

# *Star Formation and Pressure*

<http://apod.nasa.gov/apod/astropix.html>

# PHASES OF THE INTERSTELLAR MEDIUM

Component	Fractional volume	Scale Height (pc)	Temperature	Density	State of Hydrogen	Observational Technique
Molecular Clouds	< 1%	70	10 - 20	$10^2 - 10^6$	H <sub>2</sub>	Radio and infrared (molecules)
Cold Neutral Medium (CNM)	1 - 5%	100 - 300	50 - 100	20 - 50	H I	21 cm
Warm Neutral Medium (WNM)	10 - 20%	300 - 400	5000-8000	0.2 - 0.5	H I	21 cm
Warm Ionized Medium (WIM)	20 - 50%	1000	6000 - 12000	0.2 - 0.5	H II	H <sub>α</sub> pulsar (n <sub>e</sub> )
H II Regions	<1%	70	8000	$10^2 - 10^4$	H II	H <sub>α</sub>
Coronal Gas (Hot Ionized Medium (HIM))	30 - 70%	1000 - 3000	$10^6 - 10^7$	$10^{-4} - 10^{-2}$	H II metals also ionized	x-ray ultraviolet

*In which of these components can star formation take place?*

*A necessary condition is a region of gas that has greater gravitational binding energy than internal energy. (The force pulling the region together must be greater than the pressure pushing it apart.)*

*Since internal energy increases with the amount of mass that is present while binding energy increases as  $M^2$ , there is a critical mass that is bound.*

# The Jean's Mass

$$\Omega \approx \text{KE}$$

*Ignore factor of 2 in the Virial Theorem. The clouds we are envisioning have not reached equilibrium.*

$$\begin{aligned}\Omega &\approx \frac{3}{5} \frac{GM^2}{R} \approx (\text{Number of particles}) \left(\frac{3}{2} kT\right) \\ &\approx \frac{M}{m_H} \frac{3}{2} kT \quad (\text{if made of pure hydrogen}) \\ &= N_A M \frac{3}{2} kT \quad (N_A \text{ is Avogadro's Number, } 6.02 \times 10^{23})\end{aligned}$$

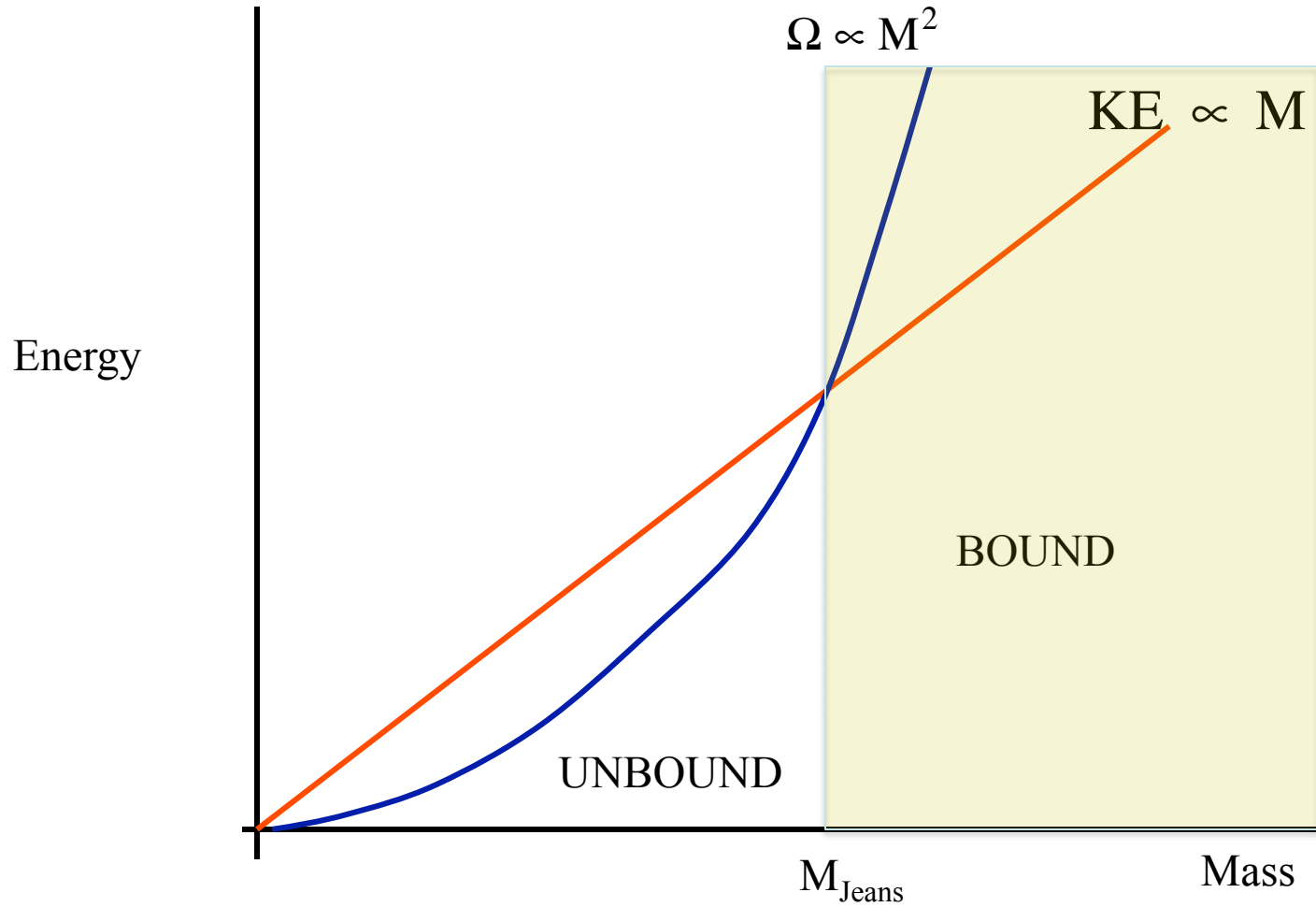
This can be solved for the "Jean's Mass",  $M_J$

$$\frac{3}{5} \frac{GM_J^2}{R} = \frac{3}{2} N_A M_J kT$$

$$M_J = \frac{5N_A kTR}{2G}$$

*Clouds of gas with radius  $R$  and temperature  $T$  that have a mass bigger than this are unstable to gravitational collapse*

For masses larger than the Jean's Mass gravitational binding energy exceeds internal energy



It is easier to measure densities and temperatures rather than radii, so the equation on the previous page can be transformed using

$$R = \left( \frac{3M}{4\pi\rho} \right)^{1/3} \quad \text{assume sphere, constant density} \quad M = \frac{4}{3}\pi R^3 \rho$$

previous page

$$M_J = \frac{5N_A kTR}{2G} = \frac{5N_A kT}{2G} \left( \frac{3M_J}{4\pi\rho} \right)^{1/3}$$

$$M_J^{2/3} = \frac{5N_A k}{2G} \left( \frac{3}{4\pi} \right)^{1/3} \left( \frac{T^3}{\rho} \right)^{1/3}$$

$$M_J = \left( \frac{5N_A k}{2G} \right)^{3/2} \left( \frac{3}{4\pi} \right)^{1/2} \left( \frac{T^3}{\rho} \right)^{1/2}$$

$$= 8.5 \times 10^{22} \text{ gm} \left( \frac{T^{3/2}}{\rho^{1/2}} \right) = 4.2 \times 10^{-11} \left( \frac{T^{3/2}}{\rho^{1/2}} \right) M_\odot$$

It is more frequent that one finds the density in this context expressed as atoms/cm<sup>3</sup> rather than gm/cm<sup>3</sup>.

If  $n = \rho N_A$  (actually true only for H I), then

$$M_J = 8.5 \times 10^{22} \frac{T^{3/2} N_A^{1/2}}{n^{1/2}} \text{ gm}$$

$$M_J = 34 \frac{T^{3/2}}{n^{1/2}} M_\odot$$

where  $n$  is the density in atoms cm<sup>-3</sup>.

By this criterion, only molecular clouds and possibly portions of the coldest neutral medium (depending on mass) are unstable to collapse.

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**Example:** Molecular cloud;  $T = 20 \text{ K}$ ,  $n = 10^4 \text{ atoms cm}^{-3}$

$$\begin{aligned} M_J &= 34 \frac{T^{3/2}}{n^{1/2}} \\ &= 34 \frac{(20)^{3/2}}{(10^4)^{1/2}} = 34 \frac{89.4}{100} \\ &= 30 M_\odot \end{aligned}$$

Any cloud with this temperature and density and a mass over 30 solar masses is unstable to collapse

## How long does the collapse take?

$$v_{esc} = \sqrt{\frac{2GM}{R}} \quad \tau_{ff} \approx \frac{R}{v_{esc}} = \sqrt{\frac{R^3}{2GM}}$$

but,  $\rho$ , the density, is given by

$$\rho = \frac{3M}{4\pi R^3} \Rightarrow \frac{R^3}{M} = \frac{3}{4\pi\rho}$$

so,

$$\tau_{ff} \approx \sqrt{\frac{3}{8\pi G\rho}} \approx 1000 \text{ seconds} / \sqrt{\rho}$$

*Denser regions  
collapse faster*

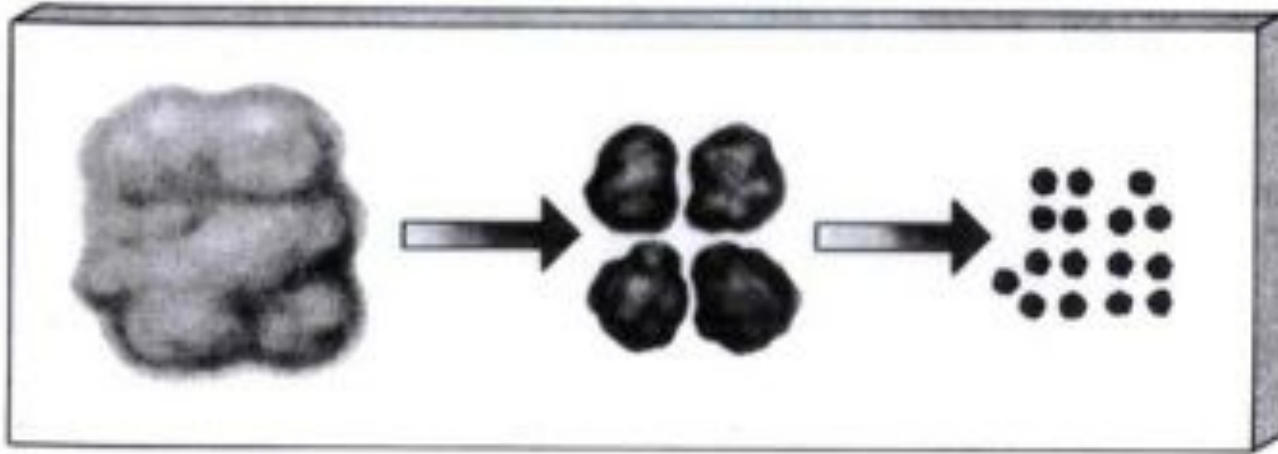
but  $\rho \approx n / N_A$ , so

$$\tau_{ff} \approx 10 \text{ million years} / \sqrt{n}$$

where  $n$  is the number of atoms per cubic cm.

1 million years if  $n = 10^4$  atoms/cm<sup>3</sup>

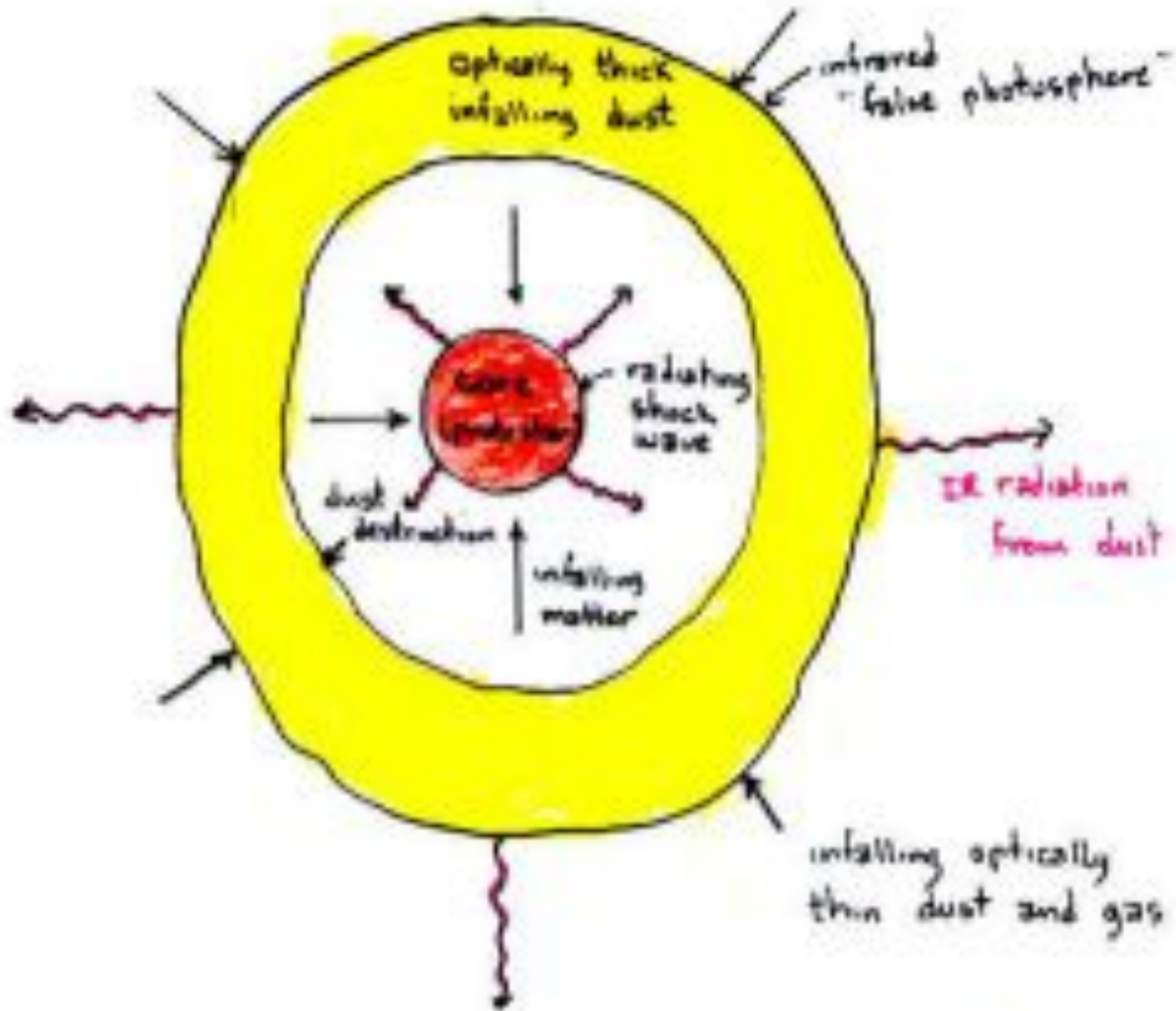
# Fragmentation



## Complications:

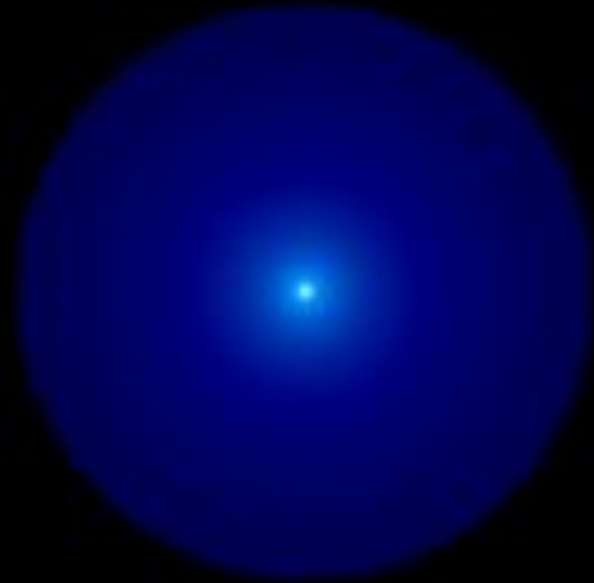
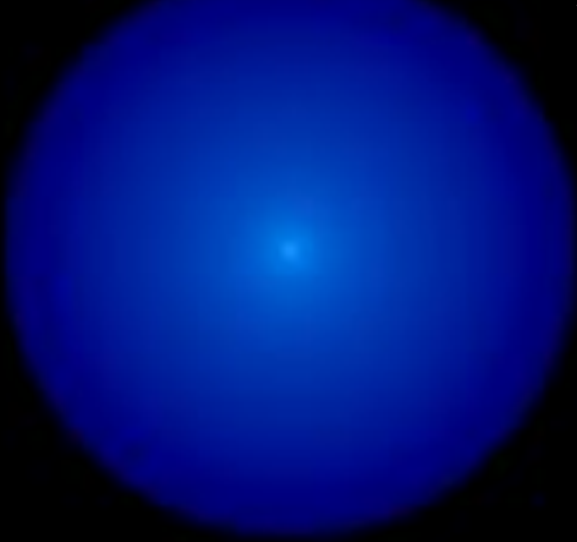
Rotation

Magnetic fields



Star formation is inefficient. Even of the collapsing gas only 10 – 20 % of the gas ends up in the star, and overall an even smaller fraction of the cloud collapses to protostars.

Formation of a massive binary



Krumholz et al (2000)

Collapse of a 500 solar mass cloud  
2.6 ly across 285,000 years  
Mathew Bate et al

UK Astrophysical  
Fluids Facility

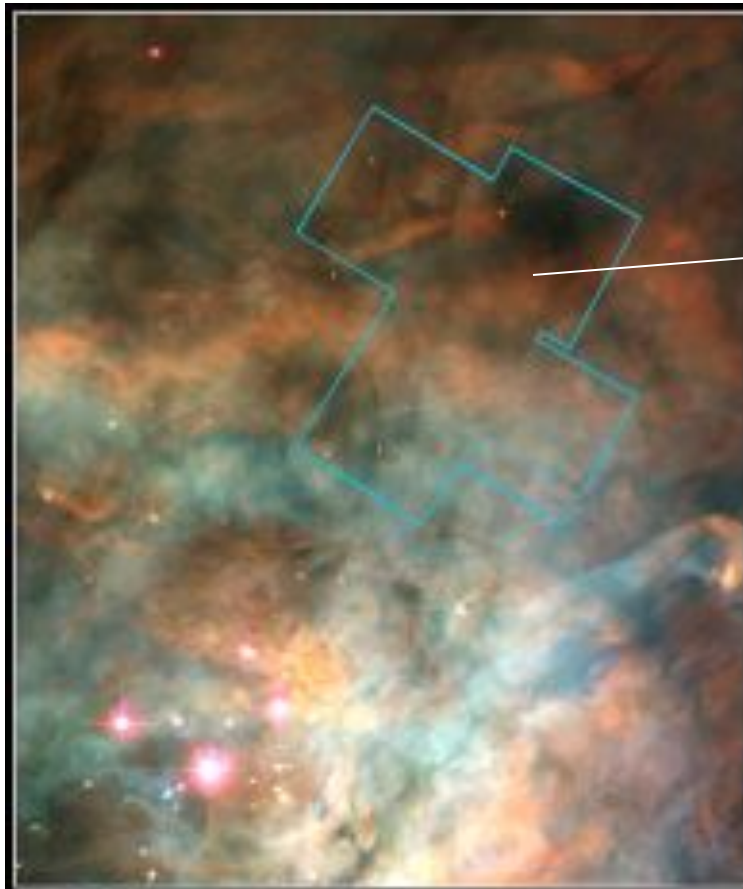


Matthew Bate



<http://www.astro.ex.ac.uk/people/mbate/Cluster/cluster3d.html>

# Power of observing in the infrared

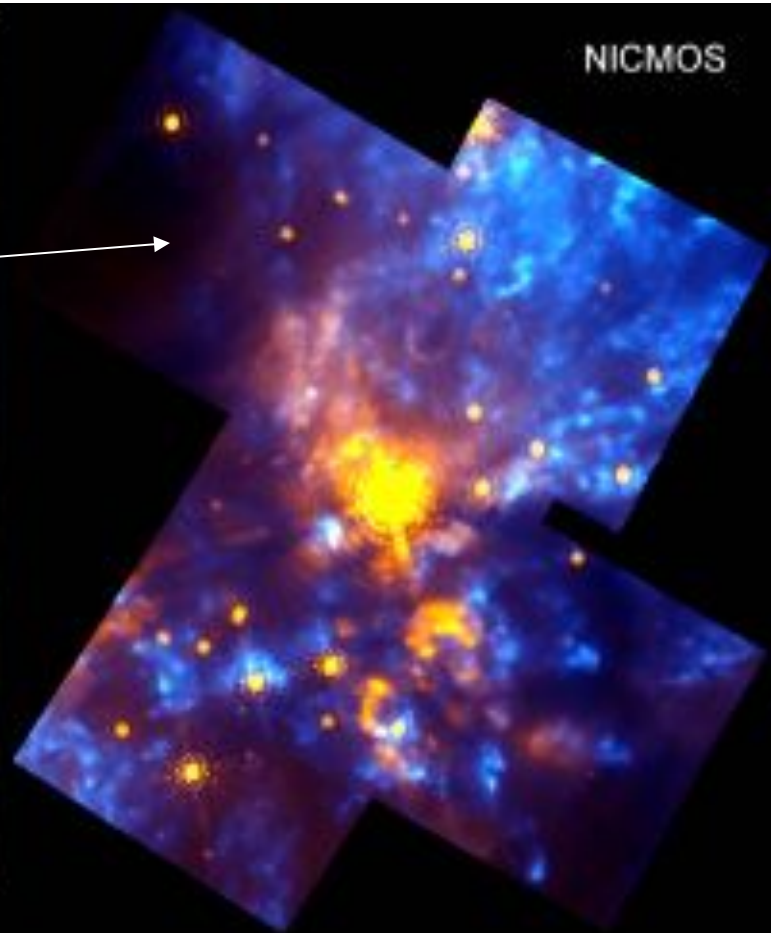


WFPC2

**Orion Nebula • OMC-1 Region**

PRC97-13 • ST ScI OPO • May 12, 1997

R. Thompson (Univ. Arizona), S. Stolovy (Univ. Arizona), C.R. O'Dell (Rice Univ.) and NASA



NICMOS

Hubble Space Telescope



*LH 95  
A Stellar  
Nursery in the  
Large Magellanic  
Cloud (HST)*





*The star formation region N11B in the LMC taken by  
WFPC2 on the NASA/ESA Hubble Space Telescope*

# T-Tauri Stars

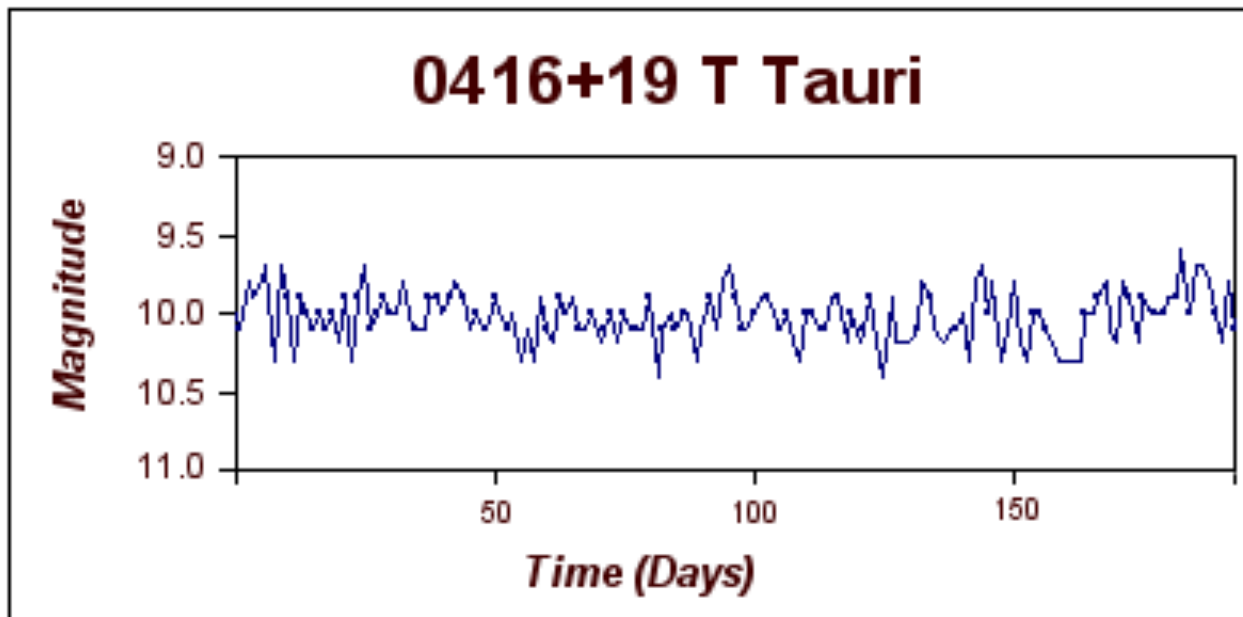


T-Tauri discovered by John Hind in 1852 as a 10<sup>th</sup> magnitude star. A faint nebula was subsequently discovered nearby (“Hind’s nebula”). Both the star and nebula had variable brightness. The nebula was a “reflection” nebula, shining from the reflected light of T-Tauri.

*T-Tauri – in Taurus close to the Pleiades*

By 1861 the nebula disappeared from view and by 1890 T-Tauri itself had faded to 14<sup>th</sup> magnitude, about the limit of telescopes then. A faint nebula at the site of T-Tauri itself was observed at that time,

Over the next 10 – 20 years, T-Tauri brightened back to 10<sup>th</sup> magnitude and its local nebula became invisible against the glare. T-Tauri has remained at about 10<sup>th</sup> magnitude since (but varies).





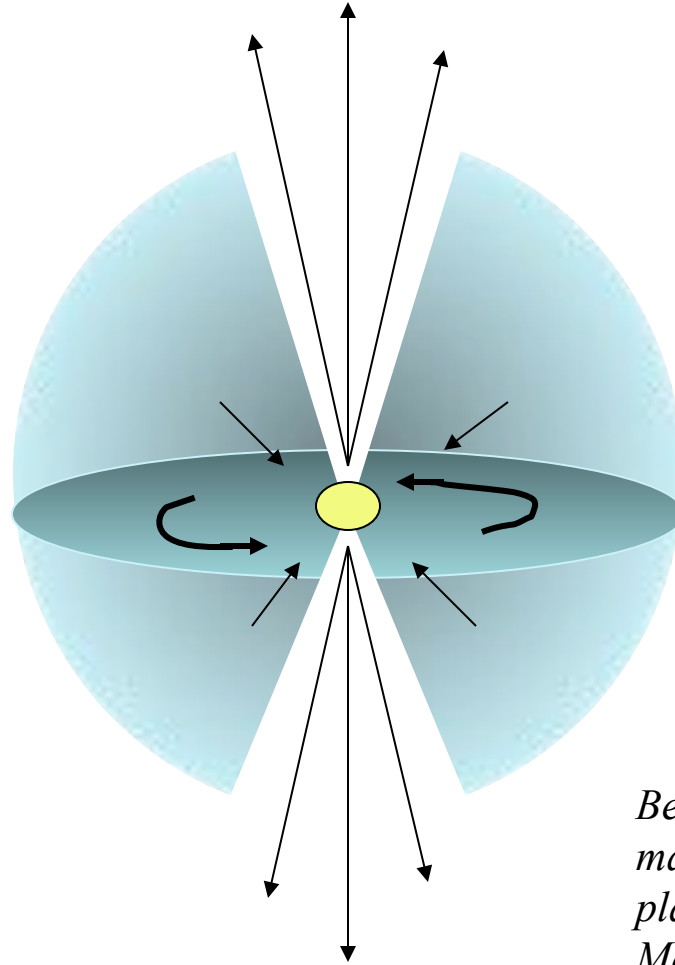
T-Tauri - about 400 ly away at the edge of a molecular cloud.

FOV here is 4 ly at the distance of T-Tauri <http://apod.nasa.gov/apod/ap071213.htm>

## T-Tauri Stars

- Short lived phase in life of stars under 2 solar masses. Heavier stars evolve quicker and start burning by the time the star is visible. Above 2 solar masses the objects evolve rapidly and are rarely seen - “Herbig Ae and Be stars”.
- Accretion disks and jets are common features
- Emission and absorption lines
- Powered by gravitational contraction, not nuclear burning
- May be forming planetary systems
- High lithium abundance
- Embedded in dense, dusty regions
- Can be highly variable

When the star first becomes visible it may still be surrounded by the gas and dust from which it formed. Often jets are seen.

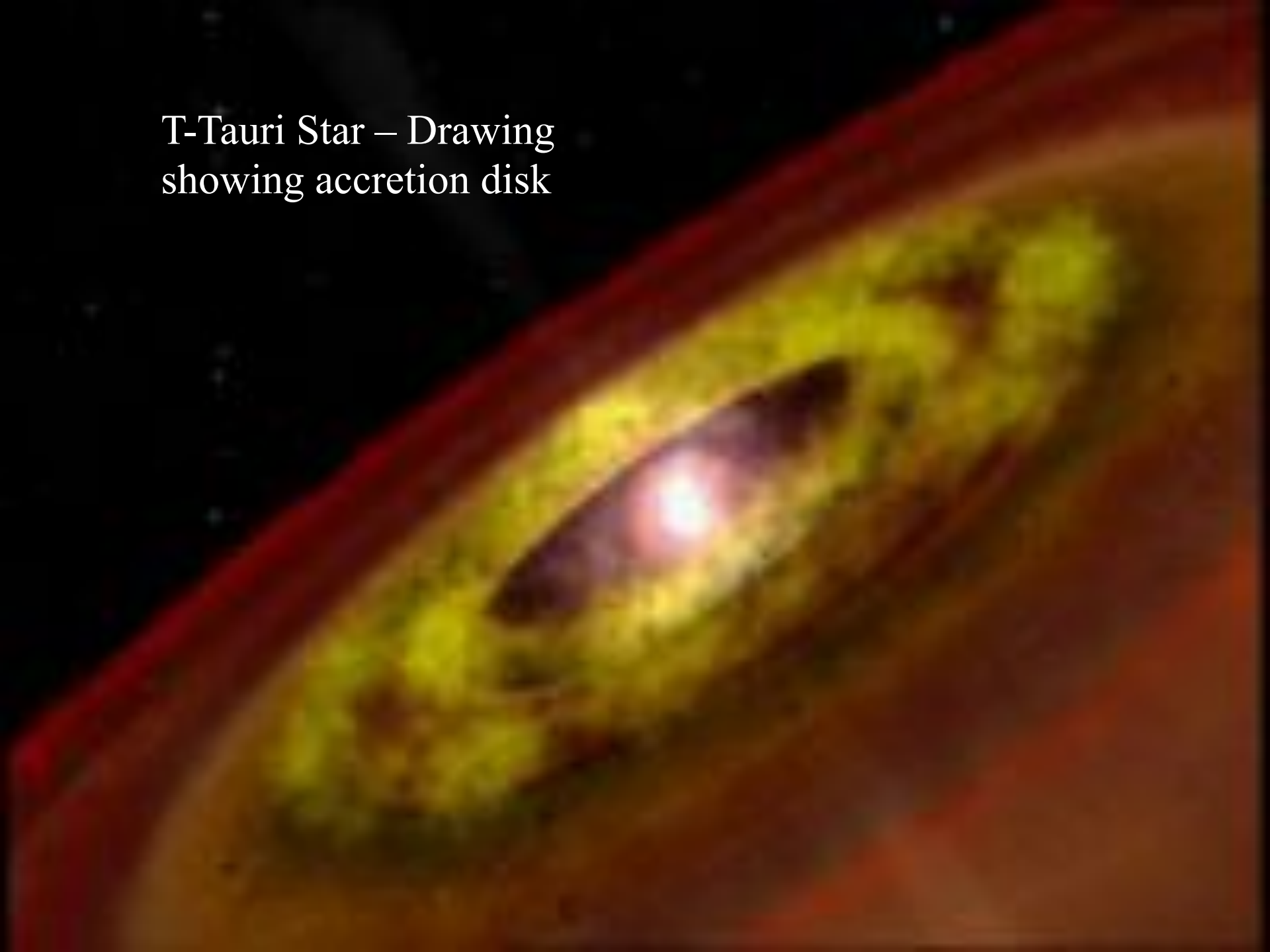


*Because of rotational support matter hangs up in the equatorial plane forming an “accretion disk”. Matter first rains down on the poles, but then later reverses direction in a strong collimated outflow called a “jet”.*

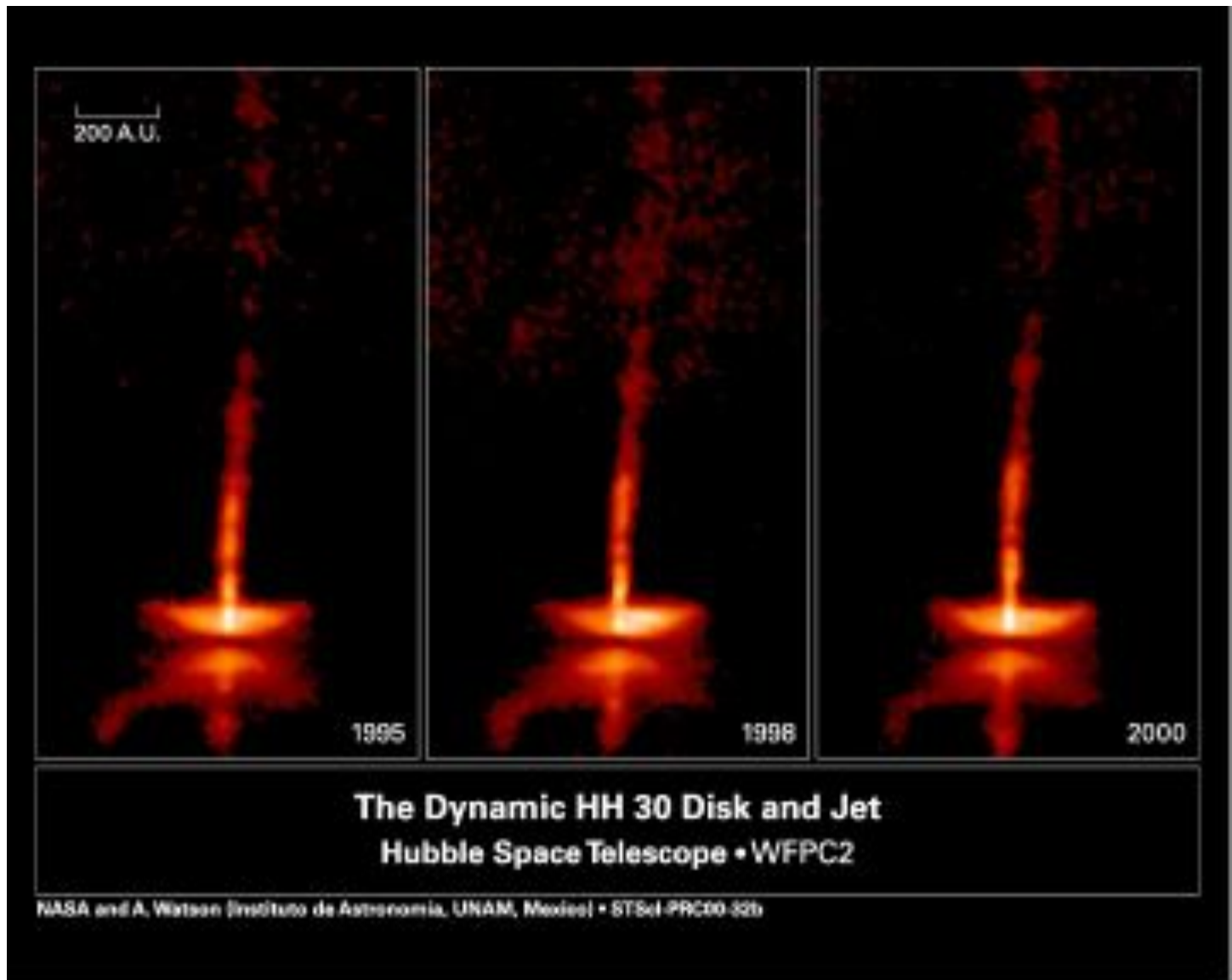
Protoplanetary disks orbit over half of T-Tauri stars.  
This shows 5 such stars in the constellation Orion.  
Picture using HST - field is about 0.14 ly across  
[http://en.wikipedia.org/wiki/T\\_Tauri\\_star](http://en.wikipedia.org/wiki/T_Tauri_star)



T-Tauri Star – Drawing  
showing accretion disk

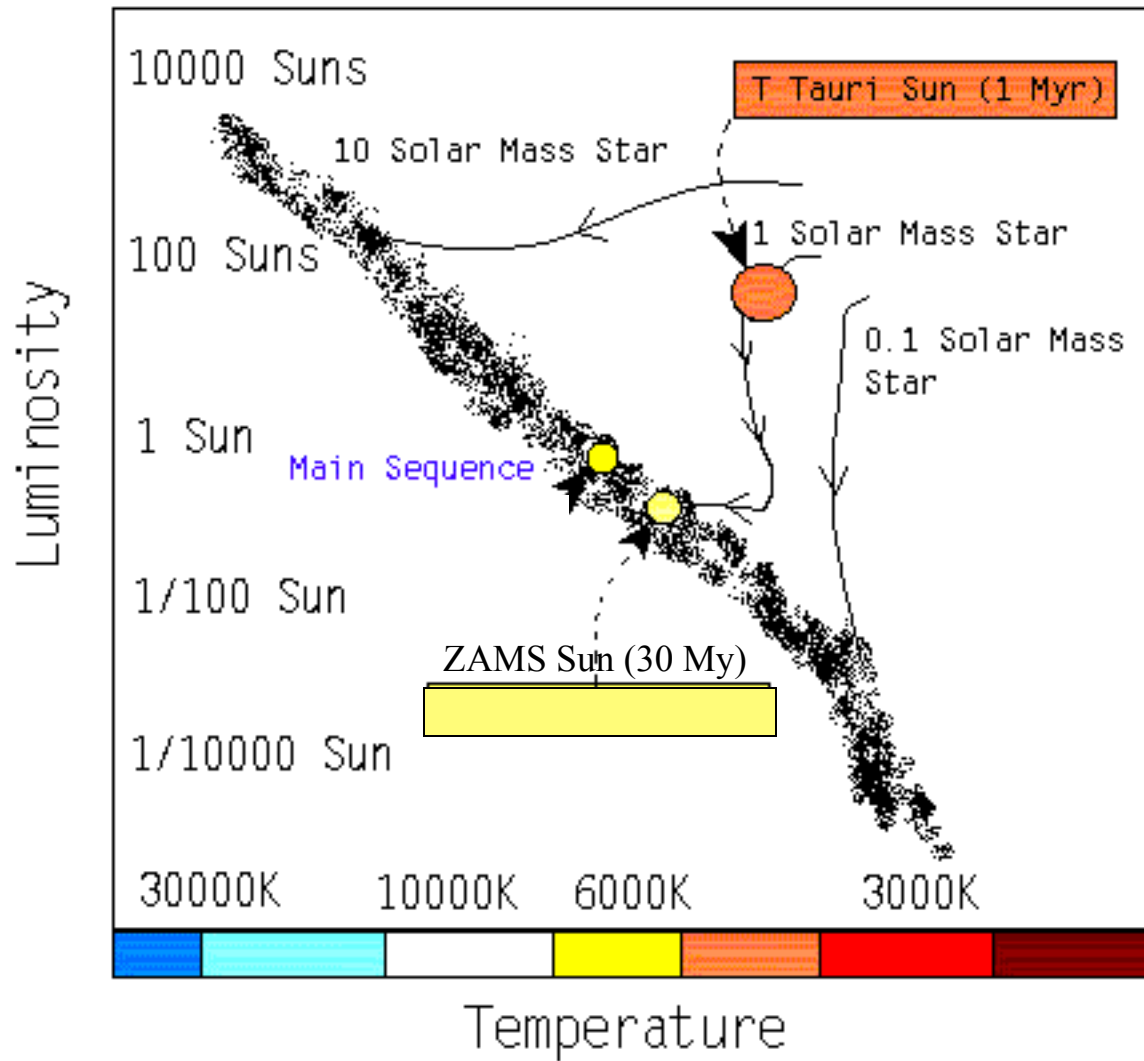






30" west of the brightest point in Hind's nebula is a disk-jet system, Herbig-Haro 30. At the center of this is probably another T-Tauri like star.

# Hertzprung-Russell Diagram

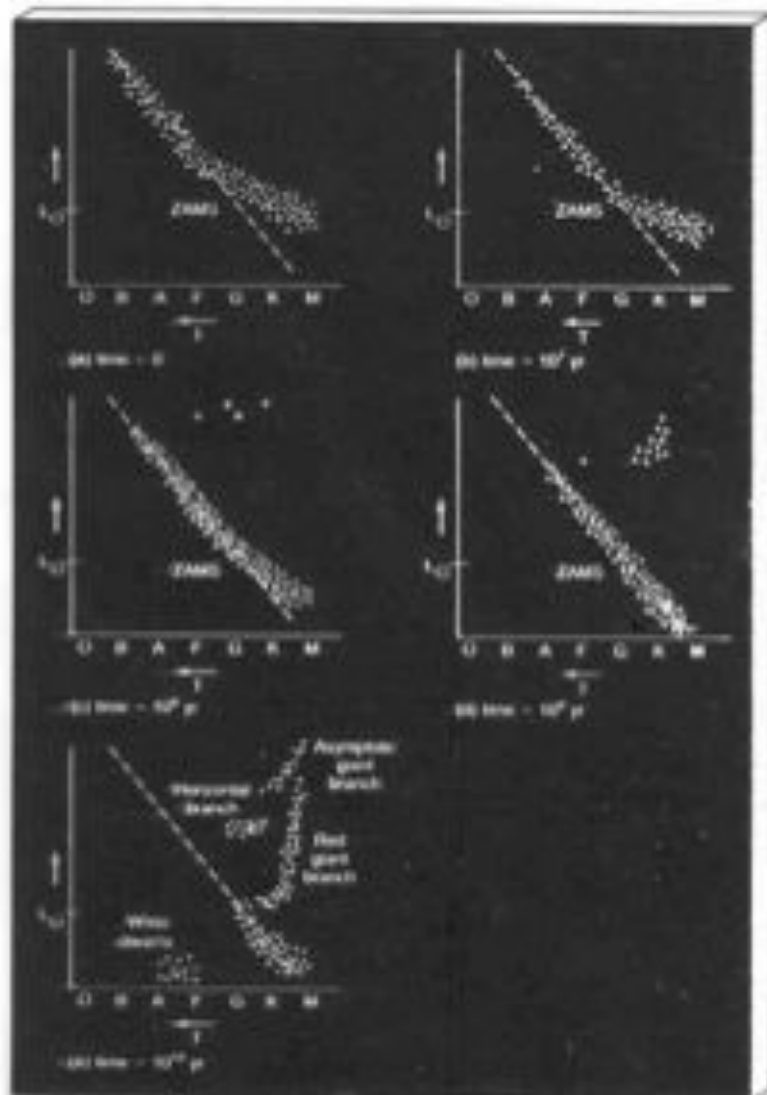


Eric Mamajek (PSU) Apr. 1998

*Protostars start off with very large radii because they begin as contracting clouds of gas. They additionally have high luminosities because they are fully convective (more about this later) and able to transport the energy released by gravitational contraction efficiently to their surface.*

*Most of the time is spent close to the main sequence.*

TM 22-4 HR evolution of hypothetical cluster



$$T_{\text{cl}} \propto \frac{M^2}{RL}$$

$$\propto M^2$$

$$\propto t \text{ with } M$$

$$\therefore T_{\text{cl}} \propto$$

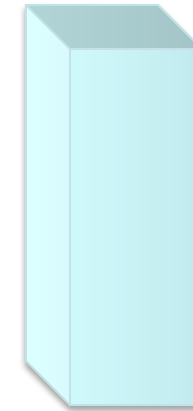
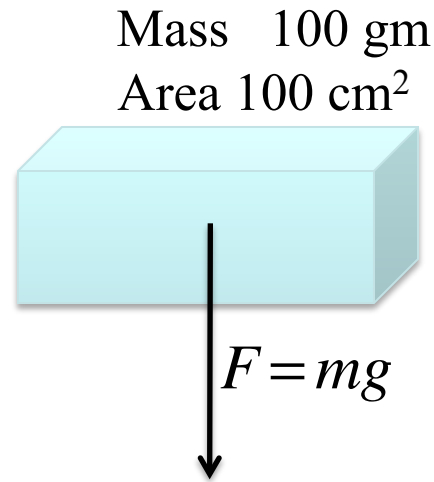
$$\propto M^2$$

*Stellar Interiors - Kinds  
of Pressure*

# Pressure is force *per unit area*

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

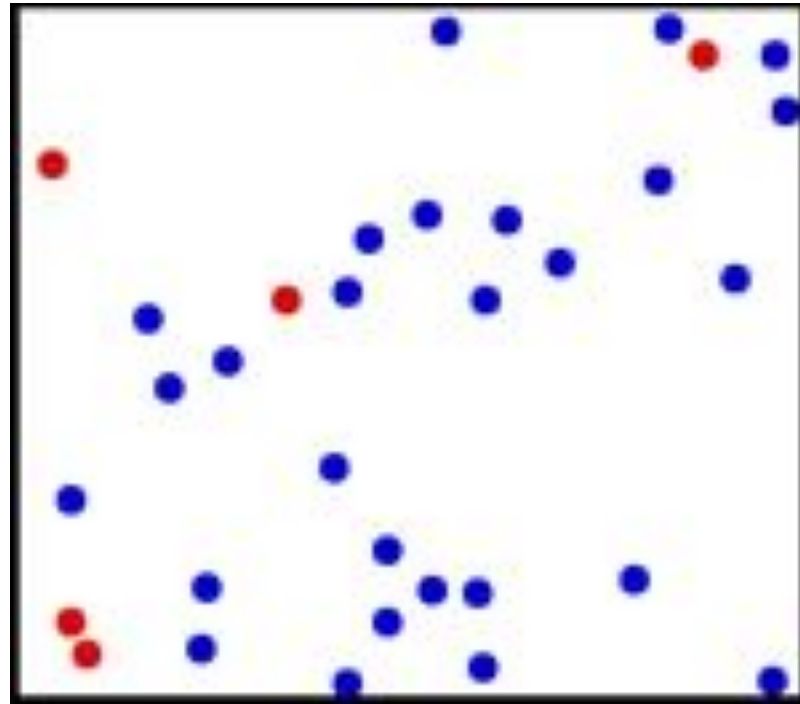
$$g = \frac{GM_{\text{earth}}}{R_{\text{earth}}^2} = 980 \text{ cm s}^{-2}$$



Area 10 cm<sup>2</sup>

$$P = \frac{mg}{A} = \frac{(100)(980)}{100} = \frac{(100)(980)}{10}$$

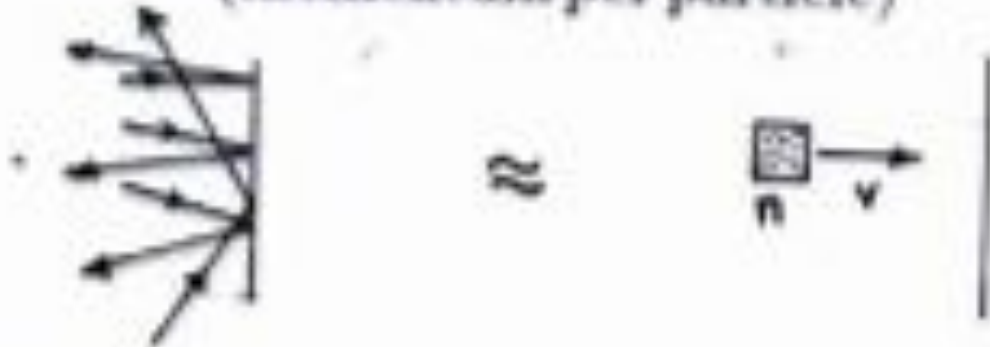
980 dyne cm<sup>-2</sup>                      9800 dyne cm<sup>-2</sup>



<http://intro.chem.okstate.edu/1314f00/laboratory/glp.htm>

## Qualitatively

$$P \sim (\text{number density})(\text{mean velocity}) \\ (\text{momentum per particle})$$



Each particle delivers  
a “kick” =  $2 m \Delta v_x$   
where  $\Delta v_x$  is the change  
in x-velocity

Approximate this with  
a group of particles  $n$  in one  
cubic cm all moving to the  
right with  $v_x = v$ . The particle  
flux then =  $n$  times  $v$  and each  
particle imparts momentum  
of roughly  $mv$

$$P \approx (mv)(nv) = n m v^2$$

For a gas, pressure is defined as

$$P = \frac{1}{3} \int \frac{dn}{dp} v p dp$$

where  $(dn/dp) * dp$  is the number density (per  $\text{cm}^3$ ) of particles having momentum between  $p$  and  $p+dp$ , and  $v$  is their speed. The  $1/3$  is from an integral over angles.

Pressure thus has units

$$\frac{1}{\text{cm}^3} \frac{\text{cm}}{\text{s}} \frac{\text{gm cm}}{\text{s}} = \frac{\text{gm cm}}{\text{cm}^2 \text{s}^2} = \frac{\text{dyne}}{\text{cm}^2}$$



Approximation:

suppose  $p$  (and therefore  $v$ ) is constant and

Then

$$\frac{1}{3} \int \frac{dn}{dp} v p dp \approx \frac{1}{3} v p \int \frac{dn}{dp} dp = \frac{1}{3} v p n$$

where  $n$  is the total number density of particles per cubic cm. When one integrates over a distribution of momenta, the  $1/3$  out front changes.

# IDEAL GAS PRESSURE

- Due to thermal motion of particles such as electrons, atoms, and ions. Particles are assumed to only interact during their collisions.

Qualitatively

$$P \sim (n)(v)(mv) = nmv^2 \sim nkT$$

In fact,

$$P = nkT$$

where  $n$  is the total number of particles per  $\text{cm}^{-3}$  (both ions and electrons) and  $T$  is the temperature in Kelvins. For pure unionized H I,  $n = N_A \cdot \rho$ . In general,

$$P_{\text{ideal}} = \overbrace{8.31 \times 10^7}^{N_A k} \rho T F \text{ dyne cm}^{-2}$$

$N_A = 6.023 \times 10^{23} \text{ gm}^{-1}$   
 $k = 1.38 \times 10^{-16} \text{ erg K}^{-1}$

where  $F = 1$  for pure H I, 2 for pure ionized hydrogen (H II), and 1.69 for a mixture of 75% H and 25% He, both completely ionized.

nb units

$$\frac{\text{gm cm}^2}{\text{cm}^3 \text{ s}^2} = \frac{\text{gm cm}}{\text{s}^2} \frac{1}{\text{cm}^2}$$

$$= \frac{\text{dyne}}{\text{cm}^2}$$

$$\left( \text{also} = \frac{\text{dyne cm}}{\text{cm}^3} = \frac{\text{erg}}{\text{cm}^3} \right)$$

**Example:** *What is the pressure of a fully ionized gas composed of 75% H and 25% He at a temperature of  $10^7$  K and a density of  $100 \text{ g cm}^{-3}$  (similar to solar center when the sun was born)?*

In each  $\text{cm}^3$  of gas there are 75 gm of hydrogen and 25 gram of helium.

In 75 gm of hydrogen there are

$$\frac{75 \text{ gm}}{m_{\text{proton}}} = 75 N_A = 75 \times 6.02 \times 10^{23} = 4.52 \times 10^{25} \text{ protons}$$

In 25 gram of helium there are (the helium nucleus weighs approximately  $4 m_{\text{proton}}$ )

$$\frac{25 \text{ gm}}{4m_{\text{proton}}} = 6.25 N_A = 6.25 \times 6.02 \times 10^{23} = 3.76 \times 10^{24} \text{ helium nuclei}$$

In addition, each hydrogen contributes to the ionized plasma 1 electron and each helium contributes two

$$N = 4.52 \times 10^{25} + \overbrace{4.52 \times 10^{25}}^{\text{electrons}} + 3.76 \times 10^{24} + \overbrace{2(3.76 \times 10^{24})}^{\text{electrons}}$$
$$= 1.02 \times 10^{26} \text{ particles per cm}^3$$

continued...

$$P = nkT = \left( \frac{1.02 \times 10^{26}}{\text{cm}^3} \right) \left( \frac{1.38 \times 10^{-16} \text{ erg}}{\text{K}} \right) (10^7 \text{ K})$$

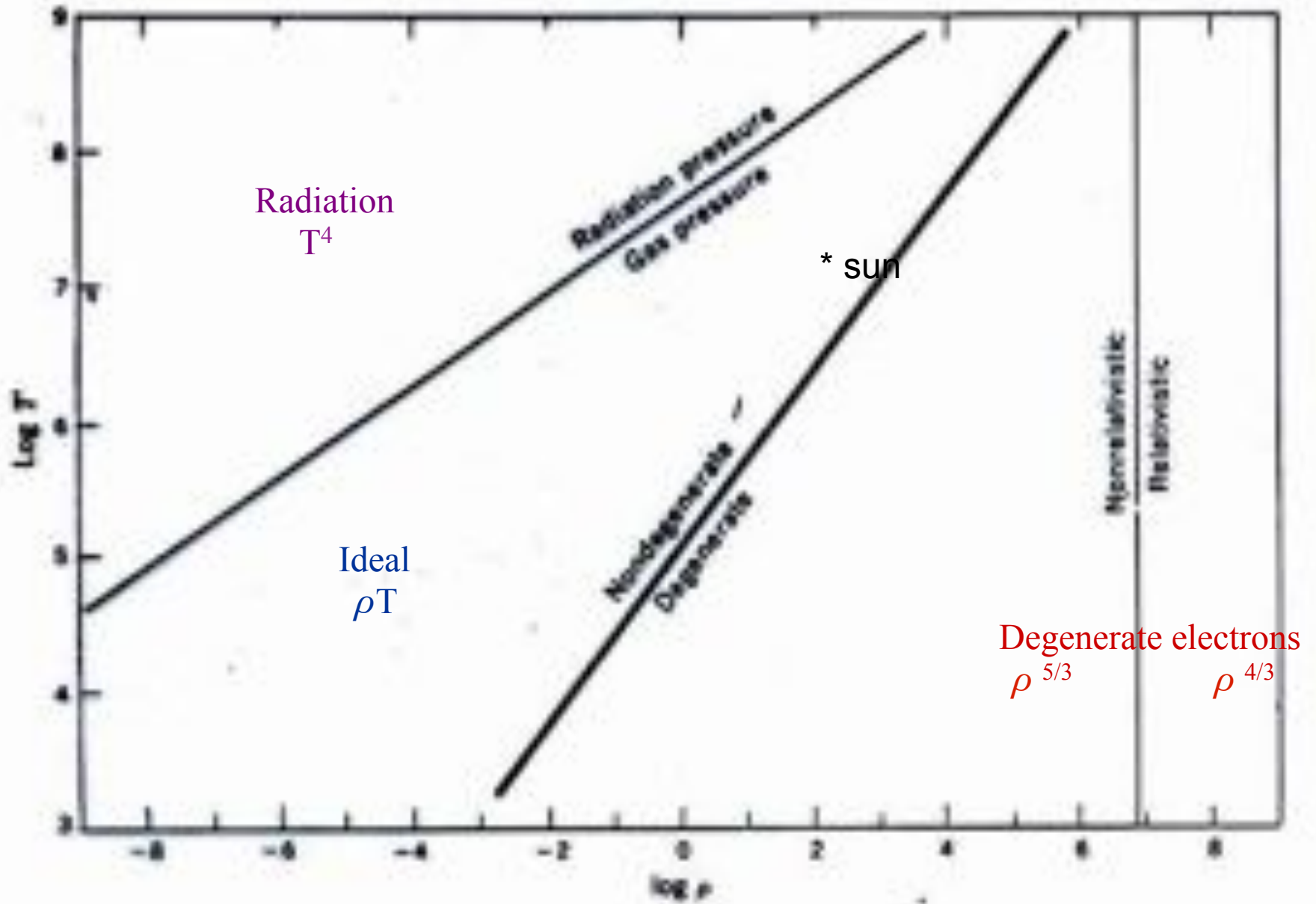
k

$$= 1.40 \times 10^{17} \text{ dyne cm}^{-2} \quad \frac{\text{erg}}{\text{cm}^3} = \frac{\text{dyne cm}}{\text{cm}^3}$$

But it is easier and just as accurate to use the formula

$$P = 8.31 \times 10^7 \rho T F$$
$$= (8.31 \times 10^7)(100)(10^7)(1.69)$$
$$= 1.40 \times 10^{17} \text{ dyne cm}^{-2}$$

Most main sequence stars have pressures that are dominantly ideal gas pressure



# DEGENERACY PRESSURE

Pressure due entirely to the Uncertainty Principle

Suppose one packs as many electrons with momentum  $p$  into a volume,  $V$ , as are quantum mechanically allowed (“Pauli exclusion principle”)

$$\Delta x \cdot p \approx h$$

Each pair of electrons occupies a cell of size  $\sim (\Delta x)^3$ , but  $\Delta x \sim h/p$

$$\text{Number of cells in volume } V \sim \frac{V}{(\Delta x)^3} = \frac{Vp^3}{h^3}$$

$$\text{Number of electrons, } N, \text{ in volume } V = \frac{2Vp^3}{h^3}$$

$$\text{Number of electrons per unit volume } n_e = \frac{N}{V} = \frac{2p^3}{h^3}$$

$$\text{So, } p_F \sim \left( \frac{n_e h^3}{2} \right)^{1/3}$$

*This is commonly called the “Fermi Momentum”*

[http://en.wikipedia.org/wiki/Electron\\_degeneracy\\_pressure](http://en.wikipedia.org/wiki/Electron_degeneracy_pressure)

[http://en.wikipedia.org/wiki/Pauli\\_exclusion\\_principle](http://en.wikipedia.org/wiki/Pauli_exclusion_principle)

# DEGENERACY PRESSURE

Now the pressure

$$P \sim n_e p_F v = n_e p_F \frac{mv}{m} = \frac{n_e p_F^2}{m}$$

$$\sim \frac{n_e}{m} \left( \frac{n_e h^3}{2} \right)^{2/3}$$

$$P_{\text{deg}} \sim \frac{h^2 n_e^{5/3}}{2^{2/3} m}$$

The contribution of electrons, when present, is much larger than from neutrons or protons because of the  $1/m$

As  $n_e$  goes up the speed of each electron rises

$$p_F = m_e v \approx \left( \frac{n_e h^3}{2} \right)^{1/3} \quad \text{more accurately} \quad \left( \frac{3}{8\pi} n_e h^3 \right)^{1/3}$$

$$v = \frac{1}{m_e} \left( n_e \left( \frac{3h^3}{8\pi} \right) \right)^{1/3} \quad n_e \approx \frac{1}{2} \rho N_A \quad \text{for elements other than H}$$

$$v = \left( \frac{3\rho N_A h^3}{16\pi m_e^3} \right)^{1/3} \approx 2 \times 10^{10} \left( \frac{\rho}{10^6 \text{ gm cm}^{-3}} \right)^{1/3} \text{ cm s}^{-1}$$

At around  $10^7 \text{ gm cm}^{-3}$  the electrons will move close to the speed of light.



"non – relativistic" degeneracy pressure =  $P_{NRD}$

$$P_{NRD} \sim \frac{n_e p_F^2}{m_e} = \frac{n_e \left( \frac{n_e h^3}{2} \right)^{2/3}}{m}$$
$$= \frac{h^2}{2^{2/3} m_e} n_e^{5/3}$$

A more accurate calculation gives

$$P_{NRD} = \frac{1}{20} \left( \frac{3}{\pi} \right)^{2/3} \frac{h^2}{m_e} n_e^{5/3}$$

<http://scienceworld.wolfram.com/physics/ElectronDegeneracyPressure.html>

For charge neutrality, number of electrons = number of protons and for pure hydrogen,  $n_e = N_A \rho$ .

For other compositions,  $n_e = N_A \rho Y_e$  where  $(Y_e)^{-1}$  is the number of electrons per atomic mass unit in the neutral atom. E.g.,  $Y_e = 1$  for hydrogen, 0.5 for  ${}^4\text{He}$ ,  ${}^{12}\text{C}$ , etc, and 0.88 for 75% H and 25% He.

Then

usually where  $P_{deg}$  is important  
 $Y_e = 0.5$

$$P_{deg}^{NR} = 1.00 \times 10^{13} (\rho Y_e)^{5/3} \text{ dyne cm}^{-2}$$

$$\rho \approx 10^7 \frac{\text{g}}{\text{cm}^3}$$

Note that the degeneracy pressure depends only on the density and not on the temperature

## RELATIVISTIC DEGENERACY PRESSURE

The above remains true only so long as  $v$  of the electrons remains  $\ll c$ . As  $v$  approaches  $c$

$$P_{deg} \sim (n_e)(c)(p) \sim n_e^{4/3} \quad (p \propto n_e^{1/3})$$

and in fact

$$P_{deg}^R = 1.24 \times 10^{15} (\rho Y_e)^{4/3} \text{ dyne cm}^{-2} \quad \rho \approx 10^7 \text{ g/cm}^3$$

Once the electrons move near the speed of light, the pressure does not increase as rapidly with density as before.

## RADIATION PRESSURE

Because electromagnetic radiation (light) carries energy, it also carries momentum. In general, for non-relativistic motion ( $v \ll c$ ), momentum = (2)(kinetic energy)/(velocity), e.g.,  $2(1/2mv^2)/v = mv = p$ . For photons the relation is a little different.

$$p = E/c = h\nu/c$$

So to get momentum flux just divide energy flux by  $c$ .

# THE “PRESSURE” OF SUNLIGHT

From the sun, at the earth’s orbit (1AU), we receive a flux of radiation

$$\begin{aligned}\phi &= \frac{L}{4\pi d^2} = \frac{L_{\odot}}{4\pi(AU)^2} \\ &= 1.37 \times 10^6 \text{ erg cm}^{-2} \text{ s}^{-1}\end{aligned}$$

This corresponds to a momentum flux, or pressure of

$$\begin{aligned}P &= \frac{\phi}{c} = \frac{(1.37 \times 10^6)}{(3.00 \times 10^{10})} \frac{\text{erg}}{\text{cm}^2 \text{ s}} \frac{(\text{s})}{(\text{cm})} \\ &= 4.57 \times 10^{-5} \frac{\text{dyne}}{\text{cm}^2} \quad \text{since } (\text{dyne})(\text{cm}) = \text{erg}\end{aligned}$$

(1 square meter ( $10^4 \text{ cm}^2$ ) would be accelerated  $0.46 \text{ cm/s}^2$  if it weighed 1 gm; would reach  $c$  in about 1000 years)

ion tail  
pushed back by  
solar wind

dust tail  
accelerated by  
radiation pressure



1997 Comet Hale Bop

The pressure is then

$$\begin{aligned} P &\sim (\text{number flux})(\text{momentum per photon}) \\ &= \left( \frac{\text{E flux}}{\text{energy per photon}} \right) (\text{momentum per photon}) \\ &= \left( \frac{\sigma T^4}{h\nu} \right) \left( \frac{h\nu}{c} \right) \\ &= \frac{\sigma}{c} T^4 \end{aligned}$$

In fact the correct expression is

$$P_{rad} = \frac{4\sigma}{3c} T^4 = \frac{1}{3} a T^4$$

where  $a = 7.56 \times 10^{-15} \text{ dyne cm}^{-2} (\text{K})^{-4}$ .  $= \frac{4\sigma}{c}$

## IN SUMMARY

There are 3 kinds of pressure:

- Ideal gas pressure ( $\propto \rho$  and  $T$ )
- Radiation pressure ( $\propto T^4$ )
- Degeneracy pressure ( $\propto \rho^n$ )  $4/3 < n < 5/3$

The total pressure is given by

$$P_{\text{tot}} = P_{\text{ion}} + P_{\text{rad}} + P_e$$

Except in neutron stars,  $P_{\text{ion}}$  is ideal.  $P_e$  can be quite complex (semidegenerate, semirelativistic) which can lead to some difficult math (Fermi integrals) which we will not consider.



Most main sequence stars have pressures that are dominantly ideal gas pressure

