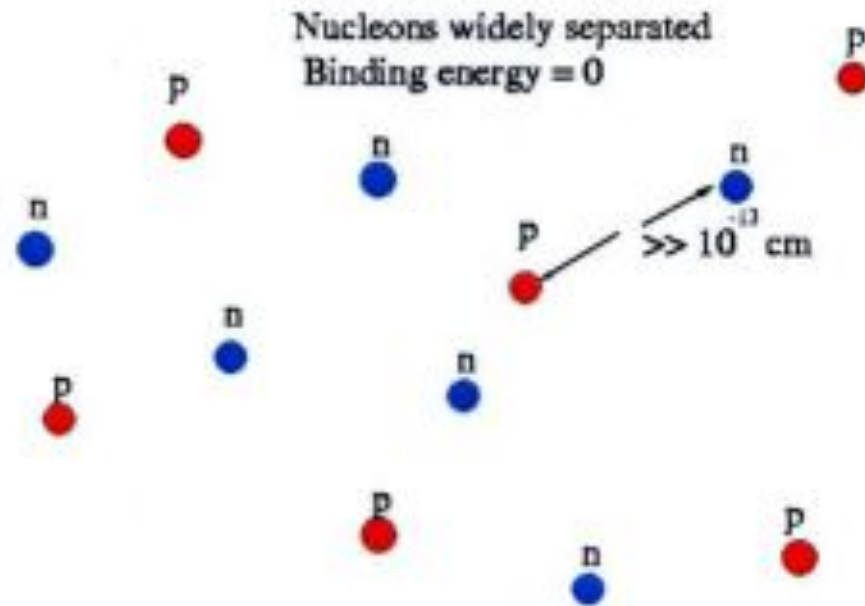


Lecture 16:

Iron Core Collapse, Neutron Stars, and Nucleosynthesis

<http://apod.nasa.gov/apod/astropix.html>

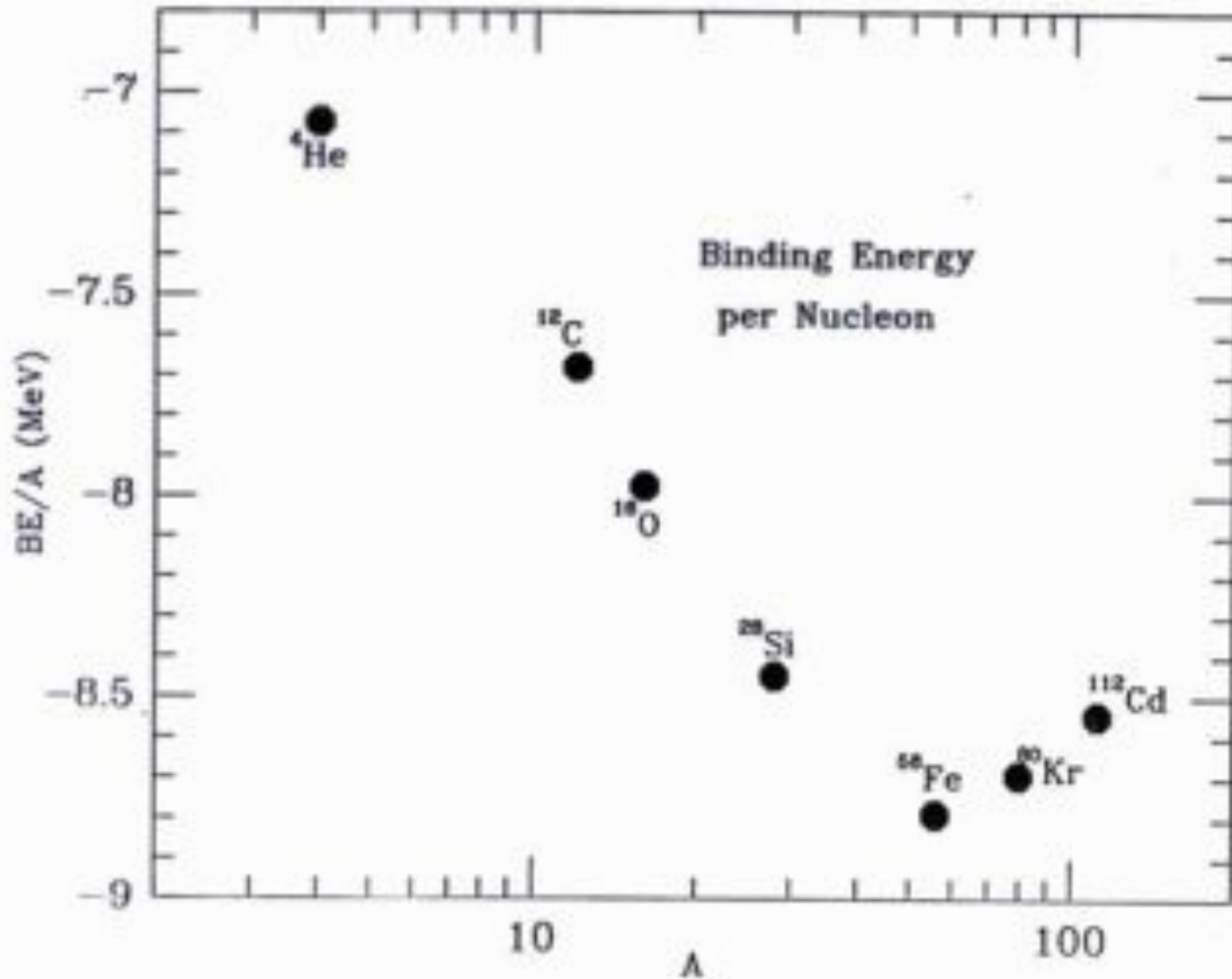
Nuclear Binding Energy



^{12}C

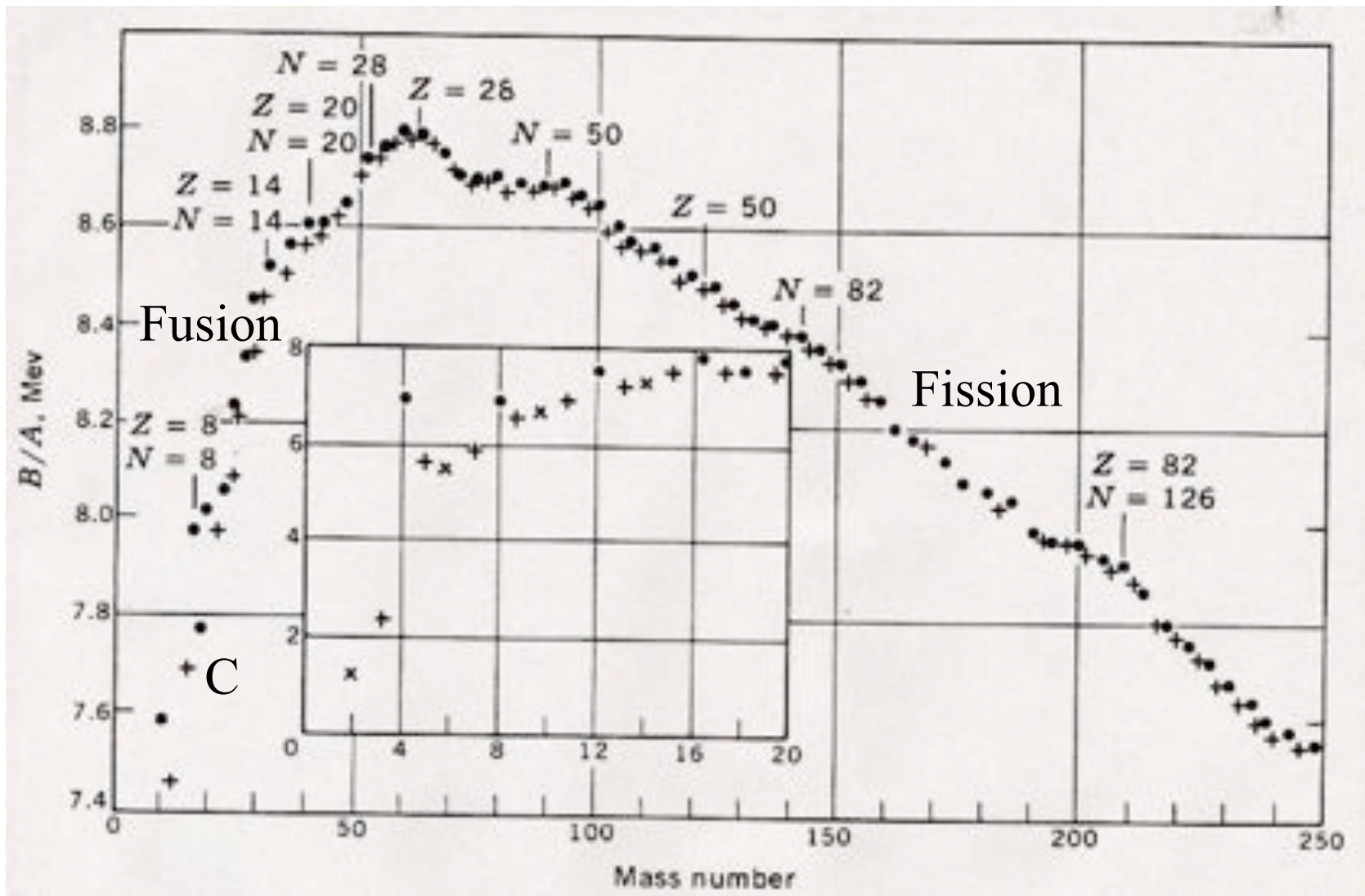
Binding energy = -92.16 MeV
= 1.48×10^{-4} erg
= 7.68 MeV/nucleon

Below iron can repack the nucleons into heavier nuclei and gain energy (fusion) but this stops at iron. Above iron, fission of e.g., ^{235}U to lighter nuclei can release energy



*
 ^{235}U
7.59 MeV

In greater detail.....



Qualitative description of the nucleus

The nucleus is composed of neutrons and protons.

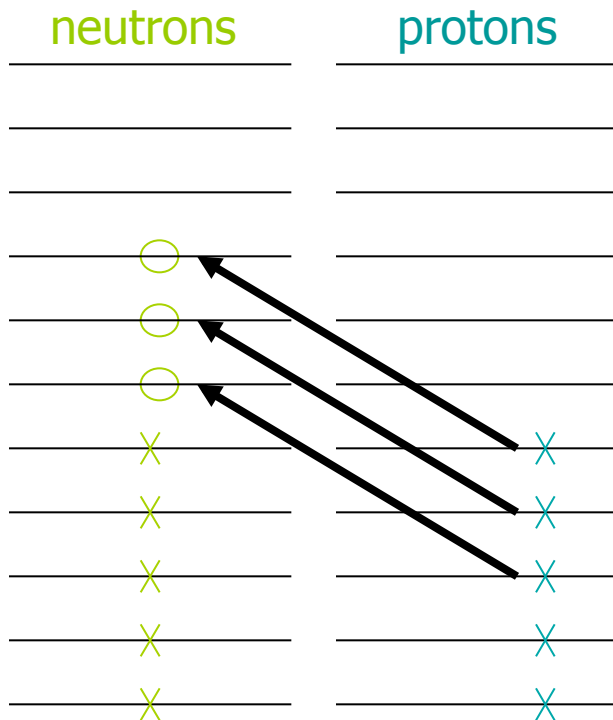
The neutron and proton “gases” are both highly degenerate and the main task of the strong force is to bind the nucleus against its degeneracy pressure (the positive charge of the protons is also important, but not dominant). Nuclei with $Z = N$ are more tightly bound.

The nuclear range is short range. The nucleons on one side of a large nucleus do not feel attracted by nucleons on the other side, only to their neighbors

As the mass of the nucleus increases above some value, the strong force has greater difficulty binding the large collection of neutrons and protons and the electrical repulsion becomes important. The nucleus can fission.

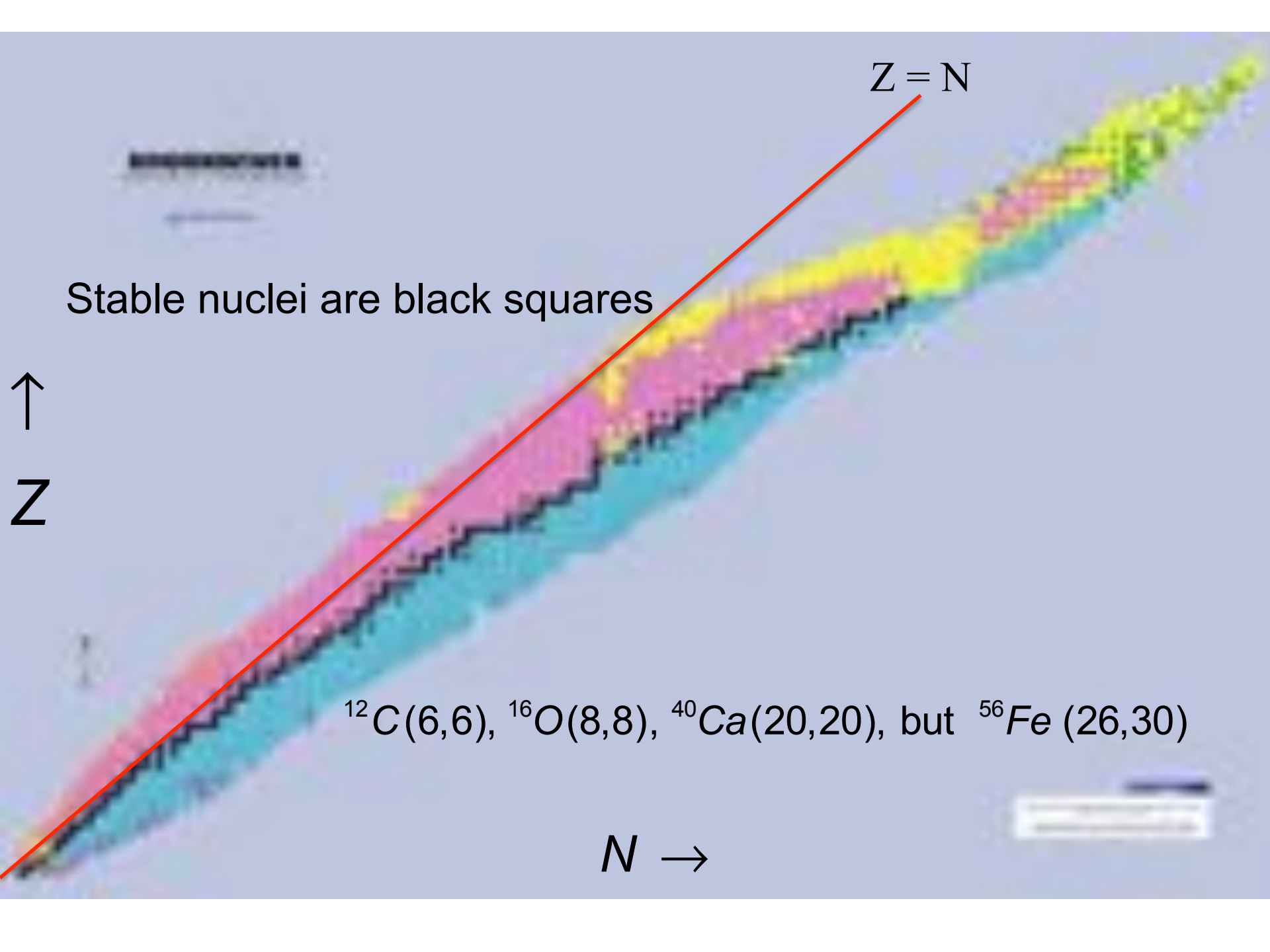
Asymmetry Term

- Neutrons and protons are spin $\frac{1}{2}$ fermions \rightarrow obey Pauli exclusion principle.
- If all other factors were equal nuclear ground state would have equal numbers of n & p.



Illustration

- n and p states with same spacing Δ .
- Crosses represent initially occupied states in ground state.
- If three protons were turned into neutrons
- the extra energy required would be $3 \times 3 \Delta$.
- In general if there are Z-N excess protons over neutrons the extra energy is $\sim ((Z-N)/2)^2 \Delta$ relative to Z=N.



$Z = N$

Stable nuclei are black squares

↑
 Z

$^{12}\text{C}(6,6)$, $^{16}\text{O}(8,8)$, $^{40}\text{Ca}(20,20)$, but $^{56}\text{Fe}(26,30)$

$N \rightarrow$

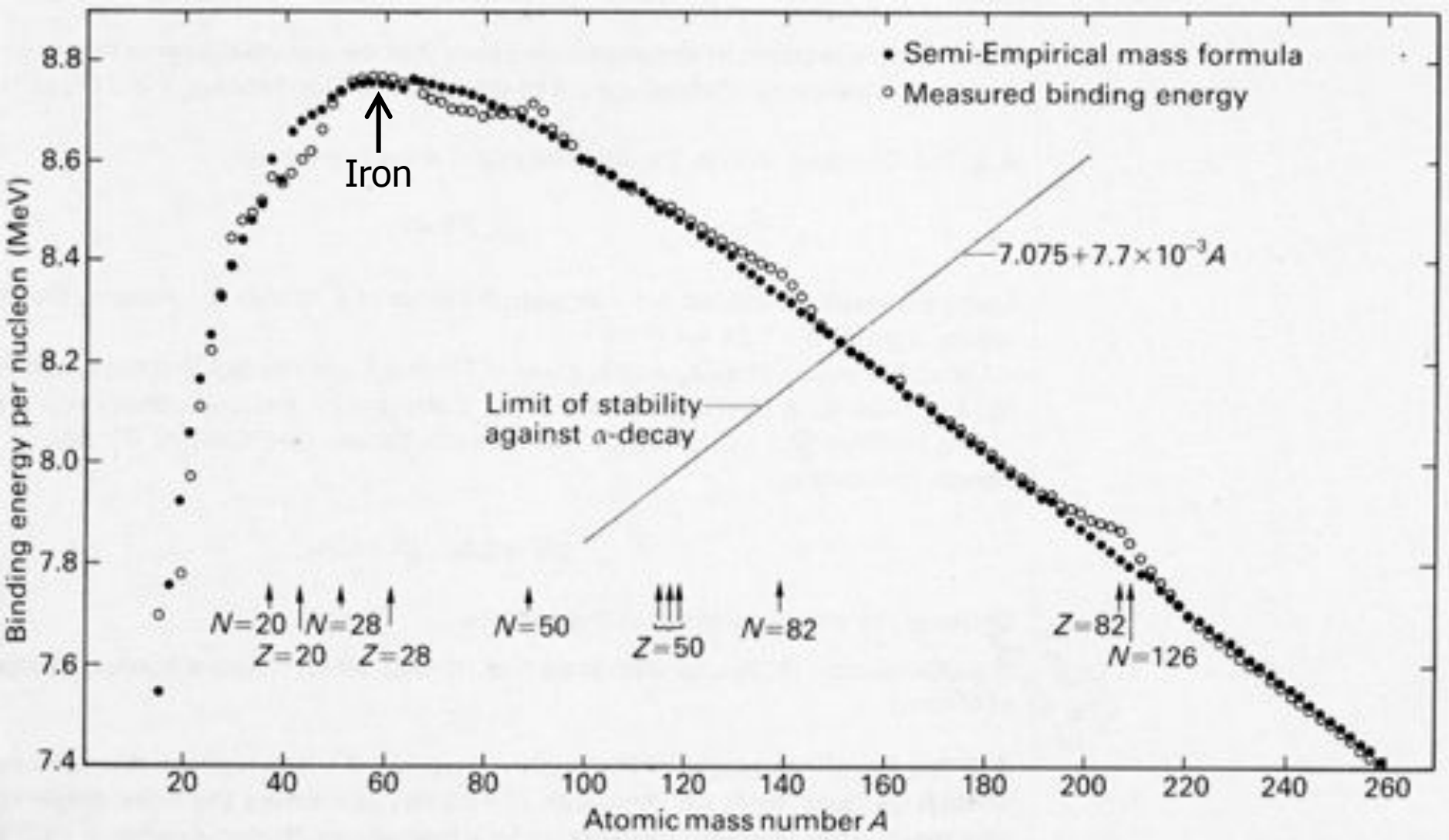
Also as go to heavier nuclei the electrical repulsion of the protons increases as Z^2 . What was negligible becomes significant and above iron actually makes the net binding per nucleon decrease.

That is not to say that the binding energy of the heavier nucleus is smaller, but that the energy per proton or per neutron is less.

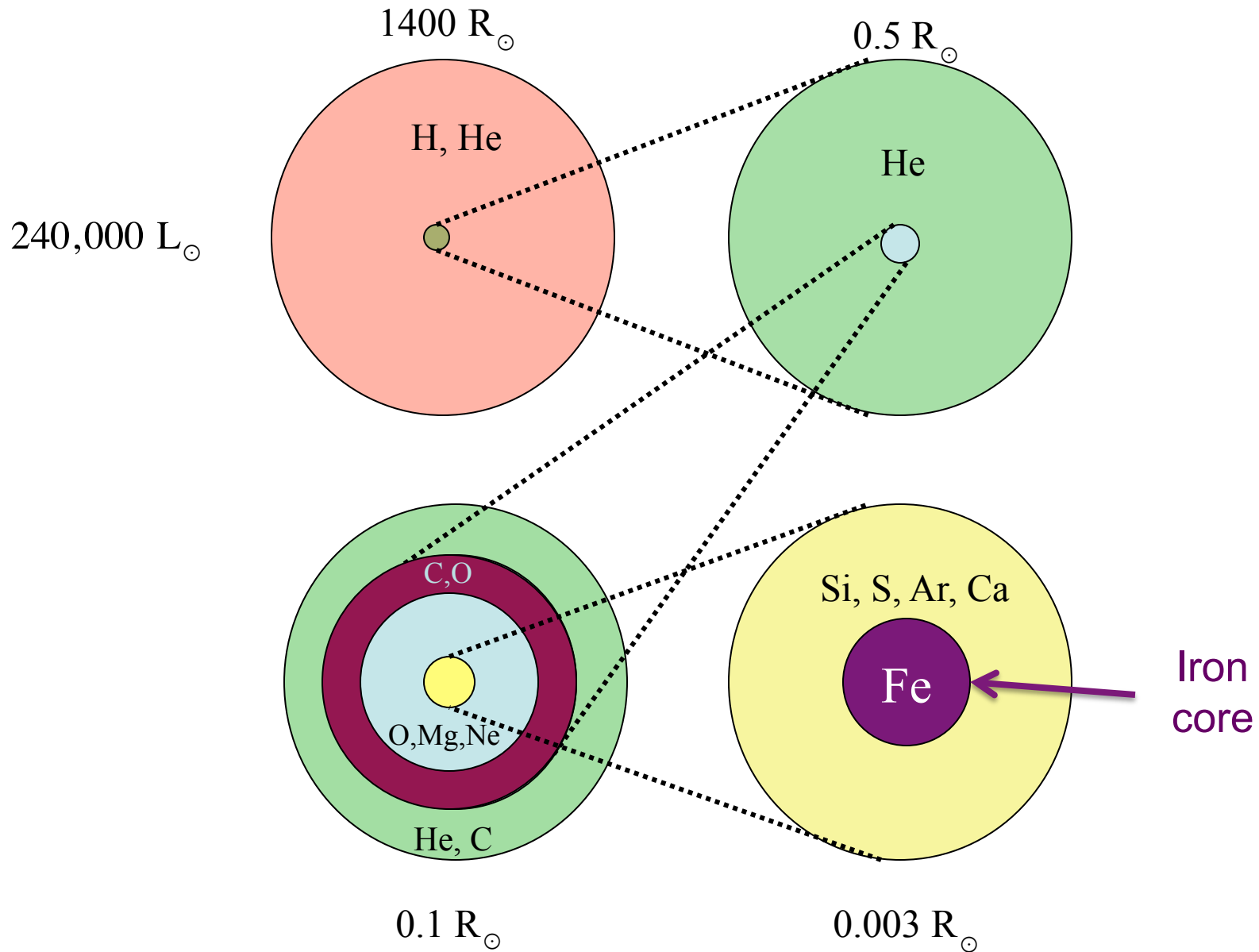
That means that having A nucleons in one nucleus is less tightly bound than having the same number of nuclei in two lighter nuclei that sum to A .

Semi Empirical Mass Formula

Binding Energy vs. A for beta-stable odd- A nuclei



25 M_⊙ Presupernova Star (typical for 9 - 130 M_⊙)



IRON CORE COLLAPSE

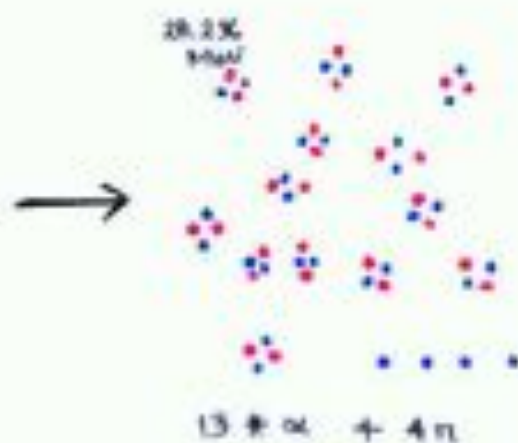
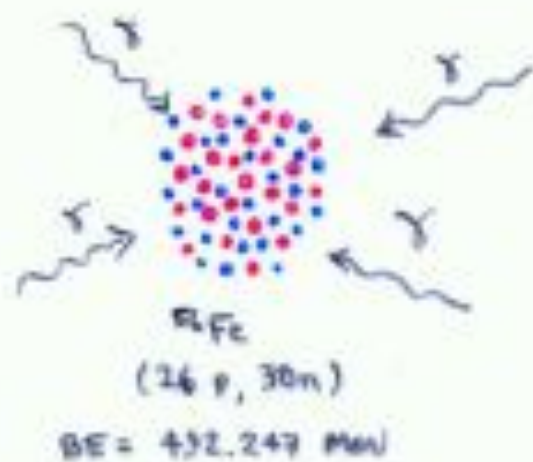
- Having exhausted silicon in the inner 1.3 to 2.0 solar masses of the star, the center of the star has no further nuclear energy resources
- It is not degenerate though, so it contracts and grows denser and hotter, looking for a new source of energy. None is found.
- As the temperature exceeds about 10×10^9 K, the typical photons on the blackbody have energy ~ 3 MeV (4 kT). Photons further out on the tail have enough energy (~ 8 MeV) to begin to rip nucleons out of the nucleus (analogue to ionization). The process does not go to completion but about 10% helium by mass is “boiled” out of the iron and this process (photodisintegration) saps energy that might have held up the star. The collapse accelerates.

- As the density goes up above $\rho \sim 10^9 \text{ g cm}^{-3}$, the Fermi energy of the electrons is also becoming several MeV. Electrons begin to capture on nuclei like ^{56}Fe turning them into nuclei with a larger neutron-to-proton ratio

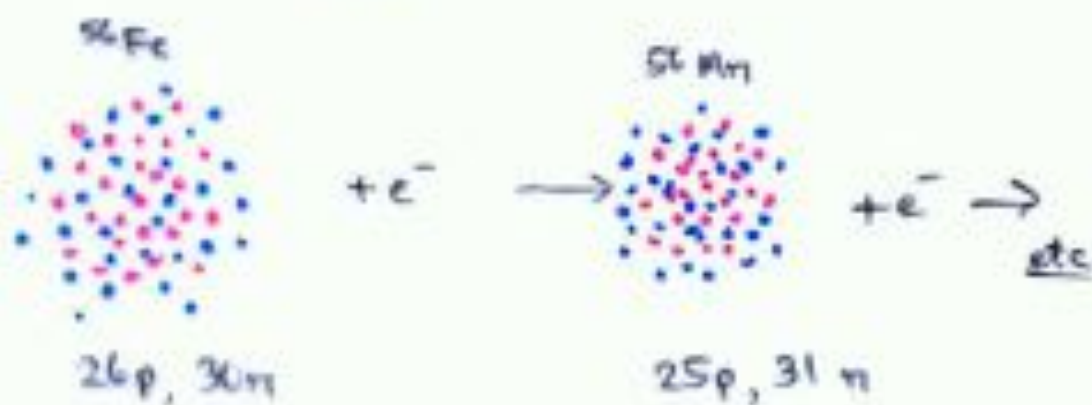


- Since these electrons were the chief source of pressure in the contracting core, their loss further accelerates the collapse.
- As the temperature and density continue to rise the emission of neutrinos by the pair process accelerates.

ALL OF THESE ARE BAD NEWS FOR THE STABILITY OF THE STAR!



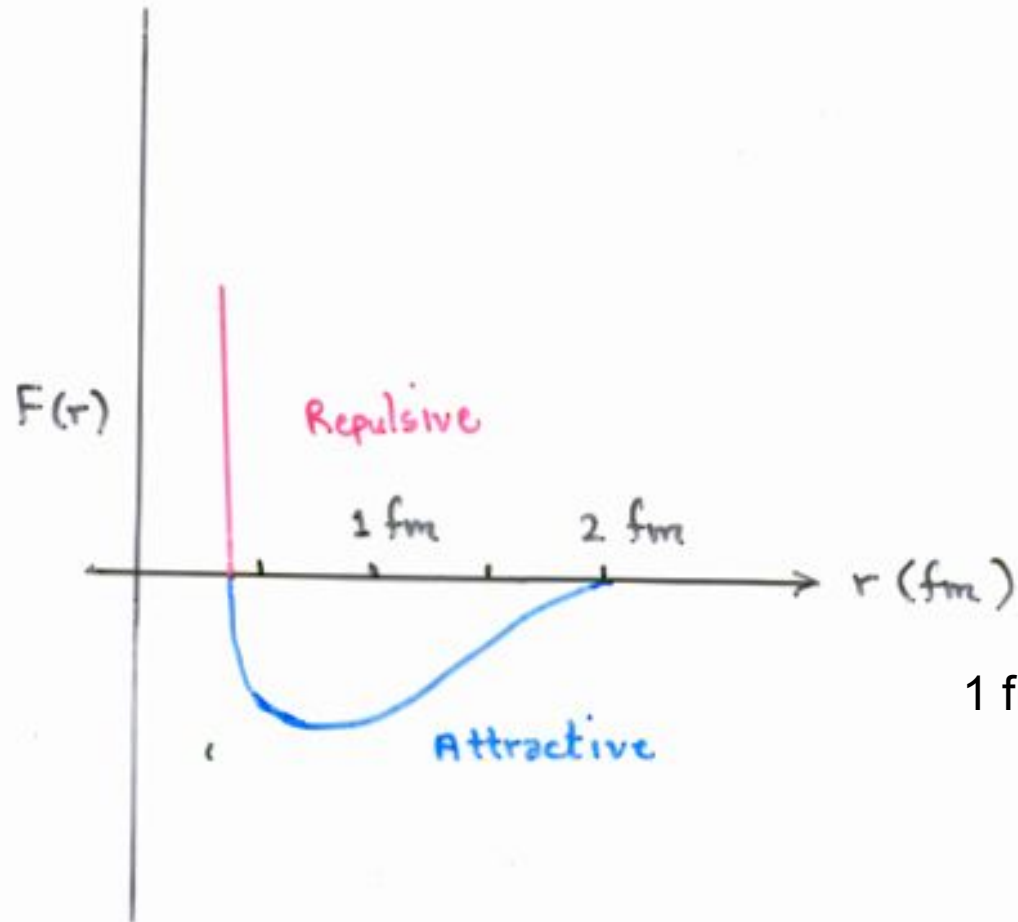
AND



COLLAPSE AND BOUNCE

- As a result of these instabilities, the iron core is soon collapsing in almost free fall
- At a density of several times 10^{11} g cm⁻³, the neutrinos start to be trapped. This provides some new pressure, but not enough to halt the collapse.
- As the density nears 2.4×10^{14} g cm⁻³, **the density of the atomic nucleus**, new forces come into play. First the attractive **and then the strongly repulsive part** of the strong force. The collapse in the central regions halts abruptly and rebounds.
- This rebounding inner core, about 0.7 solar masses, runs into the overlying collapse iron core at about 70,000 km/s. A “shock wave” forms.

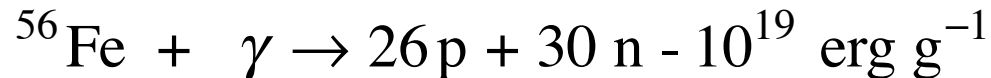
The strong force is much more complicated than e.g., the electric force and involves a repulsive component as well as an attractive one. At very short distances it is strongly repulsive.



$$1 \text{ fm} = 10^{-13} \text{ cm}$$

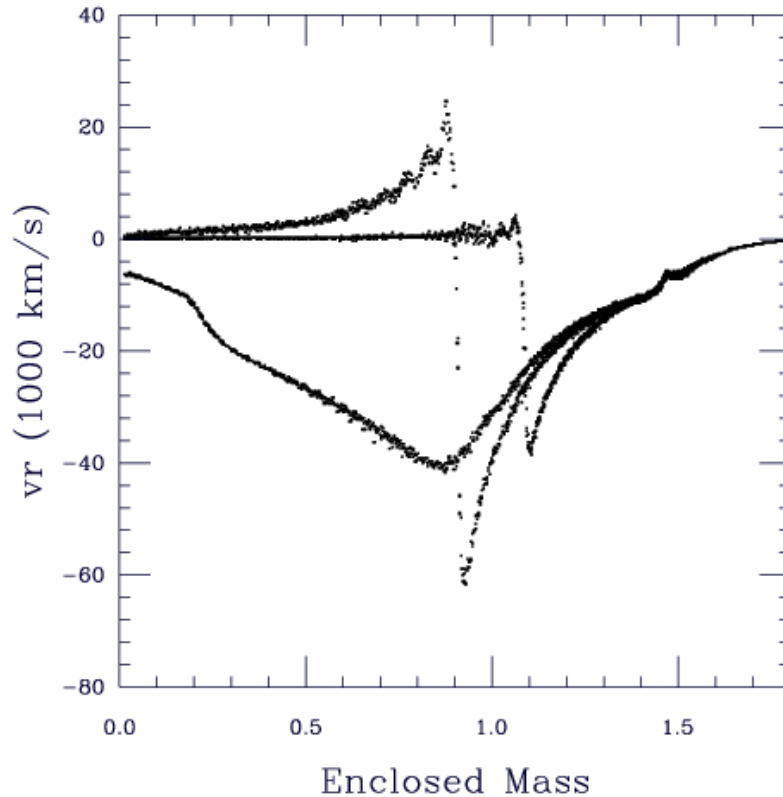
SHOCK PROPAGATION AND STALL

- In the center, temperatures rise above 10^{11}K . As the shock wave begins to move out material is halted and turned around, but it is heated to such high temperature that the iron is disintegrated to unbound neutrons and protons. This costs a lot of energy. All burning since the main sequence is undone



- The shock weakens further because of neutrino cooling of the matter inside it. By the time it has passed through all the iron core, all outward velocity has been lost.
- As matter settles to super-nuclear density, most of the protons turn to neutrons. A giant nucleus with mass ~ 1.5 solar masses called a proto-neutron star is formed. All this took about 10 milliseconds.

DEATH OF THE SHOCK



Death of the shock in a star of 15 solar masses.

The shock is born near 0.7 solar masses. Initially the bounce gives it positive kinetic energy, but for each 0.1 solar masses it traverses and photodisintegrates about 10^{51} erg of energy is lost. Additional energy is lost to neutrinos as the shock moves to low densities, $\rho \approx 10^{11}$ gm cm⁻³.

After about 10 ms the once powerful shock has stalled and become an [accretion shock](#).

REBIRTH OF THE SHOCK

- The “prompt shock” has died, or more correctly become an “accretion shock”. But now the proto-neutron star experiences its **Kelvin-Helmholtz** evolution as it contracts from $R \sim 50 \text{ km}$ to $R \sim 10 \text{ km}$.
- During this phase the neutron star radiates away its binding energy, approximately $\sim 3 \times 10^{53} \text{ erg}$, as neutrinos (of all flavors). Brighter than the rest of the universe combined!!
- Most of these neutrinos escape without interaction but a few per cent deposit energy in the neutron star atmosphere (the region between the accretion shock and the neutrino “photosphere”, or “neutrinosphere”)
- The shock is re-energized by the hot radiation and pairs created beneath it and expands outwards again

INTERLUDE - NEUTRON STARS

Neutron stars are “stars” (actually giant nuclei) supported by *neutron* degeneracy pressure and the strong force.

Redoing the derivation of non-relativistic electron degeneracy pressure for neutrons, one gets the same answer except that the mass of the neutron substitutes for the mass of the electron

$$P_{\text{deg}}^{NR} \propto \frac{n^{5/3}}{m_e \text{ or } m_{\text{neut}}}$$

For neutrons

$$P_{\text{deg}}^{NR} = 5 \times 10^9 \rho^{5/3} \text{ dyne cm}^{-2} \quad (i.e., \frac{P_{e,\text{deg}}}{1839})$$

The neutrons are more massive but move slower. The Fermi momentum, p_F , is the same but $p = mv$ so v is 1839 times slower.

The pressure goes as mv^2 . The neutrons are never relativistic.

Complication: Cannot ignore the strong force here.

NEUTRON STARS

$$P_{n,\text{deg}} \approx \frac{GM\rho}{2R} \quad \text{if constant density}$$

$$5 \times 10^9 \rho^{2/3} = \frac{GM}{2R}$$

$$5 \times 10^9 \left(\frac{3M}{4\pi R^3} \right)^{2/3} = \frac{GM}{2R}$$

$$5 \times 10^9 \left(\frac{3}{4\pi} \right)^{2/3} \frac{M^{2/3}}{M} \frac{2}{G} = R$$

$$R \approx 4.6 \text{ km} \left(\frac{M_{\odot}}{M} \right)^{1/3}$$

Actually

$$R \approx 10 \text{ km} \left(\frac{1.4 M_{\odot}}{M} \right)^{1/3}$$

note – shrinks
as M rises

NEUTRON STAR ALMOST A BLACK HOLE

The Schwarzschild radius for a 1.4 solar mass black hole is

$$R_S = \frac{2GM}{c^2}$$

or 4 km. Neutron stars are close to being black holes. Their escape speed is about $1/3 c$ and their binding energy is about 20% mc^2

The average density of a neutron star, $3M/4\pi R^3$, is $\sim 10^{15} \text{ g cm}^{-3}$, greater than the density of an atomic nucleus

The gravitational binding energy of the neutron star is enormous, a significant fraction even of its rest mass $Mc^2 = (1.4)(2 \times 10^{33})(3 \times 10^{10})^2 = 2.5 \times 10^{54}$ erg

$$\Omega \approx \frac{3}{5} \frac{GM^2}{R} = 0.6 \frac{(6.7 \times 10^{-8})(2.8 \times 10^{33})^2}{10^6}$$
$$= 3.1 \times 10^{53} \text{ erg}$$

Actually $2 - 3 \times 10^{53}$ erg when the realistic structure is used.

Where does all this energy go?

In the end the neutron star is supported by degeneracy and the strong force not heat. All the energy diffuses out as neutrinos. A neutrino mediated Kelvin Helmholtz stage

- During its roughly 3 second Kelvin-Helmholtz time, the luminosity of the neutron star in (all flavors of) neutrinos is $L \sim 1 \times 10^{53} \text{ erg s}^{-1}$. Contrast that to the luminosity of the Milky Way galaxy, about $10^{44} \text{ erg s}^{-1}$, or the luminosity of the entire observable universe.

$$\begin{aligned} L_{\text{univ}} &\sim 3 \times 10^{10} \text{ galaxies} \times 10^{43} \text{ erg s}^{-1} \text{ per galaxy} \\ &\sim 3 \times 10^{53} \text{ erg s}^{-1} \end{aligned}$$

(very approximate)

The measured light density of the universe is $2 \times 10^8 L_{\text{sun}}$ per cubic megaparsec – gives a similar number

- These neutrinos are emitted equally in the 6 different flavors - $\nu_e, \bar{\nu}_e, \nu_\tau, \bar{\nu}_\tau, \nu_\mu, \bar{\nu}_\mu$ roughly like a blackbody

$$L_{\nu_e} \approx \frac{7}{8}(4\pi R^2 \sigma T_\nu^4) \\ \approx 10^{52} \text{ erg s}^{-1}$$

which can be solved for the temperature, $T_\nu \approx 6 \times 10^{10}$ K, or energy about 5 MeV (1 MeV = 11.605 billion K).

- Just as there is a Chandrasekhar mass limit for white dwarfs there is also a limiting mass for the heaviest neutron star that can exist. This is a much less certain number but is about $2 M_\odot$.

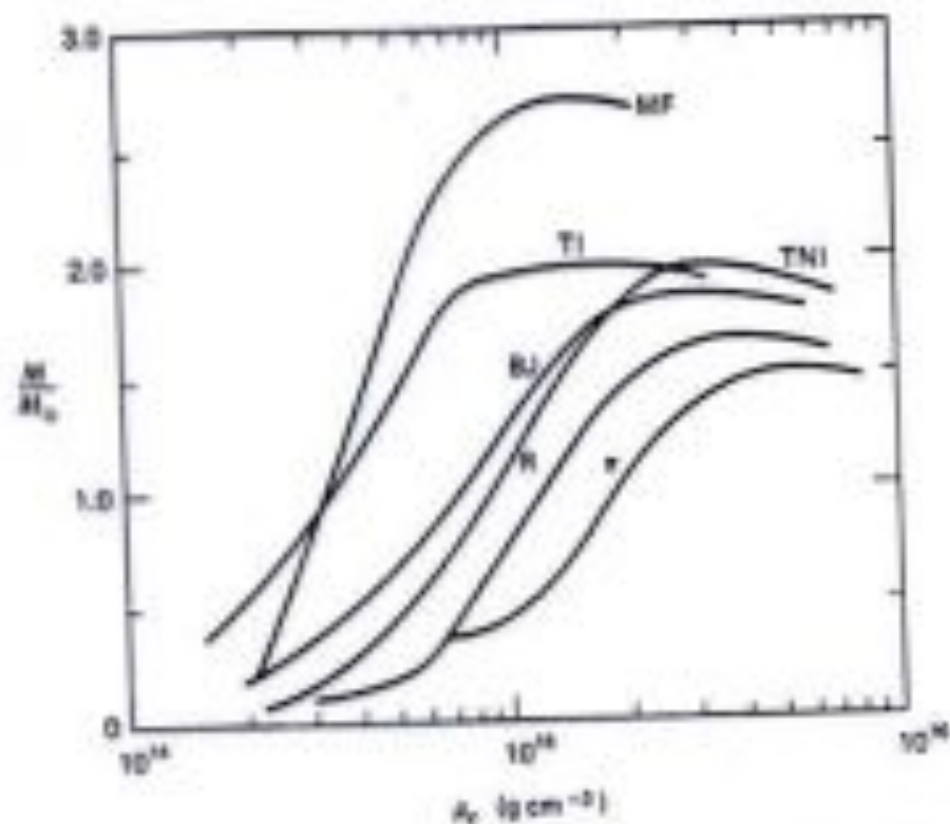
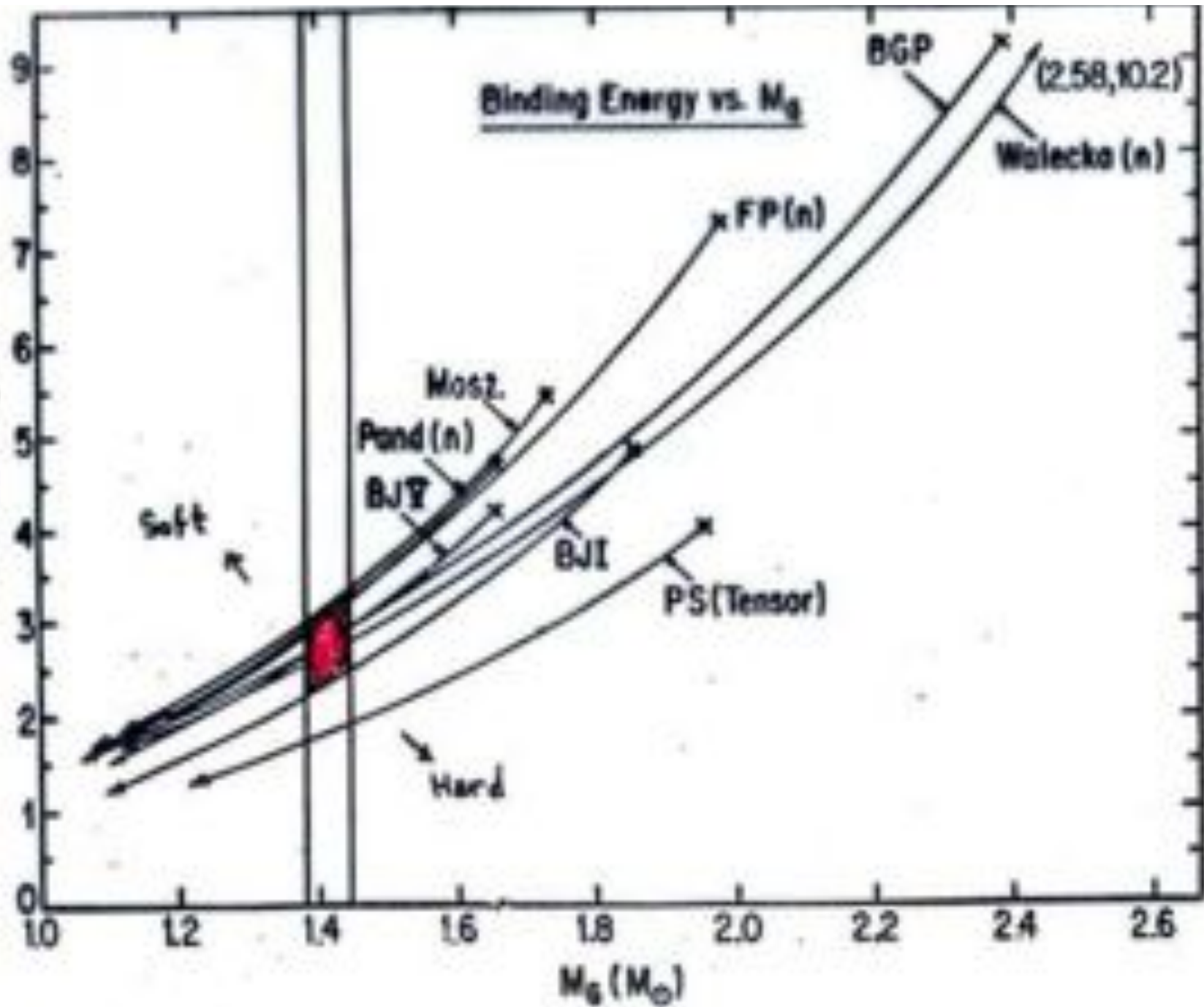


Figure 9.2 Gravitational mass vs. central density for various equations of state. The letters labeling the different curves are identified in Table 8.2 with the exception of *e*, which denotes a Reid equation of state modified by charged-pion condensation. The ascending portions of the curves represent stable neutron stars. [After Baym and Pethick (1979). Reproduced with permission, from the *Annual Review of Astronomy and Astrophysics*, Vol. 17. © 1979 by Annual Reviews Inc.]

Binding Energy (10^{53} erg)



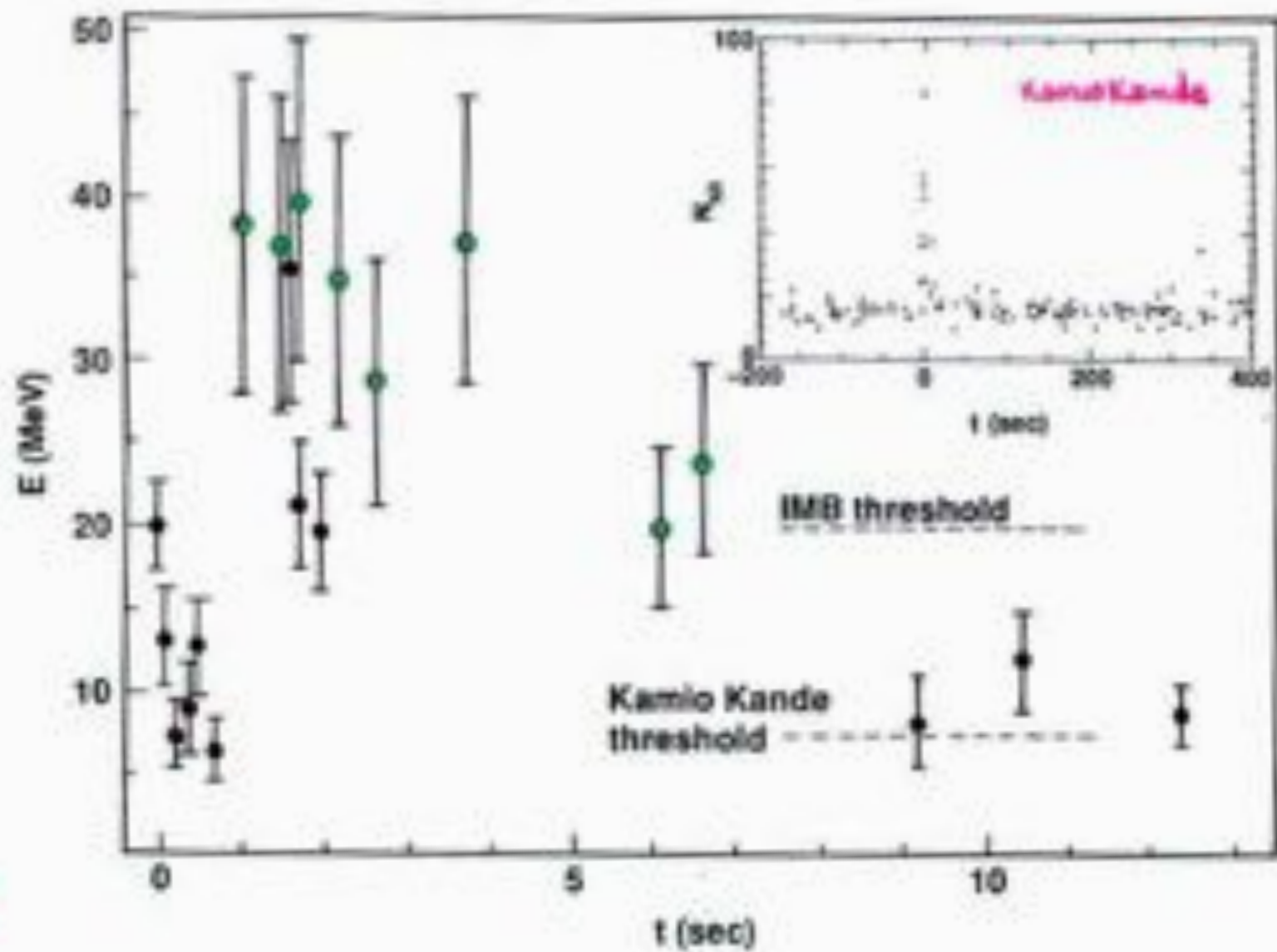
The heaviest neutron star detected yet (PSR J1614-2230) has a mass of 1.97 ± 0.04 solar masses. This rules out many of the “softer” equations of state.

Demorest et al, Nature, 467, 1081, (2010)

NEUTRINO BURST SN 1987A

FEBRUARY 23, 1987

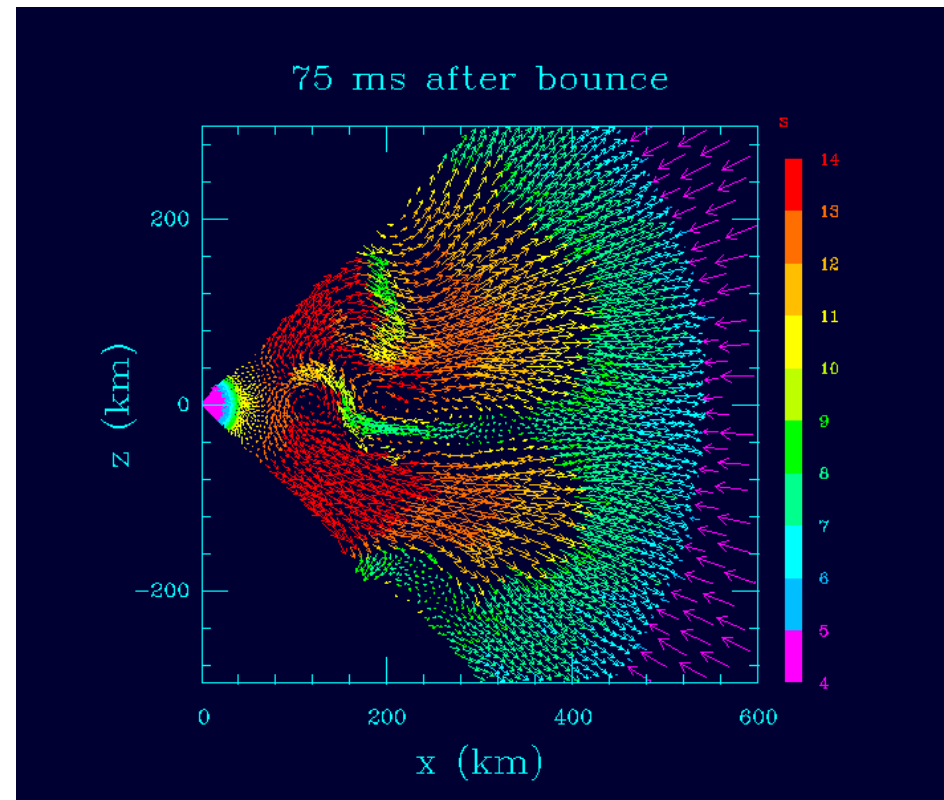
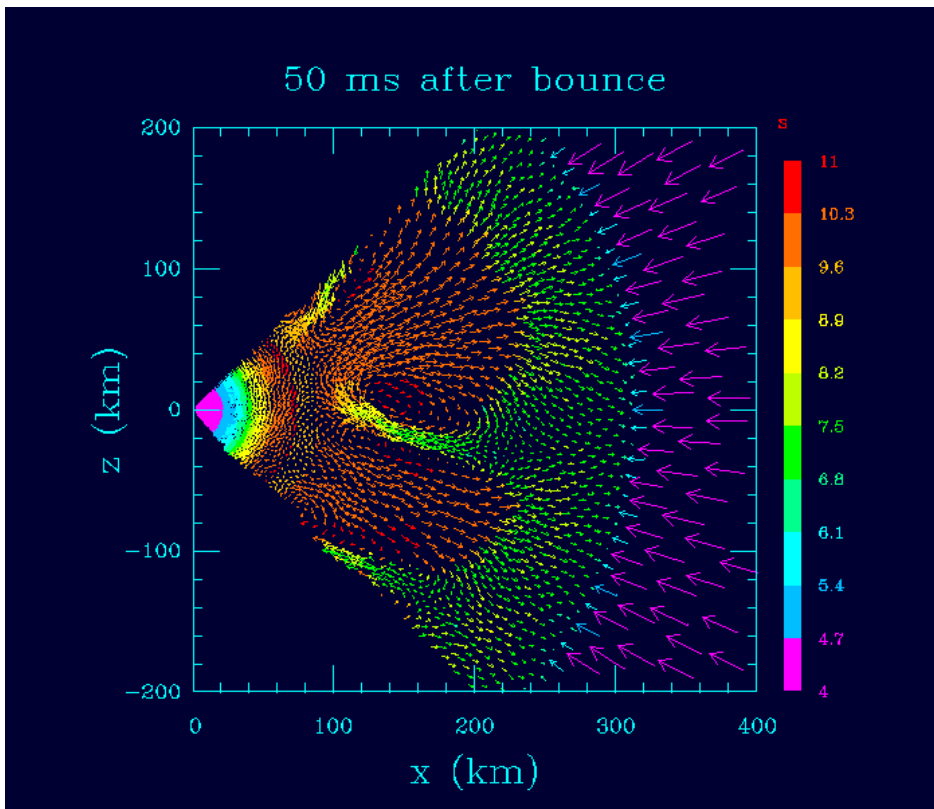
- Originated from SN 1987A in the Large Magellanic Cloud about 55 kpc from here – first signal from the supernova though the optical light was detected first (about 6 hr later) and the neutrino signal only discovered by processing data about a week later
- Detected in three locations – Kamiokande (Japan), IMB (Cleveland), and Baksan (USSR) – all in northern hemisphere. The neutrinos had come through the earth
- Observed at Kamiokande and IMB – 18 neutrino events from 8 to 40 MeV. Inferred neutrino temperature 5 MeV – or about 60 billion K



(2) Kam.
(3) IMB

- Total energy inferred at the source about 2 to 5×10^{53} erg. Duration about 10 seconds with most emission occurring during the first 3 seconds
- Neutrino flux at the earth about 5×10^{10} neutrinos $\text{cm}^{-2} \text{s}^{-1}$.
- Arrival time at same time as the light (within 6 hours) after traveling for 160,000 years put limits on the mass of the neutrino. The neutrinos had to travel very close to the speed of light
- Properties of the burst in overall good agreement with the theory.

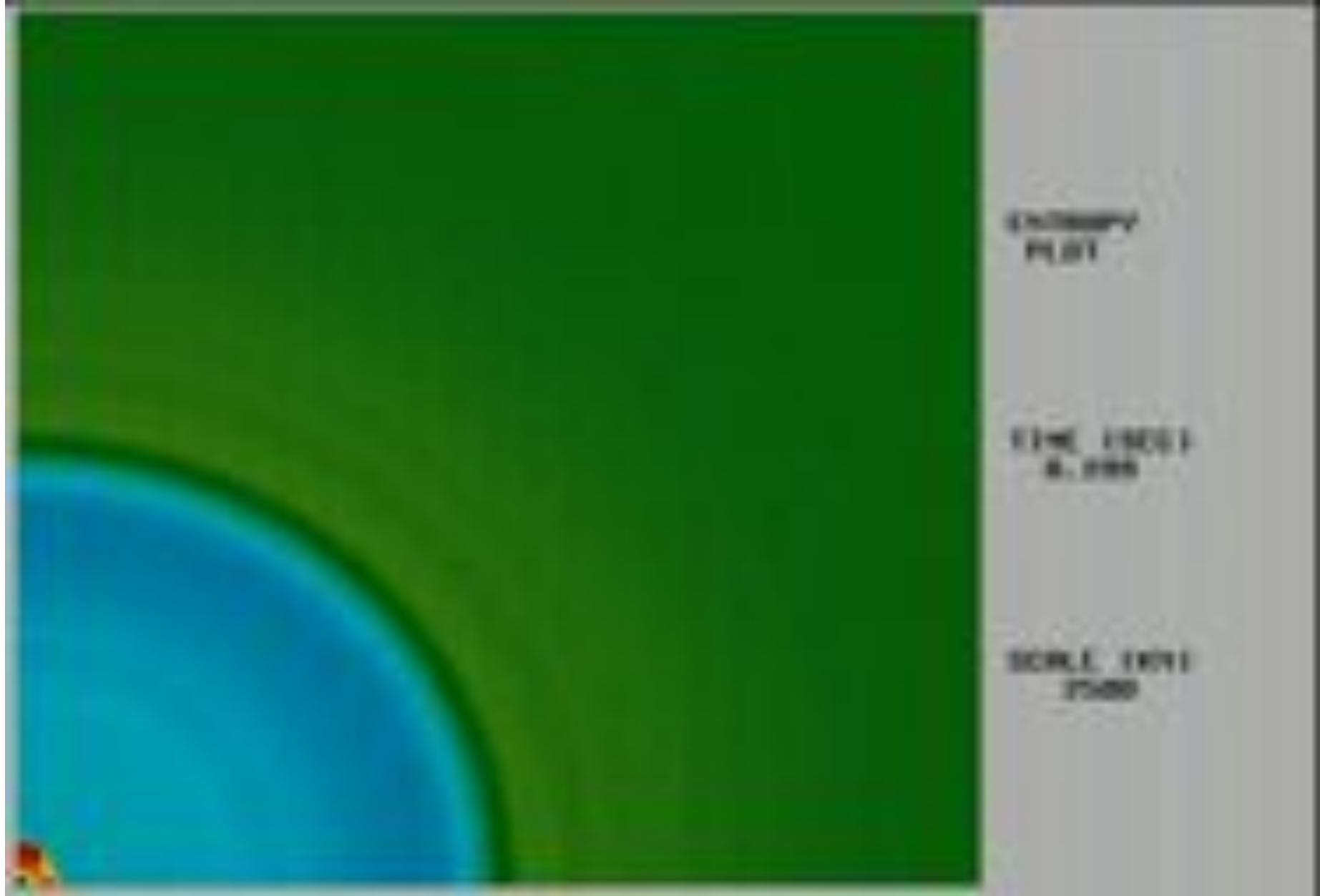
BACK TO THE SHOCK



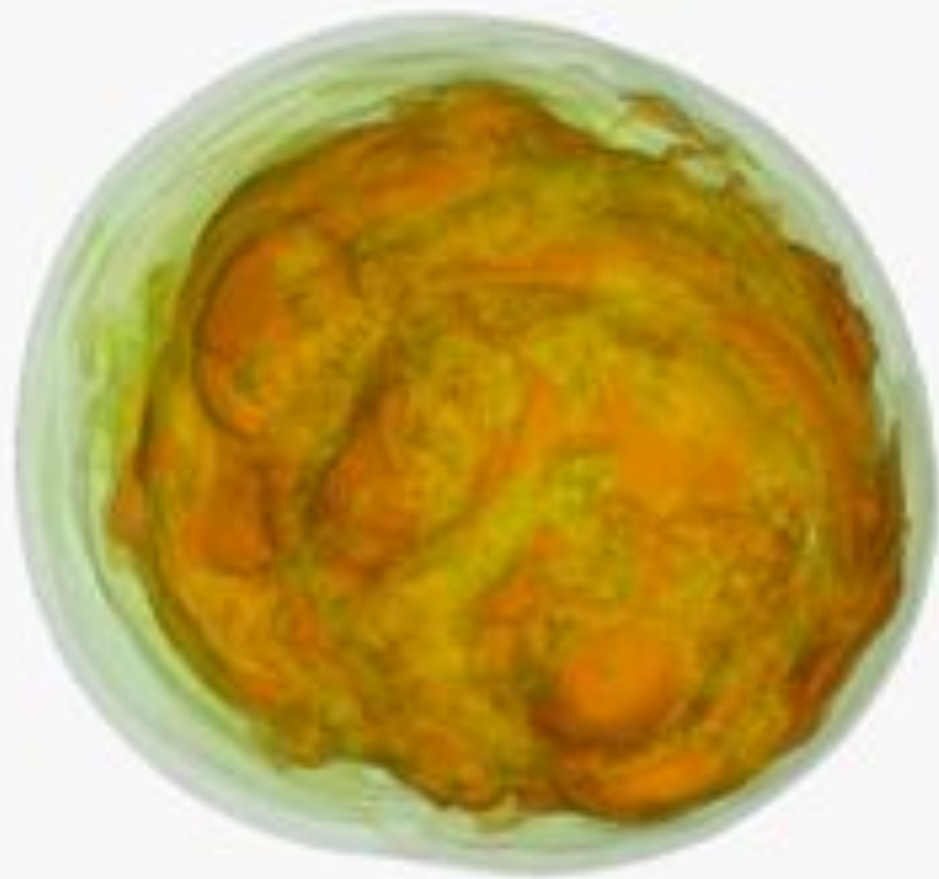
Herant and Woosley, 1995. 15 solar mass star.
successful explosion.
(see also Herant, Benz, & Colgate (1992), *ApJ*, **395**, 642)



15 Solar masses – explodes with an energy of order 10^{51} erg.



At 408 ms, $KE = 4.2 \times 10^{50}$ erg, stored internal energy is 3.8×10^{50} erg, and the total explosion energy is still growing at 4.4×10^{51} erg s^{-1} .



Burrows and Nordhaus (2011)
3D explodes easier than 2D





ismod2p_r04k
B-Field
Time = -178.5 ms
Radius = 100.00 km

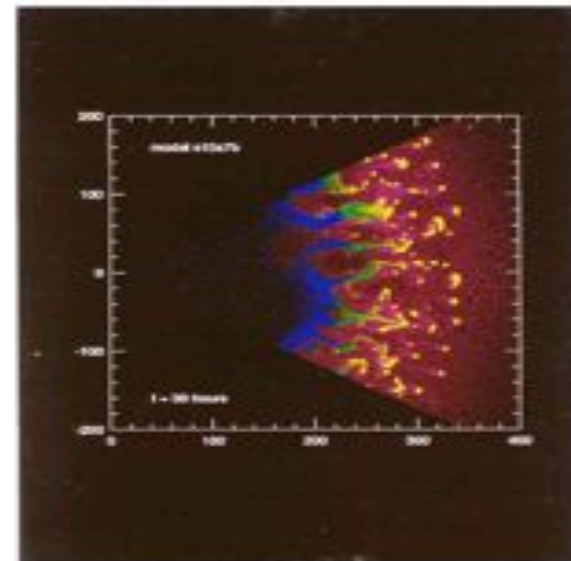
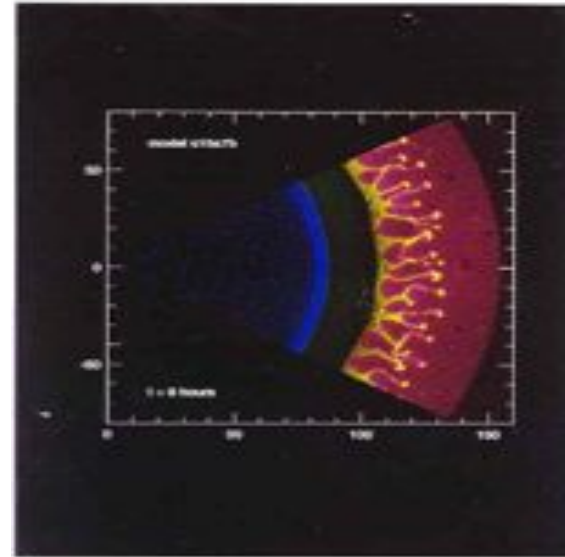
Explosion

- The “shock” moves out through the star. Everything external to the neutron star is ejected (though in larger stars ($M > 30 M_{\odot}$) some material, up to several solar masses may fall back, depending on the explosion energy). The final kinetic energy of the ejecta is about 10^{51} erg.
- All of the heavy elements made by nuclear reactions during the star's life (external to the neutron star) are also ejected as well as some new elements made in the explosion itself. One very important species made in the explosion is ^{56}Ni . This nucleus decays with a half-life of 6.1 days to ^{56}Co which in turn decays (77.1 days) to ^{56}Fe , the most abundant form of the element iron.
- As the shock erupts from the surface of the star, one day later, the supernova display begins.
- Much later when all the debris has expanded away, all that is left is a neutron star or black hole - both are stable for essentially forever.

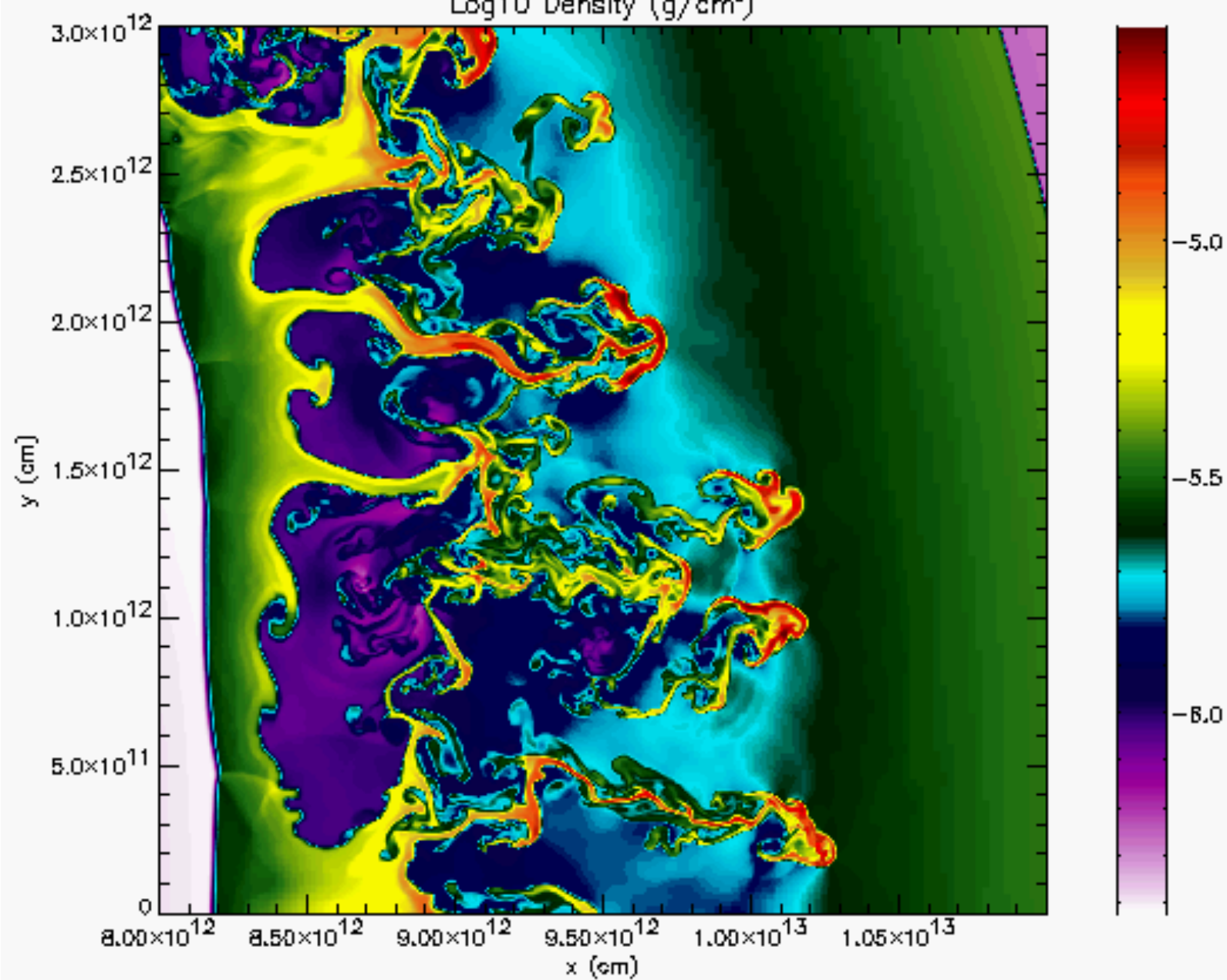
As the expanding helium core runs into the massive, but low density hydrogen envelope, the shock at its boundary decelerates. The deceleration is in opposition to the radially decreasing density gradient of the supernova.

Rayleigh-Taylor instability occurs.

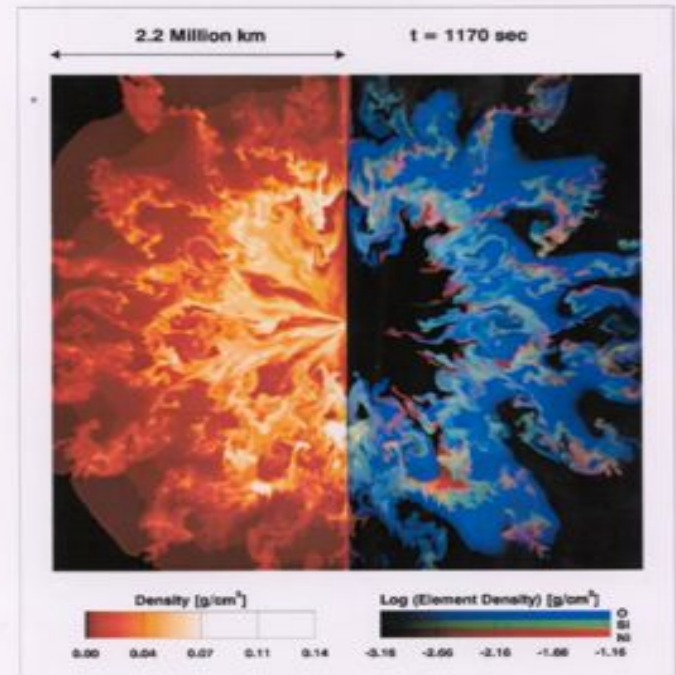
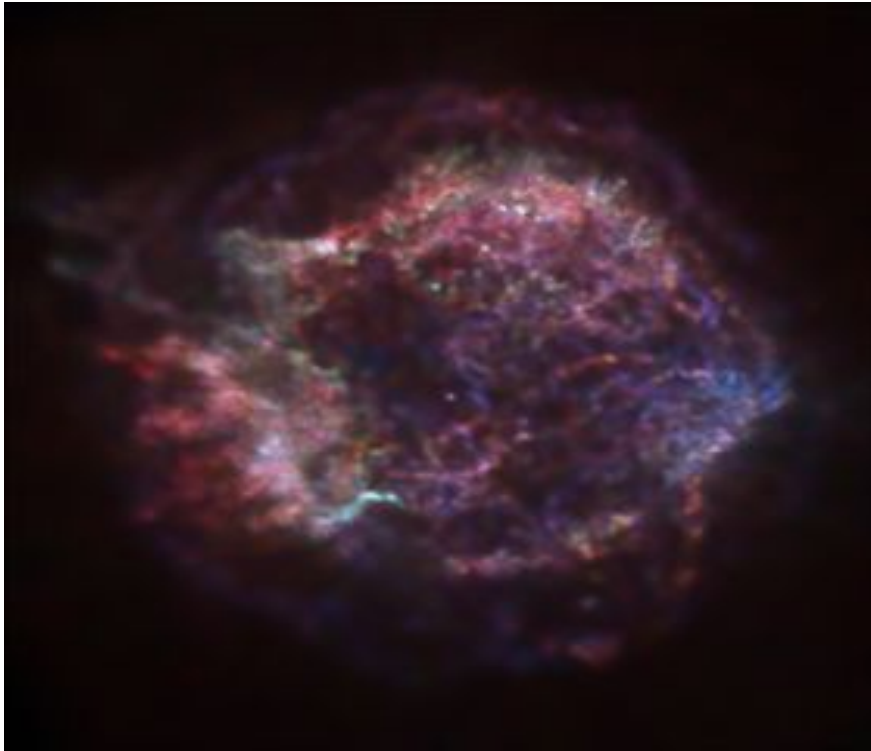
Red is hydrogen, yellow is helium, green is oxygen, and blue is iron. Radius is in solar radii.



Log10 Density (g/cm^3)



Kifonidis et al. (2001), *ApJL*, 531, 123



Left - Cas-A SNR as seen by the Chandra Observatory Aug. 19, 1999

The red material on the left outer edge is enriched in iron. The greenish-white region is enriched in silicon. Why are elements made in the middle on the outside?

Right - 2D simulation of explosion and mixing in a massive star - Kifonidis et al, Max Planck Institut fuer Astrophysik

*Explosive
Nucleosynthesis
in More Detail*

Explosive Nucleosynthesis

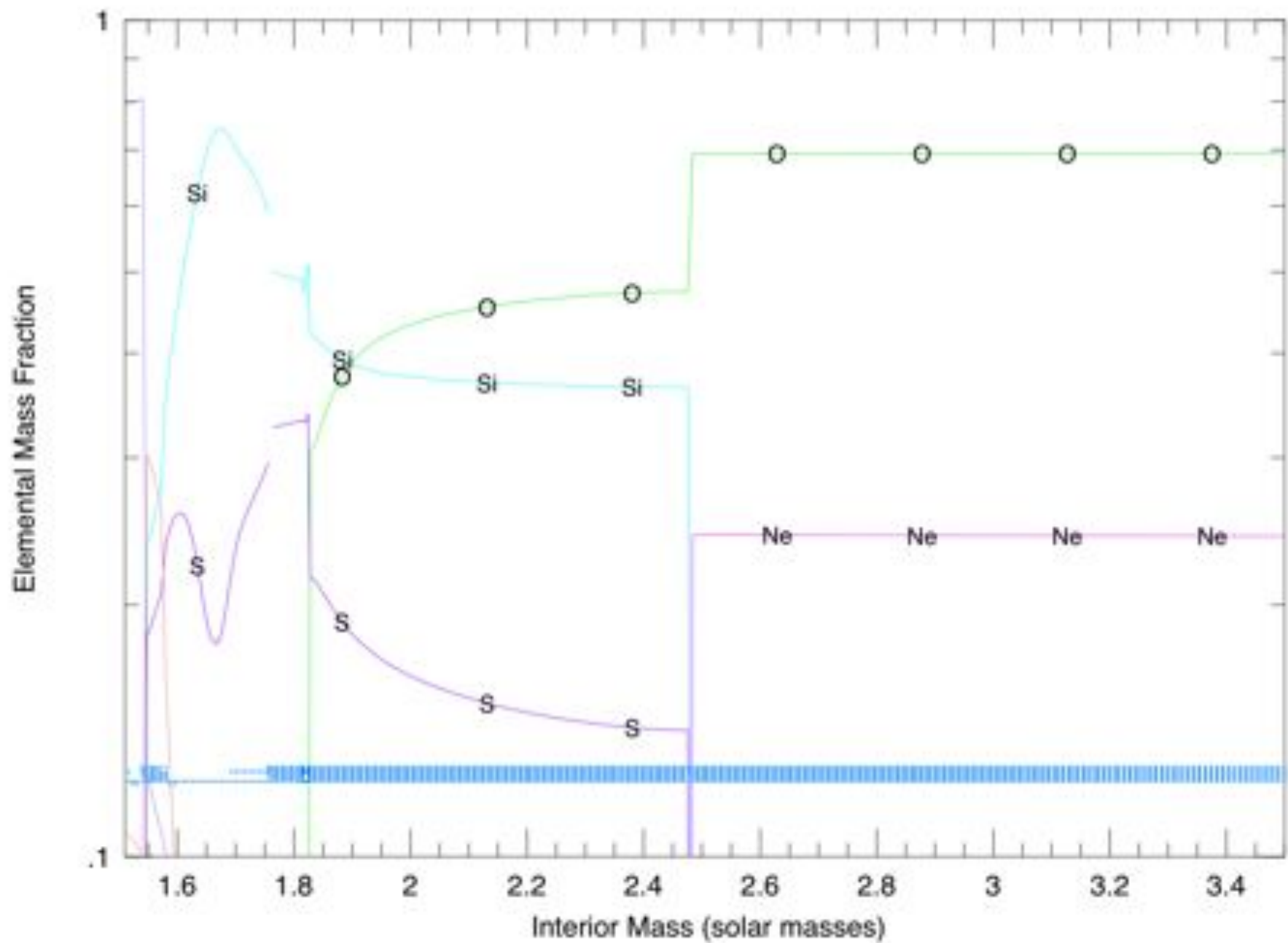
- As the shock wave propagates through the inner layers of the supernova, matter is abruptly raised to a higher temperature. Since nuclear reactions occur at rates that are very sensitive to the temperature, this causes an increase in the burning. New elements are created in seconds that it might otherwise have taken weeks and months to synthesize.
- Material heated to above 5 billion K is turned into “iron” (where the nuclear binding energy is maximal). Because there is no time for weak interactions, however, the nucleus produced has equal numbers of neutrons and protons, just like the fuel that burned. ^{56}Ni ($Z = N = 28$) is produced copiously
- Beyond the carbon burning shell, material is pushed off without much explosive processing

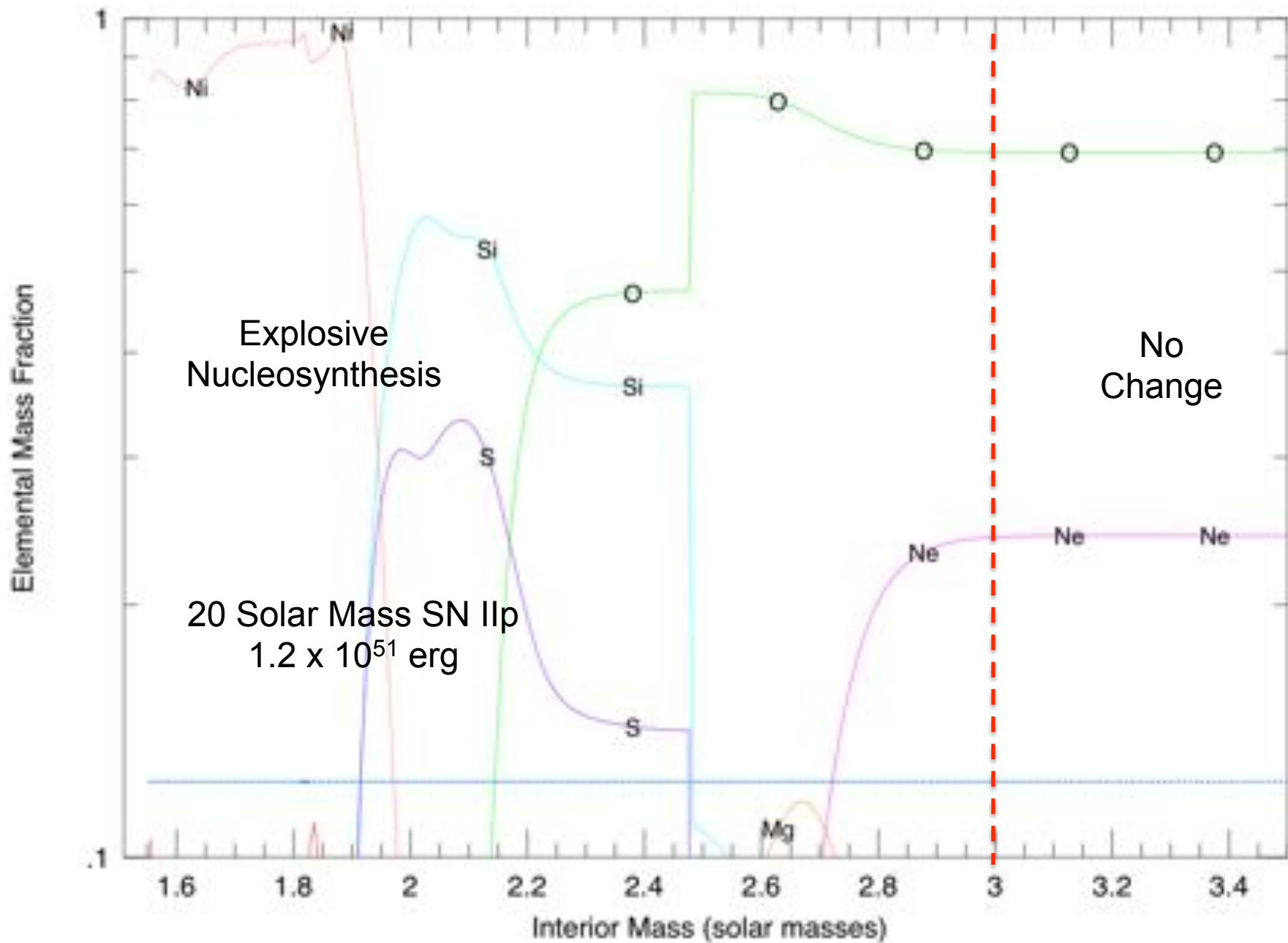
Explosive nucleosynthesis

$$\tau_{nuc} \leq \tau_{HD} = \frac{446}{\sqrt{\rho}} \text{ sec} \sim 1 \text{ sec}$$

τ_{nuc} is very temperature sensitive - proportional to the inverse of the reaction rates hence

$$\frac{1}{T^n} \quad \text{with } n \gg 1$$



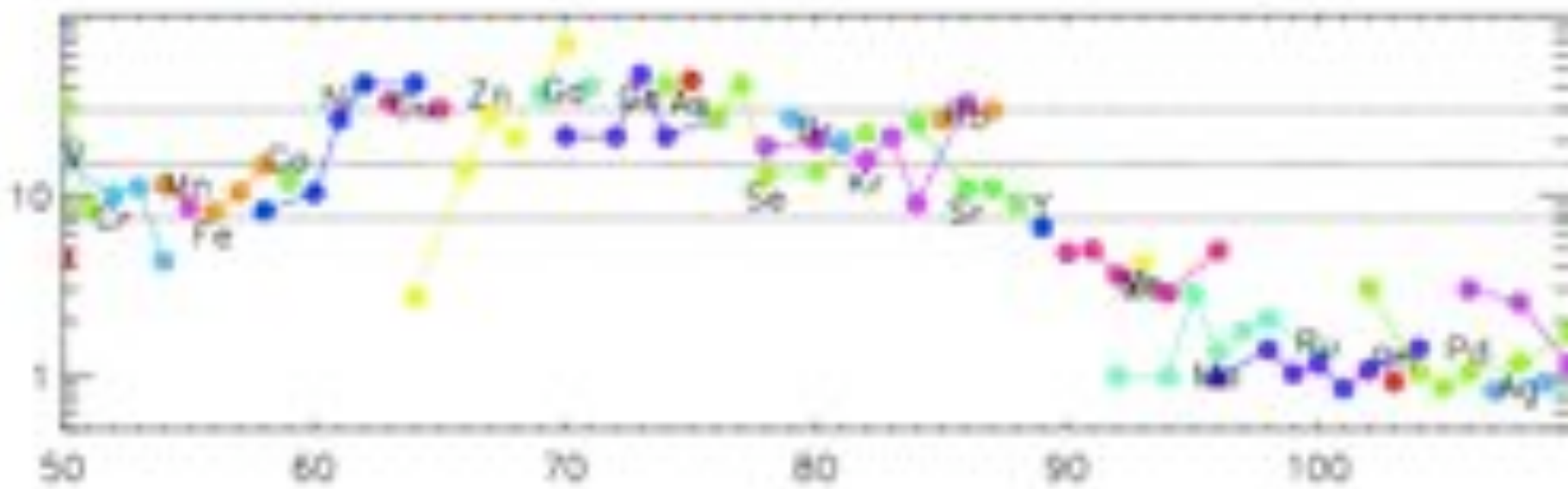
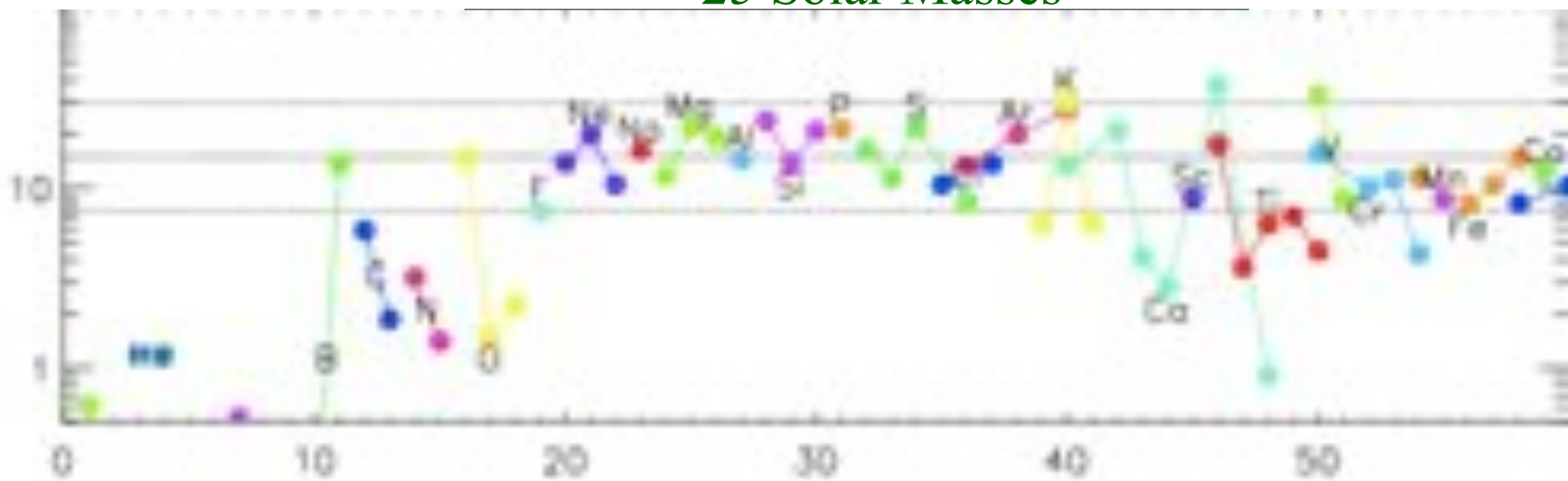


EXPLOSIVE NUCLEOSYNTHESIS

Fuel	Main Products	Secondary Products	Temperature (10^9 K)	Time (sec)
Si, O	^{56}Ni	Iron Group	> 4	0.1
O	Si, S	Ar, Ca, Cl, K	3 - 4	1

A single 25 solar mass supernova ejects 3 solar masses of oxygen, enough for several million Earths

25 Solar Masses



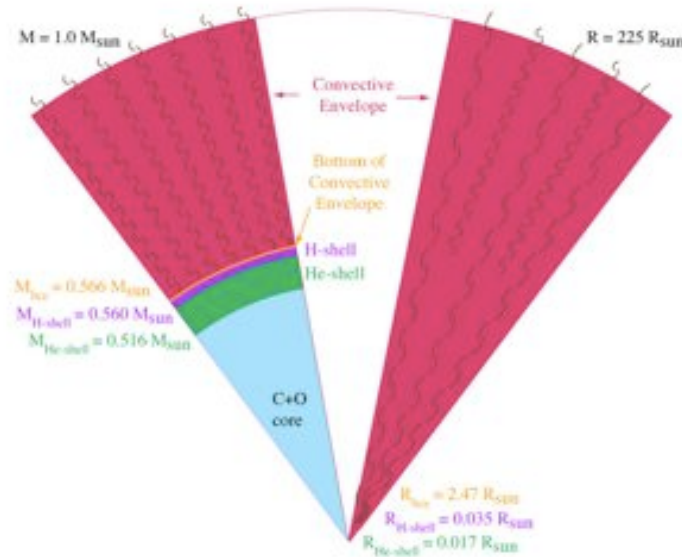
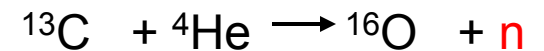
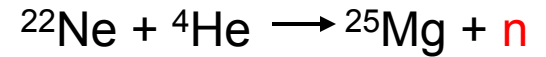
A 25 solar mass supernova ejects:

1.1 million Earth masses of	oxygen *
160,000	carbon *
26,000	nitrogen *
6,200	sodium
76,000	magnesium
100,000	silicon
1,000	phosphorus *
35,000	sulfur
244	chlorine
107	potassium
3,500	calcium *
51,000	iron
0.01	silver
0.006	gold

plus about 70 other elements

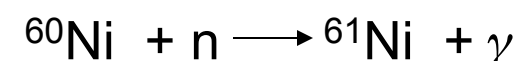
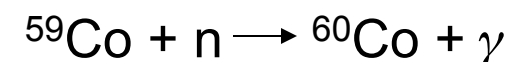
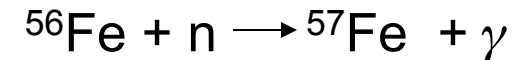
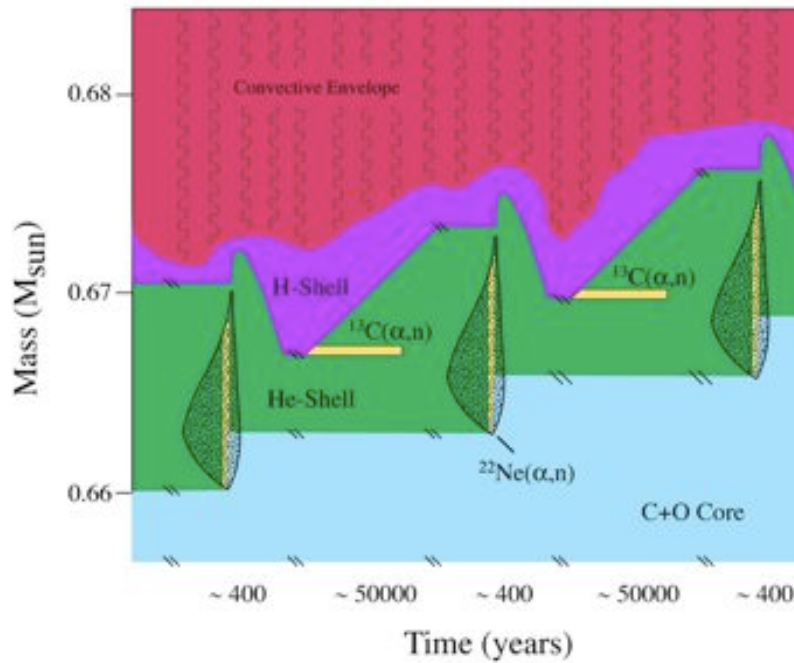
The “s-process”

During helium burning in a star of about 1.5 to 8 solar masses



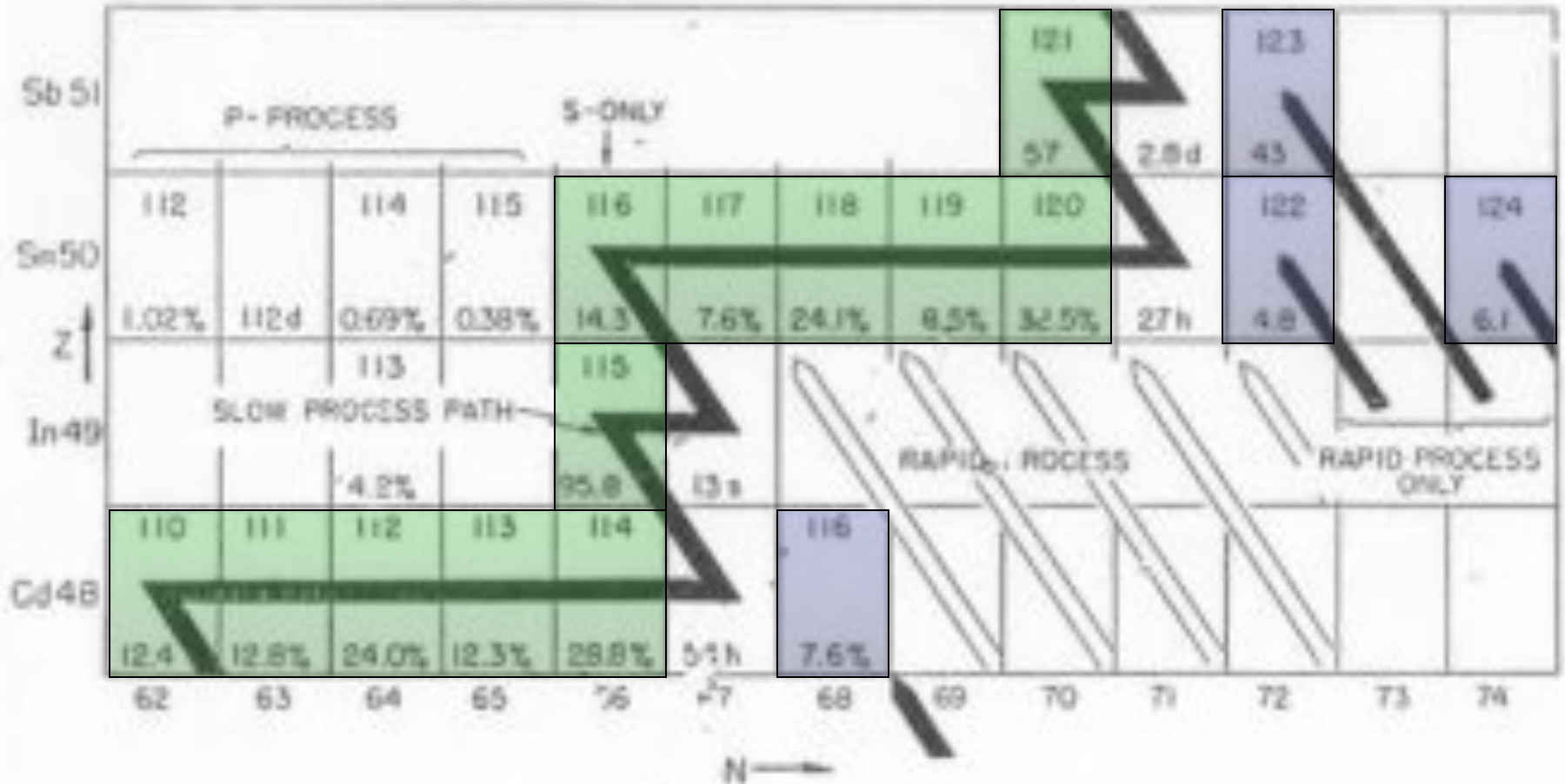
mass coordinates

radius coordinates

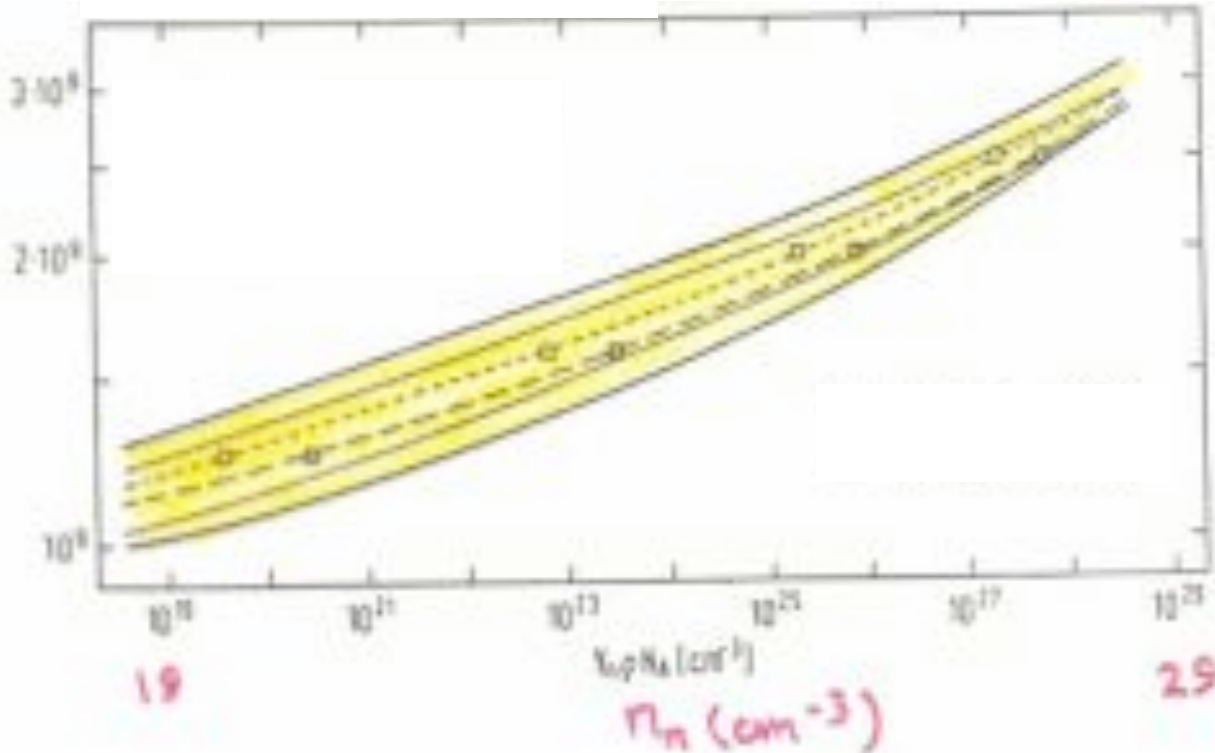


etc..... to lead

The *r*-Process



Optimal conditions for the r-process



For example, at $T_9=2.5$,
 $n_n \sim 10^{27} \text{ cm}^{-3}$ or about
a kilogram of neutrons
per cubic cm.

r-Process Site #1: The Neutrino-powered Wind

