LECTURE 15:
WHITE DWARFS
AND THE ADVANCED EVOLUTION
OF MASSIVE STARS

http://apod.nasa.gov/apod/astropix.html

White Dwarfs

- Low mass stars are unable to reach high enough temperatures to ignite elements heavier than carbon in their core become white dwarfs.
- Hot exposed core of an evolved low mass star.
- Supported by electron degeneracy pressure. This is the tendency of atoms to resist compression.
- The more massive a white dwarf, the smaller it is. A solar mass white dwarf is about the size of the Earth.
- As white dwarfs radiate energy, they become cooler and less luminous gradually fading into oblivion, but it can take a long time….

A white dwarf is the remnant of stellar evolution for stars between 0.08 and 8 solar masses (below 0.08 one can have brown dwarfs). They can be made out of helium, or more commonly carbon and oxygen (rarely NeOMg).
For objects made of normal matter, radius tends to increase with mass.
More massive white dwarf stars are denser for the radius.

\[
\rho = \frac{3M}{4\pi R^3} = \frac{(3)(1.99 \times 10^{33})}{(4\pi)(5 \times 10^8)^3} \left( \frac{M}{M_\odot} \right)^2 = 4 \times 10^8 \left( \frac{M}{M_\odot} \right)^2 \text{ g cm}^{-3}
\]

\* Note implication: As M goes up, R gets smaller and \( \rho \) gets larger.

White dwarfs are known with temperatures ranging from 4000 K to 200,000 K.

**Mass distribution**
Most WDs cluster around 0.6 \( M_\odot \). Narrow mass distribution

**Luminosity function**

**LF of disk white dwarfs**

Madej et al. 2004
**Maximum white dwarf mass**

- As mass increases, electron speed approaches $c$. Pressure becomes due relativistic electrons. Proportional to $\rho^{4/3}$

- Electron degeneracy cannot support a white dwarf heavier than 1.4 solar masses, the “Chandrasekhar limit”.

\[
M \approx \left( \frac{\hbar^3 c^3}{G^3 m_p^4} \right)^{1/2} Y_e^2
\]

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**THE CHANDRASEKHAR MASS**

As $M$ gets larger and the radius decreases, the density rises. Eventually at $\rho$ greater than about $10^7 \text{ g cm}^{-3}$ electrons in the *central* part of the white dwarf start to move close to the speed of light. As the mass continues to grow, a larger fraction of the star is supported by relativistic electron degeneracy pressure.

Consider the limit:

\[
P_{\text{deg}}^R = 1.24 \times 10^{15} \left( \rho Y_e \right)^{4/3} = \frac{GM\rho}{2R}
\]
As usual examine the constant density case for guidance

\[ \rho = \left( \frac{3M}{4\pi R^3} \right) \]

\[ 1.24 \times 10^{15} \rho Y_e^{4/3} \left( \frac{3M}{4\pi R^3} \right)^{1/3} = \frac{GM\rho}{2R} = P_{\text{central}} \]

For density \( Y_e \)

\[ M^{2/3} = 1.24 \times 10^{15} Y_e^{4/3} \left( \frac{3}{4\pi} \right)^{1/3} \frac{2}{G} \]

\[ M^{2/3} = 2.3 \times 10^{22} Y_e^{4/3} \]

\[ M = 3.5 \times 10^{33} Y_e^2 \text{ g.m.} = 1.75 Y_e^2 \text{ M}_\odot \]

Actually

\[ M = 5.7 Y_e^2 \text{ M}_\odot = 1.4 \text{ M}_\odot \text{ if } Y_e = 0.5 \]

Aside:

This result extends beyond white dwarfs.

There can be no stable star whose pressure depends on its density to the 4/3 power.

What happens to a star more massive than 1.4 solar masses?

1. There aren’t any
2. They shrink to zero size
3. They explode
4. They become something else
EVOLUTION OF WHITE DWARF STARS

For a WD of constant mass, $R = \text{constant}$

Crystallization in white dwarfs

When the interior temperature declines to $\sim 5000$ K, the carbon and oxygen start to crystallize into a lattice. This crystallization releases energy and provides a source of luminosity that slows the cooling.

The number counts pile up.

Hansen et al (2007)
NGC 6397 - globular cluster

Critical Masses

$0.08 \ M_\odot$

Contracting protostars below this mass do not ignite hydrogen burning on the main sequence. They become brown dwarfs or planets.

$0.50 \ M_\odot$

Stars below this mass are completely convective on the main sequence

““““ do not ignite helium burning

$2.0 \ M_\odot$

Stars below this mass (and above 0.5) experience the helium core flash
Stars above this mass are powered by the CNO cycle (below by the pp-cycles)
Stars above this mass have convective cores on the main sequence (and radiative surfaces)

$8 \ M_\odot$

Stars below this mass do not ignite carbon burning. They end their lives as planetary nebulae and white dwarfs. Stars above this mass make supernovae.

$\sim 150 \ M_\odot$

Population I stars much above this mass pulse apart on the main sequence.
No heavier stars exist.

The coolest, faintest white dwarfs still have a surface temperature of $\sim 4000$ K. The universe is not old enough for “black dwarfs” to have formed yet.

E.g., 0.59 solar mass WD - like the sun will make - takes about 1.5 billion years to cool to 7140 K and another 1.8 billion years to cool to 5550 K.

http://en.wikipedia.org/wiki/White_dwarf

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The Evolution and Explosion of Massive Stars

Because of the increasing dominance of radiation pressure, stars much above 100 solar masses become pulsationally unstable and experience episodes of violent mass ejection (not Cepheids nor supernovae or planetary nebulae, but a lot of fast mass loss).

No star can be supported by 100% radiation pressure:

\[ P_c = \frac{GM\rho}{2R} \approx \frac{1}{3} \alpha T^4 \quad \text{if supported by } P_{\text{radiation}} \]

but \( \rho \sim \left( \frac{3M}{4\pi R^3} \right) \Rightarrow \frac{3GM^2}{8\pi R^4} \approx \frac{1}{3} \alpha T^4 \)

so for a fixed M, \( T^4 \propto \left( \frac{1}{R} \right)^4 \propto \rho^{4/3} \)

\[ P \propto \rho^{4/3} \quad \text{which is known to have no stable solution} \]

MAXIMUM MASS STAR

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Most luminous star in our galaxy (that we can study well), several million times more luminous than the sun, bigger than the solar system.

\[ \text{dist} \sim 8000 \text{ ly} \]
\[ \text{diam} \sim 10 \text{ billion miles} \]

Probably a supernova in the next 100,000 years, maybe sooner.

Peculiar star Eta Carina in Carina

1677 – discovered Edmond Halley – 4th magnitude star
1730 – brightness had reached 2nd magnitude
1801 – brightened again then faded back to 4th magnitude by 1811
1820 – began to brighten again
1822 – reached 2nd magnitude
1827 – reached 1st magnitude began to fade back to 2nd magnitude
for about 5 years, then rose to magnitude 0 faded slightly
then rose again
1843, April – magnitude -0.8 second brightest star in sky after Sirius,
then faded continuously
1868 – became invisible
1900 – had faded to 8th magnitude, stayed there till 1941, then began
to brighten again
1953 – 7th magnitude
early 1990’s – 6th magnitude
1998-99 – brightened by a factor of 2

Eta Carina is about 8,000 light years away and one of the most
massive stars in the sky (120 to 150 times the mass of the sun). 99%
of its luminosity is in the infrared.
Observations suggest a cutoff around 150 Msun. Controversial claims of heavier stars come and go.

<table>
<thead>
<tr>
<th>Star Name</th>
<th>Mass (solar masses)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R136a1</td>
<td>265?</td>
</tr>
<tr>
<td>WR101e</td>
<td>150 - 160</td>
</tr>
<tr>
<td>HD 269810</td>
<td>150</td>
</tr>
<tr>
<td>Peony Nebula Star</td>
<td>150</td>
</tr>
<tr>
<td>LBV 1806 - 20</td>
<td>130</td>
</tr>
</tbody>
</table>

Overview of Evolution (150 > M > 8 Solar Masses)

- Above 8 solar masses, stars ignite carbon burning stably after helium depletion. They avoid becoming degenerate in their centers and go on to burn heavier fuels culminating in the production of an iron core.
- Such massive stars have very high luminosities and short lives. They are all (presently) of Population I.
- During the red giant stage the very high luminosities of these stars (and their large radii) imply that the surface layers are very loosely bound. Extensive mass loss occurs.
- For stars above about 35 solar masses the entire hydrogen envelope is lost during helium burning. The star becomes a Wolf-Rayet star and even then mass loss continues at a rapid pace.

Convective history of 15 $M_\odot$ and 25 $M_\odot$ stars

- On the main sequence such massive stars have convective cores and are powered by the CNO cycle. Their surfaces are not convective. After burning hydrogen they ignite helium burning non-degenerately (no “helium flash”).
- Evolution beyond helium burning is greatly accelerated by thermal neutrino losses, especially from electron-positron pair annihilation (TBD).
- The massive stars that keep part of their hydrogen envelope become Type II supernovae. Those that lose their envelope (either in binaries or single stars above 40 solar masses) become Type Ib or Ic supernovae.
Post-Helium Burning Evolution

**Why the big speed up?**

**Pair Neutrino Losses**

After helium burning the core contracts and the temperature rises. The most abundant fuel with the lowest charge is carbon ($^{12}$C). In order to get two carbons to fuse, a temperature of almost a billion K is required (actually 0.8 billion).

At such high temperatures, a new energy loss mechanism comes into play.

Gamma rays ($\gamma$) $\leftrightarrow$ $e^+ + e^-$

Very rarely though $e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$

For $T \sim 10^9$ K, $kT = 86$ keV  
$m_e c^2 = 511$ keV

number $e^+ \sim$ number $e^- \sim T^4$

Because the number of electron-positron pairs is very sensitive to the temperature, the energy loss rate due to neutrino losses also depends on a high power of the temperature.

For temperatures over about $2 \times 10^9$ K

$$\epsilon_{\nu,\text{pair}} \approx -\frac{2 \times 10^{15}}{\rho} \left(\frac{T}{10^9 \text{K}}\right)^9 \text{erg g}^{-1} \text{s}^{-1}$$

For carbon burning and other later burning stages, these losses greatly exceed those due to radiative diffusion and convection.

Because the amount of energy released by each stage is roughly constant, the lifetime at each stage goes down very roughly as $1/T^9$. A higher $T$ is required to burn each fuel.
CARBON BURNING

At a temperature $T \approx 8 \times 10^8$ K and a density $\rho \approx 10^5$ g cm$^{-3}$, carbon fusion provides energy at a rate that balances losses due to neutrinos. A little bit of extra energy powers convection and keeps the core hot. Simply carbon $\rightarrow$ neon and magnesium but in greater detail, the chief reaction is the fusion of two $^{12}$C nuclei to produce isotopes of neon, sodium and magnesium

\[
^{12}\text{C} + ^{12}\text{C} \rightarrow ^{23}\text{Na} + p + 2.24 \text{ MeV}
\]

\[
^{12}\text{C} + ^{12}\text{C} \rightarrow ^{20}\text{Ne} + \alpha + 4.62 \text{ MeV} \quad (\alpha \equiv ^4\text{He})
\]

\[
^{12}\text{C} + ^{12}\text{C} \rightarrow ^{23}\text{Mg} + n - 2.63 \text{ MeV} \quad \text{(rarely)}
\]

OXYGEN BURNING

- Similar to carbon burning; at $T \approx 2.0 \times 10^9$K, $\rho \approx 10^6$ g cm$^{-3}$

\[
^{16}\text{O} + ^{16}\text{O} \rightarrow ^{28}\text{Si} + \alpha
\]

\[
^{16}\text{O} + ^{16}\text{O} \rightarrow ^{31}\text{P} + p
\]

\[
^{16}\text{O} + ^{16}\text{O} \rightarrow ^{31}\text{S} + n
\]

and a host of secondary reactions

- The net result is

$^{16}$O, $^{20}$Ne, $^{24}$Mg $\rightarrow$ abundant isotopes of silicon, sulfur, chlorine, argon potassium and calcium. Most abundant ashes - $^{28}$Si and $^{32}$S

\[ q_{\text{nuc}} = 5.0 \times 10^{17} \Delta X_{16} \quad \text{erg g}^{-1} \]

\[ \epsilon_{\text{nuc}} \propto T^{33} \]

CARBON BURNING

The neutrons, protons and alpha-particles (helium nuclei) react with other species that are there so that following the composition becomes complicated (but calculable)

\[
^{23}\text{Na} + p \rightarrow ^{24}\text{Mg} + \gamma \quad ^{23}\text{Na} + \alpha \rightarrow ^{27}\text{Al} + \gamma
\]

\[
^{20}\text{Ne} + \alpha \rightarrow ^{24}\text{Mg} + \gamma \quad ^{23}\text{Mg} + n \rightarrow ^{25}\text{Mg} + \gamma
\]

\[
^{24}\text{Mg} + n \rightarrow ^{25}\text{Mg} + \gamma \quad \text{etc.}
\]

The net result is that $4 \times 10^{17} \Delta X_{12}$ erg g$^{-1}$ are released and the most abundant isotopes of neon, sodium, magnesium and aluminum are created. Oxygen also survives with a slightly increased abundance. $\Delta X_{12} \approx 0.2$

Note the gradual decrease in energy yield from $6 \times 10^{18}$ erg g$^{-1}$ for hydrogen burning to about $1 \times 10^{18}$ erg g$^{-1}$ for helium burning to about $10^{17}$ erg g$^{-1}$ for carbon burning.
After each burning stage the core contracts, heats up and ignites another fuel.

\[ \rho \propto T^3 \]

25 M\(_{\odot}\) Presupernova Star (typical for 9 - 130 M\(_{\odot}\))

1400 R\(_{\odot}\) (6 AU)

240,000 L\(_{\odot}\)

0.1 R\(_{\odot}\)

0.003 R\(_{\odot}\)
Neutrino emission dominates the energy budget after helium depletion in the center of the star...

<table>
<thead>
<tr>
<th>Fuel</th>
<th>$n_0$ (g cm$^{-3}$)</th>
<th>$T_e$ (10$^6$ K)</th>
<th>$\tau$ (yr)</th>
<th>$L_{\text{phot}}$ (erg s$^{-1}$)</th>
<th>$L_\nu$ (erg s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>5.6(0)</td>
<td>0.040</td>
<td>1.0(7)</td>
<td>2.7(38)</td>
<td>—</td>
</tr>
<tr>
<td>Helium</td>
<td>9.4(2)</td>
<td>0.19</td>
<td>9.5(5)</td>
<td>5.3(38)</td>
<td>&lt;1.0(36)</td>
</tr>
<tr>
<td>Carbon</td>
<td>2.7(5)</td>
<td>0.81</td>
<td>3.0(2)</td>
<td>4.3(38)</td>
<td>7.4(39)</td>
</tr>
<tr>
<td>Neon</td>
<td>4.0(6)</td>
<td>1.7</td>
<td>3.8(1)</td>
<td>4.4(38)</td>
<td>1.2(43)</td>
</tr>
<tr>
<td>Oxygen</td>
<td>6.0(6)</td>
<td>2.1</td>
<td>5.0(1)</td>
<td>4.4(38)</td>
<td>7.4(43)</td>
</tr>
<tr>
<td>Silicon</td>
<td>4.9(7)</td>
<td>3.7</td>
<td>2 days</td>
<td>4.4(38)</td>
<td>3.1(45)</td>
</tr>
</tbody>
</table>

In the HR diagram, massive stars evolve at nearly constant luminosity off the main sequence and eventually explode as red or blue supergiants.

Most massive stars die as red supergiants. This one made a transition back to the blue just before dying.