Lecture 3

Basic Physics of Astrophysics - Force and Energy

http://apod.nasa.gov/apod/

Forces

Momentum is the product of mass and velocity - a vector

$$\vec{p} = m\vec{v}$$

(generally *m* is taken to be constant)

An *unbalanced force* is capable of producing a change in momentum

$$\vec{F} = \frac{d\vec{p}}{dt}$$

(n.b., a mass times an acceleration)

$$\vec{F} = m\left(\frac{d\vec{v}}{dt}\right) = m\vec{a}$$

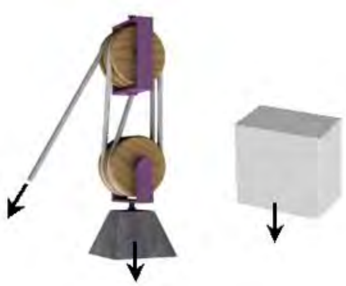
Units:

$$\text{dyne} = \frac{\text{gm cm}}{\text{s}^2}$$

1 dyne = 2.248 x 10^{-6} pounds force

Intuitively, force is the "push or pull" on an object.

A force doesn't always produce motion. It may be balanced by an equal opposite force.



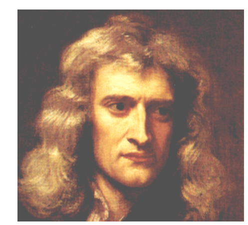
Note that a force may also be required to balance another force even when nothing is moving. For example a block setting on a table is pulled downwards by gravity but supported by the table.

NEWTON'S THREE LAWS

1) In the absence of an external force, The product of mass and (vector) velocity is a constant. Also may be stated "Objects at rest tend to remain at rest; objects in motion tend to remain in motion in the same straight line". This defines *momentum*, p = m **v**.

2) When a force acts on a body it produces a change in the momentum in the direction of and proportional to the applied force.

3) For every action there is an equal but opposite reaction.



Conservation of momentum in the absence of forces

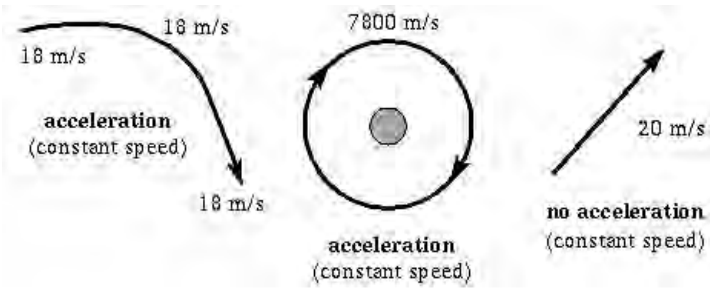
Definition of force

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$$

Action-reaction

Figure from Nick Strobel's site

Examples



A force is required to change either the speed of an object *or the direction in which it is going.*

FORCES IN THE UNIVERSE

There are four fundamental interactions that exist, each of which is associated with a force.

• The weakest but most commonly experienced force is **GRAVITY**. This force is exerted by anything that has mass or energy on all other masses. The force is proportional to each of the two masses that interact and declines as $1/r^2$. It thus can be experienced, albeit weakly, over very large distances (infinite range force).

$$F = \frac{GMm}{r^2}$$

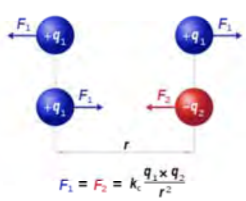
$$G = 6.673 \times 10^{-8} \frac{dynes\ cm^2}{gm^2}$$

• The next to strongest (skipping one for now) is the **ELECTRIC** or **ELECTROMAGNETIC** force, the most obvious example being the interaction between two charges. The force is mathematically very similar to gravity (though fundamentally different in physical nature). It is proportional to the two interacting charges and also declines a $1/r^2$

$$F = \frac{e^2 Z_1 Z_2}{r^2}$$

$$e^2 = 2.307 \times 10^{-19}\ dynes\ cm^2$$

$q_1 = Z_1$; $q_2 = Z_2$



All of chemistry is due to the electric force. So are the strength of solids and all processes related to the emission and absorption of light.

Two other forces have only been recognized during the last century. They only affect phenomena on the scale of nuclei and individual particles, i.e., they are *"short ranged"*.

The *strong* force is responsible for binding together the neutrons and protons in the atomic nucleus. On an even smaller scale, the strong force binds together quarks to make the neutron and proton and other particles on the sub-atomic scale. The typical range of the strong force is 10^{-13} cm.

The strong force is strong enough at short range to overcome the repulsion of electrically charged protons in the nucleus (as well as the "degeneracy" energy of the nucleus itself). But outside the nucleus it falls rapidly to zero. At very short range the nuclear force is actually repulsive.

The *weak interaction* (actually much stronger than gravity but weaker than the strong or electric interaction) is in some sense analogous with the electric force, but is a short range interaction that acts on a quantum mechanical property called isospin. It allows one kind of quark to turn into another.

Its chief effects are that it allows neutrinos to be produced and to interact with matter and it allows neutrons to change into protons and vice versa if energy conservation allows it.

A free neutron outside the nucleus will decay into a proton in 10.3 minutes by the weak interaction

(udd) (uud)

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Note conserved quantities – charge, baryon number, lepton number, energy, momentum

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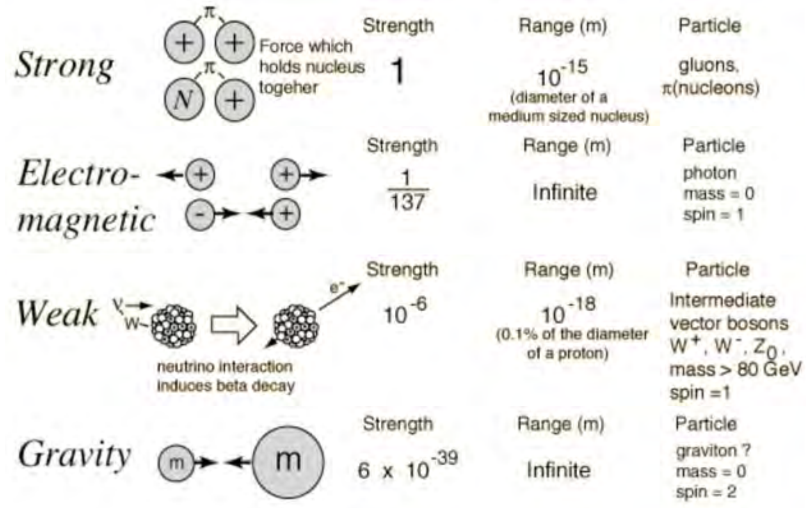


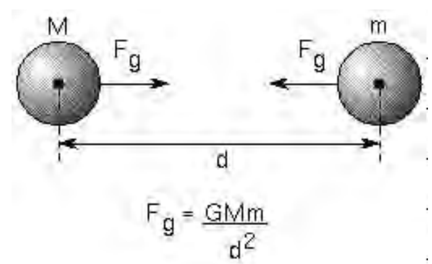
http://hyperphysics.phy-astr.gsu.edu/hbase/particles/expar.html

SUMMARY

FORCE	STRENGTH	RANGE	EXAMPLE
Strong	1	10^{-13} cm	nucleus
Electric	10^{-2}	$1/r^2$	chemistry
Weak	10^{-6}	$< 10^{-13}$ cm	$n \rightarrow p + e^- + \bar{\nu}_e$
Gravity	10^{-38}	$1/r^2$	binds earth to sun

Figure from Nick Strobel's electronic text. See his website.

Gravity



For spherically symmetric objects, gravity acts as if all the mass were concentrated at the center of the sphere (more generally at the center of mass). [often we Use r instead of d for distance]

GRAVITY

- Examples:

Your weight (assume m = 70 kg):

$$F = -\frac{GM_{Earth}(m)}{R^2_{Earth}}$$
$$= -\frac{6.67\times10^{-8}\,\text{dyne cm}^2(5.9? \times 10^{27}\,\text{gm})(7\times10^4\,\text{gm})}{\text{gm}^2(6.38\times10^8\,\text{cm})^2}$$
$$= -6.9\times10^7\ \text{dyne}$$

= 154 pound force

1 dyne = 2.248 x 10^{-6} pound fprce

The moon's force:

$$F = -\frac{GM_{Moon}m}{R^2_{Moon}}$$
$$= -233\ \text{dynes}$$

The sun's force:

$$F = -\frac{GM_{\odot}m}{AU^2}$$
$$= -4.1\times10^4\ \text{dynes}$$

almost 200 times that of the moon

Force from sun at 4.4 ly = 5×10^{-7} dyne

Force from Jupiter at nearest point = 2.2 dynes

Force from another peson at 1 m = 0.033 dynes

The calculations on the previous page would suggest that the sun is actually more influential on the Earth (tides, etc.) than the moon, but the Earth is in free fall around the sun.

What matters is the differential force because the Earth has finite size.

$$\frac{dF}{dr} = \frac{2GMm}{r^3} \qquad \text{i.e., } \frac{d}{dr}\left(-\frac{1}{r^2}\right) = \frac{2}{r^3}$$

For an object of diameter d, the difference in force from one side to the other is

$$\boxed{\Delta F = \frac{dF}{dr}(d) = F\left(\frac{2d}{r}\right)}$$

The distance (r in this equation) to the sun is 390 times that to the moon so the tidal forces from each are comparable

$$F_{tide}(moon) = \left(\frac{GM_{moon}m}{r^2_{moon}}\right)\left(\frac{2d_{earth}}{r_{moon}}\right)$$

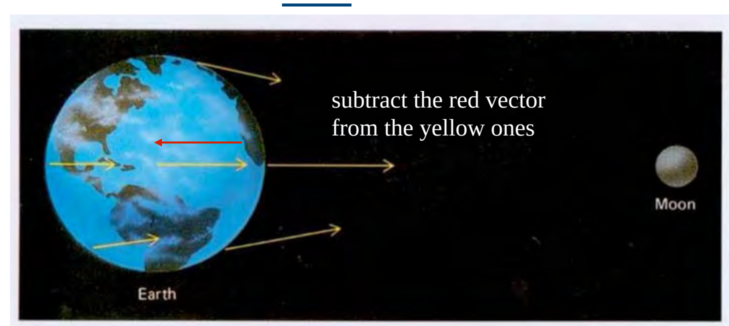
$$F_{tide}(sun) = \left(\frac{GM_{sun}m}{r^2_{sun}}\right)\left(\frac{2d_{earth}}{r_{sun}}\right)$$

distance to sun

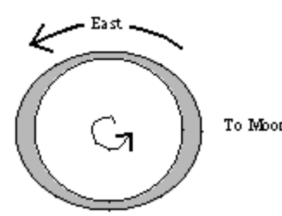
$$\left(\frac{F_{tide}(moon)}{F_{tide}(sun)}\right) = \left(\frac{F_{grav}(moon)}{F_{grav}(sun)}\right)\left(\frac{r_{sun}}{r_{moon}}\right) = \left(\frac{233}{4.1\times10^4}\right)\left(\frac{1.5\times10^{13}}{3.84\times10^{10}}\right)$$
$$= \left(\frac{390}{176}\right) = 2.2$$

distance to moon

Tides



The moon pulls on all parts of the Earth. It pulls strongest on the part that is closest, less on the center, and least of all on the far side. Subtracting the force at the center of mass from all components leads to a bulge in the oceans both on the near and far side of the Earth.



Q: Are there tides at the earth's poles?

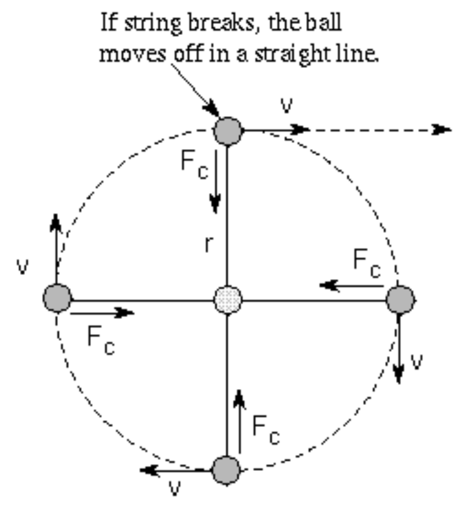
A: Yes. But the biggest tides are in the tropics and near the equator.

Remember that it is the sun and moon causing the tides. These are not generally in the celestial equator. The path of the sun is the ecliptic, which we discussed. The moon's orbit is inclined by about 5º to the ecliptic.

The biggest tides are when the moon and sun are in the same direction or in opposite directions, i.e. at new moon and full moon.

Because two bodies are responsible and their position with respect to the equator changes during the year there is no place on earth with no tides. There are also small forces due to the rotation of the earth.

CENTRIFUGAL FORCE



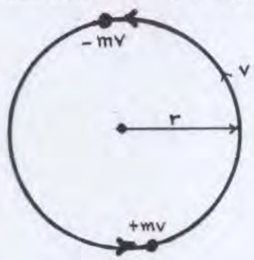
$F_c = m\,v^2 / r$

Just enough speed that centrifugal force balances the force on the string.

Centrifugal force is sometimes called a "fictitious" force or an "inertial" force.

CENTRIFUGAL FORCE

Even when an object moves at constant speed a force is required to continually change its direction.



Approximately:

half

At the top of the circle the momentum is -mv. At the bottom it is +mv. Every orbit the momentum changes by 2mv. The time tc execute one orbit is $2\pi r/v$. The change in momemtum per unit time is then approximately

$$F = \frac{dp}{dt} = \frac{\Delta p}{\Delta t} = 2mv\left(\frac{v}{\pi r}\right)$$
$$= \frac{2}{\pi}\frac{mv^2}{r}$$

When derived correctly the $2/\pi$ is not there.

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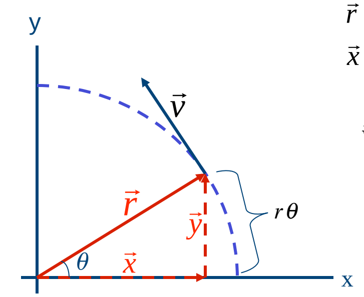
Centrifugal Force



$$\vec{r} = \vec{x} + \vec{y}$$

$$\vec{x} = r\cos\theta\,\hat{x} \qquad \vec{y} = r\sin\theta\,\hat{y}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = r(-\sin\theta)\frac{d\theta}{dt}\hat{x} + r(\cos\theta)\frac{d\theta}{dt}\hat{y}$$

Assume:

$$\frac{d\theta}{dt} = \text{constant}, \qquad r = \text{constant}$$

$$v = r\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = \frac{v}{r}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = r(-\cos\theta)\left(\frac{d\theta}{dt}\right)^2\hat{x}$$
$$+ r(-\sin\theta)\left(\frac{d\theta}{dt}\right)^2\hat{y}$$
$$= -(\vec{x}+\vec{y})\left(\frac{d\theta}{dt}\right)^2 = -\vec{r}\left(\frac{d\theta}{dt}\right)^2$$

$$\boxed{\vec{F} = m\frac{d^2\vec{r}}{dt^2} = -m\vec{r}\left(\frac{d\theta}{dt}\right)^2 = -m\vec{r}\left(\frac{v^2}{r^2}\right) = -\left(\frac{mv^2}{r}\right)}$$

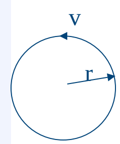
(directed along r.)

Combining the definition of centrifugal force and Newton's equation for gravitational attraction we get

$$\boxed{F_{ceni} = \frac{mv^2}{r}}$$

- Kepler's Third Law

$$\frac{GM_{\odot}m}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{P}\right)^2 \qquad v = \frac{2\pi r}{P}$$



Therefore

$$\frac{GM_{\odot}}{r} = \left(\frac{2\pi r}{P}\right)^2 \Rightarrow \boxed{P^2 = \frac{4\pi^2}{GM_{\odot}} r^3}$$

ORBITS
KEPLER'S THREE LAWS

http://en.wikipedia.org/wiki/Kepler's_laws_of_planetary_motion

Soulution of

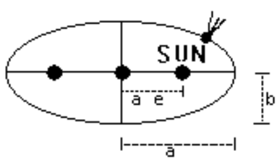
$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{GM}{r^3}\mathbf{r}$$

general equation – not circular orbits

- The planets orbit the sun in elliptical orbits with the sun at one focus of the ellipse.
- A line connecting the planet and the sun sweeps out equal areas in equal times
- $P^2 \propto r^3$ as we derived

In Ay 12 we shall presume circular orbits to be a good approximation

Kepler's First Law



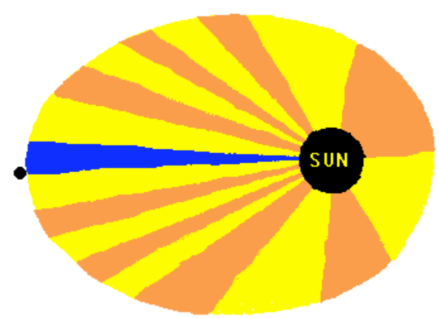
The orbit of a planet around the sun is an ellipse with the sun at one focus of the ellipse. a is called the "semi-major axis" of the ellipse. e is the "eccentricity" For earth e = 0.0167. For Mercury e = 0.206.

$$\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = \text{constant}$$

The equation for an ellipse

The equation for a circle is $x^2 + y^2 = r^2$

Kepler's Second Law



A line connecting the orbiting both with one focus of the ellipse (e.g., the sun) sweeps out equal areas in equal times.

Kepler's Third Law

The squares of the periods of the planets are proportional to the cubes of their semi-major axes

Planet	P(yr)	a(AU)	P^2	a^3
Mercury	0.24	0.39	0.058	0.058
Venus	0.62	0.72	0.38	0.38
Earth	1.0	1.0	1.0	1.0
Mars	1.88	1.52	3.52	3.52

$$P^2 = \frac{4\pi^2}{GM} r^3$$

for the Earth

$$(1\,yr)^2 = \frac{4\pi^2}{GM_\odot} (AU)^3$$

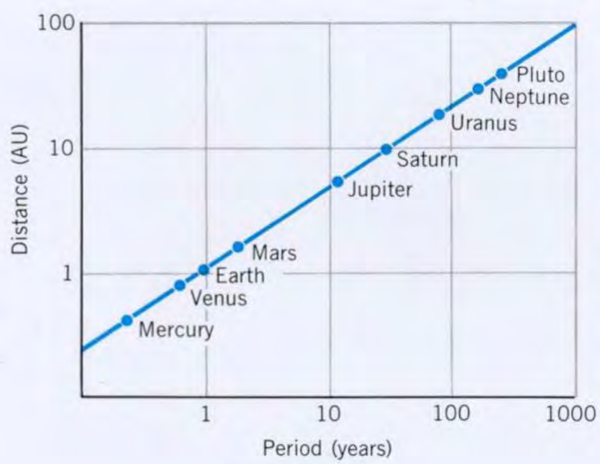
So for any object orbiting the sun

$$\frac{P^2}{1\,yr^2} = r^3/(AU)^3$$

$$r\ (\text{in AU}) = \left[P\ (\text{in yr})\right]^{2/3}$$

$$\frac{P^2}{(1\,yr)^2} = \frac{\left(\frac{4\pi^2}{GM}\right)}{\left(\frac{4\pi^2}{GM_\odot}\right)} \left(\frac{r}{1\text{ AU}}\right)^3$$

This only works when comparing small masses, each of which orbits the same big mass, M (e.g., planets around the sun).



Kepler's third law.

THE ASTRONOMICAL UNIT (AU)

The astronomical unit (AU) is the distance from the earth to the sun. It is measured from the center of the earth to the center of the sun.

Actually the distance from the earth to the sun varies about 1.7% during the year since its orbit is elliptical. Prior to 1976 the AU was defined as the semi-major axis of the earth's orbit but in 1976 the IAU redefined it to be the *average* distance to the sun over a year.

Its standard (1996) value is 1.495978707 x 10^{13} cm

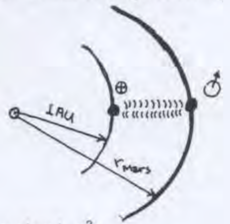
http://en.wikipedia.org/wiki/Astronomical_unit

APLLICATIONS OF KEPLER'S THIRD LAW

$$P^2 \propto r^3$$

- Distance to sun

The modern way. Bounce radar off of Mars when Mars is in "opposition". Determine distance between earth and mars (or Venus or a spacecraft) to high accuracy.



$$\frac{P^2_{Mars} = Constant\ r^3_{Mars}}{P^2_{Earth} = Constant\ (AU)^3}$$

assumes Mar's orbit is in the same plane as the Earth's.

Know $P_{Mars} = 686.98$ days $= 1.881$ years.
Therefore $r_{Mars} = (1.881)^{2/3} = 1.524$ AU

And since 0.524 AU has been measured by radar we can solve for the AU ($1.49597870 \times 10^{13}$ cm ± 5 km).

from center of mass of the earth to center of mass of the sun

For our purposes usually AU $= 1.50 \times 10^{13}$ cm.

WEIGHING THE UNIVERSE

$$P^2 = \frac{4\pi^2}{GM} r^3$$

$$\boxed{M = \frac{4\pi^2}{GP^2} r^3}$$

and the length of the year and AU are known

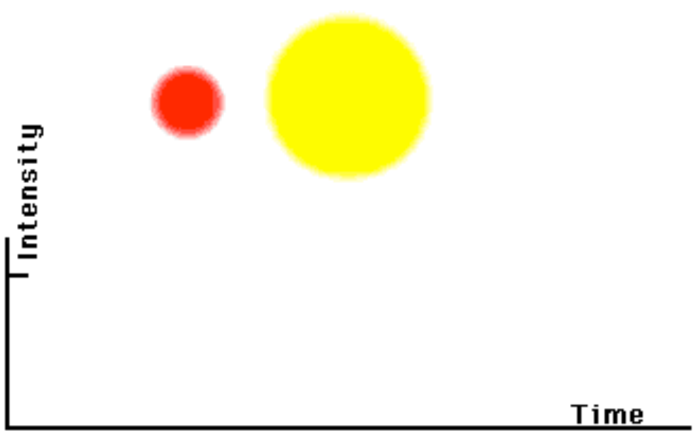
$$M_\odot = \frac{4\pi^2}{G(1\ yr)^2} (AU)^3$$

$$= \frac{(4)(3.14)^2 (\text{gm})^2}{(6.67\times 10^{-8})(\text{dyne cm}^2)(1\text{ yr})^2}(1\text{ AU})^3 \left(\frac{1.50\times 10^{13}\text{ cm}}{1\text{ AU}}\right)^3 \left(\frac{\text{dyne s}^2}{\text{gm cm}}\right)\left(\frac{1\text{ yr}}{3.16\times 10^7\text{ s}}\right)^2$$

$$= \left(\frac{(4)(3.14)^2(1.50\times 10^{13})^3}{(6.67\times 10^{-8})(3.16\times 10^7)^2}\right)\text{ gm}$$

$$= 2.00\times 10^{33}\text{ gm}$$

Later we shall see how to get the masses of other stars (and planets) if those stars are in binary systems



- The Earth

The moon has a distance (radar) of 3.84×10^{10} cm (semi-major axis).
The month is 27.32 days $= 2.36 \times 10^6$ seconds.

$$M = \left(\frac{4\pi^2}{GP^2}\right) r^3$$

$$M_E = \frac{4\pi^2}{G(1\ mo)^2} r^3_{Moon}$$

$$= \frac{(4)(3.14)^2}{(6.67\times 10^{-8})(2.36\times 10^6)^2}(3.84\times 10^{10})^3$$

$$= 6.0 \times 10^{27}\text{ gm}$$

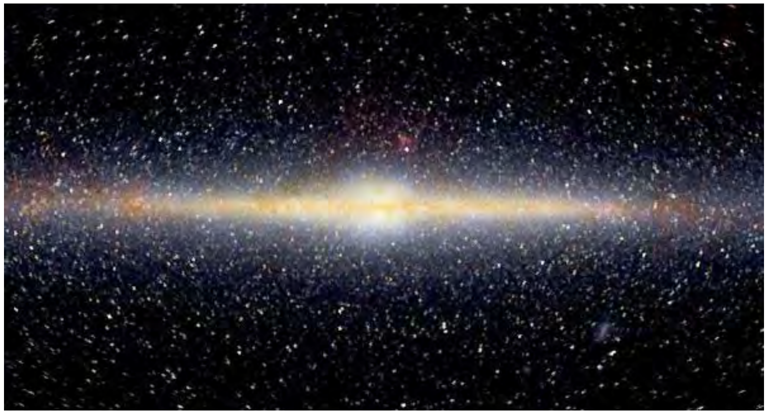
(5.977×10^{27})

- Other planets that have satellites (all but Mercury + Venus)
- The moon
- Other stars that are in binary systems
- Lest we forget Kepler's orginal intent, if we know the period of other planets their semi-major axes follow immediately. E.g. Jupiter

Jupiter year $= 11.9$ Earth Years

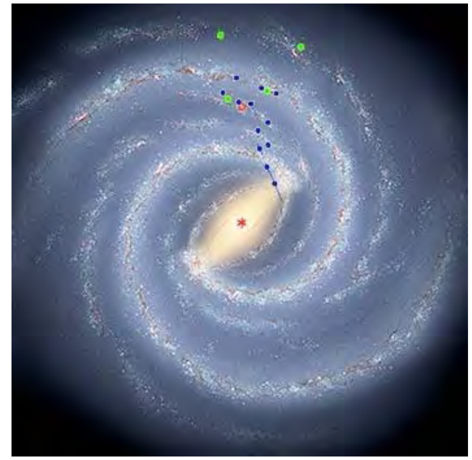
Therefore its semi-major axis around the sun is $(11.9)^{2/3} = 5.2$ AU

How massive is the Milky Way Galaxy?
(and how many stars are in it?)



COBE (1990) - the galaxy as seen in far infrared
1.25, 2.2, and 3.5 microns (optical = 0.4 - 0.7 microns)
stars are white; dust is reddish

THE MASS OF THE MILKY WAY GALAXY



Measurements of the parallax of numerous star forming regions using radio (Very Long Baseline Array) have given (2009) accurate measurements of the distance from the galactic center to the earth (28,000 ly) and the speed of the solar system in its orbit around the center (254 km/s; used to be 220 km/s prior to January, 2009)

http://www.nrao.edu/pr/2009/mwrotate/

Treat Milky Way as a disk. Gravitationally, the sun “sees” only the mass interior to its orbit and that mass acts as if it were all located at the center of mass, i.e., the center of the Galaxy.

Milky Way interior to sun’s orbit

The matter outside the sun’s orbit exerts zero net force on the sun.

http://www.merlyn.demon.co.uk/gravity1.htm - FiSSh

THE MASS OF THE MILKY WAY GALAXY

$$M_{MW} = \left(\frac{4\pi^2}{GP^2}\right) r^3$$

$$P = \left(\frac{2\pi r}{v}\right)$$

Period not known but do know the speed and the distance (period is actually about 200 My)

$$M_{MW} = \left(\frac{(4\pi^2)(v^2)}{G(4\pi^2)(r^2)}\right) r^3$$

$$= \frac{v^2 r}{G}$$

$$= \frac{(2.56\times10^7)^2 (2.65\times10^{22})}{6.67\times10^{-8}}$$

28,000 ly is 2.65×10^{22} cm
(1 ly = 9.46×10^{17} cm)

$$= 2.51\times10^{44}\ \text{gm}$$

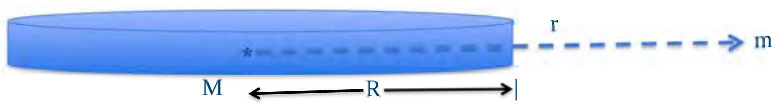
$$\boxed{= 1.29\times10^{11}\ M_\odot}$$

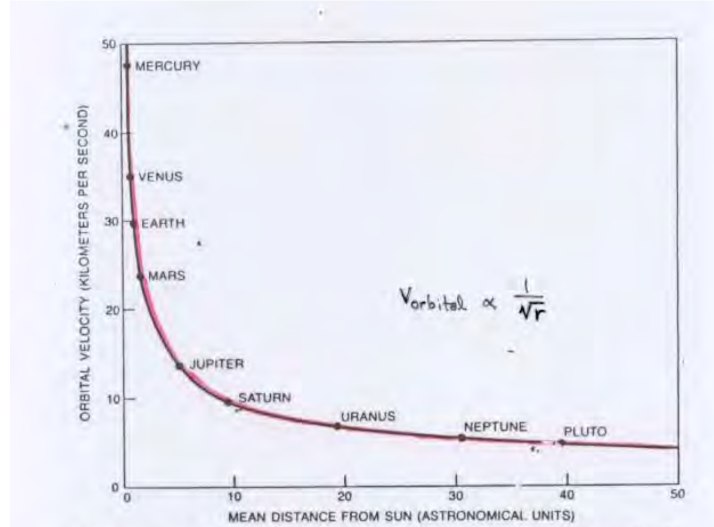
interior to sun’s orbit

Where is the edge of the Milky Way, i.e. where does the amount of mass enclosed reach a constant?

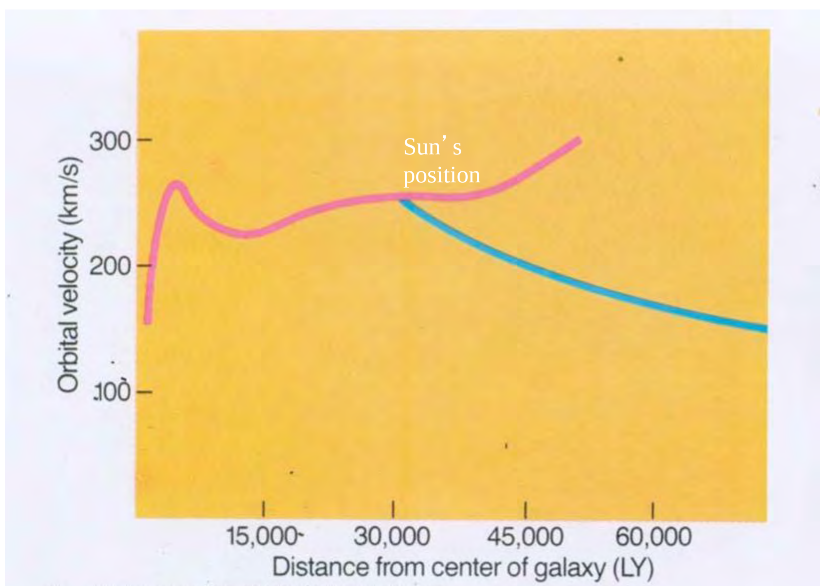
Once M = constant then as r increases

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v \propto 1/\sqrt{r} \qquad r > R$$

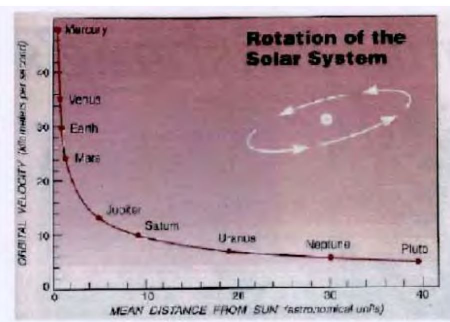


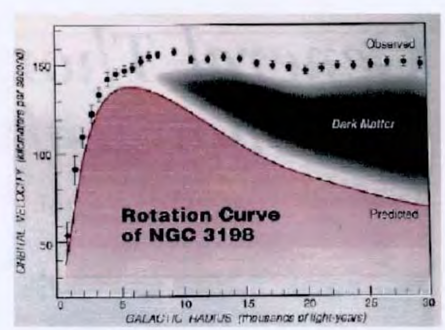


KEPLER'S LAW for the orbital velocity of planets in the solar system, in which more than 99 percent of the total mass resides in the sun, yields this plotted curve. Orbital velocity decreases inversely as the square root of *r*, the planet's mean distance from the sun. The distance is shown here in astronomical units; one A.U. equals the mean distance between the earth and the sun. Pluto, at 39.5 A.U., lies 100 times farther from the sun than Mercury, at .39 A U. Mercury's orbital velocity is about 47.9 kilometers per second; Pluto's velocity is accordingly slower by a factor of 10, or 4.7 kilometers per second ($47.9 \times 1/\sqrt{100}$). The author's results show that the orbital velocities of stars in a spiral galaxy depart strongly from a Keplerian distribution.



'93. Abell/Morrison/Wolff: EXPLORATION OF THE UNIVERSE, 6/E





$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = const. \Rightarrow M \propto r.$$

If density were independent of r,

$M \propto r^3$ (sphere), $M \propto r^2$ (disk)

so the density is indeed declining

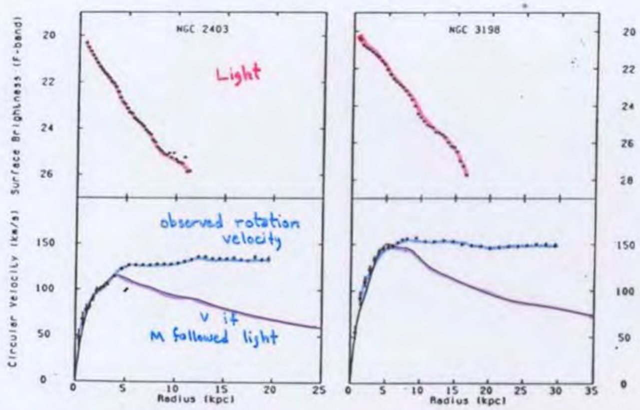
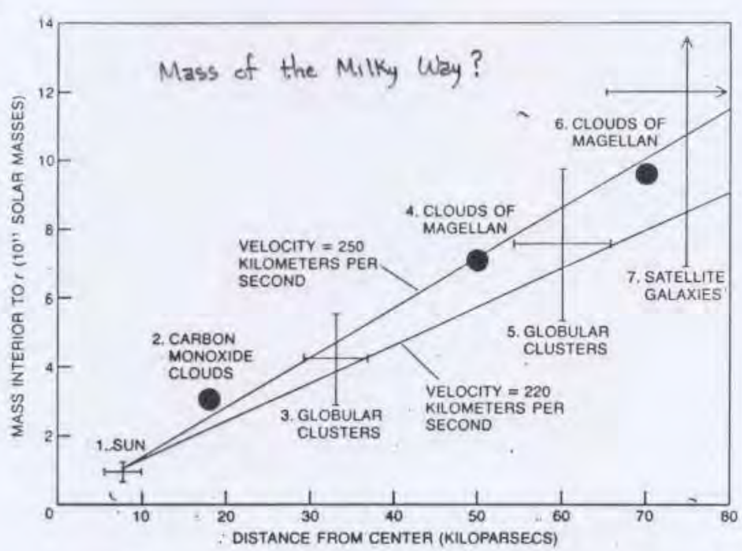
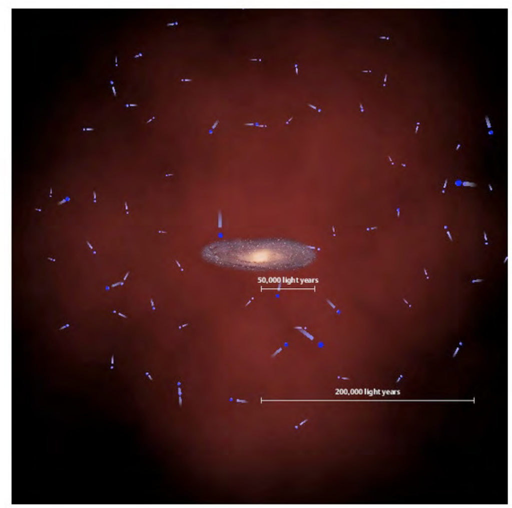


Figure 3.12. Luminosity profiles and rotation curves for spiral galaxies (van Albada and Sancisi 1986). The numerical value of the radius is based on the Hubble parameter $h = 0.75$.

That is the orbital velocity should decrease with increasing distance from the center of the Milky Way.

- In fact the orbital velocity measured by radio observations does not fall off but tends to stay nearly constant outside the sun's orbit, maybe even increase
- But the optical luminosity of our galaxy outside the sun's orbit does decline dramatically. Therefore there is a lot of non-luminous mass. The mass interior to 75,000 ly is about 3 times that interior to the sun's orbit.
- Note that the Milky Way does not rotate as a "rigid" body. Stars in the disk closer to the center execute an orbit quicker than stars farther out. Note also that v stays about constant interior to the suns orbit.





http://www.sdss.org/news/releases/20080527.mwmass.html

May 27, 2008 - Sloan sky survey using a sample of over 2400 Population II (blue horizonal branch) stars infers a mass of slightly less than 10^{12} solar masses

What is Dark Matter?

Anything with a large mass to light ratio

Baryonic dark matter: (made of neutrons, protons and electrons)

- White dwarfs
- Black holes (large and small)
- Neutron stars
- Brown dwarfs and planets
- Gas either in small cold clouds or a hot inter-galactic cluster gas

But Big Bang nucleosynthesis limits the amount of baryonic mass.

Big Bang nucleosynthesis implies several times more baryonic matter than we see in galaxies and stars. Where is the rest?

Probably in an ionized hot intergalactic medium.

But observations of the dynamics of galactic clusters suggest even more matter than that.

*

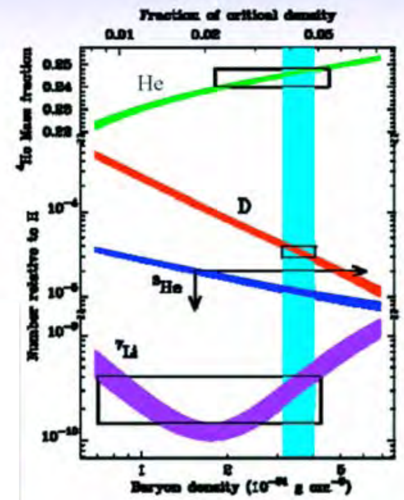
Baryon (p and n) - Density

The Ratio between Proton- and Neutron- Abundance at the Time of Nucleosynthesis (as it is today) depends on the Baryon-Density

The Abundance of light Elements (He, D, ^{3}He) reflects the n/p-Ratio

Light Elements
=> Baryon-Density

The value inferred is less than that required to bind galaxies and especially clusters of galaxies together.



*

Dark Matter in the Universe

Hot Gas in Galaxy Clusters
'to much' kinetic Energy
(measured by X-Ray Emission)

\+

large Scale Motion
of Galaxies at 'too high' Speed

About 5 times more mass than BB nucleosynthesis suggests

Best indications are that dark matter is composed of two parts “baryonic dark matter” which is things made out of neutrons, protons and electrons, and non-baryonic dark matter, which is something else.

Of the *baryonic* matter, stars that we can see are at most about 50% and the rest (the dark stuff) is in some other form. Part of the baryonic matter may be in the hot intergalactic medium and in ionized halos around galaxies (50%?).

The non-baryonic matter has, in total, 5 times more mass than the total baryonic matter. I.e. baryonic matter is 1/6 of matter and non-baryonic matter is 5/6.

More Exotic

*

Anything with a large mass to light ratio

Non-baryonic dark matter:

- Massive neutrinos
- Axions (particle that might be needed to understand absence of CP violation in the strong interaction)
- Photinos, gravitinos, ...
- WIMPs
- unknown...

In general, particles that have mass but little else

- *The universe was not born recently (i.e., there is nothing special about the present*
- *We are not at the center of the solar system*
- *Our solar system is not at the center of the Milky Way Galaxy*
- *Our galaxy is not at the center of the universe (and there are many other galaxies)*
- *We are not made of the matter which comprises most of the universe (as thought <30 years ago)*
- *[Still to come – matter is not the dominant constituent of the universe]*