the cluster, a Type Ia supernova or a Type II? Explain your reasoning. [Hint: Think about the lifetimes of the progenitor stars for each type of supernova.]

6) Where in Nature - by what event and or process(es) is each of the following species made and how is it ejected into the interstellar medium? Use complete phrases like "carbon burning in a massive star ejected during a supernova": a) hydrogen b) carbon c) nitrogen d) oxygen e) silicon f) iron.