AY12 Homework #1 Solutions

Winter 2016

Longer Problems

1. a) As discussed in class, the declination of the Sun on vernal equinox is $\boxed{\delta = 0^\circ}$ Declination $\boxed{\text{does not depend on your location on Earth}}$.

   b) The angle between the celestial equator and the horizon is $90^\circ - L$, where L is your latitude. The angle between the celestial equator and the Sun is 0°. The sum of these two angles is 55°. Thus $55^\circ = (90^\circ - L) + 0^\circ$, and hence $\boxed{\text{XXXXX}}$. 35 N

   c) The Sun will rise $\boxed{\text{due east}}$.

   d) On the summer solstice, the Sun's declination is $\delta = +23.5^\circ$, so the maximum altitude reached$=(90^\circ - 35) + 23.5^\circ$, or $\boxed{78.5^\circ}$.

2. a) To answer this question, let's determine the declination of a star that just barely rises above the horizon for each location. If Alpha Centauri's declination is greater than this, then we know we can observe it from that location.

   The declination of a star that just barely rises is $\delta = -(90^\circ - L)$, where L is your latitude. For Lick Observatory, $L = 37^\circ$ N, which gives $\delta = -(90^\circ - 37^\circ) = -53^\circ$. Since Alpha Centauri's declination is $\delta = -60^\circ < -53^\circ$, it never rises above the horizon at Lick.

   At Keck, doing the same calculation gives $\delta = -(90^\circ - 19.8^\circ) = -70.2^\circ$, where you can convert $L = 19^\circ 50'$N into degrees by $19^\circ + 50' \times \frac{1^\circ}{60'} = 19.8^\circ$ This means Alpha Centauri does rise above the horizon, since its declination is greater than -70.2°. The astronomer $\boxed{\text{should use a telescope at Keck.}}$

   b) Your longitude can be determined if you know two things: 1) the right ascension of stars currently on your meridian, and 2) the sidereal time at Greenwich. Your longitude is the difference between 2) and 1). For this problem, 1) $= 6^\text{h}45^\text{m}$ and 2) $= 11^\text{h}45^\text{m}$. Thus your longitude is 5^h. In degrees, this is $5^\text{h} \times \frac{15^\circ}{1^\text{h}} = 75^\circ$. More precisely, it is $\boxed{75^\circ \text{ W of Greenwich}}$ because you are 5 hours *behind* Greenwich.

3. The luminosity of a star is equal to the energy radiated divided by the time it radiates it, $L = E/t$. In this problem we are solving for the lifetime of a main sequence star in terms of its mass. This means we solve the above equation for t (or τ): $\tau = E/L$. Now the luminosity is proportional to the mass cubed, $L \propto M^3$, and the energy available is proportional to the mass, $E \propto M$. Thus
$$\tau = \frac{E}{L} \propto \frac{M}{M^3} \propto \frac{1}{M^2} = M^{-2}.$$
   The exponent $\boxed{n = -2}$, which means that the more massive a star is, the shorter its main sequence lifetime is.

   To compute actual ages, we rewrite the relation $\tau \propto M^{-2}$ as an equation by scaling to solar values,
$$\frac{\tau}{10^{10}\,\text{yr}} = \left(\frac{M}{M_\odot}\right)^{-2} \Rightarrow \tau = 10^{10} \times \left(\frac{M}{M_\odot}\right)^{-2} \text{ yr}.$$

We substitute $0.8M_\odot$ in for M in the equation to find the lifetime of the star:

$$\tau \approx 10^{10} \times \left(\frac{M}{M_\odot}\right)^{-2} \text{yr} = 10^{10} \times \left(\frac{0.8\cancel{M_\odot}}{\cancel{M_\odot}}\right)^{-2} \text{yr} = 10^{10} \times 0.8^{-2}\,\text{yr} = \boxed{15.6 \times 10^9\,\text{yr}}.$$

If a $0.8\text{-}M_\odot$ star is the most massive star still on the main sequence in a globular cluster, then the cluster's age is equal to the lifetime of that $0.8\text{-}M_\odot$ star, which you calculated to be $\boxed{\text{16 billion years}}$. The cluster has the same age.

4. a) The tidal force on an object of mass m and length d located a distance r from another mass M is given by

$$F_{\text{tidal}} = \frac{2GMmd}{r^3}.$$

Here, $d = 200\,\text{cm}$, $r = 100\,\text{km}$, $m = 60\,\text{kg}$, and $M = 10\,M_\odot$. First we convert the masses into grams, then we substitute the values into the equation above. You should get the tidal force on the astronaut to be $\boxed{F_{\text{tidal}} = 3.2 \times 10^{13}\ \text{dyne}}$. To convert this answer to *pounds*, use the conversion factor given:

$$3.1 \times 10^{13}\,\cancel{\text{dyne}} \times \frac{2.2 \times 10^{-6}\,\text{lb}}{1\,\cancel{\text{dyne}}} = \boxed{7.0 \times 10^7\,\text{lb}}.$$

The poor astronaut feels a tug at her feet that is 70 million pounds greater than at her head!

b) If the distance is halved to $r = 200$ km, then the tidal force *decreases by a factor of 8.* This is because $F_{\text{tidal}} \propto 1/r^3$, and doubling the distance gives $F_{\text{tidal}} \propto 1/(2r)^3 \propto (1/8r^3)$, i.e., the new tidal force is an eighth of the original force. Thus $\boxed{F_{\text{tidal}} = 4.0 \times 10^{12}\ \text{dyne} = 8.8 \times 10^6\ \text{lb}}$.

5. a) It's easier to calculate how fast the telescope is moving first. The orbital speed of an object is

$$v_{\text{orb}} = \sqrt{\frac{GM}{r}},$$

where M is the mass of the central body (the Earth), and r is the distance *from the center of the Earth* to the satellite. Since the telescope is 600 km above the Earth's surface, $r = 600 + 6380 = 6980$ km. Converting km to cm and plugging into the above equation, we find that $v_{\text{orb}} = 7.6 \times 10^5$ cm/s $= \boxed{7.6\ \text{km/s}}$.

To calculate how many minutes each orbit takes, we use the following definition of orbital speed:

$$v_{\text{orb}} = \frac{2\pi r}{P},$$

where r is the orbital distance, and P is the orbital period. Substituting in for r and v_{orb}, we find that $P = 5800$ sec, or $\boxed{97\ \text{min}}$.

b) In geosynchronous orbit, $P = 24$ hours. In order to calculate the radius of the orbit, we can use Kepler's 3rd law in the form:

$$r^3 = \frac{GM}{4\pi^2}P^2,$$

where M is the mass of the object you're orbiting (the Earth, in this case). Substituting in values, we find

$$r = \sqrt[3]{\frac{(6.67 \times 10^{-8})(5.98 \times 10^{27}\,\mathrm{g})}{4\pi^2}(24\,\mathrm{h} \times 3600\,\mathrm{sec/h})^2} = 4.2 \times 10^9\,\mathrm{cm} = 4.2 \times 10^4\,\mathrm{km}.$$

The question asks for the distance *above the Earth's surface*, so we subtract the radius of the Earth from r, obtaining $\boxed{3.6 \times 10^4\ \mathrm{km}}$, or $\boxed{5.6\ \text{Earth radii}}$ above the Earth's surface.

6. a) The gravitational binding energy for a sphere of constant density is

$$\Omega = \frac{3}{5}\frac{GM^2}{R},$$

and substituting the mass of the Sun (1.99×10^{33} g) for M and the radius of the Sun (6.96×10^{10} cm) for R, we find that $\boxed{\Omega = 2.3 \times 10^{48}\,\mathrm{erg}}$.

b) The virial theorem says that as a star collapses, $\boxed{\text{one-half}}$ of the gravitational binding energy must be radiated away as light. (The other half remains and heats up the star.)

c) The luminosity of the Sun equals the energy available divided by the time to radiate it away, $L = E/t$. We are interested in how long it would take the Sun to radiate away half its binding energy, assuming its luminosity remained constant at its current value:

$$t = \frac{\Omega/2}{L} = \frac{0.5 \times 2.3 \times 10^{48}\ \mathrm{erg}}{3.83 \times 10^{33}\ \mathrm{erg/s}} = 3 \times 10^{14}\ \mathrm{s} = \boxed{10^7\ \mathrm{yr}}.$$

So gravitational contraction could power the Sun for only 10 million years.

d) Since the Sun is over 4.6 billion years old, it would be impossible for gravitational contraction to power the Sun, since the latter can only last for about 10 million years.

7. a) The basic formula for density is

$$\rho = \frac{M}{V},$$

where ρ (the Greek letter rho) is the density. For the spherical asteroid, $V = \frac{4}{3}\pi R^3$. Thus the mass of the asteroid is found by multiplying the density by its volume:

$$M = \rho \cdot \frac{4}{3}\pi R^3 = (5\,\mathrm{g\,cm^{-3}}) \cdot \frac{4}{3}\pi \left(3\ \mathrm{km} \times \frac{10^5\ \mathrm{cm}}{1\ \mathrm{km}}\right)^3 = \boxed{5.7 \times 10^{17}\,\mathrm{g}},$$

where I converted the radius to cm.

b) For the case of a small body falling toward a much more massive body, we can use conservation of energy to derive a formula for the speed of the small body once it collides with the larger object:

$$v = \sqrt{\frac{2GM}{R}},$$

where M is the mass of the the Earth and R is the radius of the Earth. Substituting in values, we get that $\boxed{v = 11\,\mathrm{km/s}}$. Note that this answer is just the escape velocity from the Earth and does not depend on the asteroid's mass.

c) Once the asteroid collides with the Earth, all its kinetic energy will be transformed into heat and kinetic energy of the ejected debris. Thus all we need to do is calculate the asteroid's kinetic energy just when it hits the Earth:

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(5.7 \times 10^{17}\,\mathrm{g})(1.1 \times 10^6\,\mathrm{cm\,s^{-1}})^2 = \boxed{5.5 \times 10^{29}\,\mathrm{erg}}.$$

In megatons, this corresponds to an energy of $\boxed{1.3 \times 10^7\ \mathrm{MT}}$! An atomic bomb has a yield of "only" 0.5 MT…

8. The kinetic energy of the Earth orbiting the Sun is given by

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}M_E v_E^2 = \frac{1}{2}M_E\left(\sqrt{\frac{GM_\odot}{1\,\mathrm{AU}}}\right)^2 = \boxed{\frac{1}{2}\frac{GM_E M_\odot}{1\,\mathrm{AU}}},$$

where we have substituted the orbital velocity $v_{\mathrm{orb}} = \sqrt{GM/r}$ for the Earth's velocity. The gravitational potential energy between the Sun and Earth is given by

$$PE = -\frac{Gm_1m_2}{r} = \boxed{-\frac{GM_E M_\odot}{1\,\mathrm{AU}}}.$$

The $\boxed{\text{gravitational potential energy}}$ is larger in magnitude, by a factor of two:

$$\left|\frac{PE}{KE}\right| = \frac{GM_E M_\odot/(1\,\mathrm{AU})}{GM_E M_\odot/2(1\,\mathrm{AU})} = \boxed{2}.$$

NB. We get the same result if we apply the virial theorem.

Short Questions

1.

Low Mass Stars	High Mass Stars
a) Common	a) Rare
b) Less luminous	b) Luminous
c) Long lived	c) Short lived
d) Red	d) Blue

2. The age of the Earth is approximately 4.6 billion years, or $\boxed{4.6 \times 10^9\ \mathrm{yr}}$. The Sun's age is $\boxed{\text{roughly the same as the Earth's}}$. The two most abundant elements in the Sun are $\boxed{\text{Hydrogen and Helium}}$. The element that constitutes most of the mass in your body is $\boxed{\text{Oxygen}}$. The approximate diameter of the bright disk of the Milky Way Galaxy is $\boxed{\text{100,000 light years}}$.

3. On the autumnal equinox, the Sun's declination is $\boxed{\delta = 0^\circ}$. Its right ascension is $\boxed{\mathrm{RA} = 12^{\mathrm{h}}}$.

4. On the equinoxes, the Sun's declination is $\delta = 0°$. Thus the Sun is located right on the *celestial* equator. For the Sun to be directly overhead on these days, the celestial equator must be directly overhead (i.e., pass through zenith). This occurs only on the Earth's equator. So you must be $\boxed{\text{on the Earth's equator}}$ for the Sun to be overhead at noon on the equinoxes.

5. a) The $\boxed{\text{strong force}}$ is responsible for binding the protons and neutrons together in the nucleus.

   b) An atom is held together by the $\boxed{\text{electric force}}$ (electrical attraction between electrons and protons).

   c) Molecular bonds are formed by the $\boxed{\text{electric force}}$.

   d) $\boxed{\text{Gravity}}$ keeps planets in orbit around stars.

   e) The $\boxed{\text{electric force}}$ binds your fingers to your hands (since your body is made of molecules).

   f) You are held to the ground by the Earth's $\boxed{\text{gravity}}$.

   g) The $\boxed{\text{weak force}}$ is responsible when a subatomic particle changes from one type to another (e.g., neutron $\rightarrow$ proton).

   h) Stars in a galaxy are bound together by their $\boxed{\text{gravitational attraction}}$.

6. The density of an object is given by $\rho = M/V$. For a spherical planet, $V = \frac{4}{3}\pi R^3$. Assuming the mass is constant, then $\rho \propto 1/r^3$. If the radius of an Earth-mass planet were twice the Earth's radius, then the density is $\rho \propto 1/(2R)^3 \propto (1/8)(1/R^3)$, and the density would be $\boxed{\text{8 times smaller than the Earth's}}$.

   The escape velocity of an object in the gravitational field of a mass M whose radius is R is given by

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}.$$

   Thus $v_{\text{esc}} \propto \sqrt{1/R}$. If the radius were twice the Earth's radius, then $v_{\text{esc}} \propto \sqrt{1/(2R)} \propto \sqrt{1/2}\sqrt{1/R}$, and the escape velocity is $\boxed{\sqrt{2}\text{ smaller than from the Earth}}$.

7. We use Kepler's third law to solve this problem. For our solar system, we can write this law as:

$$P^2 = r^3,$$

   with the understanding that the orbital period P must be expressed in years and the orbital radius r in AU. We are given $r = 2$ AU, and thus the orbital period of this planet is

$$P = r^{3/2}\,\text{yr} = (2)^{3/2}\,\text{yr} = \boxed{2.8\,\text{yr}}.$$

   Because Kepler's third law *does not* depend on the planet's mass, the answer is the same $\boxed{(2.8\text{ yr})}$ if the planet were half as massive.

8. The escape velocity of an object in the gravitational field of a mass M whose radius is R is given by

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}.$$

Substituting the Moon's mass and radius into the equation, we find that $v_{\text{esc}} = 2.4 \times 10^5\,\text{cm}\,\text{s}^{-1}$. Converting to kilometers per second, $\boxed{v_{\text{esc}} = 2.4\,\text{km}\,\text{s}^{-1}}$. The Earth's escape speed is 11.2 km/s, so the Moon's escape speed is $\boxed{\text{4.7 times smaller}}$.