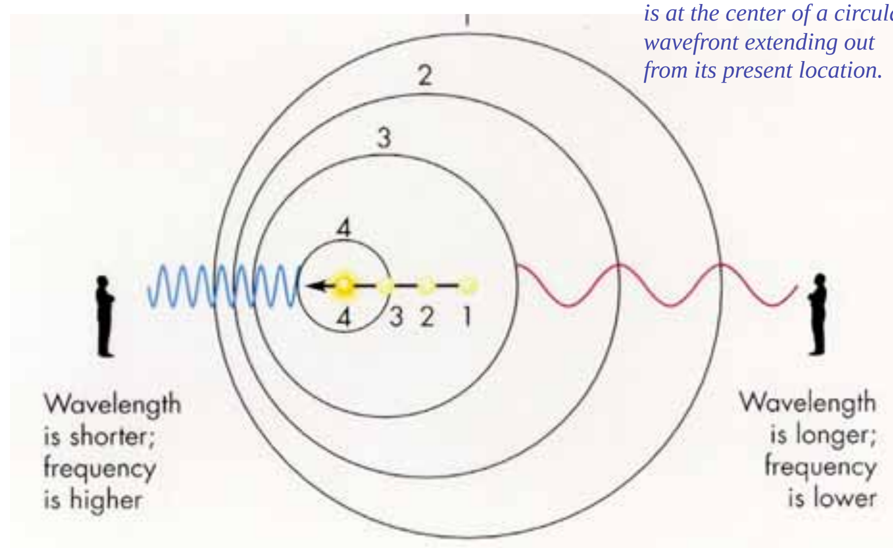
Spectroscopy, the Doppler Shift and Masses of Binary Stars

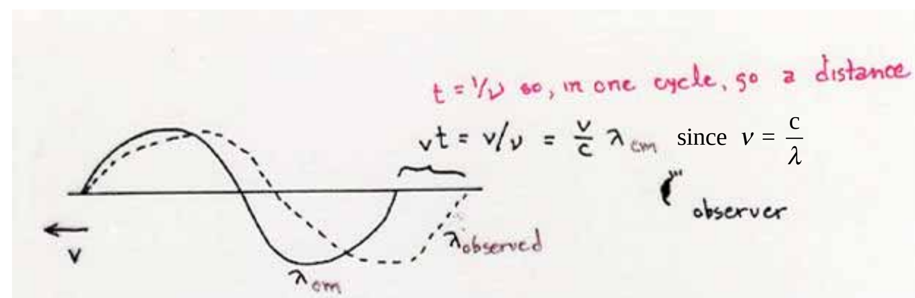
http://apod.nasa.gov/apod/astropix.html

Doppler Shift

At each point the emitter is at the center of a circular wavefront extending out from its present location.

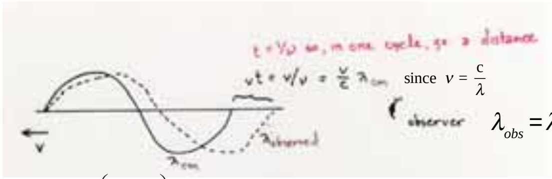


The Doppler Shift



$$\lambda_{obs} = \lambda_{emit} + \lambda_{emit}\left(\frac{v}{c}\right)$$

The Doppler Shift



since $v = \frac{c}{\lambda}$

$$\lambda_{obs} = \lambda_{emit} + \lambda_{emit}\left(\frac{v}{c}\right)$$

$\lambda_{obs} = \lambda_{emit}\left(1 + \frac{v}{c}\right)$ if moving away from you with speed v

$\lambda_{obs} = \lambda_{emit}\left(1 - \frac{v}{c}\right)$ if moving toward you with speed v

$$\Delta\lambda = \lambda_{obs} - \lambda_{emit}$$

$$= \lambda_{emit}(1 \pm \frac{v}{c} - 1)$$

$$\boxed{\frac{\Delta\lambda}{\lambda_{emit}} = \pm\frac{v}{c}}$$

if v is away from you $\Delta\lambda > 0$

if v is toward you $\Delta\lambda < 0$

This formula can only be used when v << c

Otherwise, without proof,

$$\lambda_{obs} = \lambda_{emit}\left(\frac{1 + v/c}{1 - v/c}\right)^{1/2}$$

Doppler Shift:

Note – different from a cosmological red shift!

Astronomical Examples of Doppler Shift

- A star or galaxy moves towards you or away from you (can't measure transverse motion)
- Motion of stars in a binary system
- Thermal motion in a hot gas
- Rotation of a star

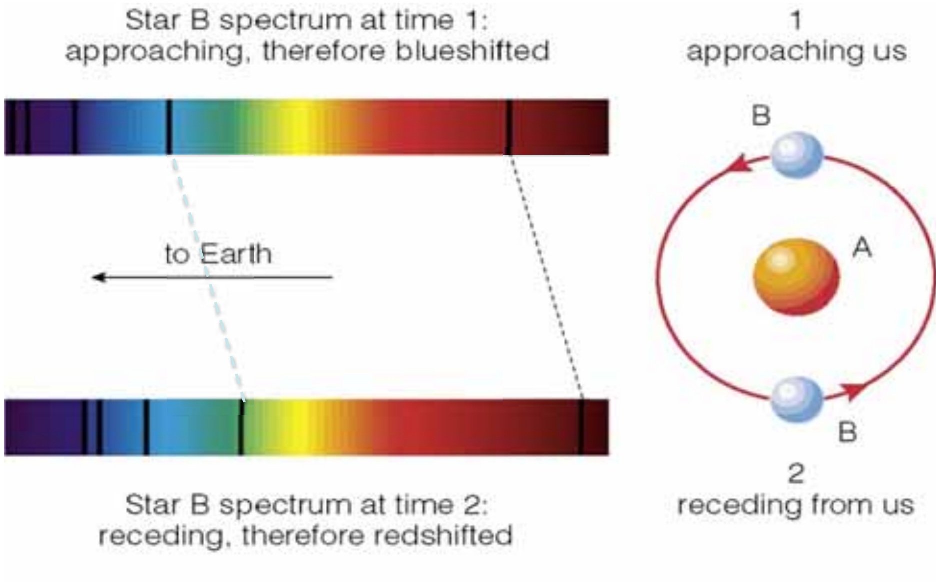
E.g. A H atom in a star is moving away from you at 3.0×10^7 cm s^{-1} = 0.001 times c.

At what wavelength will you see H_α?

$$\lambda_{obs} = 6562.8\ (1 + 0.001) = 6569.4\ \text{A}$$

Note that the Doppler shift only measures that part of the velocity that is directed towards or away from you.

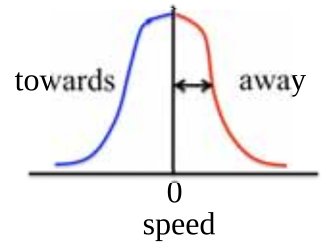
A binary star pair



Thermal Line Width

In a gas with some temperature T atoms will be moving around in random directions. Their average speed will depend upon the temperature. Recall that the definition of temperature, *T*, is

$$\frac{1}{2}\, m_{atom} \langle v^2 \rangle = \frac{3}{2}\, k\, T$$



$$v_{average} = \sqrt{\frac{3kT}{m_{atom}}}$$

where $k = 1.38 \times 10^{-16}$ erg K^{-1}
Here $\langle\,\rangle$ means "average". Some atoms will be moving faster than the average, others hardly at all. Some will be moving towards you, others away, still others across your line of sight.

Thermal Line Broadening

The full range of wavelengths, hence the width of the spectral line will be

$$\frac{\Delta\lambda}{\lambda} = 2\frac{v_{average}}{c} = \frac{2}{c}\sqrt{\frac{3kT}{m_{atom}}}$$

The mass of an atom is the mass of a neutron or proton (they are about the same) times the total number of both in the nucleus, this is an integer "A".

$$\frac{\Delta\lambda}{\lambda} = 2\left(\frac{(3)(1.38\times10^{-16})(T)}{(1.66\times10^{-24})(A)}\right)^{1/2}\left(\frac{1}{2.99\times10^{10}}\right)$$

A = 1 for hydrogen
4 for helium
12 for carbon
16 for oxygen
etc.

$$\frac{\Delta\lambda}{\lambda} = 1.05\times10^{-6}\sqrt{\frac{T}{A}}$$

where T is in K

$$\text{Full width} = \Delta\lambda = 1.05 \times 10^{-6}\sqrt{\frac{\text{T(in K)}}{A}}\;\lambda$$

Eg. H_α @ 5000 K

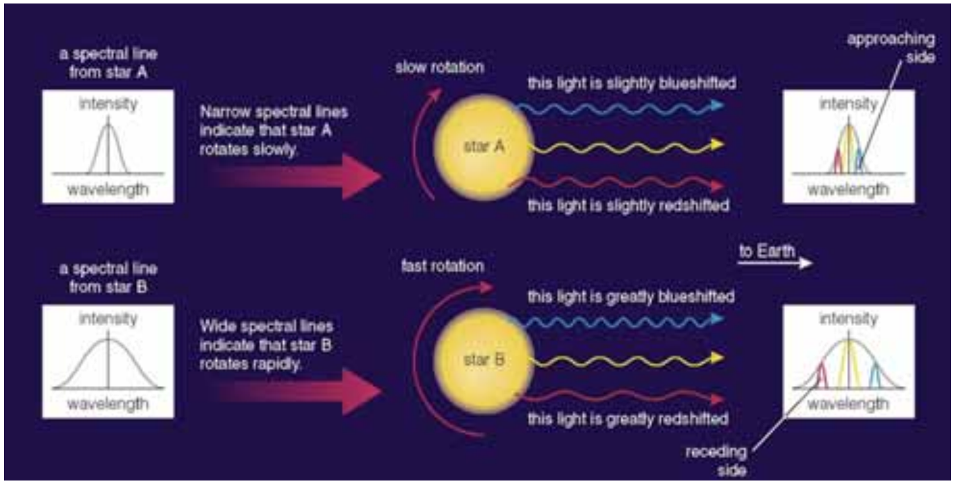
A = 1 T = 5000 $\lambda = 6563$ Å

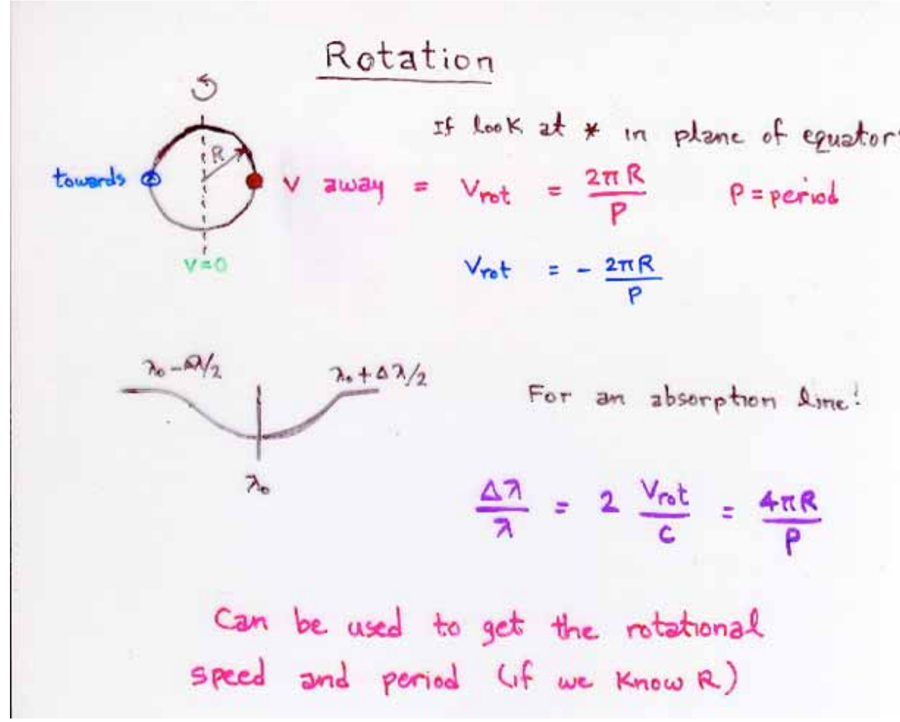
$\Delta\lambda = (6563)(1.05\times10^{-6})\left(\frac{5000}{1}\right)^{1/2}$ Å

$= 0.49$ Å

Can use this to measure the temperature. (again)

ROTATION





Note: <u>Potential</u> <u>complications</u>:

1) Star may have both thermal and rotational broadening

2) May see the star at some other angle than in its equatorial plane.

Example: H_α in a star with equatorial rotational speed 100 km/s = 10^7 cm/s

$$\text{Full width} = \Delta\lambda = 2\left(\frac{v}{c}\right)\lambda$$

$$=(2)(6563)\left(\frac{10^7}{3 \times 10^{10}}\right) = 4.4\ A$$

Average rotational velocities (main sequence stars)

Stellar Class	$v_{equator}$ (km/s)
O5	190
B0	200
B5	210
A0	190
A5	160
F0	95
F5	25
G0	12

Stellar winds and magnetic torques are thought to be involved in slowing the rotation of stars of class G, K, and M.

Stars hotter than F5 have stable surfaces and perhaps low magnetic fields.

The sun rotates at 2 km/s

Red giant stars rotate very slowly. Single white dwarfs in hours to days. Neutron stars may rotate in milliseconds

3 sources of spectral line broadening

1) Pressure or "Stark" broadening (Pressure)

2) Thermal broadening (Temperature)

3) Rotational broadening (ω, rotation rate)

SPECTROSCOPY: WHAT WE CAN LEARN

1) Temperature

Ionization stages that are present

Thermal line broadening

Wien's Law ($\lambda_{max} \propto 1/T$)

2) Radius

Blackbody $L = 4\pi R^2 \sigma T^4$

3) Rotation rate

Spectral line widths

4) Composition

From a detailed analysis of what lines are present and their strengths

5) Surface pressure

Also from line broadening. Is the star a white dwarf or a red giant or a main sequence star

6) Velocity towards or away from us

Is the star or galaxy approaching us or receding?

7) Binary membership, period, and velocity planets?

From periodic Doppler shifts in spectral lines

8) Magnetic fields

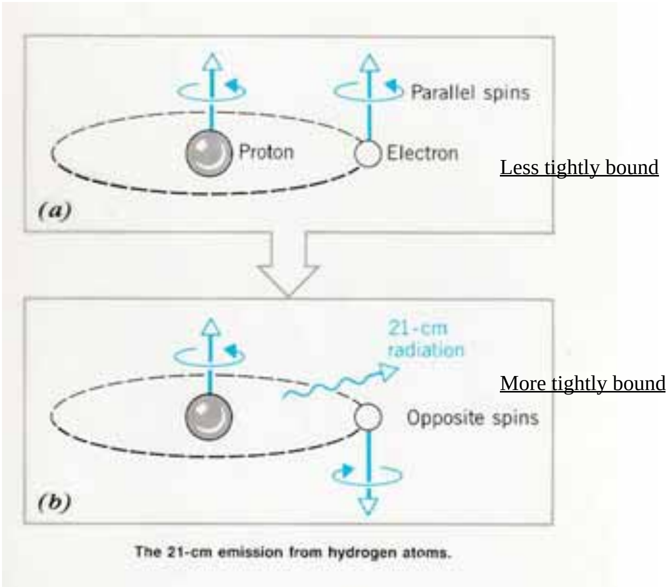
From Zeeman splitting

9) Expansion speeds in stellar winds and explosions

Supernovae, novae, planetary nebulae

10) From 21 cm - rotation rates of galaxies. Distribution of neutral hydrogen in galaxies. Sun's motion in the Milky Way.

Hyperfine Splitting   The 21 cm Line



The 21-cm emission from hydrogen atoms.

21 cm (radio)

λ= 21 cm
ν = 1.4 x 10^9 Hz
hν = (6.63 x 10^{-27})(1.4 x 10^9) = 9.5 x 10^{-18} erg
= 5.6 x 10^{-6} eV

Must have neutral H I

Emission collisionally excited

Lifetime of atom in excited state about 10^7 yr

Galaxy is transparent to 21 cm

Merits:

- Hydrogen is the most abundant element in the universe and a lot of it is in neutral atoms - H I
- It is not so difficult to build big radio telescopes
- The earth's atmosphere is transparent at 21 cm


Aerecibo - 305 m radio telescope - Puerto Rico

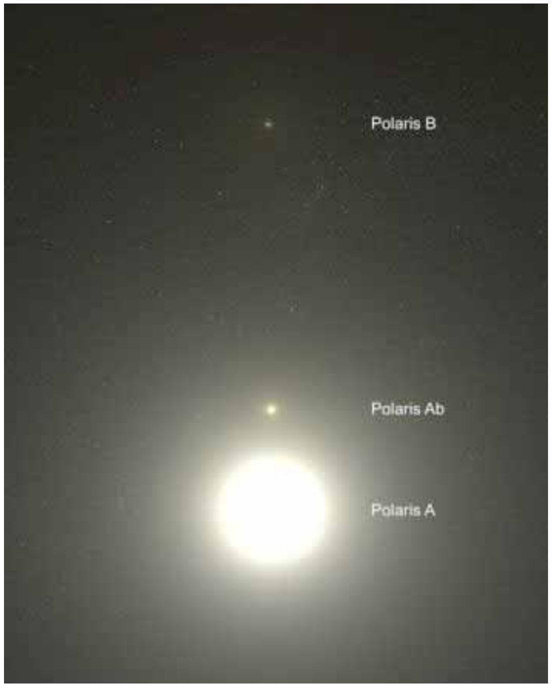
Getting Masses in Binary Systems

<u>Binary and Multiple Stars</u>
<u>(about half of all stars)</u>

Beta-Cygnus (also known as Alberio)
Separation 34.6”. Magnitudes 3.0 and 5.3.
Yellow and blue. 380 ly away. Bound?
P > 75000 y. The brighter yellow component
is also a (close) binary. P ~ 100 yr.

Alpha Ursa Minoris (Polaris)
Separation 18.3”. Magnitudes
2.0 and 9.0. Now known to be a triple.
Separation ~2000 AU for distant pair.

When the star system was born it apparently had too much
angular momentum to end up as a single star.



Polaris

1.2 Msun Polaris Ab
Type F6 - V
4.5 Msun Polaris A
Cepheid

Period 30 yr

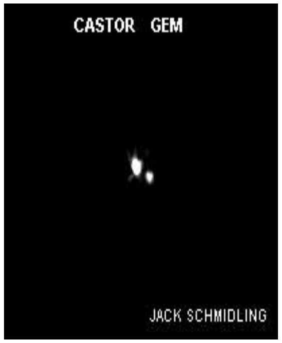
Polaris B is
F3 - V

> 10^5 years

585 years

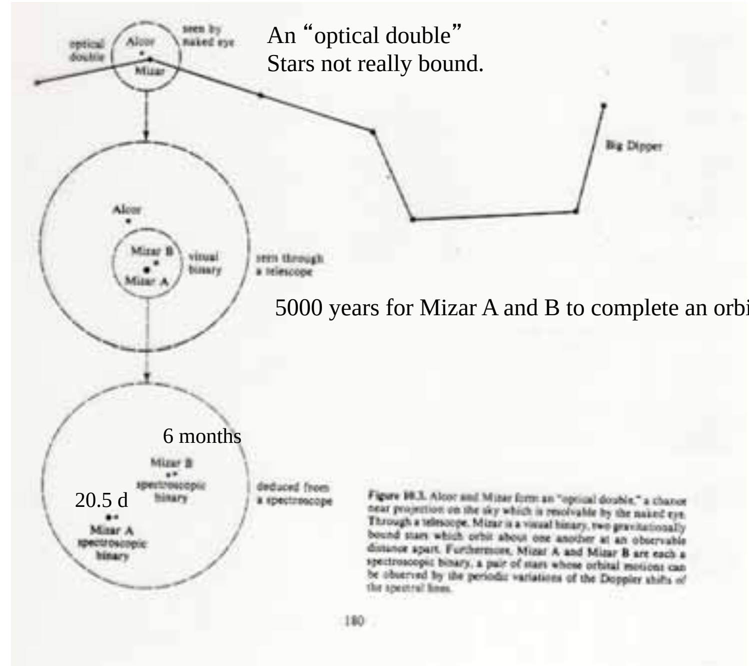
1165 years

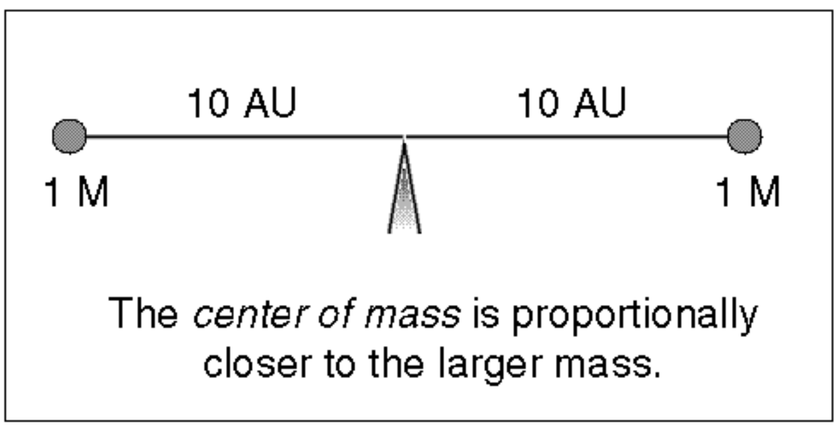
Epsilon Lyra – a double double.
The stars on the left are separated by 2.3” about 140 AU;
those on the right by 2.6”. The two pairs are separated
by about 208” (13,000 AU separation, 0.16 ly between
the two pairs, all about 162 ly distant). Each pair would
be about as bright as the quarter moon viewed from the other.



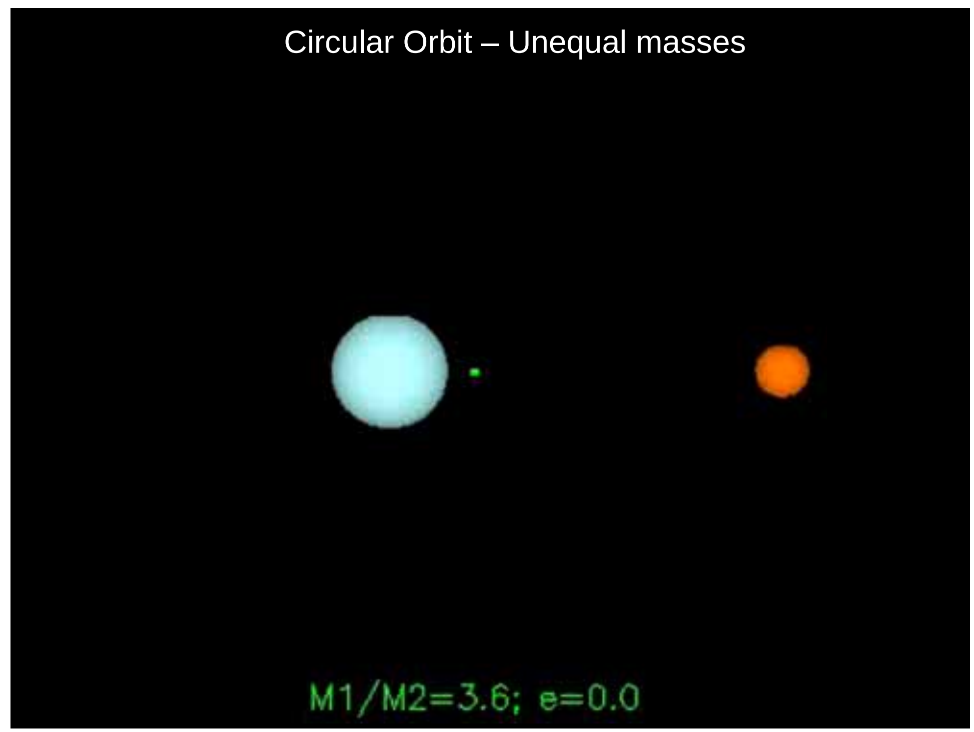
Castor A and B complete an orbit every 400 years. In their elliptical orbits their separation varies from 1.8” to 6.5”. The mean separation is 8 billion miles. Each star is actually a double with period only a few days (not resolvable with a telescope). There is actually a “C” component that orbits A+B with a period of of about 10,000 years (distance 11,000 AU).

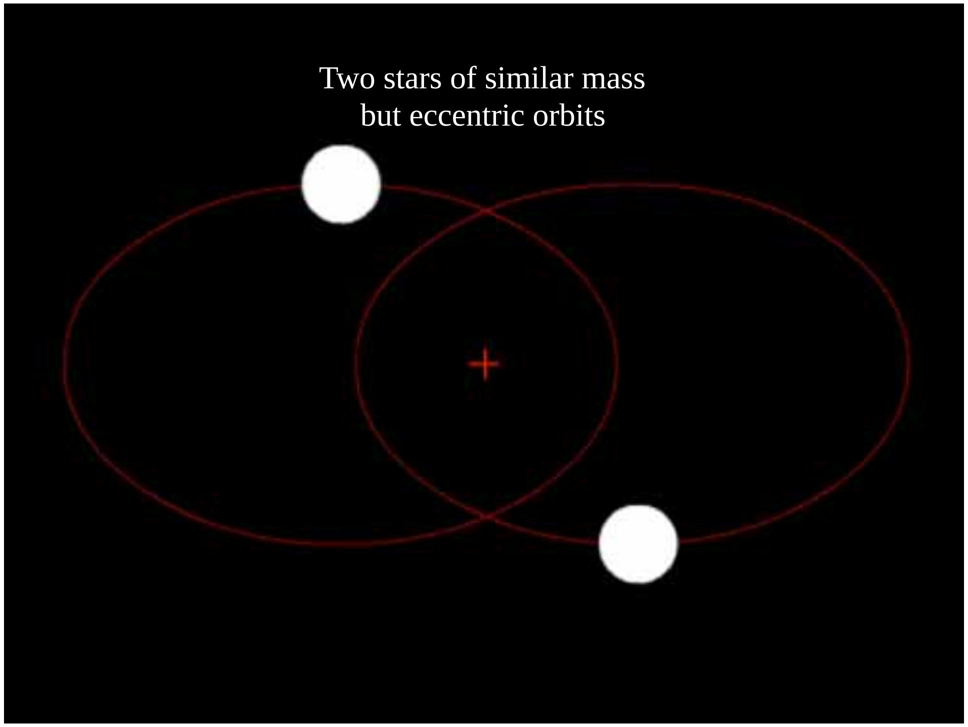
Castor C is also a binary. 6 stars in total

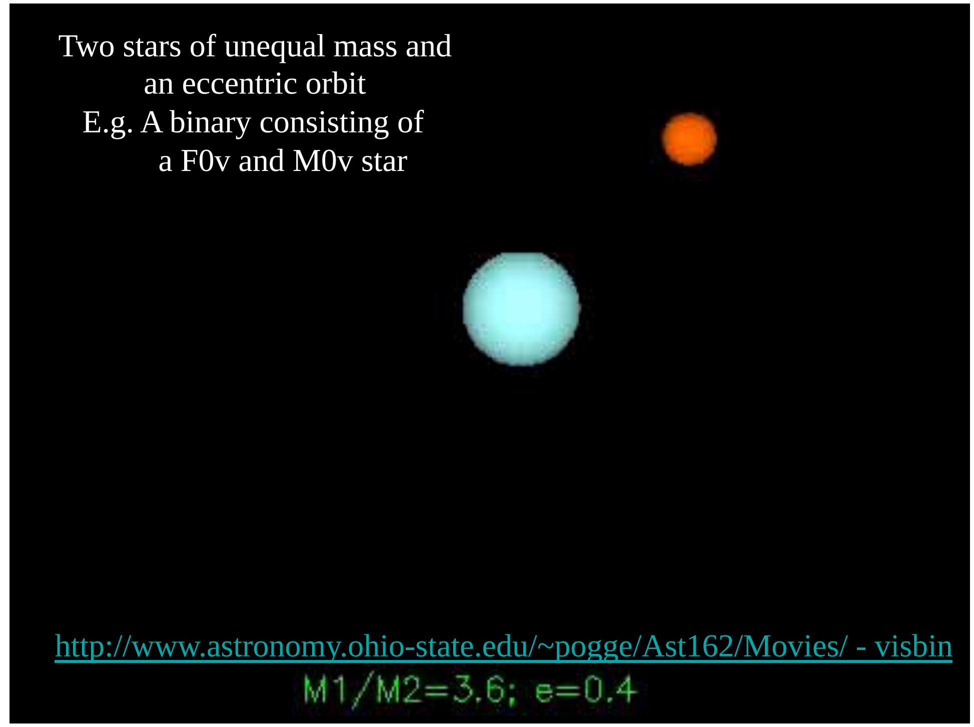




For constant total separation, 20 AU, vary the masses





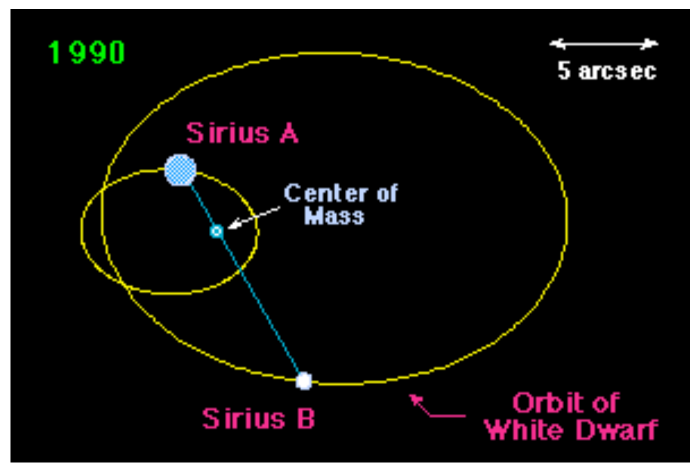


Aside

The actual separation between the stars is obviously not constant in the general case shown.

The separation at closest approach is the sum of the semi-major axes "a" times (1-e) where e is the eccentricity. At the most distant point the separation is "a" times (1+e).

For circular orbits e = 0 and the separation is constant.

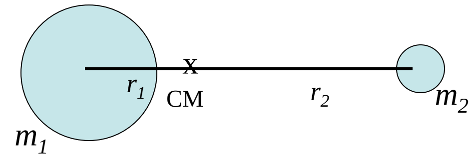


Period = 50.1 years
distance to c/m 6.4 (A) and 13.4 (B) AU

Some things to note:

- The system has only one period. The time for star A to go round B is the same as for B to go round A
- A line connecting the centers of A and B always passes through the center of mass of the system
- The orbits of the two stars are similar ellipses with the center of mass at a focal point for both ellipses
- The distance from the center of mass to the star times the mass of each star is a constant. (next page)

ASSUME CIRCULAR ORBITS



both stars feel the same gravitational attraction and thus both have the same centrifugal force

$$\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{G m_1 m_2}{(r_1 + r_2)^2}$$

$$\frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} = \text{Period}$$

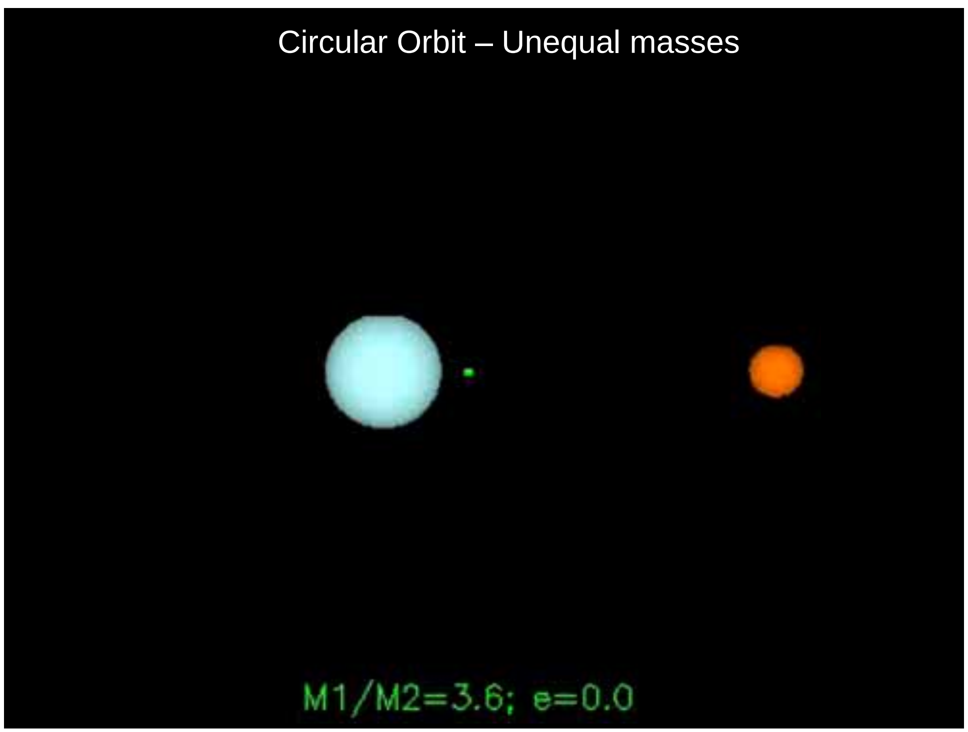
$$\therefore\ v_1 = \frac{r_1 v_2}{r_2}$$

$$\frac{m_1 r_1^2 v_2^2}{r_1 r_2^2} = \frac{m_2 v_2^2}{r_2}$$

More massive star is closer to the center of mass and moves slower.

$$m_1 r_1 = m_2 r_2$$

$$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$



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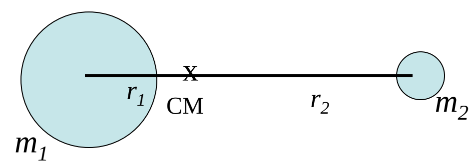

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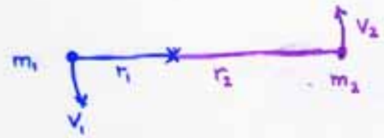
More massive star is closer to the center of mass and moves slower.

$$m_1 r_1 = m_2 r_2$$

$$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

For simplicity, assume circular motion



m_1 goes around "x" in period P

m_2 also goes around "x" in period P

$$\frac{2\pi r_1}{P} = V_1 \qquad \frac{2\pi r_2}{P} = V_2$$

$$P = \frac{2\pi r_1}{V_1} = \frac{2\pi r_2}{V_2}$$

$$\boxed{r_1 V_2 = r_2 V_1} \qquad \frac{r_1}{r_2} = \frac{V_1}{V_2}$$

$$\frac{r_1}{r_2} = \frac{v_1}{v_2}$$

So

$$\frac{m_2}{m_1} = \frac{v_1}{v_2}$$

Motion of the sun because of Jupiter

$$m_1 r_1 = m_2 r_2$$

$$M_\odot d_\odot = M_J\, d_J$$

$$d_\odot = \frac{M_J}{M_\odot}\, d_J$$

$$= (9.95 \times 10^{-4})(7.80 \times 10^{13})$$

$$= \boxed{7.45 \times 10^{10} \text{ cm}}$$

$d_\odot$ = radius of sun's orbit around center of mass

d_J = Jupiter's orbital radius

= 5.20 AU

= 7.80×10^{13} cm

$M_J = 1.90 \times 10^{30}$ gm

= $9.55 \times 10^{-4}\ M_\odot$

Can ignore the influence of the other planets.

P = 11.86 years

Doppler shift

P_J = Period Jupiter = 11.86 y

= 3.75×10^8 s

$$V_\odot = \frac{2\pi d_\odot}{P_J} = \frac{(2\pi)(7.45 \times 10^{10} \text{ cm})}{3.75 \times 10^8 \text{ s}}$$

$= 1.25 \times 10^3$ cm/s

$= 12.5$ m/s

About 40 mph

$$\frac{V}{c} = \frac{1.25 \times 10^3 \text{ cm/s}}{2.99 \times 10^{10} \text{ cm/s}} = 4.18 \times 10^{-8}$$

$$= \frac{\Delta\lambda}{\lambda}$$

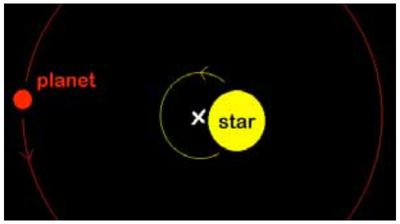
eg. Hα 6563 Å $\qquad \Delta\lambda = 2.74 \times 10^{-4}$ Å

small compared to thermal + rotational broadening

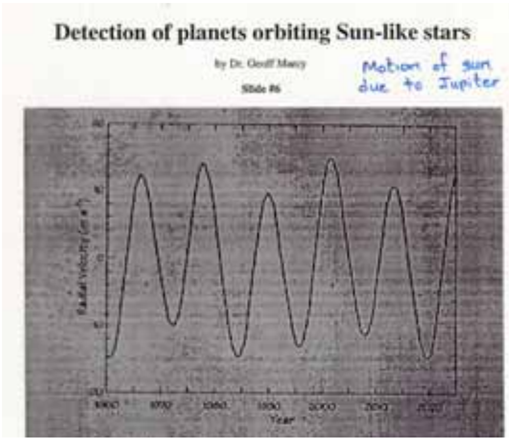
As of today – 843 extra solar planets in 665 stellar systems and the number is growing rapidly.

Many were detected by their Doppler shifts. Many more by the "transits" they produce as they cross the stellar disk.

http://exoplanet.eu/catalog.php

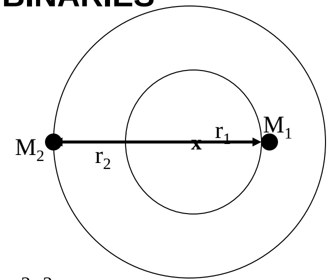


Note: "wobble" of the star is bigger if the planet is bigger or closer to the star (hence has a shorter period).



12.5 m/s
11.86 years

KEPLER'S THIRD LAW FOR BINARIES



$$\frac{G M_1 M_2}{(r_1+r_2)^2} = \frac{M_1 v_1^2}{r_1}$$

$$+\quad \frac{G M_1 M_2}{(r_1+r_2)^2} = \frac{M_2 v_2^2}{r_2}$$

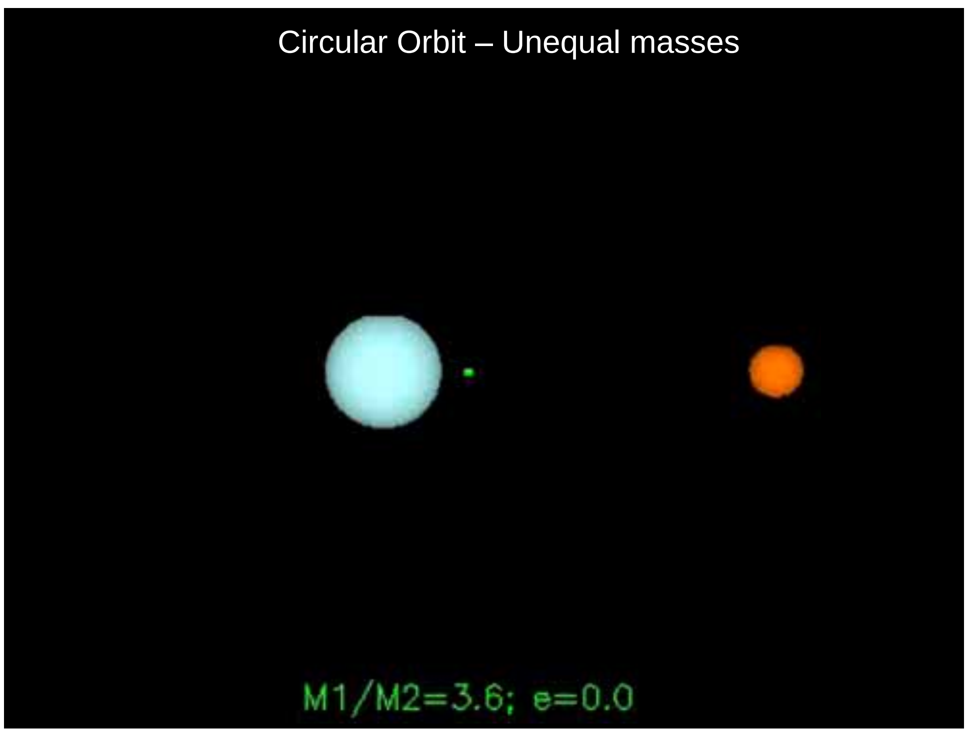
$$\frac{G(M_1+M_2)}{(r_1+r_2)^2} = \frac{v_1^2}{r_1} + \frac{v_2^2}{r_2} = \frac{4\pi^2 r_1^2}{P^2 r_1} + \frac{4\pi^2 r_2^2}{P^2 r_2}$$

$$= \frac{4\pi^2}{P^2}(r_1+r_2)$$

$$P^2 = K(r_1+r_2)^3 \qquad K = \frac{4\pi^2}{G(M_1+M_2)}$$

i.e., just like before but

$M \to M_1+M_2 \qquad R \to r_1+r_2$



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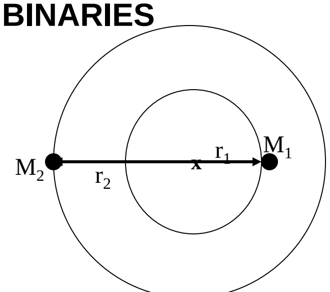

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$$= \frac{4\pi^2}{P^2}(r_1+r_2)$$

$$P^2 = K(r_1+r_2)^3 \qquad K = \frac{4\pi^2}{G(M_1+M_2)}$$

i.e., just like before but

$M \to M_1+M_2 \qquad R \to r_1+r_2$

$$(M_1 + M_2) = \frac{4\pi^2}{GP^2}(r_1 + r_2)^3$$

$$M_\odot = \frac{4\pi^2}{G(1\,yr)^2}(AU)^3$$

Divide the two equations

$$\frac{M_1 + M_2}{M_\odot} = \left(\frac{(r_1 + r_2)^3_{AU}}{P^2_{yr}}\right)$$

$$\frac{M_1}{M_2} = \frac{r_2}{r_1} \quad \text{or} \quad \frac{M_1}{M_2} = \frac{v_2}{v_1}$$

If you know r_1, r_2, or v_1, v_2, and P you can solve for the two masses.

GETTING STELLAR MASSES #1

For visual binaries measure:

- Period
- Separation
- Ratio of radii of orbits

Example: Sirius A and B

~~Maximum total~~ Average separation - 7.5"

The actual separation varies from 3 to 11 arc seconds and we are looking nearly face-on

Distance (parallax) - 2.67 pc

Sirius B twice as far from center of mass as Sirius A

Period - 50 yr

Calculation

P = 50 y

Separation in AU = d(pc) x separation in seconds of arc (follows from definition of pc and $s = r\theta$ with θ in radians.

1 pc = 206265 AU
1 radian = 206265 arc sec

$$\theta_{radian} = \frac{\theta_{arc\ sec}}{206265}$$

Separation = $(r_A + r_B) = (7.5)(2.67) = 20$ AU

For P in years and M in solar masses

$$\frac{M_A + M_B}{M_\odot} P^2(\text{yr}) = A^3(\text{AU})$$

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)}(total\ separation)^3$$

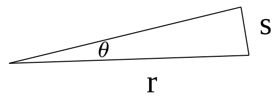
$$(Total\ M)(P^2) \propto (separation)^3$$

and so $M_A + M_B = 20^3/50^2 = 3.2\ M_\odot$, and since $M_A/M_B = r_B/r_A = 2$, the individual masses are

$$M_A = 2.13\ M_\odot$$

$$M_B = 1.07\ M_\odot$$

and since you can measure the angle of inclination of the orbit, you get the actual masses.



$$s \text{ (in pc)} = r \text{ (in pc)}\ \theta \text{ (in radians)}$$

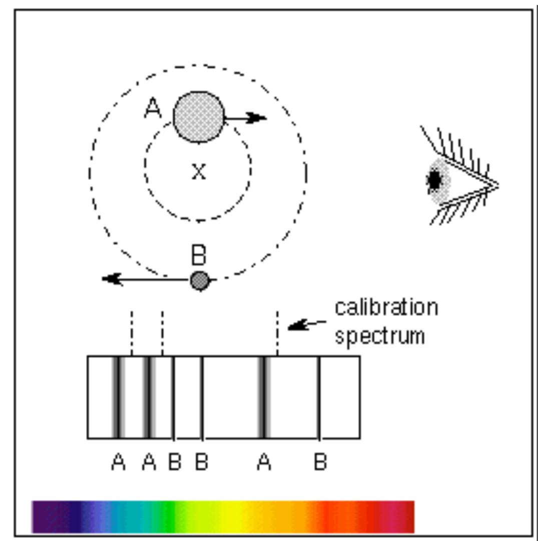
$$s \text{ (in AU)} = r \text{ (in AU)}\ \theta \text{ (in radians)}$$

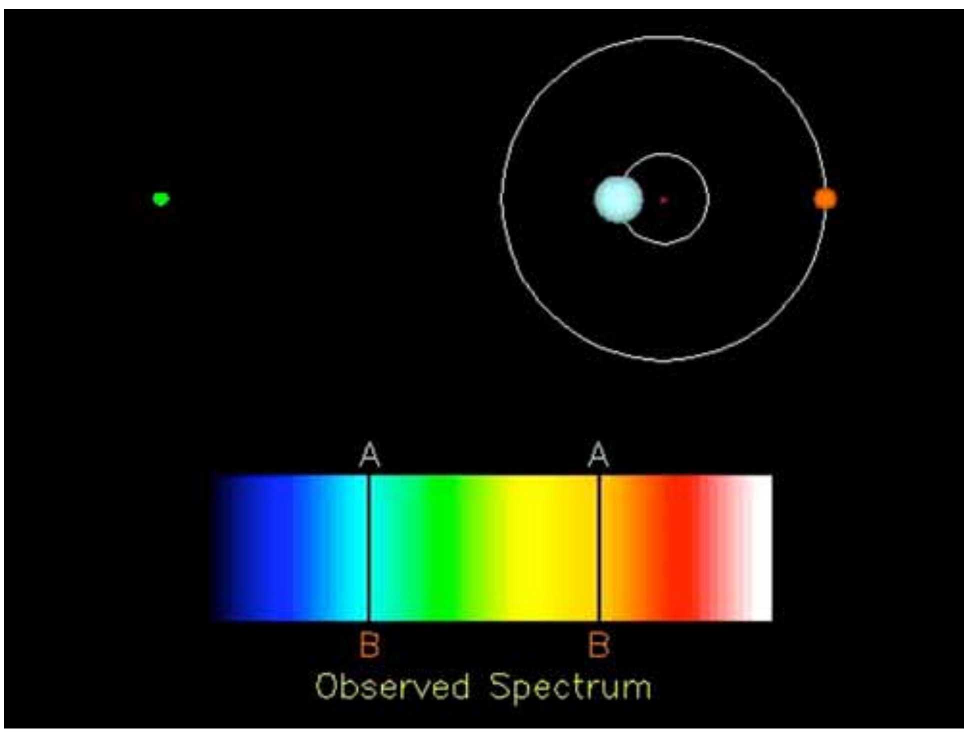
$$r \text{ (in AU)} = r \text{ (in pc)} \left(\frac{\text{number AU}}{1 \text{ pc}} \right)$$

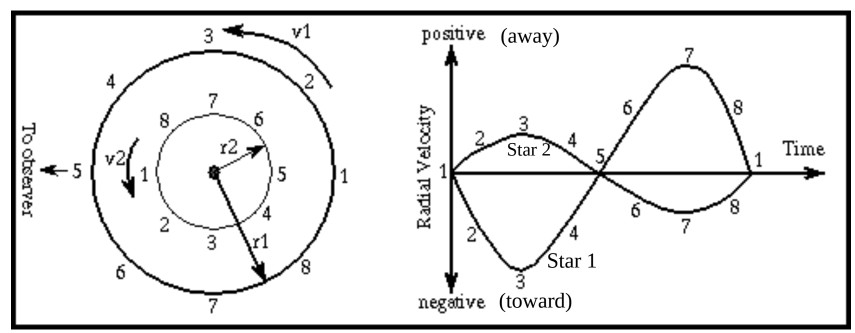
$$\theta \text{ in radians} = \theta \text{ (in arc sec)} \left(\frac{1 \text{ radian}}{\text{number arc sec}} \right)$$

$$s \text{ in AU} = r \text{ (in pc)} \left(\frac{\text{number AU}}{1 \text{ pc}} \right) \theta \text{ (in arc sec)} \left(\frac{1 \text{ radian}}{\text{number arc sec}} \right)$$

Spectroscopic Binaries







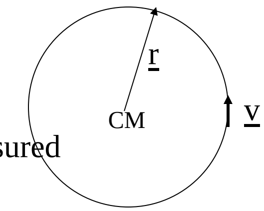
Complication:

The viewing angle

GETTING STELLAR MASSES #2

For spectroscopic binaries measure:

- Period
- Velocity of each star
- Inclination will be unknown so mass measured will be a lower bound (TBD)



CALCULATION

$$P = \frac{2\pi r}{v}$$

First get r_1 and r_2 from v_1 and v_2

$$r_i = \frac{v_i P}{2\pi}$$

Example:

$v_1 = 75 \text{ km s}^{-1}$ $\quad v_2 = 25 \text{ km s}^{-1}$

P= 17.5 days

$$A = r_1 + r_2 = \frac{P}{2\pi}(v_1 + v_2)$$
$$= \left[\frac{17.5\ \text{day}}{(2)(3.14)}\right]\left[\frac{8.64 \times 10^4\ \text{sec}}{1\ \text{day}}\right]\left[100\frac{\text{km}}{\text{sec}}\right]\left[\frac{10^5\ \text{cm}}{\text{km}}\right]\left[\frac{AU}{1.50 \times 10^{13}\ \text{cm}}\right]$$
$$= 0.16\ \text{AU}$$

$$P = \frac{2\pi r}{v} \Rightarrow r = \frac{Pv}{2\pi}$$

$$P = 17.5\,\text{d}\left(\frac{1\ \text{yr}}{365.25\ \text{d}}\right) = 0.0479\ \text{yr}$$

and can now solve as before

$$M_1 + M_2 = \frac{(0.16)^3}{(0.0479)^2} = \frac{A^3}{P^2} = 1.8\ M_\odot$$

and since $M_1/M_2 = v_2/v_1 = 1/3$, $M_1 = 0.45\ M_\odot$ and $M_2 = 1.35\ M_\odot$.

Note - the bigger the speeds measured for a given P the bigger the masses

Complication – The Inclination Angle

Let i be the angle of the observer relative to the rotation axis, i.e., i = 0 if we re along the axis.

Only if i = 90 degrees do we measure the full velocity.

Measure v Sin i which is a lower bound to v.

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)}(r_1 + r_2)^3$$

$$r_i = \frac{P v_i}{2\pi}$$

but measure $\tilde{v} = v \operatorname{Sin} i$, so we end up measuring $\tilde{r} = r \operatorname{Sin} i$ and calculate

$$\tilde{M}_1 + \tilde{M}_2 = \frac{4\pi^2}{GP^2}\left(\frac{\tilde{v}_1 + \tilde{v}_2}{2\pi}\right)^3 P^3$$

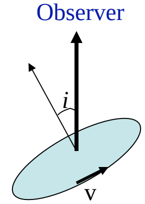
when the actual mass is

$$M_1 + M_2 = \frac{4\pi^2}{GP^2}\left(\frac{v_2 + v_2}{2\pi}\right)^3 P^3$$

hence the measurement gives a low bound on the actual mass

$$(\tilde{M}_1 + \tilde{M}_2) = (M_1 + M_2)\operatorname{Sin}^3 i$$

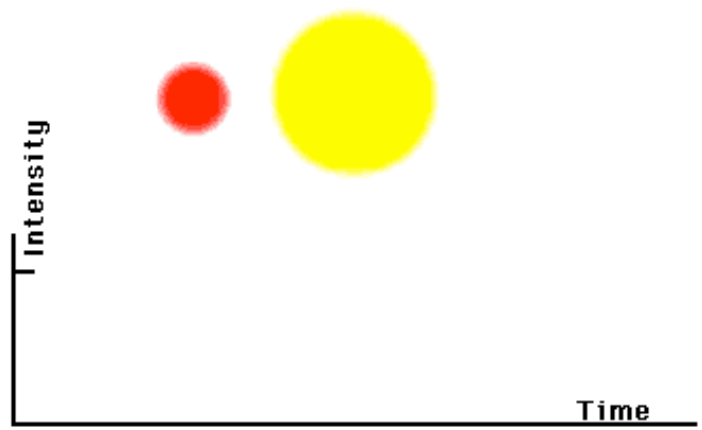
Since Sin i < 1, the measurement is a lower bound.



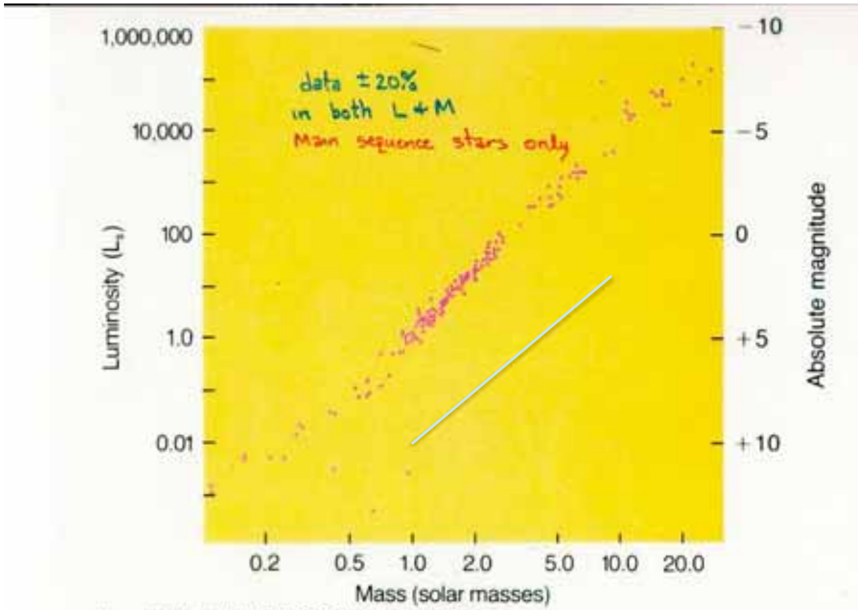
$$\langle \operatorname{Sin}^3 i \rangle = 0.59$$

But we tend to discover more edge on binaries so 2/3 is perhaps better

Eclipsing Binary



For an eclipsing binary you know you are viewing the system in the plane of the orbit. I.e., Sin i = 1



67. Abell/Morrison/Wolff: EXPLORATION OF THE UNIVERSE, 6/E

STELLAR LIFETIMES

On the main sequence:

- Luminosity determined by mass - $L \propto M^n$ $n \approx 3$ to 4
- Say star has a total energy reservoir proportional to its mass (as in a certain fraction to be burned by nuclear reactions)

$$E_{tot} = fM$$

Then the lifetime on the main sequence will be shorter for stars of higher mass;

$$\tau_{MS} \propto \frac{M^n}{fM} \quad \frac{fM}{M^n} \qquad n = 3$$

$$\tau_{MS} \approx 10^{10}\,\mathrm{yr}(M_\odot/M)^2$$

This explains some important features of the HR-diagram.