PHYSICS OF ASTROPHSYICS - Energy

http://apod.nasa.gov/apod/

ENERGY

Result of a force acting through a distance.

units = erg
= dyne cm   i.e., force x distance
= gm cm^2/sec^2

Two types:
kinetic - energy due to motion
potential - stored energy due to position

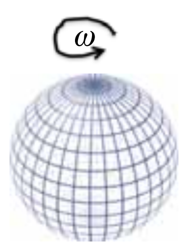
kinetic
$$E = \int \vec{F} \cdot d\vec{r}$$
$$= m \int \vec{a} \cdot d\vec{r} \qquad \frac{d\vec{r}}{dt} = \vec{v} \Rightarrow d\vec{r} = \vec{v}\, dt$$
$$= m \int_0^v \frac{d\vec{v}}{dt} \cdot \vec{v}\, dt = \frac{1}{2} m v^2 \qquad \text{(a scalar)}$$

Various Forms of Kinetic Energy

1) *Translational* as above

$$KE = \frac{1}{2} m v^2$$

2) *Rotational*



$$KE = \frac{1}{2} I \omega^2$$

I = Moment of inertia = $\frac{2}{5} MR^2$

for a rotating sphere of radius R and mass M

$\omega = \frac{2\pi}{P}$ with P, the period of rotation

Various Forms of Kinetic Energy

3) *Thermal*

$$E = \frac{3}{2} nkT$$

Here n is the number of particles, T, the temperature in Kelvins (K = C + 273) and k is Boltzmann's constant: $k = 1.38 \times 10^{-16}$ erg/degree K

The kinetic energy of a typical single particle in a thermal gas is

$$(KE / particle) = \frac{1}{2} m_{part} \langle v^2_{random} \rangle = \frac{3}{2} kT$$

This random speed is approximately equal to the speed of sound.

e.g. air in the room

$$1/2\, m\, v^2 = \frac{3}{2} k\, T$$

$$v = \sqrt{\frac{3kT}{m}}$$

$$m \approx 30\, m_H = 5x10^{-23} \text{ gm}$$

$k = 1.38 \times 10^{-16}$ erg/K

T= 293 K

$v = 4.9 \times 10^4$ cm/s

(more accurately 3 should be replaced by

γ = 1.4 for air)

The speed of sound in air at this temperature is actually 3.43×10^4 cm/s (1125 ft/s; 768 mph)

Potential Energy

r here is the distance to the center of *M*, (presumed spherical)

Must choose a reference point. Generally define potential energy between two objects to be zero when they are separated by an infinite distance.

Example: E_{grav} = Gravitational potential energy

$$E_{grav} = \int_R^\infty -\frac{GMm}{r^2}\, dr$$

$$= -GMm \int_R^\infty \frac{dr}{r^2}$$

$$= -GMm\left(-\frac{1}{r}\right)_R^\infty$$

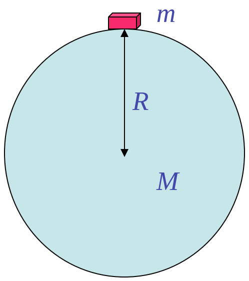
$$= -GMm\left(-\frac{1}{\infty} + \frac{1}{r}\right)$$

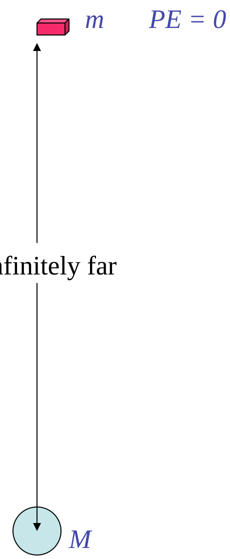
$$E_{grav} = -\frac{GMm}{R}$$

take mass, m, from R to infinity

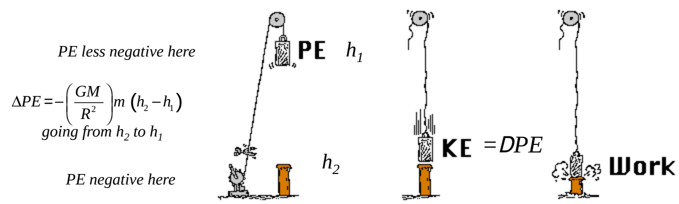
force acting through a distance

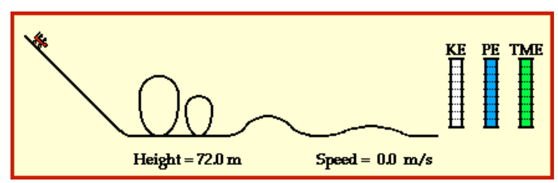
$$PE = -\frac{GMm}{R}$$





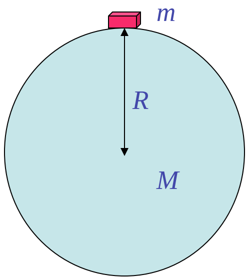
Potential energy can be changed into kinetic energy and vice versa.

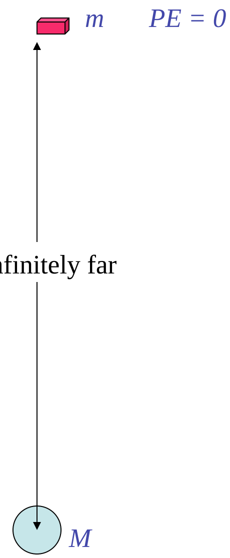




Escape Velocity

$$PE = -\frac{GMm}{R}$$





How much speed would m have to be given (straight up) to go to infinity and have no extra energy (speed) left over when it got there?

Suppose two masses "infinitely" far apart have no initial velocity with respect to one another.

$$\text{Total energy} = PE + KE = -\frac{GMm}{r} + \frac{1}{2}mv^2 = 0$$

Now release the two masses and let them fall towards one another until m, which we shall consider to be very small, strikes the surface of M at its radius, R. Total energy is "conserved", so

$$0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

Hence

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

1) eg., for Earth

$$v_{esc} = \left[\frac{(2)(6.67 \times 10^{-8})(5.98 \times 10^{27})}{6.38 \times 10^{8}}\right]^{1/2}$$
$$= 1.12 \times 10^{6} \text{ cm/sec}$$

or 11.2 km/s or about 7 mi/s.

2) eg., a neutron star ($M = 1.4\, M_\odot$; $R = 10$ km)

$$v_{esc} = \left[\frac{(2)(6.67 \times 10^{-8})(1.4)(1.99 \times 10^{33})}{1 \times 10^{6}}\right]^{1/2}$$
$$= 1.93 \times 10^{10} \text{ cm/sec}$$

or about 2/3 c. Need strong gravity to keep rapidly rotating pulsar from flying apart. ($GM/r > v_{rot}^2$)

3) eg., a black hole (original calculation due to LaPlace)

$$c = \left(\frac{2GM}{R_s}\right)^{1/2}$$
$$R_s = \frac{2GM}{c^2}$$

Light cannot escape from inside this radius which is 2.96 km for the sun and 0.89 cm for the Earth.

This is a fundamental limit on all Newtonian mechanics. We can ignore the effects of General Relativity only so long as we treat distances (and densities) such that $r/R_s >> 1$.

$R_s(\odot) = 2.96$ Km

$R_s(\oplus) = 0.89$ cm

Comets and asteroids typically impact the earth with a speed of 11.2 to 70 km/s. That's 25,000 to 156,000 miles per hour. The earth's orbital speed is 30 km/s and a asteroid could be orbiting in the opposite direction.

The oldest known fossils – of bacteria – date from 3.8 billion years ago. For almost the first billion years impacts may have made the Earth uninhabitable.

But these same collisions may have brought some of the chemicals necessary to life. (origin of oceans debated - probably not comets)

Consider the impact of even a 1 km diameter rock with density 5 gm cm^{-3} at 50 km s^{-1}. Assuming a spherical shape, the mass would be 4/3 π $r^3 \rho$, or 2.6 x 10^{15} gm. The energy,

$$\frac{1}{2}mv^2 = (0.5)(2.6x10^{15}\text{ gm})(5x10^6\text{ cm/s})^2 \qquad nb.\ r = 5\times10^4\text{ cm}$$

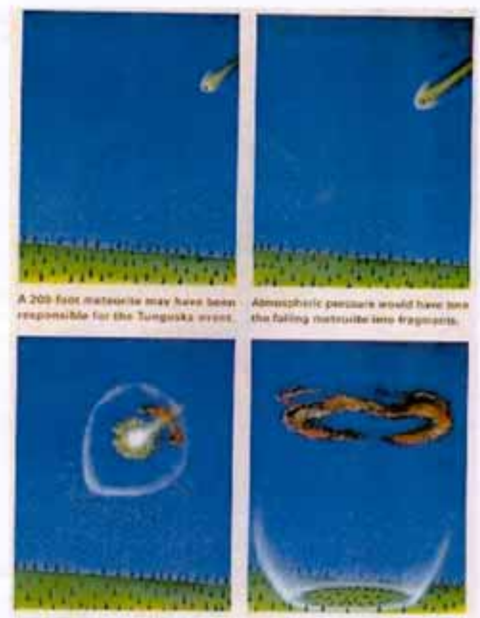
$$= 3.3x10^{28}\text{ erg}$$

equivalent to about 780,000 Megatons of high explosive.

Tunguska, Siberia 1908 (June 30)
(~200 feet)

Historical Impacts

- February 12, 1947 - Vladivostok, Siberia - about 23 tons of iron - 106 craters
- June 30, 1908 - Tunguska River, Siberia - "air burst", about 100,000 tons of material. Equivalent to 10 megaton explosion. Flattened trees in 1000 square kilometers. explosion 8 km above surface. ~50 m
- 50,000 years ago = Meteor Crater, Arizona. Object about 100 meters across. Energy comparable to Tunguska. Maybe 100 such craters world wide. Usually heavily weathered.
- 15 million years ago - Nordlingen, Germany. One billion tons. 2 to 3 km asteroid. 27 km crater.
- 65 million years ago - Yucatan, Mexico? - diameter greater than 10 km. 5 billion Hirsoshima sized bombs. 100 trillion tons of "fallout". Dinosaur killer?



A 200-foot meteorite may have been responsible for the Tunguska event.
Atmospheric pressure would have torn the falling meteorite into fragments.
The fragments in turn disintegrated quickly, creating a midair explosion.
The shock wave knocked down trees, while rising dust created high clouds.

Barringer Meteor Crater Arizona,

1.19 km crater; 49,000 years

Origin debated for decades

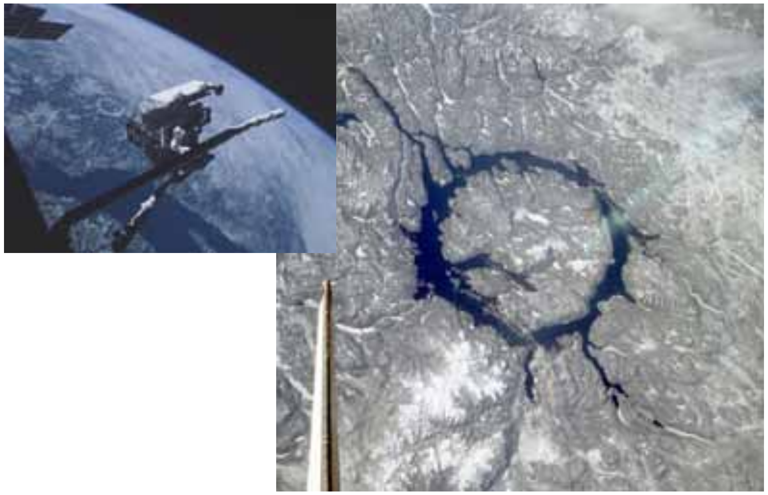
http://en.wikipedia.org/wiki/Meteor_Crater

Classic simple meteorite impact crater
~50 m impactor Ni-Fe about 10 Megatons
Meteorite mostly vaporized

Aorounga Crater, Chad Africa (Sahara Desert)
17 km in diameter; 200 million years

imaged from space, evidence for multiple impacts

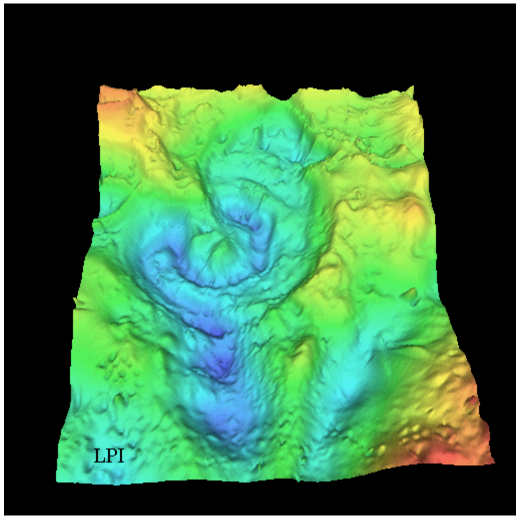
Manicouagan Crater, Quebec Canada
100 km, 212 million years



note tail of space shuttle Columbia, 1983. Lake is 70 km in diameter.

one of the largest impact craters preserved on the surface of the Earth. Outline is a lake. Glaciers have eroded much of the outer structure.

Chicxulub

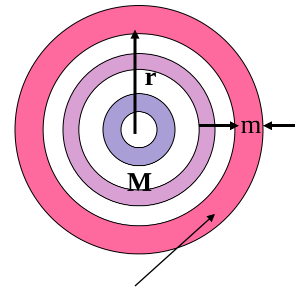


Yucatan, Peninsula, Mexico

Crater 170 km across age 64.98 million years. Buried under several hundred meters of sediment blocking it from easy view (this picture results from local gravitational and magnetic field variations). The asteroid that produced this impact crater is believed to have had a diameter of 10 to 20 km.

The impact hit a region rich in sulfur bearing rock. The sky may have been dark as night for close to a year. Temperatures would have been freezing. Half the species on earth perished.

Gravitational Binding Energy

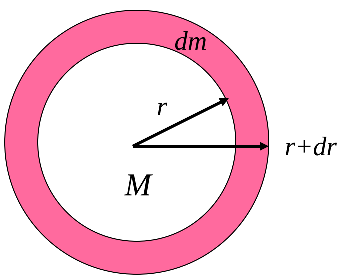


What is the potential energy of a spherical shell of mass m sitting atop a sphere of radius M and radius R?

That is how much energy would it take to remove the outer shell and take it to infinity?

$$\Delta E = -\frac{GMm}{r} \qquad m << M$$

and what is the mass of that shell *if the density is constant at all radii* and the shell is very thin with dr << r.

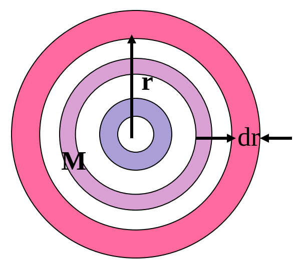


$$dm = (4\pi r^2)(dr)\rho$$

$$M(r) = \frac{4\pi}{3} r^3 \rho$$

$$dE = -\left(\frac{GM(r)\,dm}{r}\right) = \left(\frac{G}{r}\right)\left(\frac{4\pi r^3 \rho}{3}\right)\left(4\pi r^2 \rho\ dr\right)$$

To remove a shell: $dE = -\frac{G\,M\,dm}{r}$



$$dE = -\frac{G\left((4/3)\pi r^3 \rho\right)\left(4\pi r^2 \rho\ dr\right)}{r}$$

$$\Omega = -\int_0^R dE = (G)(4/3)(4)(\pi^2)(\rho^2)\int_o^R \frac{r^3 r^2}{r} dr$$

$$= (16/3)\pi^2 \rho^2 G \int_0^R r^4\, dr$$

$$= (16/3)\pi\, \rho^2 G\,(R^5/5) = (16/15) G \pi^2 R^5 \rho^2$$

Let M_{tot} be the total mass of the sphere and R its radius

but $M_{tot}^{\ 2} = (4/3)\pi R^3 \rho\,(4/3)\pi R^3 \rho$

$$= (16/9)\pi^2 \rho^2 R^6$$

so

$$\Omega = -\frac{3}{5}\frac{GM_{tot}^2}{R}$$

$$\frac{16}{15} = \frac{16}{9}\,\frac{3}{5}$$

GRAVITATIONAL BINDING ENERGY

Defined as the total potential energy of a gravitationally bound system (note there are similar concepts based on the electric and strong forces - e.g., nuclear binding energy

For the sun

$$\Omega_\odot = \frac{3}{5}\frac{GM_\odot^2}{R_\odot}$$

$$= \frac{(0.6)(6.67\times10^{-8})(1.989\times10^{33})^2}{(6.96\times10^{10})}$$

$$= 2.3\times10^{48} \text{ erg}$$

(actually 6.9 x 10^{48} erg)

How far could this go?

Suppose contract to a black hole

$$\Omega_{BH} \sim \frac{GM^2}{R_S} = \frac{GM^2c^2}{2GM} \sim Mc^2$$

There are reasons why this doesn't happen in ordinary stars.

The Kelvin-Helmholtz time scale (Lord Kelvin and Herman van Helmholtz, mid 1800's)

$$\tau_{KH} \approx \frac{\Omega}{2L}$$

$$\approx \frac{3GM_\odot^2}{5R_\odot 2L_\odot} = 0.3\frac{GM_\odot^2}{R_\odot L_\odot}$$

$$= 3.03\times10^{14} \text{ sec} = 9.6 \text{ million years}$$

(in fact, because the density is not constant, 20 to 30 million years is closer to correct)

Larger mass stars have shorter Kelvin-Helmholtz time scales because R L increases faster with M than M^3.

In fact, this extreme limit is never achieved, but it is possible in some circumstances to get 30% Mc^2.

Indeed the gravitational binding energy of a neutron star is about 1/3 Mc^2 and matter falling on neutron stars releases about this much energy. As we shall see, it is this enormous binding energy of neutron stars that powers supernovae.

Some young stars, especially T-Tauri stars, are thought to get most of their current luminosity from gravitational contraction, not nuclear fusion.

The Virial Theorem:

For a system bound together by a force that is proportional to $1/r^2$, e.g., gravity, the total potential energy is, in magnitude, equal to twice the total kinetic energy (in all forms - heat, motion, and rotation)

$$2\ KE = |PE|$$

- Always valid if the components of a gravitationally bound system have been together a long time, and are not moving close to the speed of light or are so dense as to be "degenerate"

Outline of Proof

*

http://math.ucr.edu/home/baez/virial.html

Assume: 1) A $1/r^2$ force

2) The time averages of the kinetic and potential energy are well defined

3) The positions and velocities of all particles are bounded for all time

*

$$T = \sum_i \vec{p}_i \cdot \vec{r}_i \qquad \vec{p}_i = m_i \vec{v}_i$$

$$\frac{dT}{dt} = \sum_i \frac{d\vec{p}_i}{dt} \cdot \vec{r}_i \quad + \quad \sum_i m_i \vec{v}_i \cdot \frac{d\vec{r}_i}{dt}$$

essentially T is the total net angular momentum and we assume that over long intervals of time it is not changing

but, $\frac{d\vec{p}_i}{dt} = \vec{F}_i$ and $\frac{d\vec{r}_i}{dt} = \vec{v}_i$ so

$$\frac{dT}{dt} = \sum_i \vec{F}_i \cdot \vec{r}_i \quad + \quad \sum_i m_i v_i^2 \qquad \text{since } \vec{v}_i \cdot \vec{v}_i = v_i^2$$

So,

$$\frac{dT}{dt} = \sum_i \vec{F}_i \cdot \vec{r}_i \ + \ 2KE$$

$F \cdot r$ might be e.g. $-\frac{\mathrm{GMm}}{\mathrm{r}^2} \cdot r = -\frac{GMm}{r}$

Now consider the time average of both sides over long periods of time

$$\left\langle \frac{dT}{dt} \right\rangle \rightarrow 0 = \langle PE \rangle + 2\langle KE \rangle$$

Examples:

- Orbital Motion

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow mv^2 = \frac{GMm}{r} = |PE| = 2\,KE$$

- Planets around the sun

Where did the other half of the energy go?

- Stars bound to Milky Way
- Stars in a Globular Cluster
- Thermal kinetic energy of a gravitationally bound gas

All of the gravitational energy released as a star - its total gravitational binding energy – has to go somewhere.

According to the *Virial Theorem*, half of the binding energy gets radiated away as light. the other half stays behind as heat.

Thus approximately, $KE_{particle} = \frac{1}{2} m_{particle} v^2 = \frac{3}{2} kT$

$$PE = \frac{3GM^2}{5R} \approx 2N_* \left(\frac{3}{2} kT \right) = 2\ KE$$

where N_* is the number of atoms in the star,

$$N_* \approx M / m_{atom}$$

The mass of a hydrogen atom is $1/N_A$ grams where $N_A = 6.02 \times 10^{23}\ \mathrm{gm}^{-1}$ so the number of atoms in the star, N_*, is roughly N_A M.

So

$$\frac{GM^2}{5R} \approx N_A M k T$$

$$T \approx \frac{GM}{5kRN_A}$$

(4.6×10^6 K for the sun, which is not a bad estimate for the *average* temperature. The central temperature is about three times greater.)

Note that as R gets smaller, T gets larger.

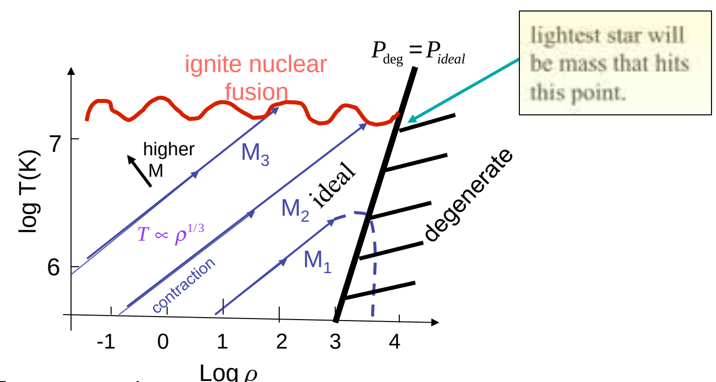
In fact this equation underestimates T because the density of the sun is not constant.

Note the implications. For star with constant mass, M, contraction occurs until T is high enough to burn a given fuel by nuclear reactions. When that fuel is gone, the star - or part of it - contracts further and the temperature goes up again.

$$T \approx \frac{GM}{5kRN_A}$$

Since $\frac{4}{3}\pi R^3 \rho \approx M$ $\qquad R \approx \left(\frac{3M}{4\pi\rho} \right)^{1/3}$

and $T \propto M^{2/3} \rho^{1/3}$ (constant density assumed to make a simple argument)



$$T \propto \frac{GM}{R} \qquad M \approx \frac{4\pi}{3} R^3 \rho$$

$$\Rightarrow R \propto \left(\frac{M}{\rho} \right)^{1/3}$$

$$T \propto \frac{M \rho^{1/3}}{M^{1/3}}$$

$$T \propto M^{2/3} \rho^{1/3}$$

$T \propto \rho^{1/3}$ for a given M

T at a given ρ is higher for bigger M