# AY 12 Homework #2 Solutions

Winter 2016

## Longer Problems

1. The "brightness" of a source is its flux. The question asks for the distance from a $10^9 L_\odot$ supernova in order to receive the same flux as the Earth does from the Sun. As an equation,

$$F_{\text{from SN}} = F_{\text{from Sun}}.$$

Substituting in the definition of $F$ given above, we get

$$\frac{L_{\text{SN}}}{4\pi d^2} = \frac{L_\odot}{4\pi d_\odot^2},$$

where the supernova's luminosity $L_{\text{SN}} = 10^9\,L_\odot$, $d$ is the distance from the star to the planet (our goal), and $d_\odot$ is the distance from the Earth to the Sun (1 AU). Solving the above equation for $d$ and substituting values we find

$$d_{\text{SN}} = \sqrt{\frac{L_{\text{SN}}}{L_\odot} d_\odot^2} = \sqrt{\frac{10^9\,\cancel{L_\odot}}{\cancel{L_\odot}}(1\,\text{AU})^2} = \sqrt{10^9\,\text{AU}^2} = 10^{4.5}\,\text{AU}.$$

Converting $10^{4.5}$ AU, or $3.16 \times 10^4$ AU, into parsecs, we get $\boxed{d_{\text{SN}} = 0.15\,\text{pc}}$.

The second question asks you to calculate $d_{\text{SN}}$ if the supernova is 10 times fainter than the Sun, i.e.,

$$F_{\text{from SN}} = 0.1 F_{\text{from Sun}}.$$

Doing the same algebra gives

$$d_{\text{SN}} = \sqrt{\frac{L_{\text{SN}}}{0.1 L_\odot} d_\odot^2} = \sqrt{10}\left(\sqrt{\frac{10^9\,L_\odot}{L_\odot} d_\odot^2}\right) = \sqrt{10}(10^{4.5}\,\text{AU}) = 10^5\,\text{AU},$$

or $\boxed{d_{\text{SN}} = 0.48\,\text{pc}}$. Alpha Centauri is about 4 light-years (1.2 pc) from Earth. The supernova is about $\boxed{\text{2.5 times closer to Earth}}$ than Alpha Centauri is. A supernova can actually be quite close to the Earth and not irradiate us to death (since it mostly emits visible light).

2. We use the formula for parallax

$$d = \frac{1}{p},$$

where $d$ is in parsec and $p$ in arcsec, to find the distance to the star. Since $p = 0.02''$, we find $d = 50$ pc. We use this distance, along with the apparent magnitude, $m = 6.0$, in the equation

$$M - m = 5 - 5\log d,$$

to find the absolute magnitude $M$:

$$M = m + 5 - 5\log d = 6.0 + 5 - 5\log 50 = 11 - 10 = \boxed{2.5}.$$

This star is $\boxed{\text{more luminous than the Sun}}$ because $M = 2.5$ is less than $M_{\text{Sun}} = 4.83$.

3. a) Using the provided period-luminosity relation graph, we find that a Cepheid with $P = 30$ days has an absolute magnitude of $M = -5.5$. The problem gives $m = 20.0$, so we can solve for $d$:

$$M - m = 5 - 5\log d \Rightarrow d = 10^{\frac{5-M+m}{5}}\ \text{pc} = 10^{\frac{5-(-5.5)+20}{5}}\ \text{pc} = \boxed{1.3 \times 10^6\ \text{pc}}.$$

   b) If the actual absolute magnitude is 1.5 magnitudes larger, $M = -5.5 + 1.5 = -4.0$. Using this in the above equation gives a revised distance to the star of $\boxed{6.3 \times 10^5\ \text{pc}}$. This is about a $\boxed{\text{factor of 2 closer}}$ than the distance found in part a).

4. a) Using the H-R diagram, a star with $B - V = 0.65$ has an absolute visual magnitude of $\boxed{M_V \approx 5}$. If a star with this absolute magnitude had an apparent magnitude of $m = 15.0$, we can use the same procedure as in Problem 2 to find that $\boxed{d = 1000\ \text{pc}}$.

   b) Wolf 28 is located in the lower part of the H-R diagram. This region of the diagram is populated by white dwarfs. Wolf 28 has a much $\boxed{\text{smaller radius}}$ than the Sun, but its $\boxed{\text{surface temperature is similar to the Sun's}}$, since both have the same color.

   c) Betelgeuse is located in the upper right hand corner of the H-R diagram. This region is populated by red (super)giants. Betelgeuse has a much $\boxed{\text{larger radius}}$ and $\boxed{\text{cooler temperature}}$ than the Sun.

5. a) Using Wien's law, I find that the peak wavelength for emission from the Earth to be:

$$\lambda_{\text{peak}} = \frac{0.289\,\text{cm} \cdot \text{K}}{T} = \frac{0.289\,\text{cm} \cdot \text{K}}{288\,\text{K}} = \boxed{10^{-3}\ \text{cm} = 10\,\mu\text{m, infrared light.}}$$

   b) The luminosity of the Earth is found using:

$$L = 4\pi R^2 \sigma T^4 = 4\pi R_E^2 \sigma T_E^4 = \boxed{2 \times 10^{24}\ \text{erg s}^{-1}},$$

   which is approximately $\boxed{\text{4000 times larger}}$ than the heat diffusing out of the Earth. $R_E$ is the radius of the Earth, and $T_E = 288\,\text{K}$ is the average temperature of the Earth's surface.

6. a) The radius of Star B must be $\boxed{\text{larger}}$ than Star A if both have the same luminosity but Star A is hotter. This follows from the blackbody formula $L \propto R^2 T^4$; in order for Star B to have the same luminosity as the *hotter* Star A, it must have a larger radius to "compensate" for its cooler temperature.

   b) Given that $L \propto R^2 T^4$, we can set up a ratio to calculate Rigel's surface temperature in terms of the Sun's temperature:

$$\frac{L_R}{L_\odot} = \left(\frac{R_R}{R_\odot}\right)^2 \left(\frac{T_R}{T_\odot}\right)^4,$$

   where the subscript $R$ represents Rigel. We can solve this equation for $T_B/T_\odot$ and substitute in $L_B = 10^5 L_\odot$ and $R_B = 75\,R_\odot$:

$$\frac{T_R}{T_\odot} = \left[\frac{L_R}{L_\odot}\left(\frac{R_R}{R_\odot}\right)^{-2}\right]^{1/4} = \left[\frac{10^5 L_\odot}{L_\odot}\left(\frac{75\,R_\odot}{R_\odot}\right)^{-2}\right]^{1/4} = 2 \Rightarrow T_R = 2T_\odot = \boxed{11{,}600\ \text{K}}.$$

c) Rigel would appear $\boxed{\text{bluer}}$ than the Sun because it has a hotter surface temperature, and hence emits most of its radiation at shorter wavelengths than the Sun.

7. a) Based on the graph, Star 1 is roughly an O star, while Star 2 is roughly a B star. Since O stars are hotter than B stars, $\boxed{\text{Star 1 is hotter}}$.

   b) Molecular lines are seen only in the coolest stars, so $\boxed{\text{Star 3}}$ is the likely candidate.

   c) The surface temperature of an F star is too low for ionized helium to be present. Much higher temperatures are required in order for the helium atoms to have sufficient energies to collide and eject their electrons. Similarly, an F star is too hot for molecules to exist; they require cooler temperatures in order to remain stable and not break apart due to collisions with other atoms or molecules.

   d) Star 1 is an O star, with $\boxed{L \approx 10^5\, L_\odot}$. Star 2 is a B star, with $\boxed{L \approx 10^3\, L_\odot}$. Star 3 is an M star, with $\boxed{L \approx 10^{-3}\, L_\odot}$. Star 4 is a K star (based on its temperature), with $\boxed{L \approx 0.3\, L_\odot}$.

**Shorter Problems**

1. This problem is similar to long problem #1. The question asks for the distance at which a planet orbiting a $0.5\, L_\odot$ star would receive the same flux as the Earth does from the Sun,

$$F_{\text{from star}} = F_{\text{from Sun}}$$

$$\frac{L_{\text{star}}}{4\pi d^2} = \frac{L_\odot}{4\pi d_\odot^2},$$

where the star's luminosity $L_{\text{star}} = 0.5\, L_\odot$, $d$ is the distance from the star to the planet (our goal), and $d_\odot$ is the distance from the Earth to the Sun (1 AU). Solving the above equation for $d$ and substituting values we find

$$d = \sqrt{\frac{L_{\text{star}}}{L_\odot} d_\odot^2} = \sqrt{\frac{0.5\, \cancel{L_\odot}}{\cancel{L_\odot}}(1\,\text{AU})^2} = \sqrt{0.5\,\text{AU}^2} = \boxed{0.7\,\text{AU}}.$$

Thus the habitable zone of this star is roughly 0.7 AU away from the star. This answer $\boxed{\text{does not depend on the radius of the planet}}$.

2. a) Cool, luminous stars are found in the $\boxed{\text{upper right}}$ of the HR diagram.

   b) Cool, faint stars are in the $\boxed{\text{lower right}}$ of the HR diagram.

   c) Hot, luminous stars are located in the $\boxed{\text{upper left}}$ of the HR diagram.

   d) Hot, faint stars are in the $\boxed{\text{lower left}}$ of the HR diagram.

   Red giants are $\boxed{\text{cool but luminous stars}}$. White dwarfs are $\boxed{\text{hot but faint stars}}$.

3. From Wien's law, we know that the peak wavelength is inversely proportional to the star's surface temperature. This means that the peak wavelength of emission from Betelgeuse is $\boxed{\text{4 times longer}}$ than Rigel's since Betelgeuse is 4 times cooler. Thus $\boxed{\text{Betelgeuse is redder}}$.

4. $\boxed{\text{M67 has the most massive main sequence stars}}$ because its main sequence extends to more luminous (and $L \propto M^3$, hence more massive) stars than NGC188.

   $\boxed{\text{NGC188 is the older cluster}}$ because its main sequence contains only lower-mass stars (which have longer lifetimes, $t \propto 1/M^2$) when compared to M67.

5. For an F5 star, $\boxed{M_V = 3.5}$. Its bolometric correction is $\boxed{BC = -0.14}$. At 10 pc, its apparent visual magnitude equals its absolute magnitude (by definition), $\boxed{m_V = M_V = 3.5}$.

6. Remember that a Roman numeral I represents a neutral atom. The table below summarizes the number of electrons stripped from the atom:

| Atom/Ion | # stripped electrons |
|---|---|
| H I | 0 |
| He I | 0 |
| H II | 1 |
| He II | 1 |
| C IV | 3 |

7. This problem requires use of the Doppler shift formula for the shift in wavelength of radiation:

$$\frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} = 1 + \frac{v}{c},$$

   where $v$ is the velocity of the moving object, and $c$ is the speed of light. Here, $\lambda_{\rm emit} = 6562.8$ Å and $v = -150$ km/s. We want to find the observed wavelength $\lambda_{\rm obs}$. Substituting values,

$$\frac{\lambda_{\rm obs}}{\lambda_{\rm emit}} = 1 + \frac{v}{c} = 1 + \frac{-150\,\text{km/s}}{3 \times 10^5\,\text{km/s}} = 0.9995 \Rightarrow \lambda_{\rm obs} = 0.9995\lambda_{\rm emit} = \boxed{6560\text{Å}}.$$

   As expected, the observed wavelength is *shorter* because the star is moving *toward* the Earth.

8. The $\boxed{\text{Balmer-alpha}}$ transition results in the longer wavelength because the electron jumps between one energy level, whereas for Balmer-beta the electron jumps between 2 levels, which is a larger energy difference (shorter wavelength photon).