Astronomical Distance Determination

http://apod.nasa.gov/apod/

For relatively nearby sources, one can measure distances by "surveying" - by measuring the very small angles that a star's position is displaced relative to very distant objects because of the motion of the Earth around the sun. Prior knowledge of the AU is essential here.

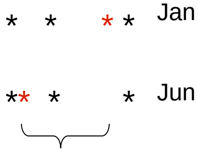
For more distant objects one uses either "standard candles" that are calibrated from nearby sources or a theoretical model.

Distance ladder (beyond the AU):

- Determine distances, d_1, for some nearby set of objects using technique 1, but then
- Find new brighter objects at distances similar to d_1.
- Use these objects, and sometimes a new technique 2, to get distances farther away at distances $d_2 >> d_1$
- etc.
- Each new distance determination inherits the errors of the earlier one. That's why it's called a "ladder"

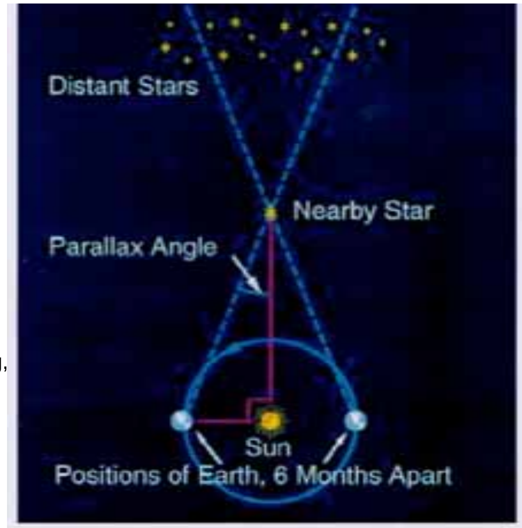
The first step is the AU which we have already covered. The next step involves the measurement of *parallaxes.*

Obtaining Distances by Parallax



This displacement is twice the parallax angle. The displacement oscillates with a period of one year.

More complicated if the stars are actually moving, but this can be adjusted for



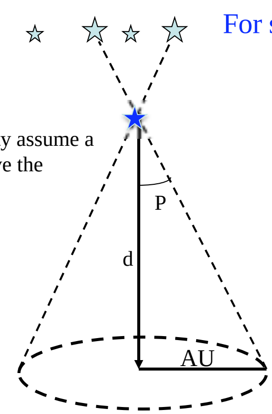
note: angles are exaggerated

The nearby star is the one that “moves” and the closer it is the more it moves

History of Parallax

- The first parallax of a star, 61 Cygni, was measured by Bessel in 1838.
- Since that time, parallax has been considered the most direct and accurate way to measure the distances to nearby stars. But the farther away they are the more technically challenging the observation becomes.

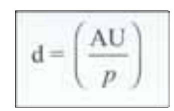
For simplicity assume a star 90º above the ecliptic



For small angles, $p \ll 1$, measured in radians

$\sin p \approx p \qquad \cos p \approx 1$

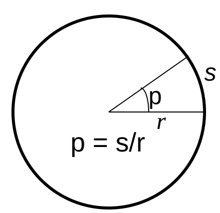
$$\frac{AU}{d} = \tan p = \frac{\sin p}{\cos p} \approx p$$

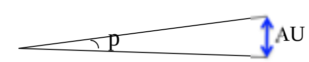


$$d = \left(\frac{AU}{p}\right)$$

if p measured in radians

1 radian = $360/2\pi$ = 57.296…º





AU = d p

$s = r\,p$ if and only if p is measured in *radians*

But astronomers actually report the angle p in seconds of arc.
1 radian is $360^o/2\pi = 57.296^o$ and each degree is
3600 arc seconds. So 1 radian = 206265 arc seconds.
Thus for p measured in seconds of arc (call it p''),

$$1 \text{ arc sec} = \frac{1}{206265} \text{ radians}$$

$$d = \frac{AU}{p \text{ (in radians)}}$$

$$d = \frac{206265\,AU}{p''}$$

$$d = \frac{1 \text{parsec}}{p''}$$

1 AU seen from one parsec away would subtend an angle of 1 arc second

p'' = parallax angle measured in seconds of arc

This defines the parsec, a common astronomical measure of length. It is equal to 206,265 AU's or 3.0856×10^{18} cm. It is also 3.26 light years.

A little thought will show that this also works for stars whose position is inclined at any angle to the ecliptic. What p measures then is the semi-major axis of the "parallactic ellipse".

Examples:

If the parallax angle of a star is 1 arc second, it is
1 parsec = 3.26 light years away

If the parallax angle is 0.5 arc sec it is 2 parsecs away

If the parallax angle is 2 arc sec (no such star) it is 0.5 parsec away etc.

Note for quite nearby stars one has to correct for the "proper motion", the continuing drift in the location of the star because it does not orbit the Milky Way at precisely the sun's speed and direction. This can be subtracted out.

To what accuracy would one have to measure angles to get distances to 1000 pc?

Hipparcos (the satellite)
(1989 - 1993)

Measured the position of 118,218 stars to a positional error of about a milli-arc second (about your size on the moon as viewed from earth)

Check out http://www.rssd.esa.int/Hipparcos/ and
http://www.rssd.esa.int/index.php?project=HIPPARCOS&page=exercises
e.g., brightest stars, closest stars, multiple stars

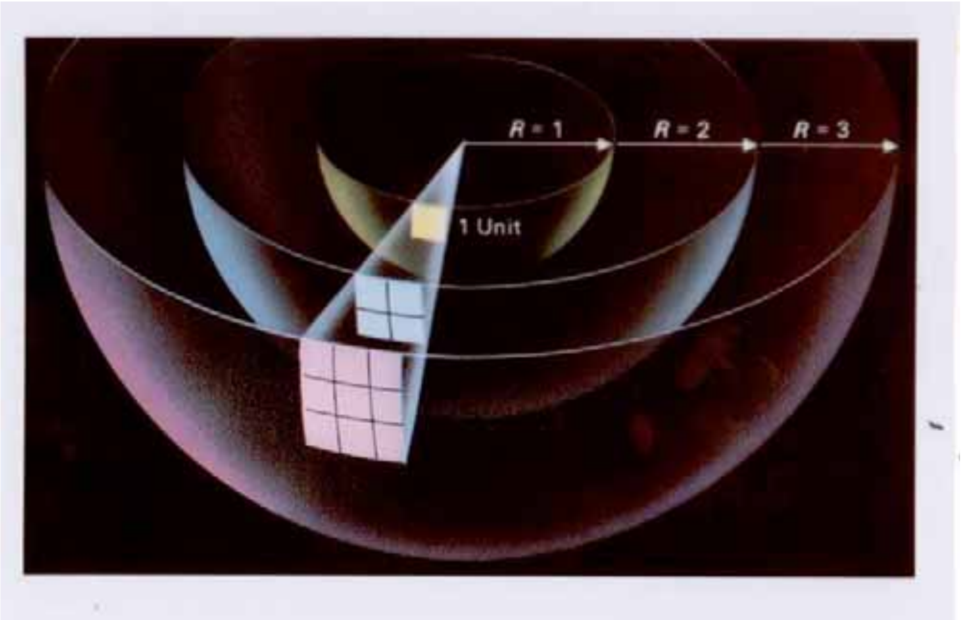
Distances measured to ~5% accuracy for about 10,000 stars to a distance of 1000 pc (including most of the stars you can see in the sky)

Some comments

Historically one used other forms of parallax – secular, statistical, moving cluster, etc., that had longer baselines than an AU, but were not very accurate and, since Hipparchos are not used anymore.

E.g. the motion of the sun around the center of the Galaxy, 250 km/s, corresponds to 53 AU/yr. Most of the nearby stars are moving along with us, but not precisely. Barnard's star "moves" 10.25 arc sec per year and hundreds of other stars move over 1 arc sec per year. The sun's average drift over a number of years compared with the local average, gives a longer baseline for estimating greater distances, but with poor precision.

To go beyond distances that can be surveyed using parallax (1 kpc), one needs "standard candles"



LUMINOSITY AND FLUX

- *Luminosity* is the total power emitted by a star. It is measured in ergs/sec. Usually we are speaking of the luminosity of light, or electromagnetic radiation of any wavelength. But one can also speak of neutrino luminosities. A synonym for luminosity is radiant *power*.

- *Flux* is a measure of how bright an object appears. Its value involves both the inherent luminosity of a source and its distance.

$$\varphi = \frac{L}{4\pi d^2}$$

SOLAR CONSTANT

The flux received by the earth from the sun:

$$\begin{aligned}\phi_\odot &= \frac{L_\odot}{4\pi(AU)^2} \\ &= \frac{3.83x10^{33}}{4\pi(1.5\,x\,10^{13})^2} \\ &= 1.35 \times 10^6 \text{ ergs}^{-1} \text{ cm}^{-2} \\ &= 0.135 \text{ watts cm}^{-2} \\ &= 1350 \text{ watts m}^{-2}\end{aligned}$$

nb. We use the symbol $\odot$ to denote the sun.

This is for 1 cm^2 (or 1 m^2) that is perpendicular to the sun's rays and ignores the effect of the earth's atmosphere.

Note that one could keep the flux constant by an appropriate adjustment of both L and d.

SOLAR CONSTANT

There are 10^7 ergs/s in one watt. One horsepower is 7.46×10^9 erg/s or 746 watts.

So the Earth when the sun is overhead on a clear day, receives about 1.8 HP per square meter of solar radiation.

If the sun were located at the distance of alpha-Centauri, the flux would be about 10^{11} times less. d = 1.3 pc.

$$\varphi = \frac{L_\odot}{4\pi d^2} = \frac{3.83 \; x \, 10^{33}}{4\pi\,(1.3)^2\,(3.08 x 10^{18})^2}$$
$$= 1.9 x 10^{-5} \text{ erg s}^{-1}\text{ cm}^{-2}$$

nb. Units of flux are those of power (erg/s) per unit area (cm^2)

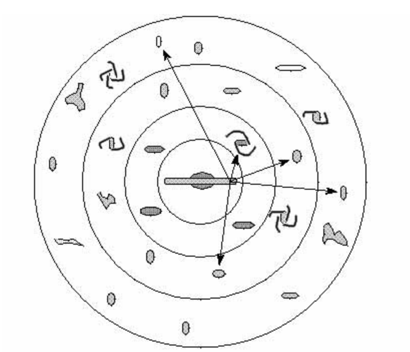
Note that if we had a "standard candle", a bright stellar source of known luminosity, L_{SC}, we could determine its distance from measuring its flux

$$\phi_{SC} = \frac{L_{SC}}{4\pi d^2}$$

$$d = \sqrt{\frac{L_{sc}}{4\pi\phi_{sc}}}$$

From Nick Strobel's Astronomy Notes

Interesting historical paradox



Olbers' Paradox: No matter what direction you look, you will eventually see a bright object. Farther away objects are fainter, but there are more of them. So each shell has the **same** overall brightness. The night sky should be bright!

Olber-Cheseaux paradox (1744)

$$\phi = \left(\frac{L}{4\,\pi\,r^2}\right)$$

$$N = \frac{4}{3}\pi r^3 \left(\frac{number}{volume}\right)$$

assume constant

$$N\,\phi \propto r \qquad diverges \; as\; r \to \infty$$

Solution??

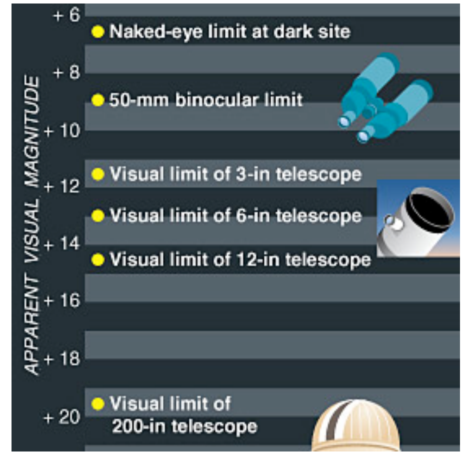
- Stars do not live forever
- Observable universe has a boundary given by how far light can have gone since the Big Bang
- Expansion of universe stretches the light and reduces its energy

Measuring Flux: Magnitudes:

- The eye is a logarithmic flux detector
- In astronomy we measure fluxes using magnitudes. Historically, a magnitude was “about a factor of two”.
- Calibrated more precisely by William Herschel in the late 18th century (see also Pogson (1856))

5 magnitudes is defined to be precisely a factor of 100 in flux. One magnitude thus corresponds to a change in flux of $(100)^{1/5} = 2.512$, i.e. $(2.512)^5 = 100$

- A sixth magnitude star is thus 100 times less “bright” than a first magnitude star. Larger magnitude is fainter.



Magnitudes

1.0	2.512
1.5	3.98
2.0	6.31
3.0	15.8
4.0	39.8
5.0	100
6.0	251
10	10^4
15	10^6
20	10^8
25	10^{10}
30	10^{12}

See http://en.wikipedia.org/wiki/Apparent_magnitude
E-ELT = 42 meter European Extremely Large Telescope (planned) Will (just barely) detect objects with m = 36. The sun in Andromeda would have m = 29.3. HST sees ~ 31.5, Venus at max = -4.9, etc.

The 10 brightest stars

Star		dist(ly)	m	M
Sun		-	-26.74	4.8
Sirius	Alpha CMa	8.6	-1.47	1.4
Canopus	Alpha Car	74	-0.72	-2.5
Rigil Kentaurus	Alpha Cen (A+B)	4.3	-0.27	4.4
Arcturus	Alpha Boo	34	-0.04	0.2
Vega	Alpha Lyr	25	0.03	0.6
Capella	Alpha Aur	41	0.08	0.4
Rigel	Beta Ori	~1400	0.12	-8.1
Procyon	Alpha Cmi	11.4	0.38	2.6
Achernar	Alpha Eri	69	0.46	-1.3

m = 0 was historically defined by the star Vega, though modern readjustments have changed m(Vega) = 0.03.

m measures “apparent magnitude”, how bright something looks.

But we also need some measurement of how luminous the star really is. In physics this is just what we have called L. But in astronomy there is another measure called the “absolute magnitude”. This is denoted M. It is not to be confused with mass.

Magnitudes, apparent and absolute

According to Herschel's definition, for fluxes ϕ_1 and ϕ_2:

$$\phi = \frac{L}{4\pi d^2}$$

$$\frac{\phi_1}{\phi_2} = 100^{\frac{m_2 - m_1}{5}}$$

$$\frac{m_2 - m_1}{5}\log(100) = \log\frac{\phi_1}{\phi_2}$$

That is, a star 5 magnitudes brighter has a flux 100 times greater.

$$\frac{2}{5}(m_2 - m_1) = \log\frac{\phi_1}{\phi_2}$$

$$m_2 - m_1 = 2.5\log\frac{\phi_1}{\phi_2}$$

So, if $\phi_1 > \phi_2$, $m_2 > m_1$. Keep in mind that bigger m means "fainter".

Apparent magnitude, *m*, is a measure of *flux*.

Absolute Magnitude

Absolute magnitude, *M*, is the magnitude a star would have if located at a certain distance – 10 pc. Since the distance is the same for all cases, *M* is a measure of the star's *luminosity*.

From these definitions of *m* and *M*, we can derive a relation which is essentially the equivalent of $\phi = \frac{L}{4\pi d^2}$

$$\varphi \leftrightarrow m$$
$$L \leftrightarrow M$$

Consider a star with luminosity L at two distances, d_1 = its real distance = d, and d_2 =10 pc. At distance d the star's magnitude is m_1. At 10 pc the star's magnitude is m_2 = *M*. From the previous page:

$$m_2 - m_1 = 2.5\log\frac{\phi_1}{\phi_2}$$

$$M - m = 2.5\log\left(\frac{L/4\pi d^2}{L/4\pi(10)^2}\right)$$

$$M - m = 2.5\log\left(\frac{10^2}{d^2}\right)$$
$$= 2.5(2.0 - 2.0\log d)$$

$$M - m = 5.0 - 5.0\log d$$

M measures the luminosity, *m*, the brightness, and d is the distance in pc.

For example, the apparent magnitude of the sun is -26.74. What is its absolute magnitude?

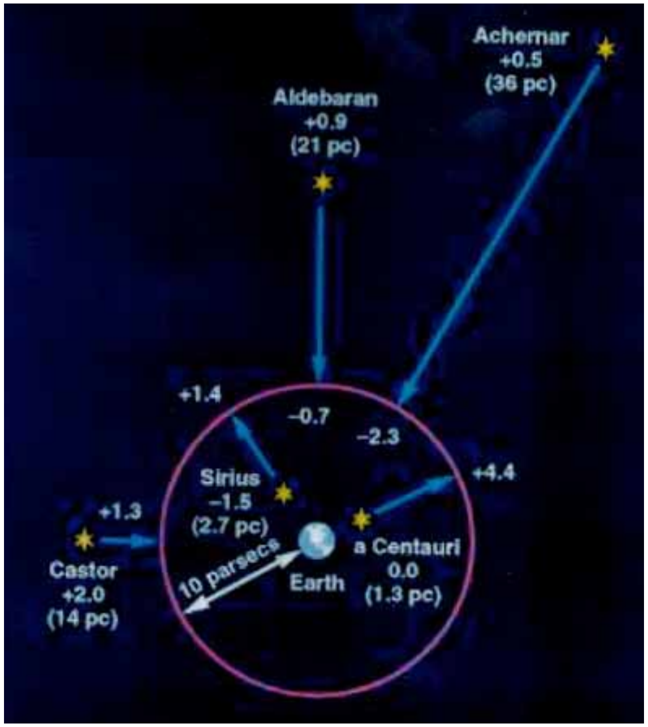
$$M = m + 5 - 5\log d(pc)$$

Here this (pc) just means that d is measured in parsecs

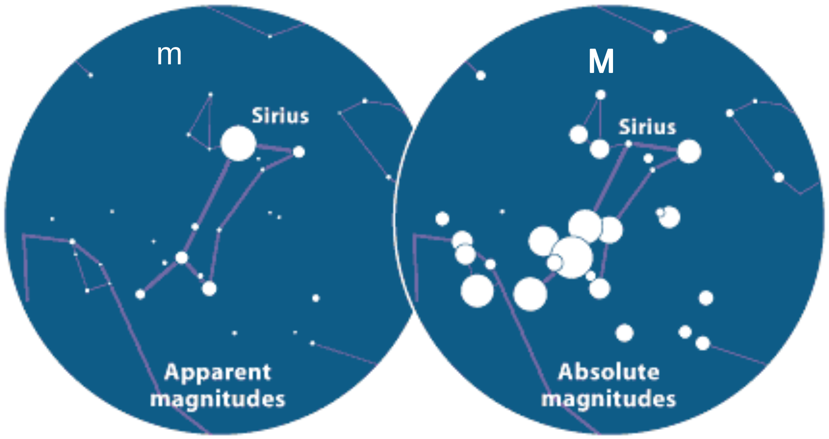
$$= -26.74 + 5 - 5\log\left(\frac{1}{206265}\right)$$
$$= -21.74 - 5(-5.51)$$
$$= 4.83$$

What would be the apparent magnitude of the sun at 10 pc? At 1.35 pc (distance to α-Centauri)?

$$4.83 - m = 5 - 5\log d$$
$$m = 4.83 - 5 + 5\log(1.35)$$
$$= -0.17 + 5\log(1.35) = -0.17 + 5(0.130)$$
$$= 0.48$$

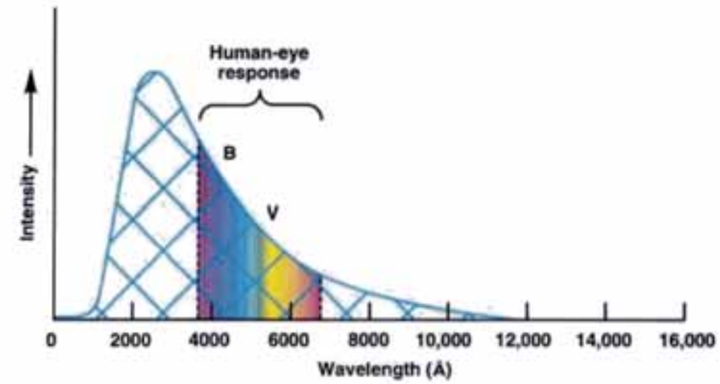


$M = m + 5 - 5 \log d$



Which stars are farther away than 10 pc and which ones are nearby?

A Complication: The Bolometric Correction



Unless otherwise indicated, m in this class, is the apparent *visual* magnitude.

BOLOMETRIC MAGNITUDE OF THE SUN

Our eyes have evolved to be most sensitive to the light emitted by the sun. Hence the bolometric correction for the missed emission in the infrared and ultraviolet is small for the sun.

The “visual” magnitude actually corresponds to the flux measured with a certain filter on the telescope. There are also blue magnitudes, red magnitudes, and others. We will discuss this later. For the sun.

$$M_{bol} = M_V - BC = 4.83 - 0.08 = 4.75$$

A similar equation would characterize apparent bolometric magnitudes, m_{bol}

Transforming Absolute (Bolometric) Magnitude to Luminosity

Two stars both at 10 pc. $r_1 = r_2$

$$M_{bol}(1) - M_{bol}(2) = 2.5 \log \frac{\phi_2}{\phi_1} = 2.5 \log \frac{L_2 / 4\pi r_2^2}{L_1 / 4\pi r_1^2}$$

$$M_{bol}(1) - M_{bol}(2) = 2.5 \log \frac{L_2}{L_1} \Rightarrow \log \frac{L_2}{L_1} = \frac{1}{2.5}\left(M_{bol}(1) - M_{bol}(2)\right)$$

Let star number 1 be the sun; let star number 2 be some star with bolometric magnitude M_{bol}. What is its luminosity, L?

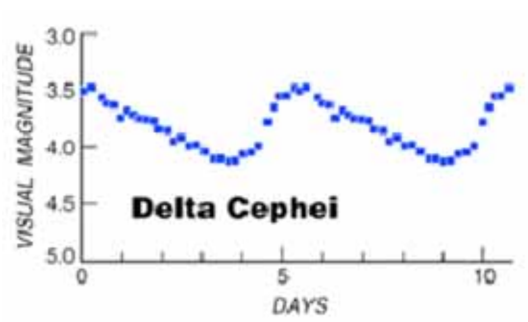
$$\log \frac{L}{L_\odot} = \frac{1}{2.5}(4.75 - M_{bol})$$

$$\log \frac{L}{L_\odot} = 1.90 - 0.4 M_{bol}$$

or

$$\frac{L}{L_\odot} = 79.4 \times 10^{-0.4 M_{bol}}$$

Standard Candles



At 900 light years as judged by Hipparchos
Delta Cephi waxes and wanes with a period of 5 days.
200 Cepheids had their distances measured by Hipparcos.

Cepheid Variables

Discovered 1794 by John Goodricke (age 19)

Delta Cephi, m = 3.6 to 4.6 in 5.4 days

A relatively nearby Cepheid (90 pc) is Polaris.
m varies from 2.0 to 2.1 every 4 days. As with all Cepheid variables, Polaris is a rather luminous star.

$$M = 5 + m - 5\log(d_{pc})$$
$$= 5 + 2.0 - 5\log(90)$$
$$= -2.77$$

Cepheid variables are large luminous stars with regular variations in brightness. The variation ranges from a few per cent to a factor of 5

Cepheids

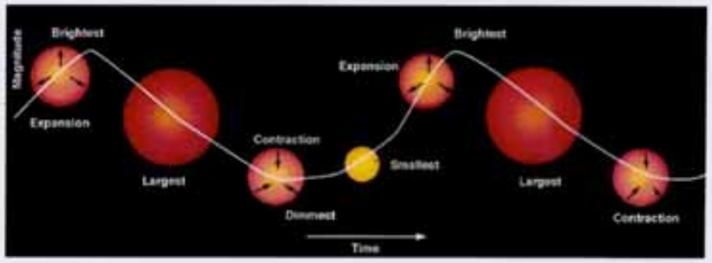
Periods of light variation are in the range 1 to 60 days and luminosities are up to 40,000 solar luminosities

The surface temperatures are similar to the sun but the star undergoes regular oscillations in size.

The radial velocity curve is almost a mirror image of the light curve, i..e., the maximum expansion velocity occurs at maximum light.

Light variation is in the range 0.5 to 2 magnitudes and radial velocities at maximum range from 30 to 60 km/s

A Cepheid variable is actually largest when its brightness is declining and smallest when it is rising.



The great merit of Cepheid variables for distance determination is that there is a clear relation between the period of the brightness variation and the average luminosity of the star.

Cepheid variables are also very bright and can be seen from far away. (They are not main sequence stars).

A complication though is that there are two populations of Cepheids and they have different period luminosity relations

Cepheids

The oscillation only occurs when the temperature structure of the star is such that the helium ionization zone lies near the stellar surface. Doubly ionized helium is more "opaque" than singly ionized helium and exists only at high temperature. The pulsation is due to properties of the envelope and does not involve the nuclear reactions in the core. More massive Cepheids are more luminous. http://www.answers.com/topic/cepheid-variable

The oscillation period depends on the surface gravity of the star and hence upon its average density.

Higher mass stars have lower density and higher luminosity. The lower density implies a longer period of variation.

$$P \propto \frac{1}{\sqrt{\rho}}$$

And so $P \leftrightarrow L$

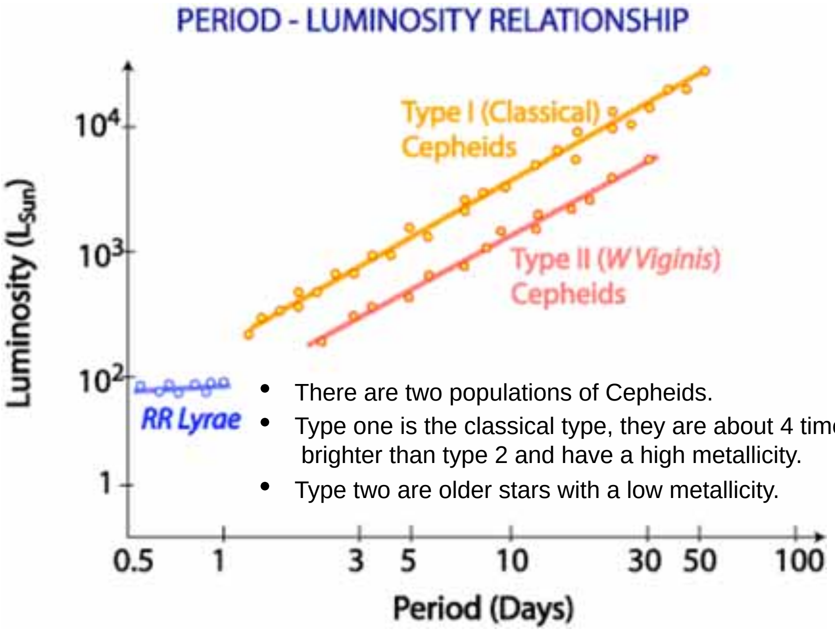
Modern Cepheids

Variable	Example	where	Period	Mass	Luminosity (Lsun)
Type I Cepheids	δ-Cephei	disk	1 – 60 d	3 – 10	300 – 40,000
Type II Cepheids (W-Virginis stars)	W-Virginis	halo globular clusters	1 - 60 d	< 1	1.5 mag less than Type I
RR-Lyrae	RR-Lyrae	globular clusters	<1 d	< 1	~100

Most stars pass through a Cepheid stage at one time or another. However the phase is short lived and only about $1/10^6$ stars are Cepheids at any one time

Cepheid variables are not main sequence stars

IN TERMS OF SOLAR LUMINOSITIES



- There are two populations of Cepheids.
- Type one is the classical type, they are about 4 times brighter than type 2 and have a high metallicity.
- Type two are older stars with a low metallicity.

How Big Is the Galaxy?

1785- Hershel - based on star counts being nearly isotropic concludes we are at the center of the the distribution. This view persisted til 1918. Size of "galaxy" (actually the universe) determined by how far away we could see stars

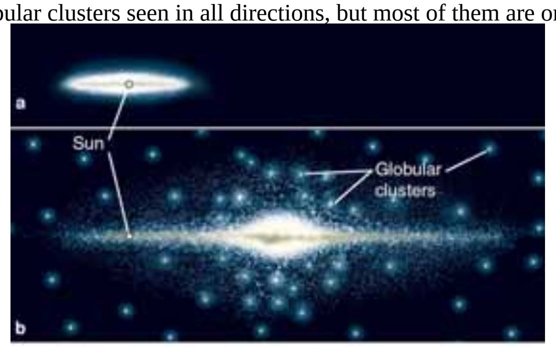
1912 - Henrietta Leavitt discovers P-L relation for Cepheid variables in the Small Magellanic Cliud

1913 - Ejnar Hertzsprung calibrates the relation using nearby (Type I Cepheids) but ignored reddening. The SMC Cepheids were thus brighter than he thought

1918 - Shapley determines disctance to galactic center by getting distances to the 93 globular clusters known at the time - got ~50,000 ly. Was looking at Type II Cepheids - which had "accidentally" been calibrated almost correctly using highly reddened nearby Type I Cepheids. Correct value is 28,000 ly. Some error due to inexact parallaxes for nearby Cepheids

Harlow Shapley's Realization… (1920s)

Globular clusters seen in all directions, but most of them are on one side of the sky!



globular cluster

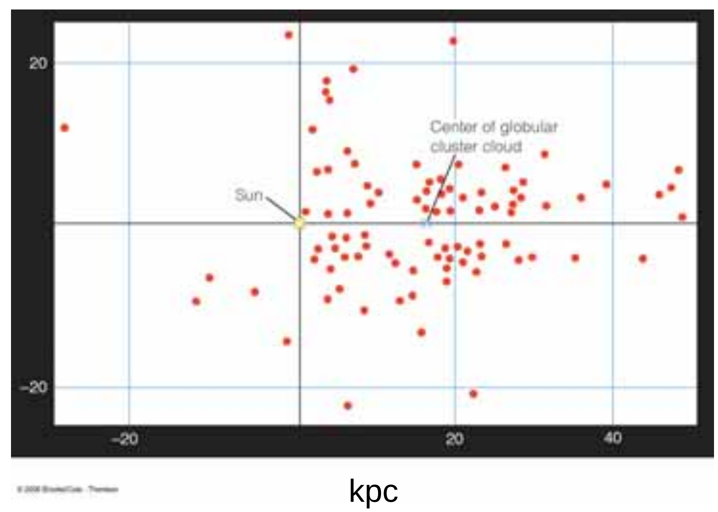
(has lots of Cepheid variable stars in it!)

Globular clusters must orbit around the center of mass of the galaxy!

Thus, assuming the clusters are distributed uniformly around the galaxy, he measured the 3D distribution of clusters (using Cepheid variables) and then assumed that the center of that distribution was where the center of the galaxy was.

He got both the direction and distance (sort of) to the galaxy center! But he had errors due to ignoring extinction and the poorly determined distance to the nearest Cepheids (statistical parallax)

Shapley's Map of the Galaxy



1918 - Shapley determines distance to galactic center by getting distances to the 93 globular clusters known at the time – got ~50,000 ly. Was looking at Type II Cepheids - which had "accidentally" been calibrated almost correctly using highly reddened nearby Type I Cepheids. Correct value is 28,000 ly

1920 – (Heber) Curtis – (Harlow) Shapley debate

1923- 5 - Hubble observes Cepheids in Andromeda - gets ~ 1 Mly

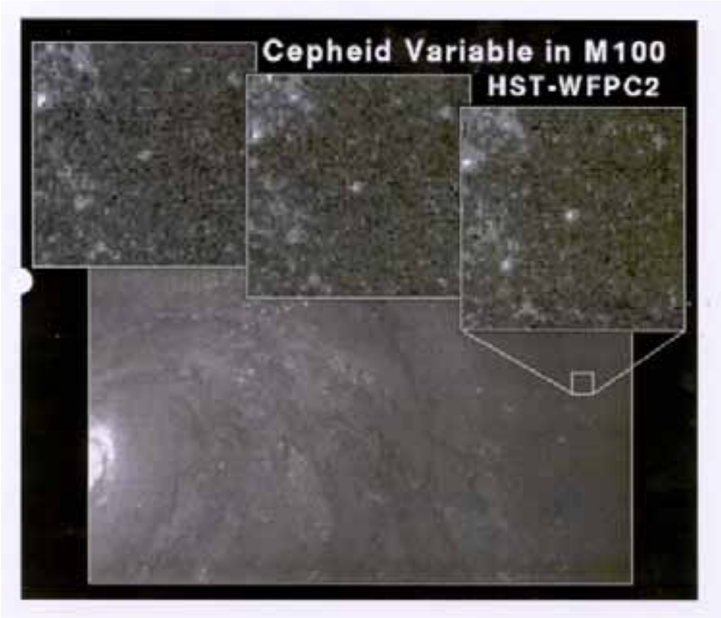
Andromeda

Early measurements of the distances to galaxies did not take into account the two types of Cepheids and astronomers underestimated the distances to the galaxies. Edwin Hubble measured the distance to the Andromeda Galaxy in 1923 using the period-luminosity relation for *Type II* Cepheids. He found it was about 900,000 light years away.

However, the Cepheids he observed were *Type I* (classical) Cepheids that were about four times more luminous than he thought. Later, when the distinction was made between the two types (Baade 1952), the distance to the Andromeda Galaxy was increased by about two times to about 2.3 million light years. Results from the Hipparcos satellite have given a larger distance near 2.5 million light years to the Andromeda Galaxy.

The Historical Problem:

- Think Type I Cepheids are fainter than they really are by 1.5 magnitudes (a factor of 4) because ignore reddening due to dust in the plane of the galaxy. End up thinking they have the same brightness as Type II Cepheids. Get distances to globular clusters right by mess up on Andromeda.
- If you see them unobscured – like in the Andromeda galaxy, you end up putting them too close (by a factor of 2)
- Then their individual stars and globular clusters, that are really much further away look too faint and too small.
- Eventually you end up thinking the universe is half as big as it actually is, and given its expansion rate, you also end up thinking it is younger than it is.



M100 is 17 Mpc (55 Mly) from the earth

- With available instrumentation, Cepheids can be used to measure distances as far as 20 Mpc to 10 - 20% accuracy.

- This gets us as far as the Virgo cluster of galaxies - a rich cluster with over 1000 galaxies.

$$M - m = 5 - 5\log(d)$$

Typical M_V for the brightest Cepheids is ~ -5

ST can easly measure fluxes to m = 28

-5 - 28 = 5 - 5 log (d)

log(d) = 38/5 = 7.6

$10^{7.6}$ = 40 Mpc

Cepheids play a critical role in bridging distance measurements in the Milky Way to other "nearby" galaxies

<u>So far:</u>

M - m = 5 - 5 log(d) d in parsecs

M measures luminosity (when corrected bolometrically and for reddening) $\frac{L}{L_\odot} = 79.4 \; x\, 10^{-0.4 M_{bol}}$

m measures flux (brightness); 5 magnitudes = factor 100

$$\phi = \frac{L}{4\pi d^2}$$

Distance "ladder" so far:

- Get AU from Kepler's 3rd and radar $P^2 \propto a^3$
- Get nearby stars from parallax $d = \frac{1}{p''}$
- Use standard candles, e.g. Cepheid variables (be careful of population) L = f (Period)
- Other standard candles... know L somehow