Spectroscopy, the Doppler Shift and Masses of Binary Stars

http://apod.nasa.gov/apod/astropix.html
Doppler Shift

At each point the emitter is at the center of a circular wavefront extending out from its present location.

Wavelength is shorter; frequency is higher

Wavelength is longer; frequency is lower
The Doppler Shift

\[ \lambda_{obs} = \lambda_{emit} + \lambda_{emit} \left( \frac{v}{c} \right) \]

since \( v = \frac{c}{\lambda} \)

\( t = \frac{v}{v} \), so, in one cycle, go a distance

\[ vt = v/v = \frac{V}{c} \lambda_{cm} \]
The Doppler Shift

\[ \lambda_{obs} = \lambda_{emit} \left(1 + \frac{v}{c}\right) \quad \text{if moving away from you with speed } v \]

\[ \lambda_{obs} = \lambda_{emit} \left(1 - \frac{v}{c}\right) \quad \text{if moving toward you with speed } v \]

\[ \Delta \lambda = \lambda_{obs} - \lambda_{emit} \]

\[ = \lambda_{emit} \left(1 \pm \frac{v}{c} - 1\right) \]

\[ \frac{\Delta \lambda}{\lambda_{emit}} = \pm \frac{v}{c} \]

if \( v \) is away from you \( \Delta \lambda > 0 \)

if \( v \) is toward you \( \Delta \lambda < 0 \)

This formula can only be used when \( v \ll c \)

Otherwise, without proof,

\[ \lambda_{obs} = \lambda_{emit} \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2} \]
Doppler Shift:

Note – different from a cosmological red shift!
Astronomical Examples of Doppler Shift

• A star or galaxy moves towards you or away from you (can’t measure transverse motion)

• Motion of stars in a binary system

• Thermal motion in a hot gas

• Rotation of a star
E.g. A H atom in a star is moving away from you at 
$3.0 \times 10^7$ cm s$^{-1}$ = 0.001 times c.

At what wavelength will you see H$_\alpha$?

$\lambda_{obs} = 6562.8 \times (1+ 0.001) = 6569.4$ A

Note that the Doppler shift only measures that part of the velocity that is directed towards or away from you.
A binary star pair

Star B spectrum at time 1: approaching, therefore blueshifted

Star B spectrum at time 2: receding, therefore redshifted
Thermal Line Width

In a gas with some temperature $T$ atoms will be moving around in random directions. Their average speed will depend upon the temperature. Recall that the definition of temperature, $T$, is

$$\frac{1}{2} m_{\text{atom}} \langle v^2 \rangle = \frac{3}{2} k T$$

where $k=1.38 \times 10^{-16}$ erg K$^{-1}$

Here $\langle \rangle$ means "average". Some atoms will be moving faster than the average, others hardly at all. Some will be moving towards you, others away, still others across your line of sight.
Thermal Line Broadening

The full range of wavelengths, hence the width of the spectral line will be

\[ \frac{\Delta \lambda}{\lambda} = 2 \frac{v_{\text{average}}}{c} = \frac{2}{c} \sqrt{\frac{3kT}{m_{\text{atom}}}} \]

The mass of an atom is the mass of a neutron or proton (they are about the same) times the total number of both in the nucleus, this is an integer "A".

\[ \frac{\Delta \lambda}{\lambda} = 2 \left( \frac{3 \times 1.38 \times 10^{-16} (T)}{1.66 \times 10^{-24} (A)} \right)^{1/2} \left( \frac{1}{2.99 \times 10^{10}} \right) \]

\[ \frac{\Delta \lambda}{\lambda} = 1.05 \times 10^{-6} \sqrt{\frac{T}{A}} \]

where \( T \) is in K

A = 1 for hydrogen
4 for helium
12 for carbon
16 for oxygen
etc.
Full width = $\Delta \lambda = 1.05 \times 10^{-6} \sqrt{\frac{T(\text{in K})}{A}} \lambda$

Eq.

Hα @ 5000 K

$A = 1$, $T = 5000$, $\lambda = 6563 \text{ Å}$

$\Delta \lambda = (6563)(1.05 \times 10^{-6}) \left( \frac{5000}{1} \right)^{1/2} \text{ Å}$

= 0.49 Å

Can use this to measure the temperature. (again)
ROTATION

A spectral line from star A

Narrow spectral lines indicate that star A rotates slowly.

Slow rotation

This light is slightly blueshifted

This light is slightly redshifted

A spectral line from star B

Wide spectral lines indicate that star B rotates rapidly.

Fast rotation

This light is greatly blueshifted

This light is greatly redshifted

To Earth

Approaching side

Receding side
Rotation

If look at * in plane of equator:

\[ v_{\text{away}} = v_{\text{rot}} = \frac{2\pi R}{P} \quad P = \text{period} \]

\[ v_{\text{rot}} = -\frac{2\pi R}{P} \]

For an absorption line:

\[ \frac{\Delta \lambda}{\lambda} = 2 \frac{v_{\text{rot}}}{c} = \frac{4\pi R}{P} \]

Can be used to get the rotational speed and period (if we know \( R \)).
Note: Potential complications:

1) Star may have both thermal and rotational broadening

2) May see the star at some other angle than in its equatorial plane.

Example: $H_\alpha$ in a star with equatorial rotational speed

$100 \text{ km/s} = 10^7 \text{ cm/s}$

Full width $= \Delta \lambda = 2 \left( \frac{v}{c} \right) \lambda$

$= (2)(6563) \left( \frac{10^7}{3 \times 10^{10}} \right) = 4.4 \text{ A}$
Average rotational velocities (main sequence stars)

<table>
<thead>
<tr>
<th>Stellar Class</th>
<th>$v_{equator}$ (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O5</td>
<td>190</td>
</tr>
<tr>
<td>B0</td>
<td>200</td>
</tr>
<tr>
<td>B5</td>
<td>210</td>
</tr>
<tr>
<td>A0</td>
<td>190</td>
</tr>
<tr>
<td>A5</td>
<td>160</td>
</tr>
<tr>
<td>F0</td>
<td>95</td>
</tr>
<tr>
<td>F5</td>
<td>25</td>
</tr>
<tr>
<td>G0</td>
<td>12</td>
</tr>
</tbody>
</table>

Stellar winds and magnetic torques are thought to be involved in slowing the rotation of stars of class G, K, and M.

Stars hotter than F5 have stable surfaces and perhaps low magnetic fields.

The sun rotates at 2 km/s

Red giant stars rotate very slowly. Single white dwarfs in hours to days. Neutron stars may rotate in milliseconds.
3 sources of spectral line broadening

1) Pressure or “Stark” broadening (Pressure)

2) Thermal broadening (Temperature)

3) Rotational broadening ($\omega$, rotation rate)
SPECTROSCOPY: WHAT WE CAN LEARN

1) Temperature

   Ionization stages that are present

   Thermal line broadening

   Wien's Law ($\lambda_{\text{max}} \propto 1/T$)

2) Radius

   Blackbody $L = 4\pi R^2 \sigma T^4$

3) Rotation rate

   Spectral line widths
4) **Composition**

From a detailed analysis of what lines are present and their strengths

5) **Surface pressure**

Also from line broadening. Is the star a white dwarf or a red giant or a main sequence star

6) **Velocity towards or away from us**

Is the star or galaxy approaching us or receding?

7) **Binary membership, period, and velocity**

From periodic Doppler shifts in spectral lines
8) Magnetic fields

From Zeeman splitting

9) Expansion speeds in stellar winds and explosions

Supernovae, novae, planetary nebulae

10) From 21 cm - rotation rates of galaxies. Distribution of neutral hydrogen in galaxies. Sun’s motion in the Milky Way.
Hyperfine Splitting  The 21 cm Line

(a) Less tightly bound

(b) More tightly bound

The 21-cm emission from hydrogen atoms.
21 cm (radio)

\[ \lambda = 21 \text{ cm} \]
\[ \nu = 1.4 \times 10^9 \text{ Hz} \]
\[ h\nu = (6.63 \times 10^{-27})(1.4 \times 10^9) = 9.5 \times 10^{-18} \text{ erg} \]
\[ = 5.6 \times 10^{-6} \text{ eV} \]

Must have neutral H I

Emission collisionally excited

Lifetime of atom in excited state about $10^7$ yr

Galaxy is transparent to 21 cm
Merits:

• Hydrogen is the most abundant element in the universe and a lot of it is in neutral atoms - H I

• It is not so difficult to build big radio telescopes

• The earth’s atmosphere is transparent at 21 cm

Arecibo - 305 m radio telescope - Puerto Rico
Getting Masses in Binary Systems
Binary and Multiple Stars (about half of all stars)

Beta-Cygnus (also known as Alberio) 
Separation 34.6”. Magnitudes 3.0 and 5.3. Yellow and blue. 380 ly away. Bound?
P > 75000 y. The brighter yellow component is also a (close) binary. P ~ 100 yr.

Alpha Ursa Minoris (Polaris) 
Separation 18.3”. Magnitudes 2.0 and 9.0. Now known to be a triple. Separation ~2000 AU for distant pair.

When the star system was born it apparently had too much angular momentum to end up as a single star.
1.2 Msun Polaris Ab
Type F6 - V
4.5 Msun Polaris A
Cepheid

Period 30 yr

Polaris B is
F3 - V
Epsilon Lyrae 1 & 2
The Double Double

October 16th 2009 • 21:40 - 22:20 BST
Celestron Omni XLT 120 • 5" f/8
6mm Celestron Omni Plössl • 166X / 0.3° FOV
RA/DE: 18h44m40s / +39°40′53″
Conditions: Clear but Hazy with Yard Lights
Antoniadi Scale: 2/5
Sketch by Ewan Bryce © 2009
Epsilon Lyra – a double double.
The stars on the left are separated by 2.3" about 140 AU; those on the right by 2.6". The two pairs are separated by about 208" (13,000 AU separation, 0.16 ly between the two pairs, all about 162 ly distant). Each pair would be about as bright as the quarter moon viewed from the other.
Castor A and B complete an orbit every 400 years. In their elliptical orbits their separation varies from 1.8” to 6.5”. The mean separation is 8 billion miles. Each star is actually a double with period only a few days (not resolvable with a telescope). There is actually a “C” component that orbits A+B with a period of about 10,000 years (distance 11,000 AU).

Castor C is also a binary. 6 stars in total
An "optical double"
Stars not really bound.

5000 years for Mizar A and B to complete an orbit

6 months

20.5 d

Figure 10.3. Alcor and Mizar form an "optical double," a chance near projection on the sky which is resolvable by the naked eye. Through a telescope, Mizar is a visual binary, two gravitationally bound stars which orbit about one another at an observable distance apart. Furthermore, Mizar A and Mizar B are each a spectroscopic binary, a pair of stars whose orbital motions can be observed by the periodic variations of the Doppler shifts of the spectral lines.
For constant total separation, 20 AU, vary the masses.
Circular Orbit – Unequal masses

\[ \frac{M_1}{M_2} = 3.6; \quad e = 0.0 \]
Two stars of similar mass but eccentric orbits
Two stars of unequal mass and an eccentric orbit
E.g. A binary consisting of
a F0v and M0v star

http://www.astronomy.ohio-state.edu/~pogge/Ast162/Movies/ - visbin

M1/M2=3.6; e=0.4
Aside

The actual separation between the stars is obviously not constant in the general case shown.

The separation at closest approach is the sum of the semi-major axes “a” times $(1-e)$ where $e$ is the eccentricity. At the most distant point the separation is “a” times $(1+e)$.

For circular orbits $e = 0$ and the separation is constant.
Period = 50.1 years
distance to c/m 6.4 (A) and 13.4 (B) AU
Some things to note:

• The system has only one period. The time for star A to go round B is the same as for B to go round A

• A line connecting the centers of A and B always passes through the center of mass of the system

• The orbits of the two stars are similar ellipses with the center of mass at a focal point for both ellipses

• The distance from the center of mass to the star times the mass of each star is a constant. (next page)
ASSUME CIRCULAR ORBITS

\[
\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} = \text{Period}
\]

\[
\frac{m_1 r_1^2 v_1^2}{v_1^2 r_1^2} = \frac{m_2 v_2^2}{v_2^2 r_2^2}
\]

More massive star is closer to the center of mass and moves slower.

\[
\frac{r_1}{r_2} = \frac{m_2}{m_1}
\]

both stars feel the same gravitational attraction and thus both have the same centrifugal force
Circular Orbit – Unequal masses

$M_1/M_2 = 3.6; e = 0.0$
ASSUME CIRCULAR ORBITS

Both stars feel the same gravitational attraction and thus both have the same centrifugal force:

\[
\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2} = \frac{G m_1 m_2}{(r_1 + r_2)^2}
\]

\[
\frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} = \text{Period}
\]

\[
\frac{m_1 r_1^2 v_1^2}{r_1^2} = \frac{m_2 v_2^2}{r_2^2}
\]

More massive star is closer to the center of mass and moves slower.

\[
\frac{r_1}{r_2} = \frac{m_2}{m_1}
\]
For simplicity, assume circular motion

\[ \frac{r_1}{r_2} = \frac{v_1}{v_2} \]

\[ m_1 \text{ goes around "x" in period } P \]
\[ m_2 \text{ also goes around "x" in period } P \]

\[ \frac{2\pi r_1}{P} = v_1 \quad \frac{2\pi r_2}{P} = v_2 \]

\[ P = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} \]

\[ r_1 v_2 = r_2 v_1 \quad \frac{r_1}{r_2} = \frac{v_1}{v_2} \]

So

\[ \frac{m_2}{m_1} = \frac{v_1}{v_2} \]
Motion of the sun because of Jupiter

\[ m_1r_1 = m_2r_2 \]
\[ M_\odot d_\odot = M_J d_J \]

\[ d_\odot = \frac{M_J}{M_\odot} \cdot d_J \]

\[ = (9.95 \times 10^{-4})(7.80 \times 10^{13}) \]

\[ = 7.45 \times 10^{10} \text{ cm} \]

Can ignore the influence of the other planets.

\[ d_\odot = \text{radius of sun's orbit around center of mass} \]
\[ d_J = \text{Jupiter's orbital radius} \]
\[ = 5.20 \text{ AU} \]
\[ = 7.80 \times 10^{13} \text{ cm} \]
\[ M_J = 1.90 \times 10^{30} \text{ gm} \]
\[ = 9.55 \times 10^{-4} M_\odot \]

\[ P = 11.86 \text{ years} \]
Doppler shift

\[ P_J = \text{period Jupiter} = 11.86 \, \text{y} = 3.75 \times 10^8 \, \text{s} \]

\[ v_0 = \frac{2\pi d_o}{P_J} = \frac{(2\pi)(7.45 \times 10^{10} \, \text{cm})}{3.75 \times 10^8 \, \text{s}} \]

\[ = 1.25 \times 10^3 \, \text{cm/s} \]

\[ = 12.5 \, \text{m/s} \]

\[ \frac{V}{c} = \frac{1.25 \times 10^3 \, \text{cm/s}}{2.99 \times 10^{10} \, \text{cm/s}} \approx 4.18 \times 10^{-8} \]

\[ \frac{\Delta \lambda}{\lambda} = \text{(some text)} \]

\[ \Delta \lambda = 2.74 \times 10^{-4} \, \text{Å} \]

\[ \text{small compared to thermal + rotational broadening} \]
As of today – 843 extra solar planets in 665 stellar systems and the number is growing rapidly.

Many were detected by their Doppler shifts. Many more by the “transits” they produce as they cross the stellar disk.

http://exoplanet.eu/catalog.php
Note: “wobble” of the star is bigger if the planet is bigger or closer to the star (hence has a shorter period).

Detection of planets orbiting Sun-like stars
by Dr. Geoff Marcy
Slide #6

Motion of sun due to Jupiter

12.5 m/s
11.86 years
KEPLER’S THIRD LAW FOR BINARIES

\[
\frac{GM_1M_2}{(r_1+r_2)^2} = \frac{M_1v_1^2}{r_1} + \frac{GM_1M_2}{(r_1+r_2)^2} = \frac{M_2v_2^2}{r_2}
\]

\[
\frac{G(M_1+M_2)}{(r_1+r_2)^2} = \frac{v_1^2}{r_1} + \frac{v_2^2}{r_2} = \frac{4\pi^2r_1^2}{P^2r_1} + \frac{4\pi^2r_2^2}{P^2r_2}
\]

\[
= \frac{4\pi^2}{P^2} (r_1+r_2)
\]

\[P^2 = K (r_1+r_2)^3 \]

\[K = \frac{4\pi^2}{G(M_1+M_2)} \]

i.e., just like before but

\[M \rightarrow M_1+M_2 \quad R \rightarrow r_1+r_2\]
KEPLER’S THIRD LAW FOR BINARIES

\[
\frac{GM_1 M_2}{(r_1 + r_2)^2} = \frac{M_N v_1^2}{r_1} \]
\[
\frac{GM_1 M_2}{(r_1 + r_2)^2} = \frac{M_N v_2^2}{r_2}
\]

\[
\frac{G(M_1 + M_2)}{(r_1 + r_2)^2} = \frac{v_1^2}{r_1} + \frac{v_2^2}{r_2} = \frac{4\pi^2 r_1^2}{P^2 r_1} + \frac{4\pi^2 r_2^2}{P^2 r_2}
\]

\[
= \frac{4\pi^2}{P^2} (r_1 + r_2)
\]

\[
P^2 = K (r_1 + r_2)^3 \quad K = \frac{4\pi^2}{G(M_1 + M_2)}
\]

i.e., just like before but

\[
M \rightarrow M_1 + M_2 \quad R \rightarrow r_1 + r_2
\]
\[(M_1 + M_2) = \frac{4\pi^2}{GP^2} (r_1 + r_2)^3\]

\[M_\odot = \frac{4\pi^2}{G(1 \text{ yr})^2} (AU)^3\]

Divide the two equations
\[
\frac{M_1 + M_2}{M_\odot} = \left( \frac{(r_1 + r_2)^3}{(A U)} \right) \frac{P_{yr}^2}{P_{yr}}
\]

\[
\frac{M_1}{M_2} = \frac{r_2}{r_1} \quad \text{or} \quad \frac{M_1}{M_2} = \frac{v_2}{v_1}
\]

If you know \( r_1, r_2, \) or \( v_1, v_2, \) and \( P \) you can solve for the two masses.
GETTING STELLAR MASSES #1

For visual binaries measure:

- Period
- Separation
- Ratio of radii of orbits

Example: Sirius A and B

Average

Maximum total separation - 7.5"

Distance (parallax) - 2.67 pc

Sirius B twice as far from center of mass as Sirius A

Period - 50 yr

The actual separation varies from 3 to 11 arc seconds and we are looking nearly face-on
Calculation

P = 50 y

Separation in AU = d(pc) x separation in seconds of arc (follows from definition of pc and s = rθ with θ in radians).

Separation = (r_A + r_B) = (7.5)(2.67) = 20 AU

For P in years and M in solar masses

\[ \frac{M_A + M_B}{M_\odot} P^2(\text{yr}) = A^3(\text{AU}) \]

and so \( M_A + M_B = 20^3/50^2 = 3.2 \, M_\odot \), and since \( M_A/M_B = r_B/r_A = 2 \), the individual masses are

\[ M_A = 2.13 \, M_\odot \]

\[ M_B = 1.07 \, M_\odot \]

1 pc = 206265 AU
1 radian = 206265 arc sec

\[ \theta_{\text{radian}} = \frac{\theta_{\text{arc sec}}}{206265} \]

\[ P^2 = \frac{4\pi^2}{G(M_1 + M_2)} \text{ (total separation)}^3 \]

\( \text{(Total M)} (P^2) \propto (\text{separation})^3 \)

and since you can measure the angle of inclination of the orbit, you get the actual masses.
\[ s \text{ (in pc)} = r \text{ (in pc)} \theta \text{ (in radians)} \]
\[ s \text{ (in AU)} = r \text{ (in AU)} \theta \text{ (in radians)} \]
\[ r \text{ (in AU)} = r \text{ (in pc)} \left( \frac{\text{number AU}}{1 \text{ pc}} \right) \]
\[ \theta \text{ in radians} = \theta \text{ (in arc sec)} \left( \frac{1 \text{ radian}}{\text{number arc sec}} \right) \]
\[ s \text{ in AU} = r \text{ (in pc)} \left( \frac{\text{number AU}}{1 \text{ pc}} \right) \theta \text{ (in arc sec)} \left( \frac{1 \text{ radian}}{\text{number arc sec}} \right) \]
Spectroscopic Binaries
Complication:

The viewing angle
GETTING STELLAR MASSES #2

For spectroscopic binaries measure:

- Period
- Velocity of each star
- Inclination will be unknown so mass measured will be a lower bound (TBD)

CALCULATION

First get $r_1$ and $r_2$ from $v_1$ and $v_2$

$$r_i = \frac{v_i P}{2\pi}$$

Example:

$v_1 = 75 \text{ km s}^{-1}$ \quad $v_2 = 25 \text{ km s}^{-1}$

$P = 17.5 \text{ days}$
\[ R = r_1 + r_2 \]
\[ = \frac{P}{2\pi} (v_1 + v_2) \]
\[ = \left[ \frac{17.5 \text{ day}}{(2)(3.14)} \right] \left[ \frac{8.64 \times 10^4 \text{ s/yr}}{1 \text{ day}} \right] \left[ \frac{100 \text{ km/s}}{1 \text{ day}} \right] \]
\[ \left[ \frac{10^5 \text{ cm}}{1 \text{ km}} \right] \left[ \frac{AU}{k \text{ km}} \right] \left[ \frac{1.50 \times 10^{13} \text{ cm}}{AU} \right] \]
\[ = 0.16 \text{ AU} \]

\[ P = 17.5 \text{ d} \left( \frac{1 \text{ yr}}{365.25 \text{ d}} \right) \]
\[ = 0.0479 \text{ yr} \]

and can now solve as before

\[ M_1 + M_2 = \frac{(0.16)^3}{(0.0479)^2} = \frac{A^3}{p^2} \]
\[ = 1.8 M_\odot \]

and since \( M_1 / M_2 = v_2 / v_1 = 1/3, M_1 = 0.45 M_\odot \)

and \( M_2 = 1.35 M_\odot \).

Note - the bigger the speeds measured for a given \( P \) the bigger the masses
Complication – The Inclination Angle

Let \( i \) be the angle of the observer relative to the rotation axis, i.e., \( i = 0 \) if we are along the axis.

Measure \( v \text{ Sin } i \) which is a lower bound to \( v \).

\[
P^2 = \frac{4\pi^2}{G(M_1 + M_2)}(r_1 + r_2)^3
\]

\[
r_i = \frac{P v_i}{2\pi}
\]

but measure \( \tilde{v} = v \text{ Sin } i \), so we end up measuring \( \tilde{r} = r \text{ Sin } i \) and calculate

\[
\tilde{M}_1 + \tilde{M}_2 = \frac{4\pi^2}{GP^2} \left( \frac{\tilde{v}_1 + \tilde{v}_2}{2\pi} \right)^3 P^3
\]

when the actual mass is

\[
M_1 + M_2 = \frac{4\pi^2}{GP^2} \left( \frac{v_2 + v_2}{2\pi} \right)^3 P^3
\]

hence the measurement gives a low bound on the actual mass

\[
(\tilde{M}_1 + \tilde{M}_2) = (M_1 + M_2) \text{ Sin}^3 i
\]

Since \( \text{ Sin } i < 1 \), the measurement is a lower bound.

Only if \( i = 90 \) degrees do we measure the full velocity.

\[\langle \text{Sin}^3 i \rangle = 0.59\]

But we tend to discover more edge on binaries so 2/3 is perhaps better
For an eclipsing binary you know you are viewing the system in the plane of the orbit. I.e., $\sin i = 1$
data ± 20% in both L + M
Main sequence stars only
STELLAR LIFETIMES

On the main sequence:

- Luminosity determined by mass - \( L \propto M^n \) \( n \approx 3 \) to 4
- Say star has a total energy reservoir proportional to its mass (as in a certain fraction to be burned by nuclear reactions)

\[ E_{\text{tot}} = fM \]

Then the lifetime on the main sequence will be shorter for stars of higher mass:

\[ \tau_{MS} \propto \frac{fM}{M^n} \quad n = 3 \]

\[ \tau_{MS} \approx 10^{10} \text{ yr} \left( \frac{M_\odot}{M} \right)^2 \]

This explains some important features of the HR-diagram.