AY 12 Homework #4 Solutions

Winter 2016

Longer Problems

1. a) To find the number of carbon nuclei in $0.7\,M_\odot$ of carbon, we convert solar masses to grams, then multiply grams by the number of carbon nuclei in one gram ($= N_A/12$):

$$N_{\rm ion} = 0.7\,M_\odot \times \frac{2\times 10^{33}\ \rm g}{1\,M_\odot} \times \frac{N_A/12\ \rm C\ nuclei}{1\ \rm g} = \boxed{7\times 10^{55}\ \rm C\ nuclei}.$$

b) The thermal energy contained in the carbon nuclei is found by using the provided formula and temperature:

$$E = N_{\rm ion}kT = (7\times 10^{55})(1.38\times 10^{-16}\ \rm erg\ s^{-1})(3\times 10^{8}\ K) = \boxed{2.9\times 10^{48}\ \rm erg}.$$

c) The relation between energy and luminosity is $L = E/t$, where L is the luminosity, E is the energy, and t is the time over which the energy is released. Since we are interested in the time to radiate away the thermal energy, we solve the above equation for t and substitute values:

$$t = \frac{E}{L} = \frac{2.9\times 10^{48}\ \rm erg}{0.01\,L_\odot} = \frac{2.9\times 10^{48}\ \rm erg}{0.01(3.83\times 10^{33}\ \rm erg\ s^{-1})} = \boxed{2.4\times 10^{9}\ \rm yr},$$

where I converted the answer from seconds to years.

d) This is the same as c), except the luminosity is one-tenth as large—thus the time is ten times longer (think about this!): $\boxed{t \approx 2.4\times 10^{10}\ \rm yr}$.

2. The luminosity of a blackbody of radius R and surface temperature T is given by $L = 4\pi R^2\sigma T^4$, where σ is the Stefan-Boltzmann constant. Since we are solving for the luminosity of the supernova, we substitute the given values for R and T and solve for L:

$$L = 4\pi(3\times 10^{15}\,\rm cm)^2\sigma(5500\,K)^4 = \boxed{5.9\times 10^{42}\,\rm erg\,s^{-1}}.$$

Converting to solar luminosities, $\boxed{L \approx 1.5\times 10^{9} L_\odot}$. Given its (relatively) cool temperature but extremely large luminosity, a supernova would be found in the $\boxed{\text{upper right}}$ of the HR diagram.

3. a) The gravitational binding energy Ω of a constant density neutron star is given by

$$\Omega = \frac{3}{5}\frac{GM^2}{R} = \frac{3}{5}\frac{G(1.4\times 2\times 10^{33}\ \rm g)^2}{(10\times 10^{5}\ \rm cm)} = \boxed{3\times 10^{53}\,\rm erg}.$$

b) This binding energy is radiated away by neutrinos in three seconds, so the luminosity in neutrinos is

$$L_\nu = \frac{3\times 10^{53}\,\rm erg}{3\,\rm sec} = \boxed{10^{53}\,\rm erg\,s^{-1}}.$$

c) The rest-mass energy is given by Einstein's famous equation:

$$E = Mc^2 = (1.4 \times 2 \times 10^{33}\,\mathrm{g})(3 \times 10^{10}\,\mathrm{cm\,s^{-1}})^2 \approx 3 \times 10^{54}\,\mathrm{erg},$$

and thus the gravitational binding energy is about 10% of the rest-mass energy.

d) The escape velocity from an object of mass M and radius R is $v_{\mathrm{esc}} = \sqrt{2GM/R}$. Substituting in $M = 1.4M_{\odot}$ and $R = 10$ km, we find that $\boxed{v_{\mathrm{esc}} = 2 \times 10^{10}\ \mathrm{cm/s}}$, which is about two-thirds the speed of light.

4. A particle on a rotating object is subject to two forces, the gravitational force pulling toward the center, and the centrifugal force pushing outward. To determine the shortest rotation period a neutron star can have without flying apart, we equate these two forces:

$$\begin{aligned} F_{\mathrm{cent}} &= F_{\mathrm{grav}} \\ \frac{mv^2}{R} &= \frac{GMm}{R^2}, \end{aligned}$$

where v is the rotational speed of the particle, m is its mass, R is the radius of the neutron star, and M is the mass of the neutron star. We know that $v = 2\pi R/P$, where P is the rotation period. Substituting this and canceling the m's, we get an expression for the period (which is just Kepler's third law):

$$P^2 = \frac{4\pi^2}{GM}R^3.$$

Substituting $R = 10$ km (converted to cm), and $M = 1.4M_{\odot}$ (converted to grams), we find that $\boxed{P = 0.0005\ \mathrm{sec}}$, or 0.5 millisecond, consistent with the fastest rotating neutron stars, which have periods of about 1 ms.

5. This problem uses the conservation of energy to solve for the kinetic energy of the ejecta (and hence its speed). The initial energy is the gravitational binding energy of the white dwarf *plus* the energy released by nuclear fusion when it explodes. This is equal to the kinetic energy of the ejecta. In mathematical terms,

$$\begin{aligned} E_{\mathrm{initial}} &= E_{\mathrm{final}} \\ E_{\mathrm{binding}} + E_{\mathrm{nuclear}} &= KE_{\mathrm{ejecta}} \\ -3 \times 10^{50}\ \mathrm{erg} + 10^{51}\ \mathrm{erg} &= \frac{1}{2}Mv^2. \end{aligned}$$

We want to solve for v. Here, M is the mass of the ejecta, which is the same as the white dwarf's mass ($1.4\,M_{\odot}$) since it is entirely destroyed in the supernova explosion. Doing the algebra, I find that $\boxed{v = 7000\ \mathrm{km\ s^{-1}}}$. This is a bit more than 2% the speed of light!

6. Below is a table comparing type Ia and type II supernovae. You only needed to discuss five properties in your answer.

Property	Type Ia	Type II
Progenitor	exploding white dwarf	exploding massive star
Remnant?	no remnant	neutron star or black hole
Speed of ejecta	$(5-30)\times 10^3$ km/s	$(2-30)\times 10^3$ km/s
Peak luminosity	10^{43} erg/s	3×10^{42} erg/s
Length of peak	2 weeks	3 months
Frequency in Milky Way	1 every 200 yr	2 every 100 yr
H in spectrum?	No	Yes
Elements produced	$\sim 2/3$ of Fe in galaxy	$\sim 1/3$ of Fe in galaxy

Shorter Questions

1. a) The *s-process* is a pathway to create heavier elements by adding neutrons to nuclei. It occurs in the outer envelopes of AGB stars. The process is "slow" to form heavier elements because the rate of neutron capture onto nuclei is lower than the rate of the nuclei decaying into lighter nuclei.

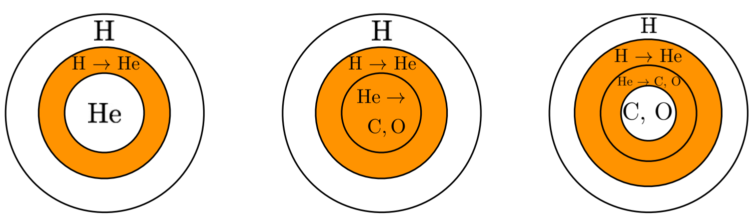
   b) The *helium flash* occurs in stars with mass less than $2\,M_\odot$ at the onset of core helium burning. Since the helium core is initially degenerate, the increase in temperature as the core contracts causes helium burning to start explosively throughout the core.

   c) *Carbon burning* occurs in stars with $M > 8M_\odot$ at a temperature of $T \sim 800$ million K and densities of $\rho \approx 10^5\,\mathrm{g\,cm^{-3}}$. Carbon nuclei fuse together to form heavier nuclei like neon and magnesium.

   d) The *Chandrasekhar mass* is the maximum mass $(1.4\,M_\odot)$ a white dwarf can have and remain stable. Electron degeneracy pressure can support a white dwarf up to this mass before gravity wins and the star collapses.

   e) Low-mass stars $(M < 8\,M_\odot)$ end their lives by ejecting their outer envelopes in the form of *planetary nebulae*. These remnants contain hydrogen, elements produced during nuclear fusion, and *s*-process elements. They last for tens of thousands of years.

2. The diagrams below show the interior structure of a red giant star, horizontal branch star, and asymptotic giant branch star. Regions colored orange are where fusion is occurring.



Red giant branch star   Horizontal branch star   Asymptotic giant branch star

3. a) A *classical nova* occurs when hydrogen and helium are accreted onto a white dwarf in a binary system. The accreted hydrogen and helium compress and heat up, igniting hydrogen burning explosively. This expels all the accreted matter and results in a dramatic increase in the white dwarf's luminosity. A white dwarf can undergo multiple novae.

b) A *pulsar* is a rapidly rotating neutron star with a strong magnetic field that is not aligned with the rotation axis. This results in the jets of radiation (due to electrons spiraling in the magnetic field) being beamed in our direction in regular pulses as the magnetic field axis sweeps across our line of sight.

c) *Binary stellar massed x-ray sources* are thought to be the result of a neutron star or black hole accreting mass from a companion star. If the x-ray emitting source can be shown to have a mass greater than $2\,M_\odot$, it must be a black hole, because there are no existing neutron stars above $2\,M_\odot$.

4. a) $0.08\,M_\odot$ is the minimum mass for a protostar to ignite hydrogen fusion. A protostar with mass *below* $0.08\,M_\odot$ is unable to fuse hydrogen and thus does not become a star. These objects become brown dwarfs.

   b) Stars with mass below $0.5\,M_\odot$ are not massive enough to ignite helium burning in their cores. These light stars will eventually evolve into helium core white dwarfs. In addition, these stars are fully convective throughout their interiors.

   c) White dwarfs below $1.44\,M_\odot$ are stable (degeneracy pressure balances gravity). Above $1.44\,M_\odot$, no stable white dwarf exists (such an object would have zero radius!).

   d) A star with $M < 2\,M_\odot$ burns hydrogen via the pp-chain. In addition, it experiences a helium flash in its core at the start of helium burning. A star above $2\,M_\odot$ burns hydrogen via the CNO cycle. Such a star is also able to transition to helium burning smoothly because its core is not degenerate.

   e) Stars less than $8\,M_\odot$ are "low-mass stars" that end their lives as white dwarfs (and planetary nebulae). Stars above $8\,M_\odot$ ("high-mass stars") end their lives as core-collapse supernovae. These stars are able to burn heavier elements for fuel, e.g., carbon, oxygen, and silicon.

   f) A star cannot become arbitrarily large. At around $150\,M_\odot$, the radiation pressure in the star becomes so great as to overcome gravity and blow the star apart. Thus, a star above $150\,M_\odot$ cannot exist.

5. A $\boxed{\text{Type II supernova}}$ would occur first in a cluster of stars. This is because Type II supernovae are due to the explosion of massive stars, which have a much shorter lifetime than low-mass stars. Type Ia supernovae can occur only after white dwarfs are formed.

6. a) Hydrogen was formed soon after the Big Bang as the universe cooled and subatomic particles could form.

   b) Carbon is formed during helium burning in post-main sequence stars with $M > 0.5M_\odot$. For low-mass stars ($M < 8M_\odot$), the carbon is ejected within the planetary nebula. For high-mass stars, carbon can be ejected during a type II supernova explosion.

   c) Nitrogen is a byproduct of the CNO cycle, which occurs in main-sequence stars with $M > 2M_\odot$. It is ejected into the ISM either in a planetary nebula or as part of the ejecta from a type II supernova.

   d) Oxygen is a byproduct of helium burning via the triple-alpha process. It is ejected via supernovae in massive stars or as part of a planetary nebula for low-mass stars.

e) Iron is formed during silicon burning in massive stars. It is ejected during a type II supernova. Additionally, iron is formed during the explosive nucleosynthesis leading up to the explosion of a white dwarf (type Ia SN).