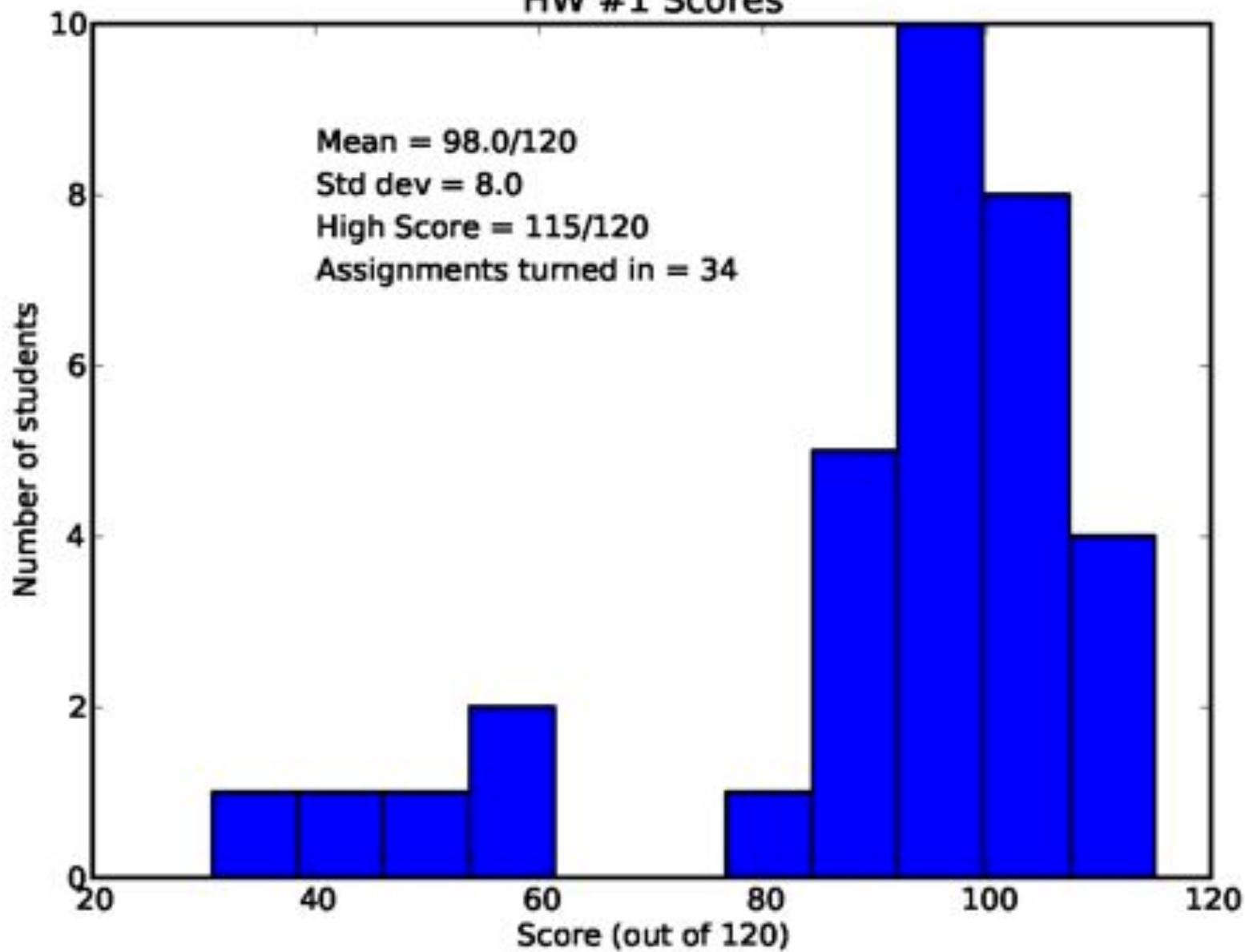


# *Quantum Mechanics and Stellar Spectroscopy*

<http://apod.nasa.gov/apod/>

### HW #1 Scores



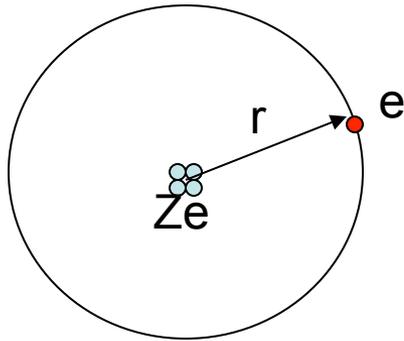
Recall the electric force. Like gravity it is a “1/r<sup>2</sup>” force/ That is:

$$F_{elec} = \frac{Z_1 Z_2 e^2}{r^2}$$

where  $Z_1$  and  $Z_2$  are the (integer) numbers of electronic charges. Similarly, the electric potential energy is

$$E_{elec} = -\frac{Z_1 Z_2 e^2}{r}$$

## Rutherford Atom (1911)



Protons in nucleus. Electrons orbit like planets. The neutron was not discovered until 1932 (Chadwick)

$$F_{elec} = F_{cent}$$

$$\frac{Ze^2}{r^2} = \frac{m_e v^2}{r}$$

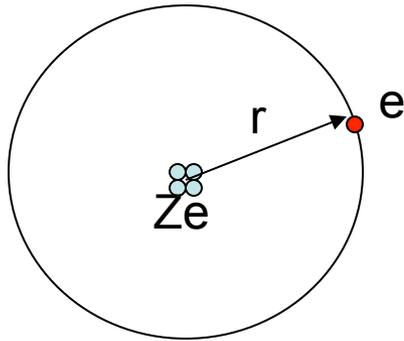
$$\Rightarrow r = \frac{Ze^2}{m_e v^2}$$

for a single electron

$Z = 1, 2, 3, \dots$   
H, He, Li, etc

*classically, any value of  $v$  or  $r$  is allowed. Much like planets.*

## Rutherford Atom (1911)



$$F_{elec} = F_{cent}$$

$$\frac{Ze^2}{r^2} = \frac{m_e v^2}{r} \Rightarrow$$

$$r = \frac{Ze^2}{m_e v^2}$$

$$Z = 1, 2, 3, \dots$$

Protons in nucleus. Electrons orbit like planets. The neutron was not discovered until 1932 (Chadwick)

Total energy:

*classically, any value of  $v$  or  $r$  is allowed.*

$$E_{tot} = KE + PE = \frac{m_e v^2}{2} - \frac{Ze^2}{r}$$

$$KE = \frac{1}{2} m_e v^2 = \frac{Ze^2}{2r}$$

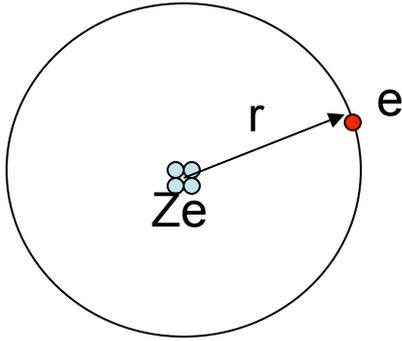
$$= \frac{Ze^2}{2r} - \frac{Ze^2}{r} = -\frac{Ze^2}{2r}$$

$$v = \sqrt{\frac{Ze^2}{m_e r}}$$

i.e.,  $2KE = -PE$  (if PE is negative)

Virial theorem still works for the electric force.

## Rutherford Atom (1911)



$$v = \sqrt{\frac{Ze^2}{m_e r}} \quad F = m_e a = \frac{m_e v^2}{r}$$

$$a = \frac{v^2}{r} = \frac{Ze^2}{m_e r^2} \quad E_{tot} = -\frac{Ze^2}{2r}$$

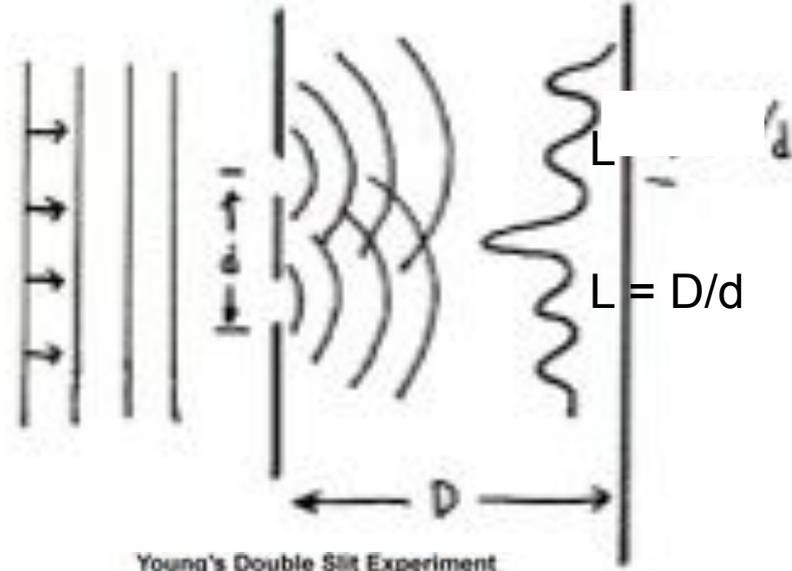
**BUT,**

As the electron moves in its orbit it is accelerated, and therefore emits radiation. Because energy is being radiated, the total energy of the system must decrease – become more negative.

This means  $r$  must get smaller and  $v$  must increase. But smaller  $r$  and larger  $v$  also imply greater acceleration and radiation.

In approximately  $10^{-6}$  s the electron spirals into the nucleus.  
Goodbye universe...

*The solution lies in the wave-like property of the electron – and of all matter*



For wavelike phenomena e.g., light, “interference” is expected

Thomas Young early 1800's for light.

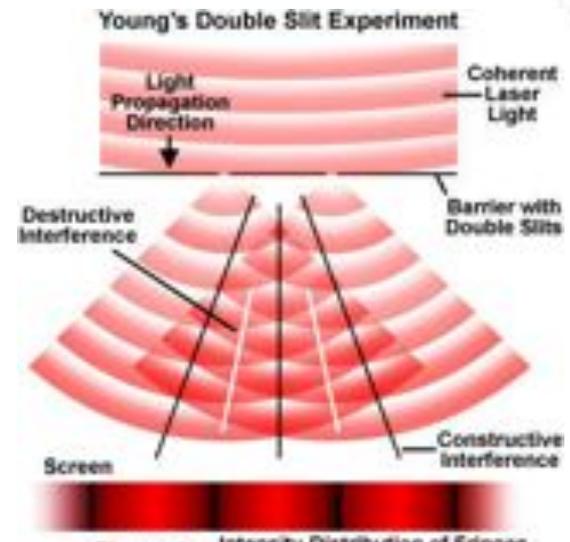
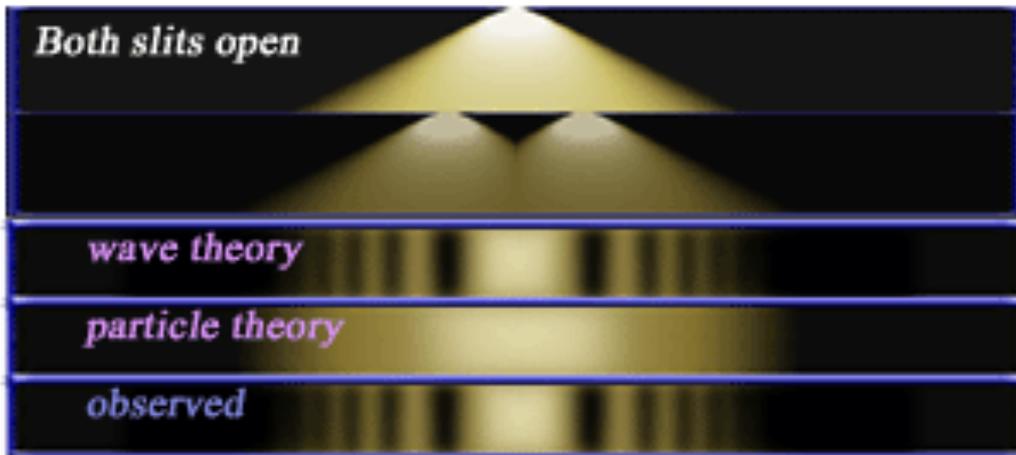
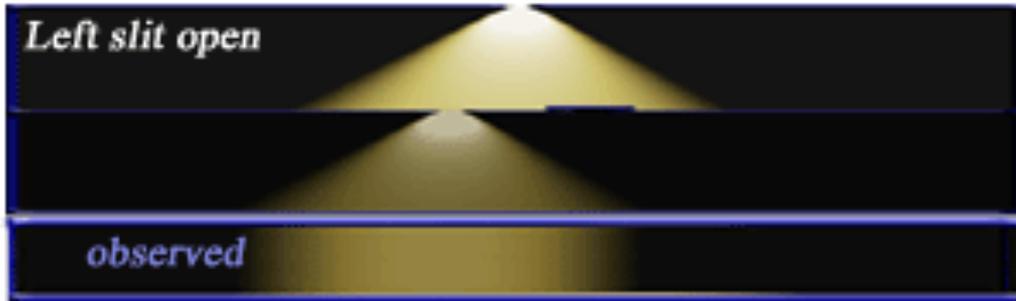
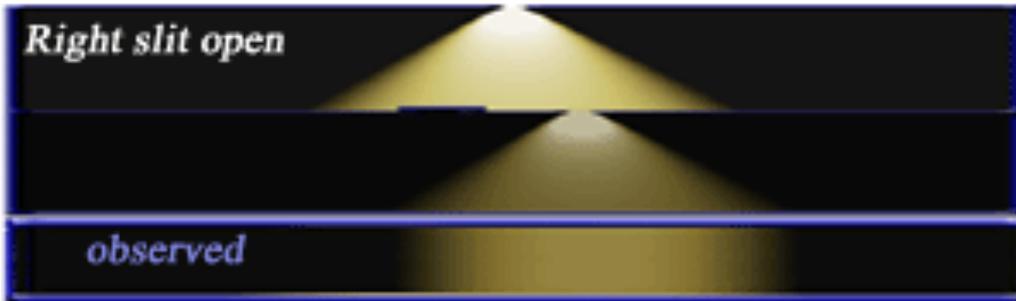
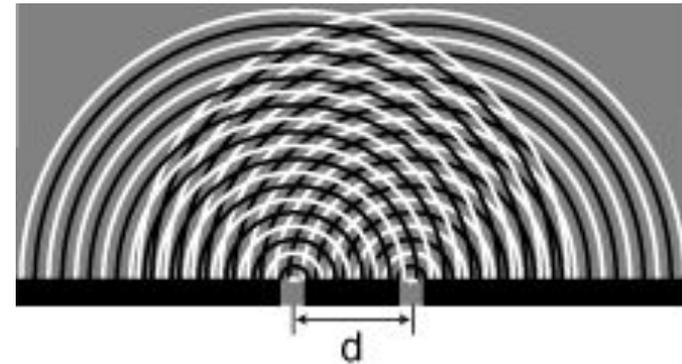


Figure 4 Intensity Distribution of Fringes

[http://en.wikipedia.org/wiki/Double-slit\\_experiment](http://en.wikipedia.org/wiki/Double-slit_experiment)



## Young's experiment

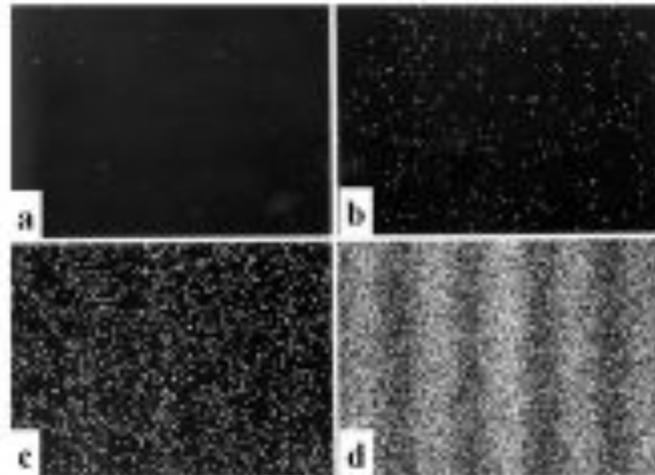


# Same basic result obtained using electrons...

$$\lambda = \frac{h}{p}$$

8e<sup>-</sup>

270e<sup>-</sup>



Single-electron Build-up of Interference Pattern

2,000e<sup>-</sup>

60,000e<sup>-</sup>

Hitachi labs (1989)

In 1924, Louis-Victor de Broglie formulated the DeBroglie hypothesis, claiming that all matter, not just light, has a wavelike nature. He related the wavelength (denoted as  $\lambda$ ) and the momentum (denoted as  $p$ )

$$\lambda = \frac{h}{p}$$

A property of our universe

This is a little like the relation we had for photons

$$E = h\nu$$
$$= hc/\lambda$$

but if

$$E = pc$$

$$\lambda = \frac{h}{p}$$

[http://en.wikipedia.org/wiki/Wave-particle\\_duality](http://en.wikipedia.org/wiki/Wave-particle_duality)

Light and particles like the electron (and neutron and proton) all have wavelengths and the shorter the wavelength the higher the momentum  $p$ . Electrons always have some motion regardless of their temperature because their wavelength cannot be zero (aka Heisenberg Uncertainty Principle)

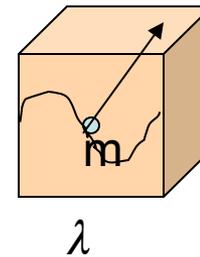
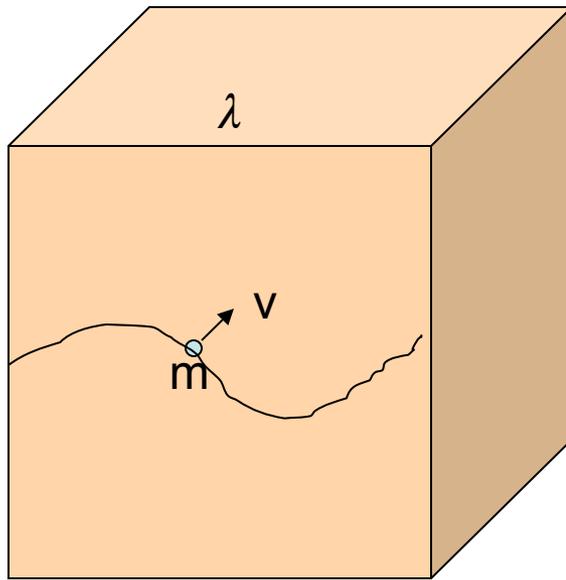
The condition that a particle cannot be localized to a region  $\Delta x$  smaller than its wavelength  $\lambda = h/p$  also implies

$$\lambda < \Delta x \Rightarrow p \Delta x > h \Rightarrow p > \frac{h}{\Delta x}$$

One cannot confine a particle to a region  $\Delta x$  without making its momentum increase

$p = \frac{h}{\Delta x}$  is the "degenerate" limit

# Consider one electron in a contracting box



As you squeeze on the box, the particle in the box has to move faster.

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\lambda \downarrow \Rightarrow v \uparrow$$

The squeezing provides the energy to increase  $v$

A little thought will show how this is going to solve our problem with the stability of matter (and also, later, the existence of white dwarfs)

As the electron is forced into a smaller and smaller volume, it must move faster. Ultimately this kinetic energy can support it against the electrical attraction of the nucleus.

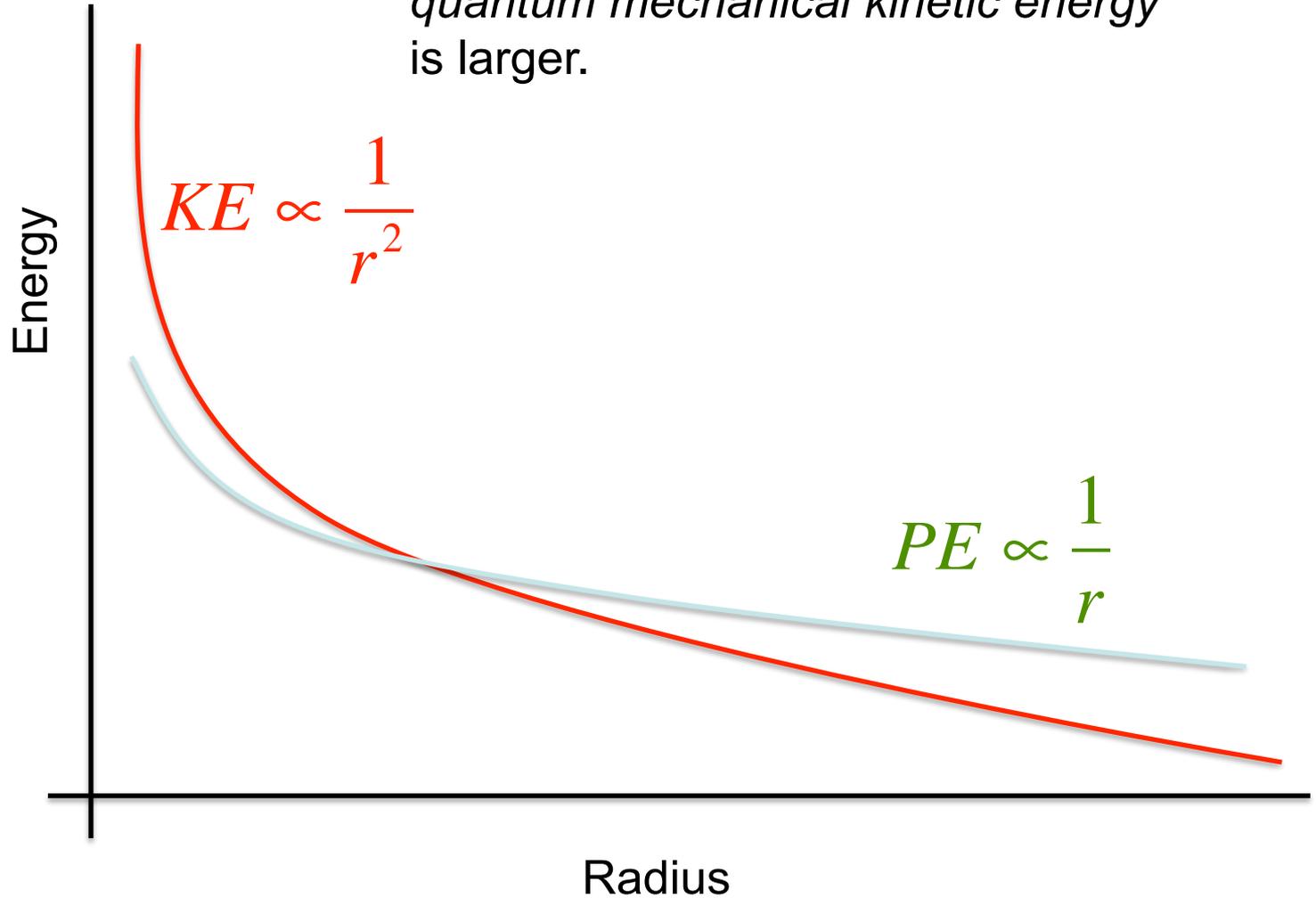
$$\text{Since } p = \frac{h}{\lambda} \Rightarrow \frac{1}{2} m_e v^2 = \frac{p^2}{2m_e} \propto \frac{1}{\lambda^2} \sim \frac{1}{r^2}$$

$$\text{but } -\frac{Ze^2}{r} \propto \frac{1}{r}$$

The kinetic energy increases quadratically with  $1/r$ , the electrical potential, only linearly.

There comes a minimum radius where the electron cannot radiate because the sum of its potential and kinetic energies has reached a minimum.

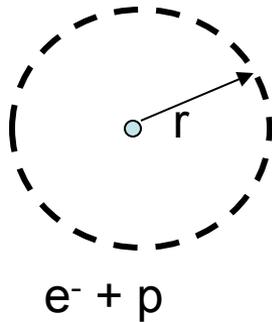
At large distance electrical repulsion dominates. At short distances the *quantum mechanical kinetic energy* is larger.



## Ground state of the hydrogen atom – Neils Bohr (1913)

(lowest possible energy state)

Must fit the wavelength of the electron inside a circle of radius  $r$ ,  
the average distance between the electron and the proton.



$$\begin{aligned}\lambda &= 2\pi r \\ &= \frac{h}{p} \\ \therefore p &= \frac{h}{\lambda} = \frac{h}{2\pi r}\end{aligned}$$

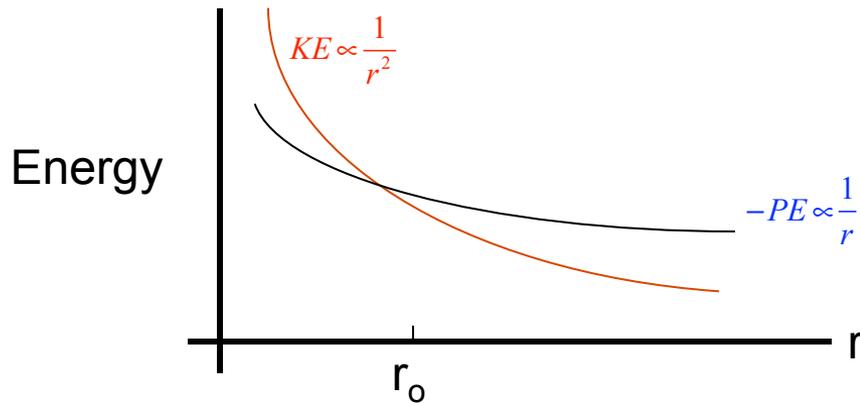
The  $2\pi$  here is rather arbitrary but gives the right answer and omits deeper discussion of “wave functions”

$$KE = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{p^2}{2m} = \frac{h^2}{2m(4\pi^2r^2)}$$

new QM energy

$$PE = -\frac{e^2}{r} \quad \text{as before}$$

Note that PE goes as  $1/r$  and KE goes as  $1/r^2$



For a single electron bound to a single proton, i.e., hydrogen.

At  $r_0$   $KE = -\frac{1}{2} PE$  *i.e.*,  $p = \frac{h}{2\pi r}$  and  $KE = \frac{1}{2} m_e v^2 = \frac{(m_e v)^2}{2m_e}$

$$\frac{h^2}{2m_e (4\pi^2 r_0^2)} = \frac{Ze^2}{2r_0} = \frac{p^2}{2m_e}$$

$$\frac{h^2}{2m_e Ze^2} = \frac{4\pi^2 r_0^2}{2r_0}$$

$$r_0 = \frac{h^2}{4\pi^2 Ze^2 m_e}$$

Energy would have to be provided to the electron to make it move any closer to the proton (because it would have to move faster), more energy than  $e^2/r$  can give.

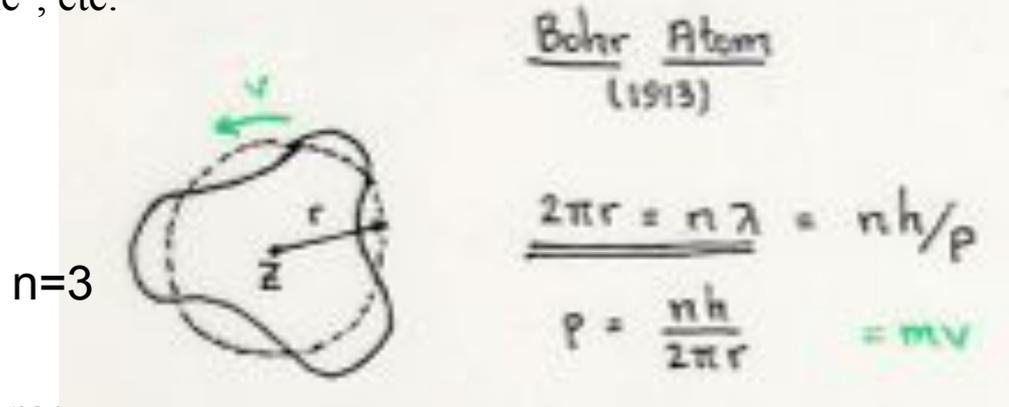
For  $Z=1$  (hydrogen)  $r_0 = 0.529189379 \overset{\circ}{\text{A}} = 5.29189379 \times 10^{-9} \text{ cm}$

This is the (average) radius of the “ground state” of the hydrogen atom, 0.529189... Å. It is permanently stable. There is no state with lower energy to make a transition to.

However, there also exist “excited states” of atoms that have a transitory existence.

For atoms with a single electron – H, He<sup>+</sup>, etc.

## Bohr's First Postulate



*The only possible states of the electron are those for which*

$$mvr = \frac{nh}{2\pi}$$

Solve as before:

$$r = \frac{n^2 h^2}{4\pi^2 Z e^2 m_e} = 0.53 \frac{n^2}{Z} \text{ Angstroms}$$

$$E_{tot} = -\frac{Ze^2}{2r} = -\frac{2\pi^2 Z^2 e^4 m_e}{n^2 h^2}$$

$$E_{tot} = -13.6 \text{ eV} \left( \frac{Z^2}{n^2} \right)$$

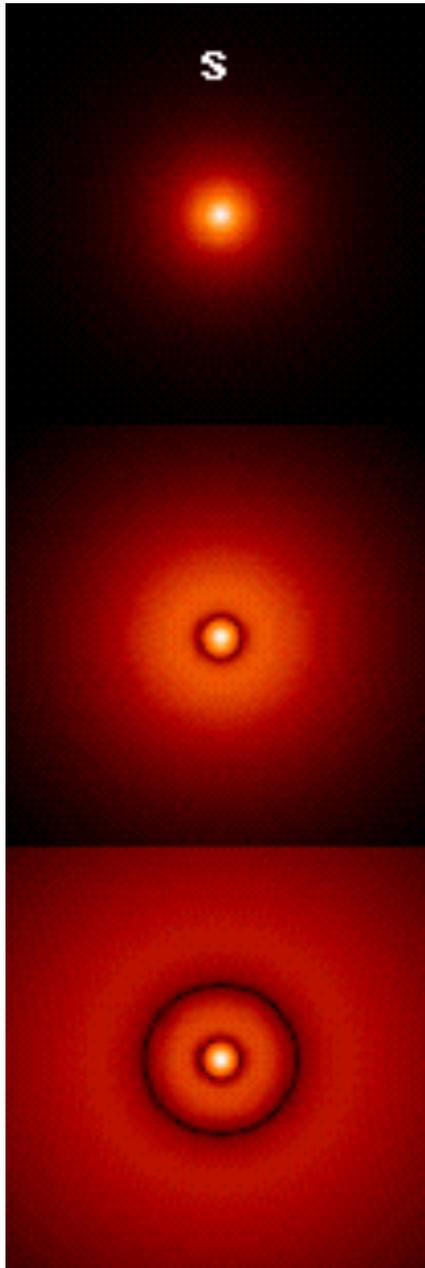
$$1 \text{ eV} \equiv 1.602 \times 10^{-12} \text{ erg}$$

n = 1 is the "ground state"

For atoms with only a single electron.

For hydrogen Z = 1

\*



$n = 1$

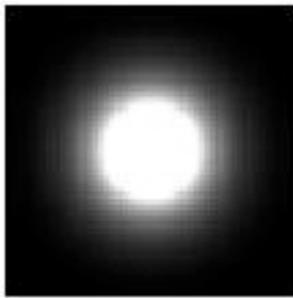
In the full quantum mechanical solution the electron is described by a “wave function” that gives its probability for being found at any particular distance from the nucleus.

$n=2$

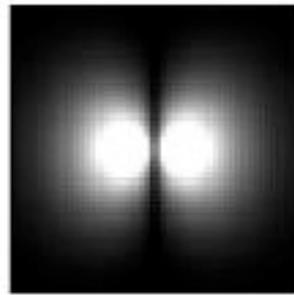
In the simplest case these distributions are spherical.

$n=3$

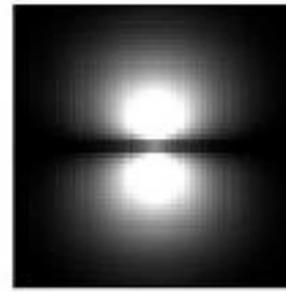
The radius in the Bohr model is the average radius but the energy is precise.



$n=1$

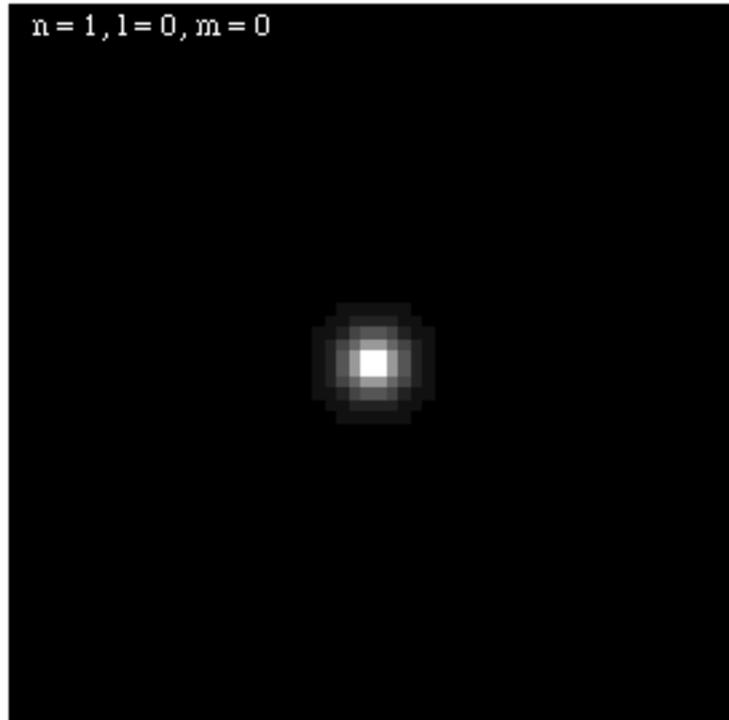


$n=2, l=1, m=0$



$n=2, l=1, m=1$

\*

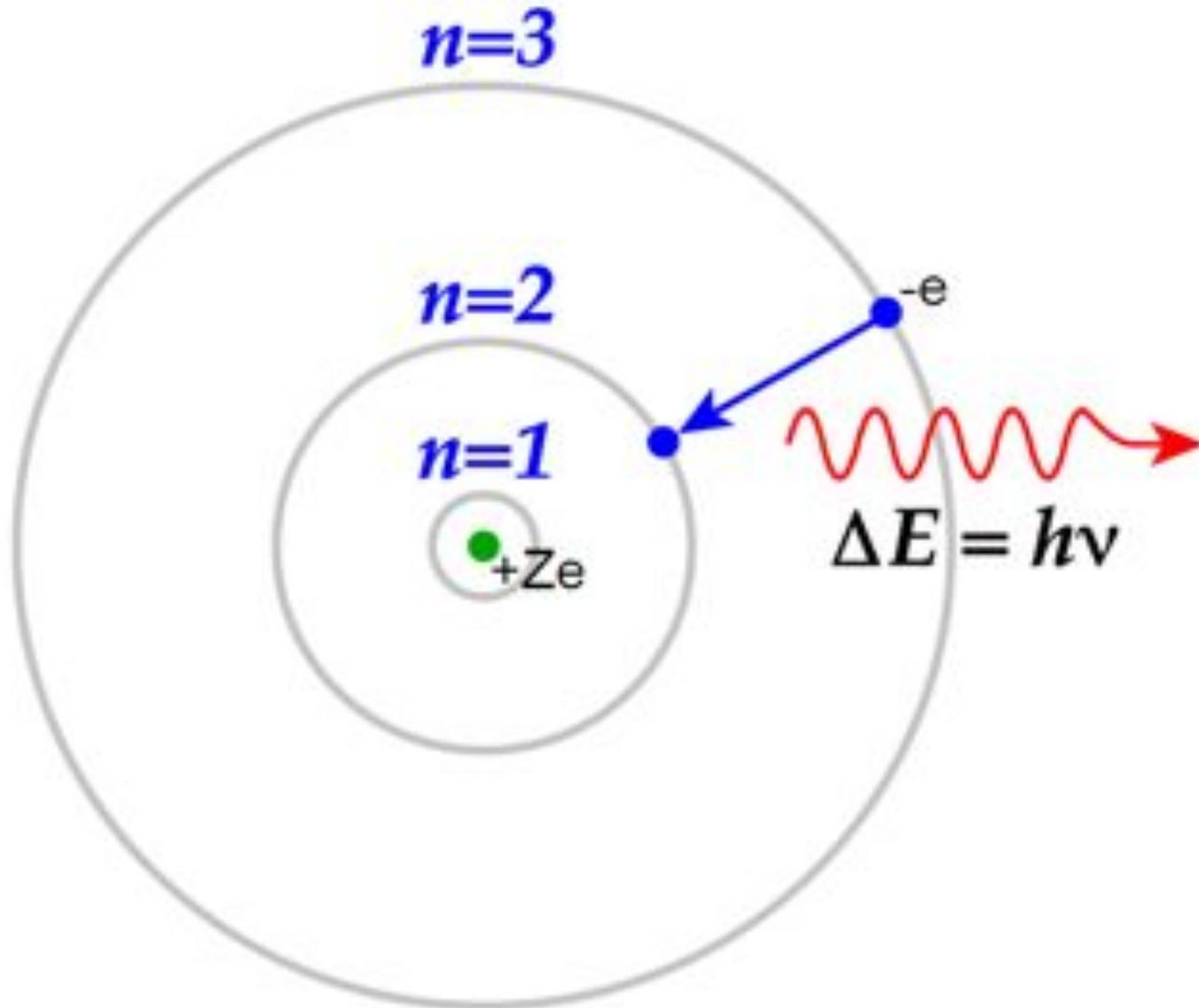


All orbitals from  $n = 1$  through 4  
Number electrons per shell is  $2n^2$ , but  
don't always completely fill one shell  
before starting on the next.

2, 8, 18

2, 10, 18, 36  
He, Ne, Ar, Kr

## Bohr's Second Postulate



Only the “ground state”,  $n = 1$ , is permanently stable

## Bohr's Second Postulate

Radiation in the form of a single quantum (photon) is Emitted (or absorbed) as the electron makes a transition From one state to another. The energy in the photon is the Difference between the energies of the two states.

$$\begin{array}{ccc} \text{emission} & & \text{absorption} \\ E_m \rightarrow E_n + h\nu & (\text{or } E_n + h\nu \rightarrow E_m) & m > n \end{array}$$

$$h\nu = \frac{hc}{\lambda} = E_m - E_n$$

$$\frac{1}{\lambda} = \frac{E_m - E_n}{hc} = \frac{2\pi^2 Z^2 e^4 m_e}{h^3 c} \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$$

$$\frac{1}{\lambda_{mn}} = 1.097 \times 10^5 Z^2 \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \text{ cm}^{-1}$$

$$\lambda_{mn} = \frac{911.6 \text{ \AA}}{Z^2} \left( \frac{1}{n^2} - \frac{1}{m^2} \right)^{-1}$$

(for atoms with only one electron)

$$\lambda_{mn} = \frac{911.6 \text{ \AA}}{Z^2} \left( \frac{1}{n^2} - \frac{1}{m^2} \right)^{-1}$$

E.g.,

$$m = 2, n = 1, Z = 1$$

$$\begin{aligned} \lambda &= 911.6 \text{ \AA} \left( \frac{1}{1^2} - \frac{1}{2^2} \right)^{-1} = 911.6 \left( \frac{3}{4} \right)^{-1} \\ &= 911.6 \left( \frac{4}{3} \right) = 1216 \text{ \AA} \end{aligned}$$

$$m = 3, n = 1, Z = 1$$

$$\begin{aligned} \lambda &= 911.6 \left( \frac{1}{1^2} - \frac{1}{3^2} \right)^{-1} = 911.6 \left( \frac{8}{9} \right)^{-1} \\ &= 911.6 \left( \frac{9}{8} \right) = 1026 \text{ \AA} \end{aligned}$$

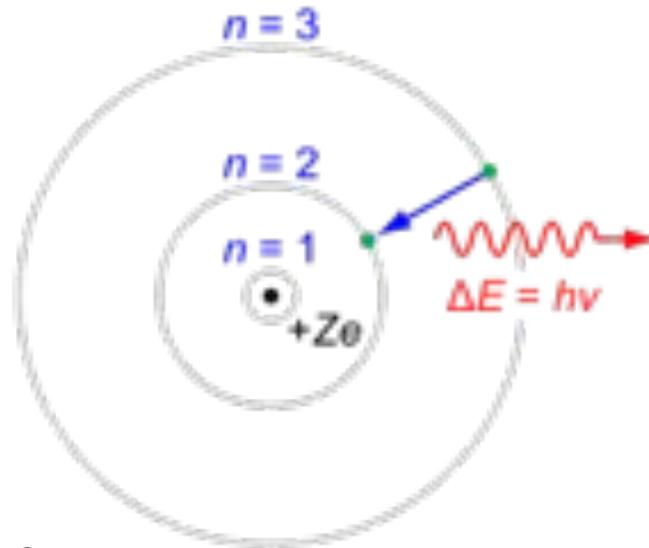
$$m = 3, n = 2, Z = 1$$

$$\begin{aligned} \lambda &= 911.6 \left( \frac{1}{2^2} - \frac{1}{3^2} \right)^{-1} = 911.6 \left( \frac{1}{4} - \frac{1}{9} \right)^{-1} \\ &= 911.6 \left( \frac{5}{36} \right)^{-1} = 911.6 \left( \frac{36}{5} \right) = 6563 \text{ \AA} \end{aligned}$$

Lines that start or end on  $n=1$  are called the “Lyman” series. All are between 911.6 and 1216 \AA.

Lines that start or end on  $n=2$  are called the “Balmer” series. All are between 3646 and 6564 \AA.

# BALMER SERIES



$5 \rightarrow 2$

$4 \rightarrow 2$

$3 \rightarrow 2$

$H_\gamma$

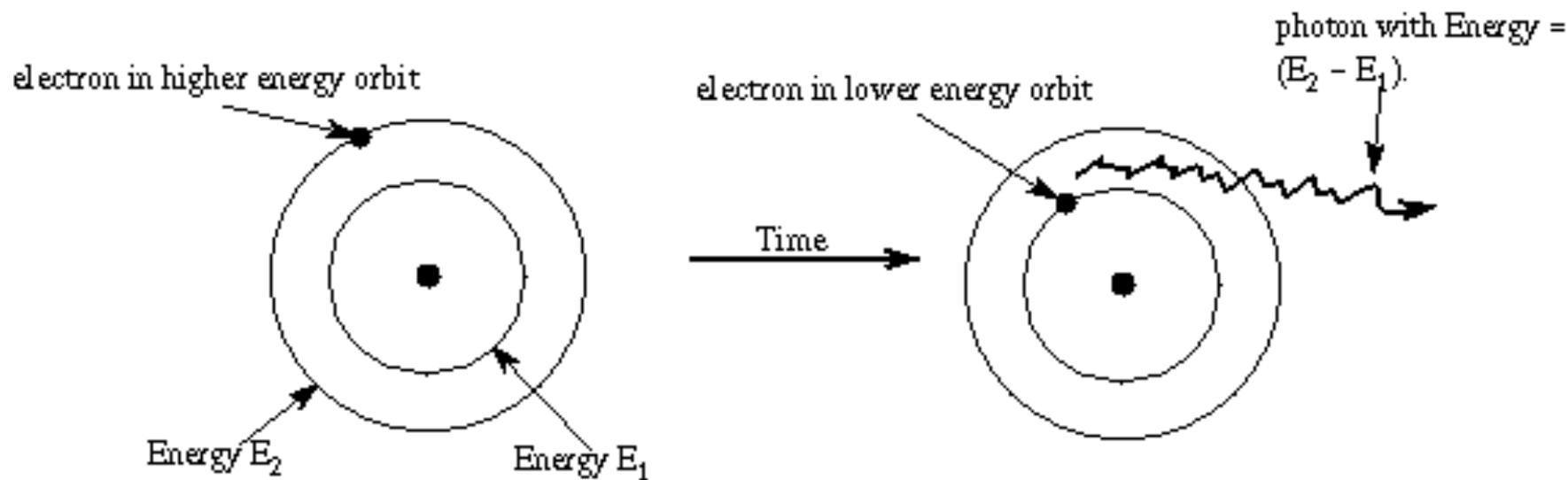
$H_\beta$

$H_\alpha$

Hydrogen emission line spectrum  
Balmer series



## Emission line



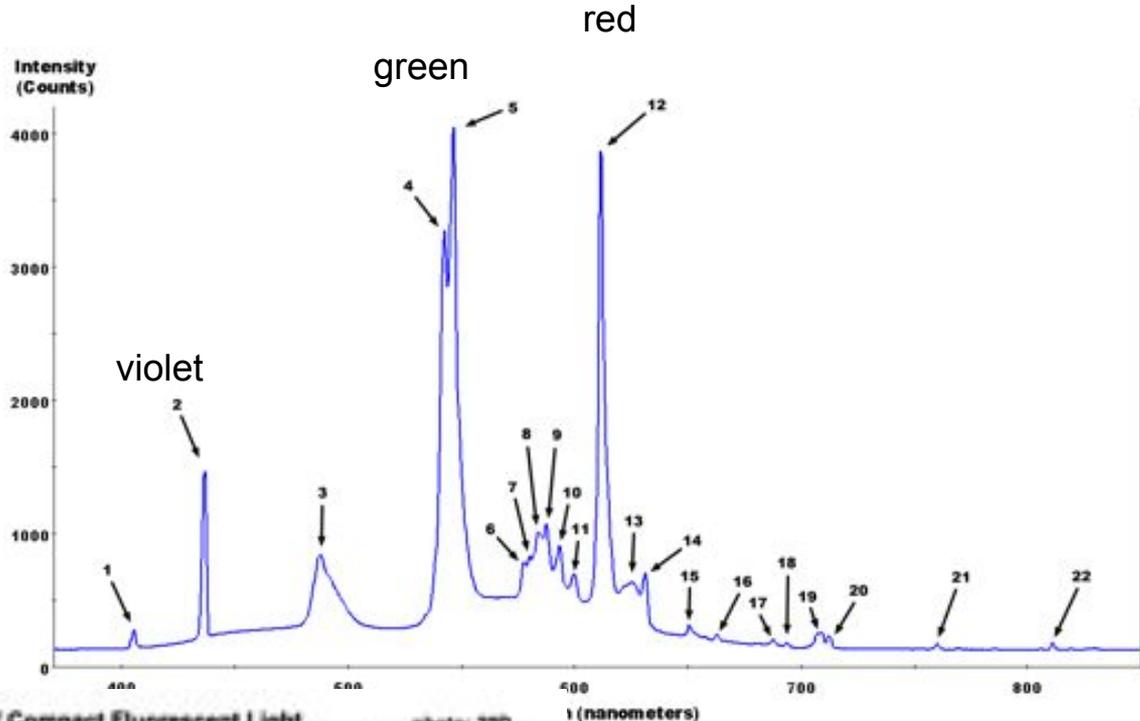
Atom was hit/bumped by another atom and gained some energy =  $(E_2 - E_1)$ . Electron in higher energy orbit ( $E_2$ ).

Emission line produced!

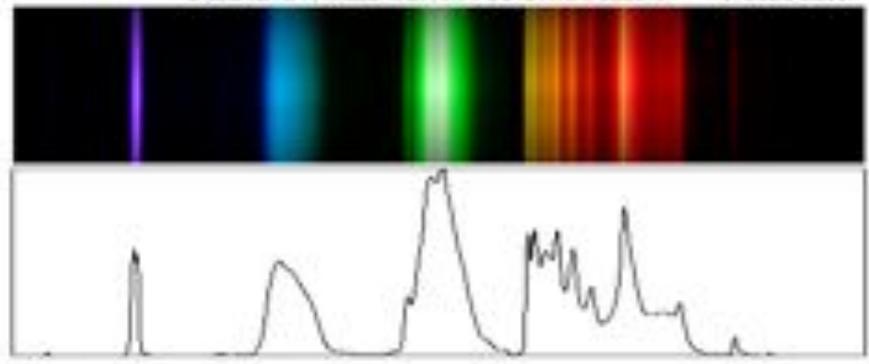


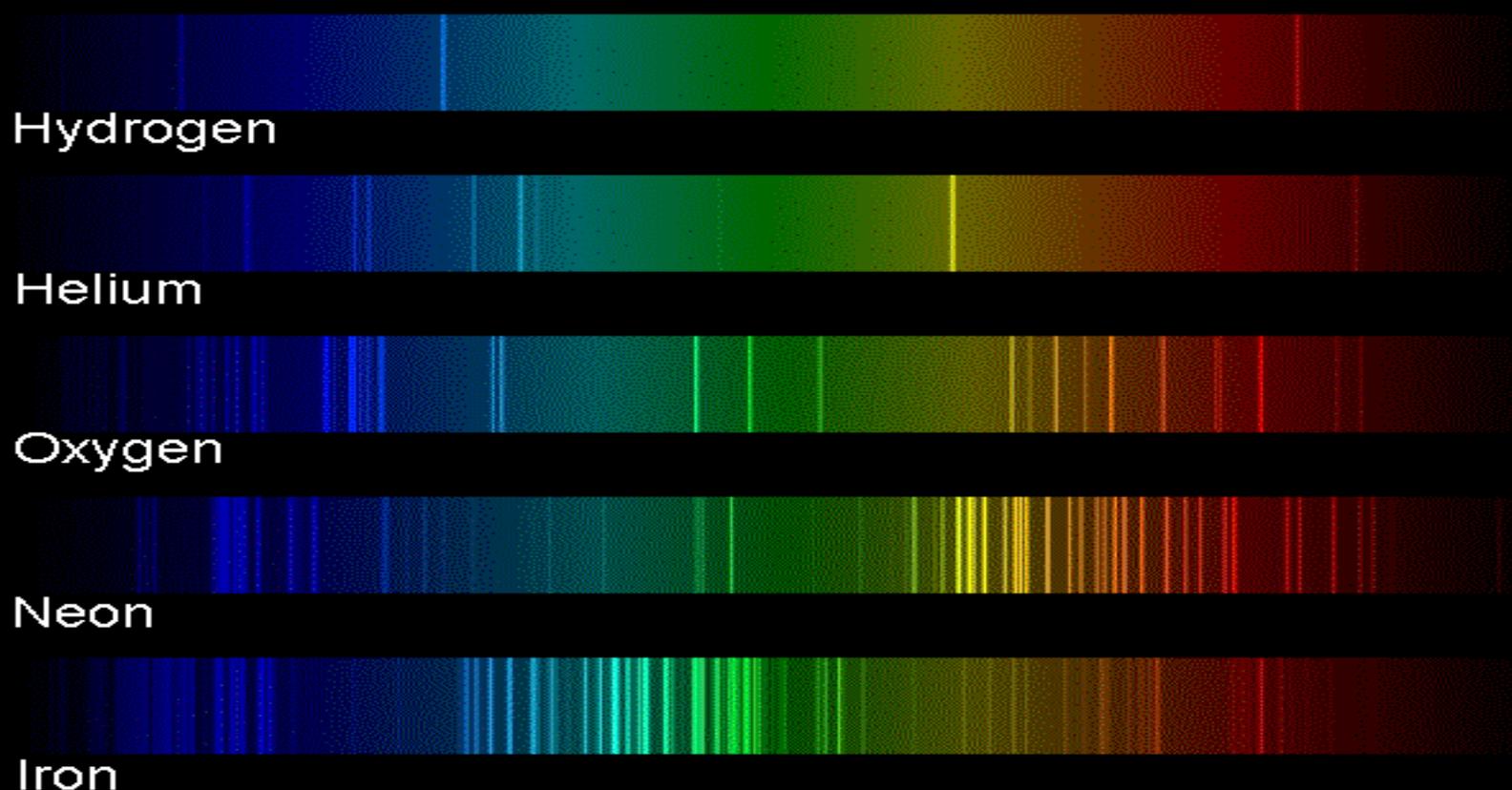
# Fluorescent Light Fixture

Peak number	Wavelength of peak (nm)	Species producing peak
1	405.4	<a href="#">mercury</a>
2	436.6	mercury
3	487.7	<a href="#">terbium</a> from $Tb^{3+}$
4	542.4	<a href="#">terbium</a> from $Tb^{3+}$
5	546.5	mercury
6	577.7	possibly mercury
7	580.2	mercury or <a href="#">europium</a> in $Eu^{+3}:Y_2O_3$ or <a href="#">terbium</a> likely $Tb^{3+}$
8	584.0	possibly <a href="#">terbium</a> from $Tb^{3+}$
9	587.6	likely <a href="#">europium</a> in $Eu^{+3}:Y_2O_3$
10	593.4	likely <a href="#">europium</a> in $Eu^{+3}:Y_2O_3$
11	599.7	likely <a href="#">europium</a> in $Eu^{+3}:Y_2O_3$
12	611.6	<a href="#">europium</a> in $Eu^{+3}:Y_2O_3$
13	625.7	likely <a href="#">terbium</a> from $Tb^{3+}$
14	631.1	likely <a href="#">europium</a> in $Eu^{+3}:Y_2O_3$
15	650.8	likely <a href="#">europium</a> in $Eu^{+3}:Y_2O_3$
16	662.6	



Spectrum of Compact Fluorescent Light photo: 200 GREENLITE 18W/LS-M 2700K FCC ID: N6AFJETS404 J. Basile 9/2007





Hydrogen

The image displays five horizontal spectral emission line patterns, each corresponding to a different element. The patterns are arranged vertically and labeled on the left. Each pattern consists of a continuous background of color (from blue on the left to red on the right) with several sharp, vertical lines of varying intensity and color. The lines represent the characteristic emission spectra of each element. The Hydrogen spectrum shows a few prominent lines, while the Iron spectrum shows a very dense collection of lines.

Helium

Oxygen

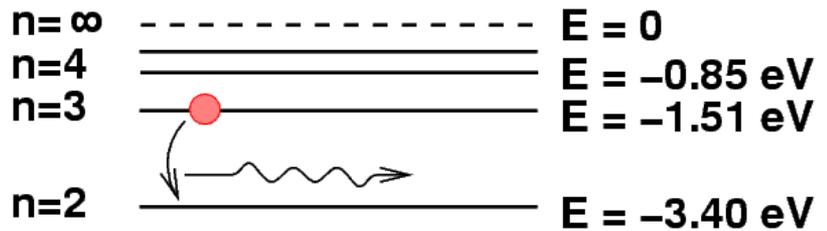
Neon

Iron

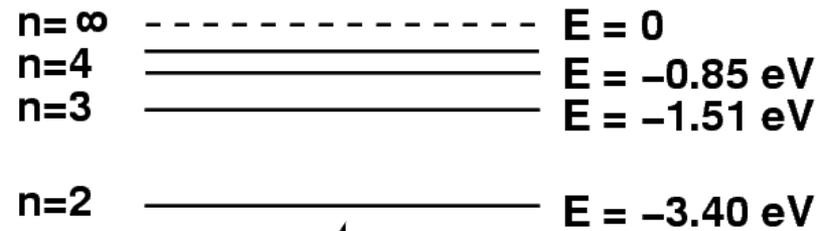
How are excited states populated?

- Absorb a photon of the right energy
- Collisions
- Ionization - recombination

[http://spiff.rit.edu/classes/phys301/lectures/spec\\_lines/Atoms\\_Nav.swf](http://spiff.rit.edu/classes/phys301/lectures/spec_lines/Atoms_Nav.swf)

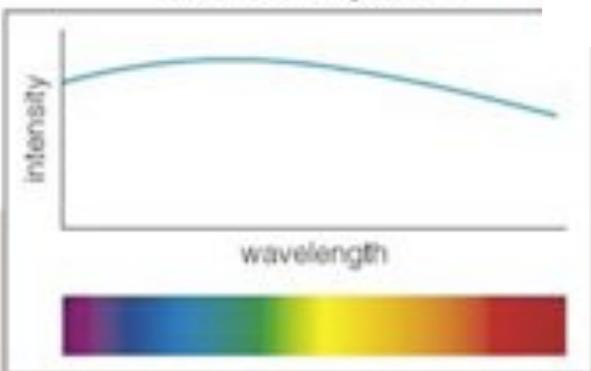


Emission – H-alpha

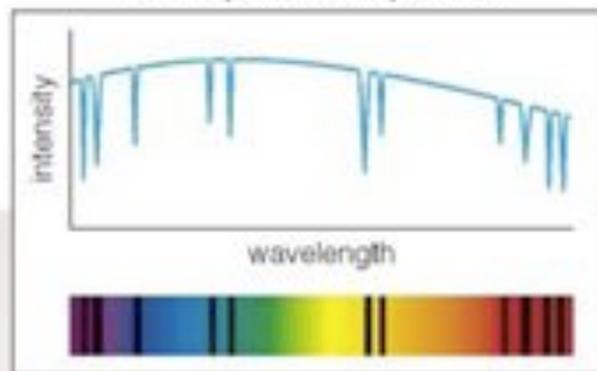


Absorption – Ly-alpha

**Continuous Spectrum**

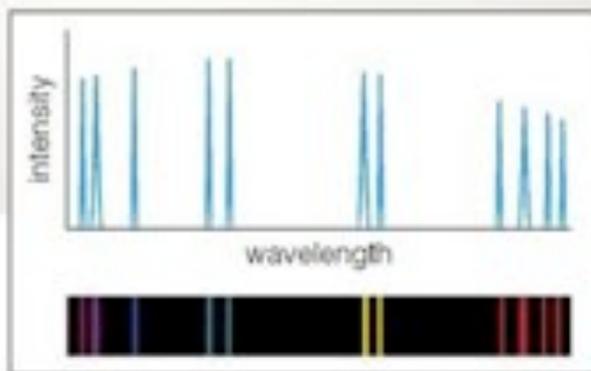


**Absorption Line Spectrum**

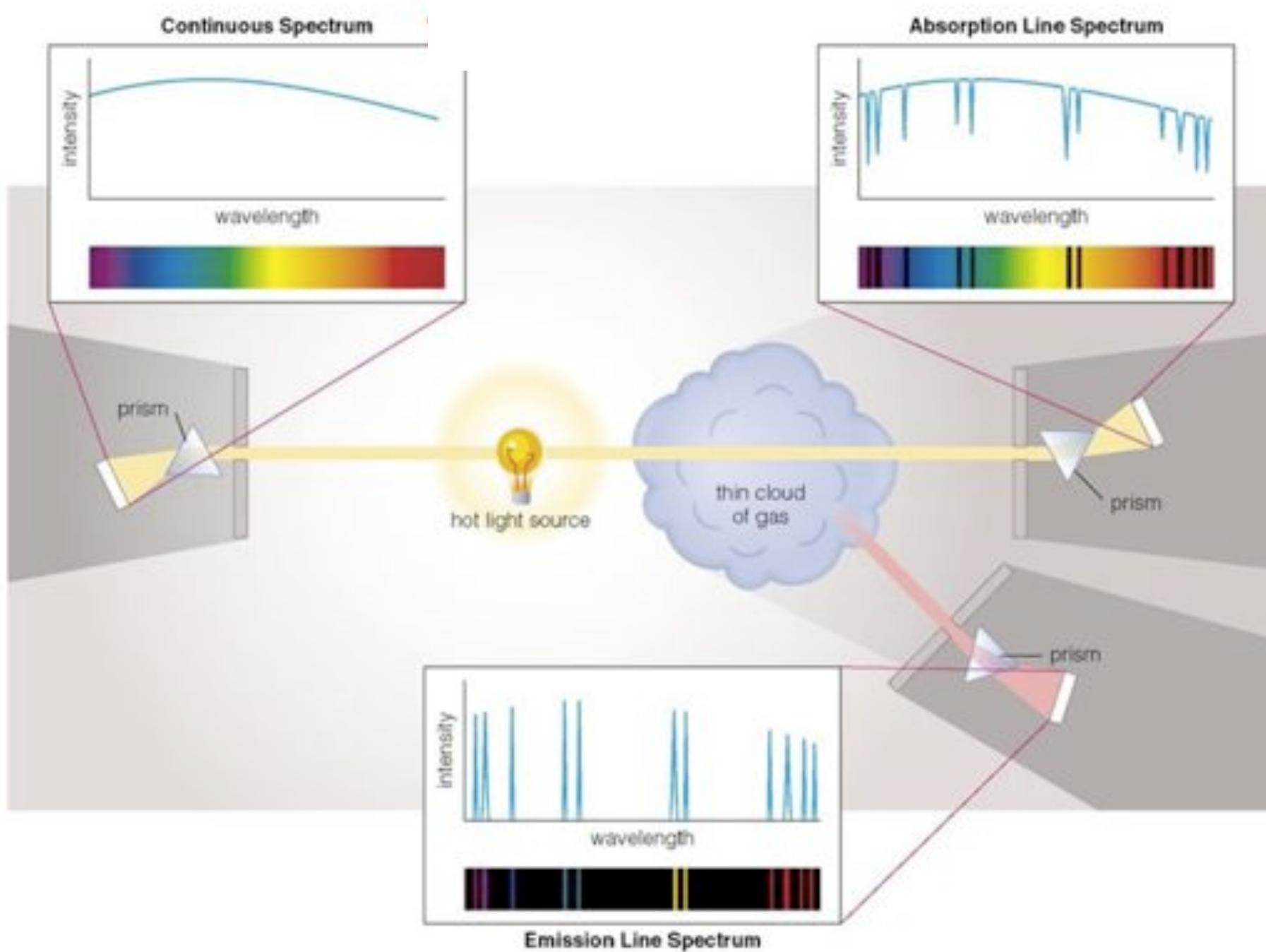


hot light source

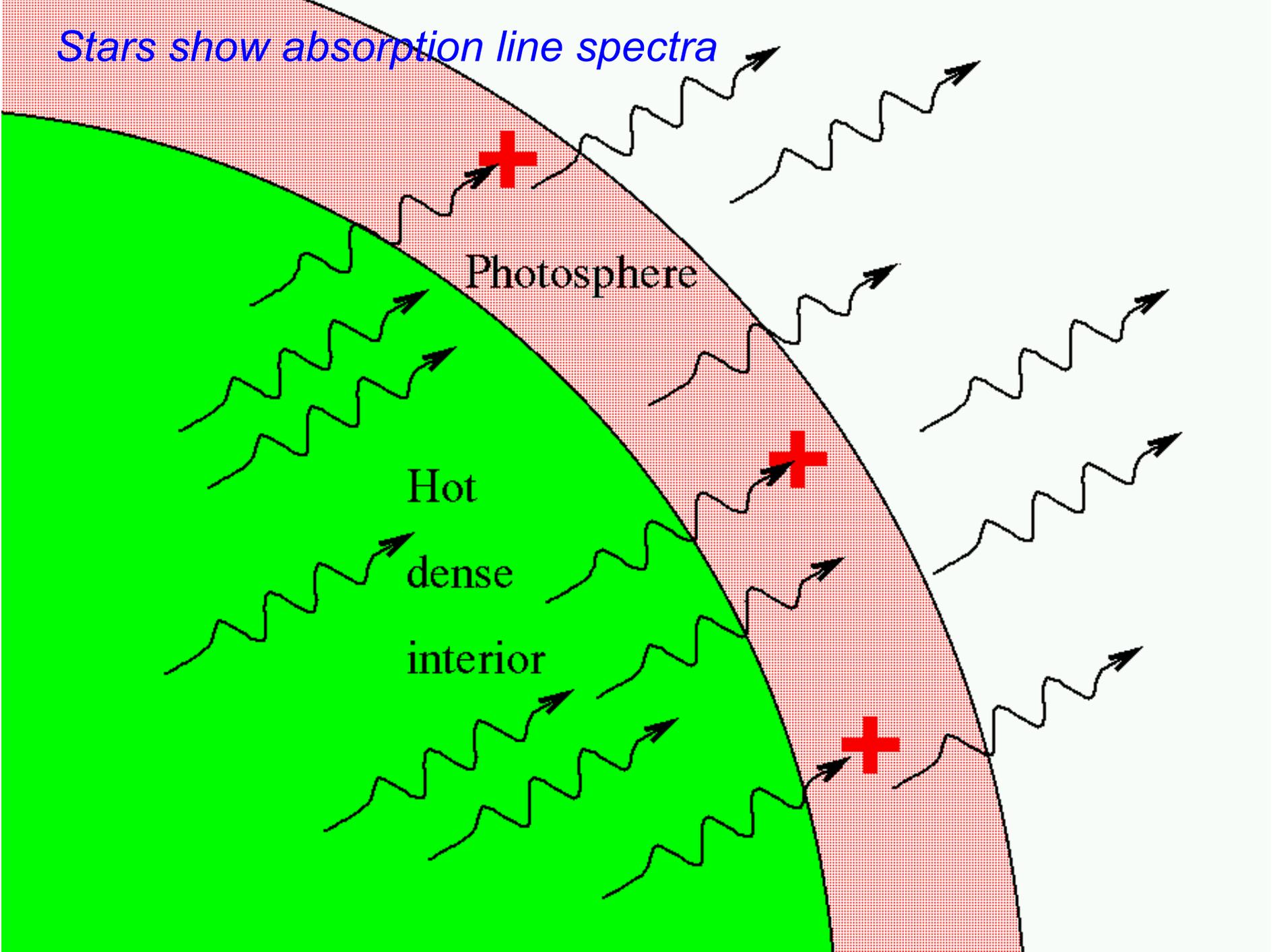
thin cloud of gas



**Emission Line Spectrum**



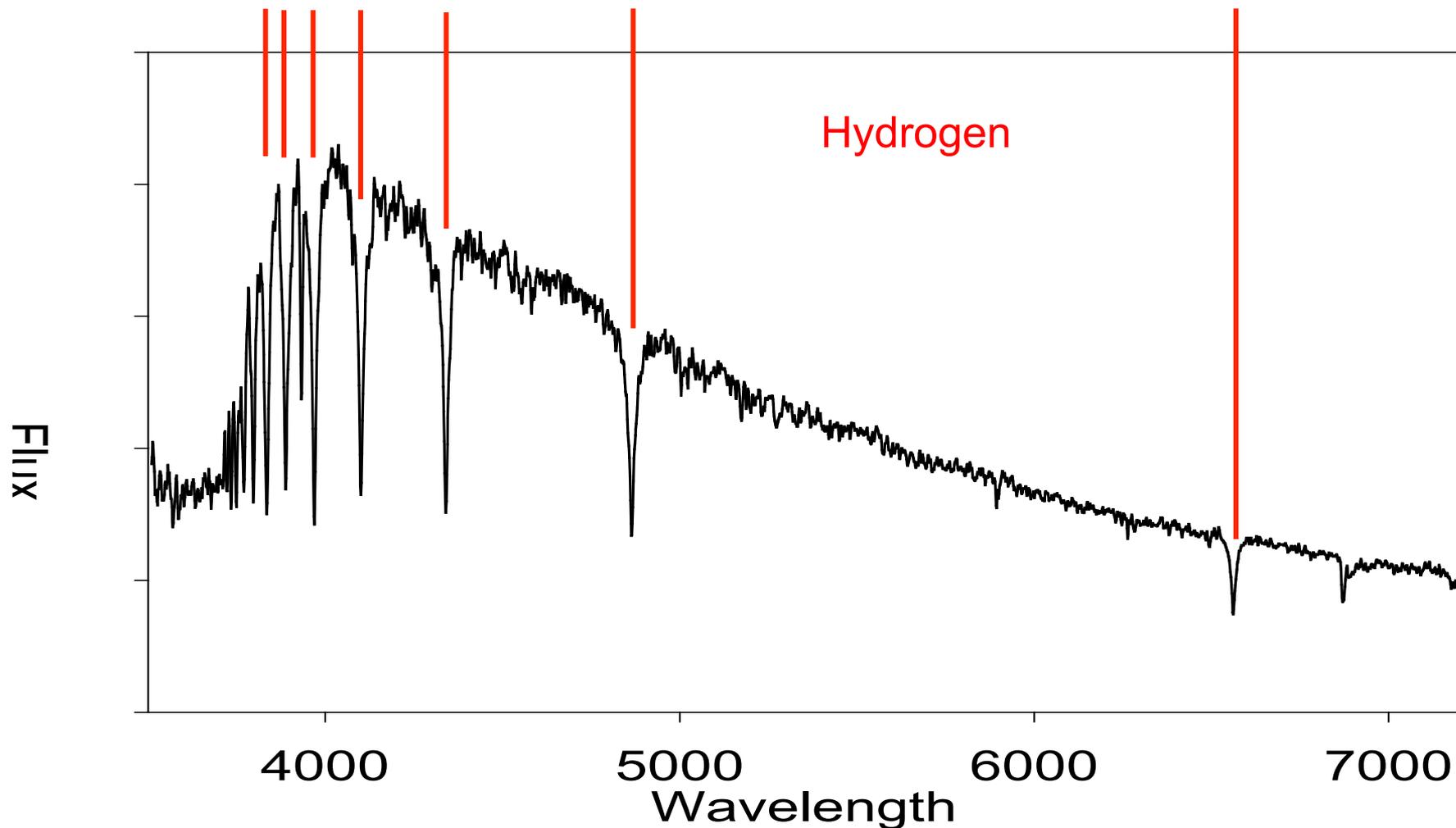
*Stars show absorption line spectra*



Photosphere

Hot  
dense  
interior

# Absorption Line Spectrum (not the sun)

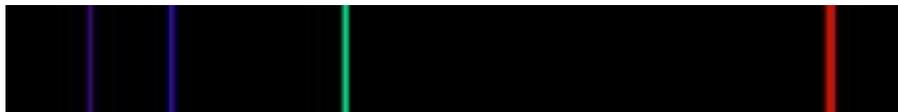


When we examine the spectra of stars, with a few exceptions to be discussed later, we see blackbody spectra with a superposition of *absorption* lines.

The identity and intensity of the “spectral lines” that are present reflect the temperature, density and composition of the stellar photosphere.



Blackbody



Emission line spectrum



Absorption line spectrum

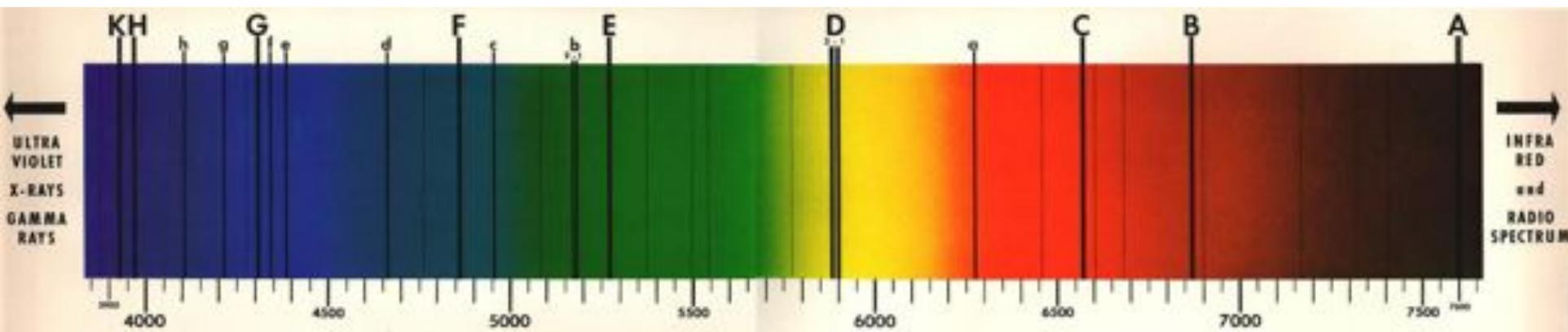
**TABLE P.1 THE COSMICALLY ABUNDANT ELEMENTS**

Element	Symbol	Number of Atoms per Million Hydrogen Atoms
Hydrogen	H	1,000,000
Helium	He	68,000
Carbon	C	420
Nitrogen	N	87
Oxygen	O	690
Neon	Ne	98
Magnesium	Mg	40
Silicon	Si	38
Sulfur	S	19
Iron	Fe	34

Determined from spectral analysis but the most abundant elements (H) do not always have the strongest lines as we shall see

# The solar spectrum

C = Balmer alpha  
F = Balmer beta  
f = Balmer gamma  
B = oxygen  
D = sodium  
H, K = singly ionized calcium  
others = Fe, Mg, Na, etc.



Wollaton (1802) discovered dark lines in the solar spectrum. Fraunhofer rediscovered them (1817) and studied the systematics

As the temperature in a gas is raised, electrons will be removed by collisions and interactions with light. The gas comes *ionized*.

The degree of ionization depends on the atom considered and the temperature.

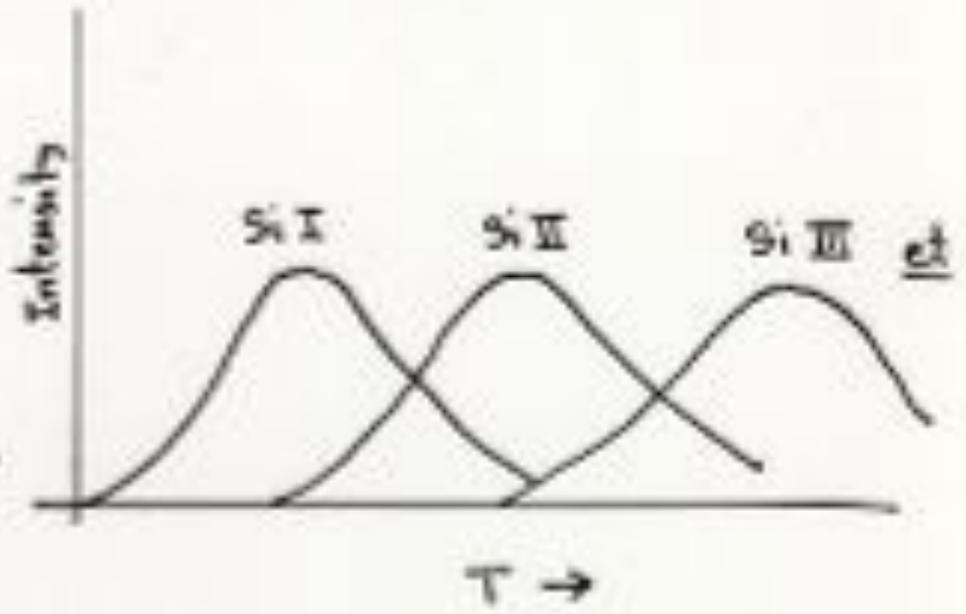
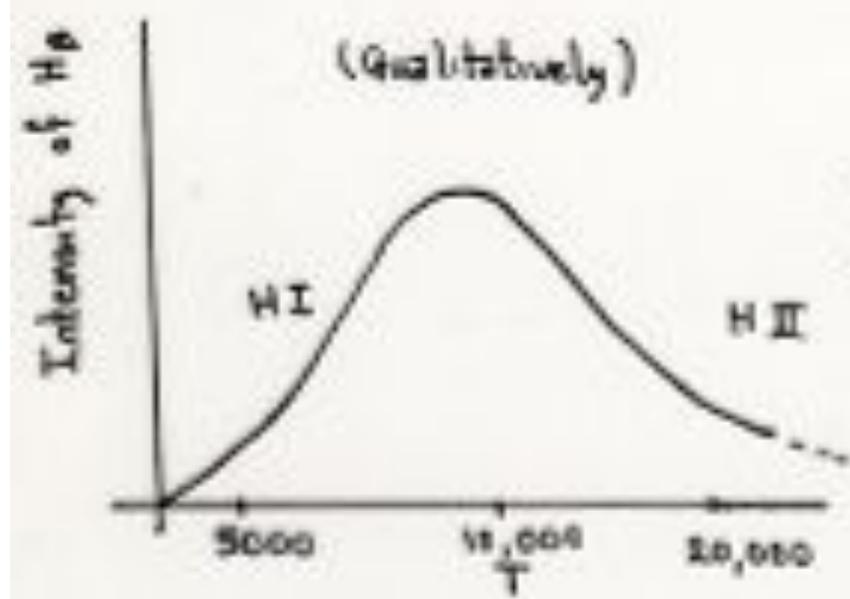
## Notation: Ionization stages

H I	neutral hydrogen	1 p	1 e
H II	ionized hydrogen	1 p	0 e
He I	neutral helium	2 p	2 e
He II	singly ionized helium	2 p	1 e
He III	doubly ionized helium	2 p	0 e
C I	neutral carbon	6 p	6 e
C II	C <sup>+</sup>	6 p	5 e
C III	C <sup>++</sup>	6 p	4 e
etc.			

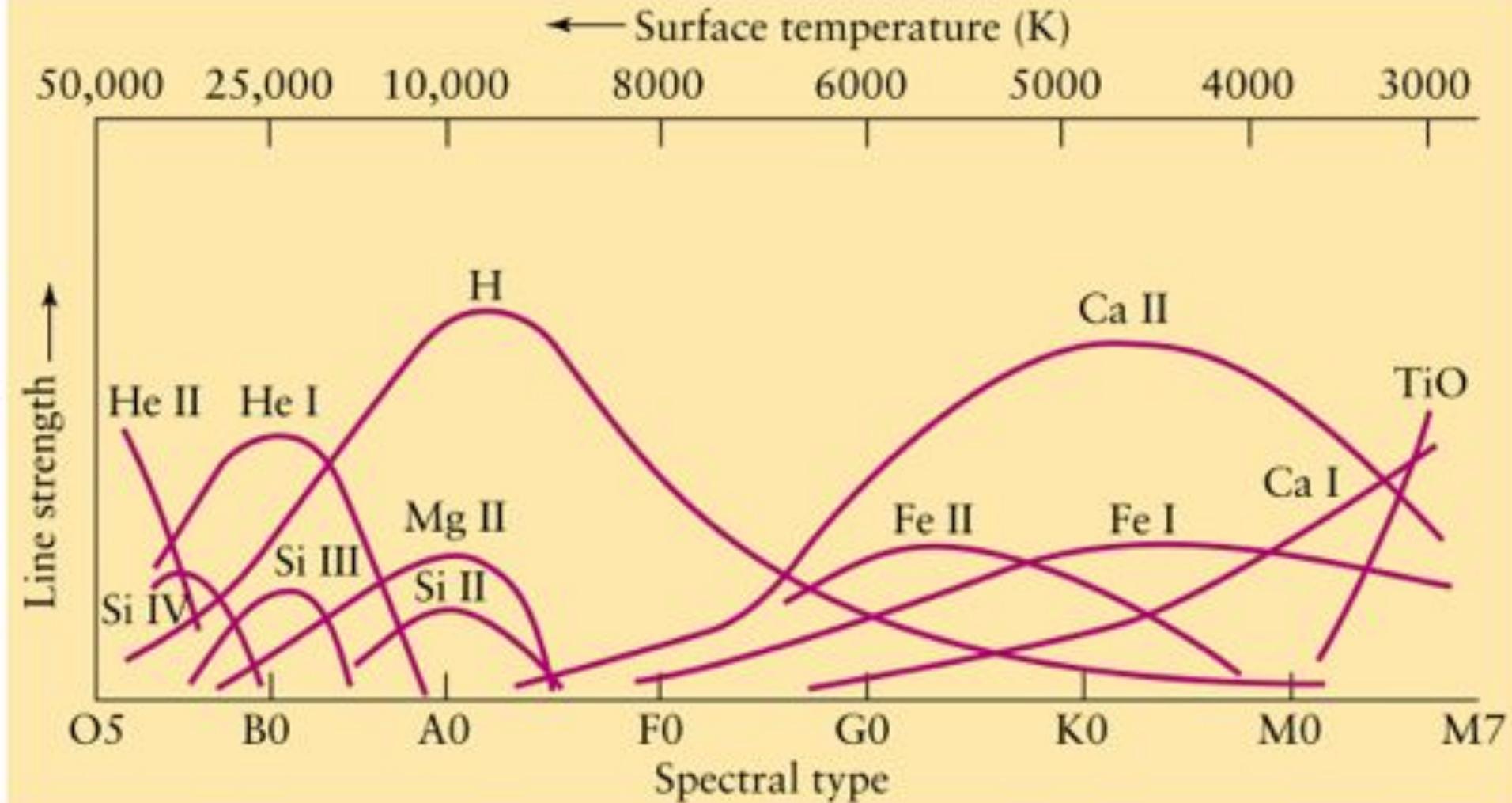
The ionization energy is the energy required to remove a single electron from a given ion. The excitation energy is the energy required to excite an electron from the ground state to the first excited state.

Ion	Excitation energy (eV)	Ionization energy (eV)
H I	10.2	13.6
He I	20.9	24.5
He II	40.8	54.4
rare Li I	1.8	5.4
Ne I	16.6	21.5
Na I	2.1	5.1
Mg I	2.7	7.6
Ca I	1.9	6.1

Li is He plus one proton, Na is Ne plus 1 proton, Ca is Ar plus 2 protons. The noble gases have closed electron shells and are very stable.



# Some of the stronger lines in stars



# The Spectral Sequence



Hottest  
50,000K

Coollest  
4300K

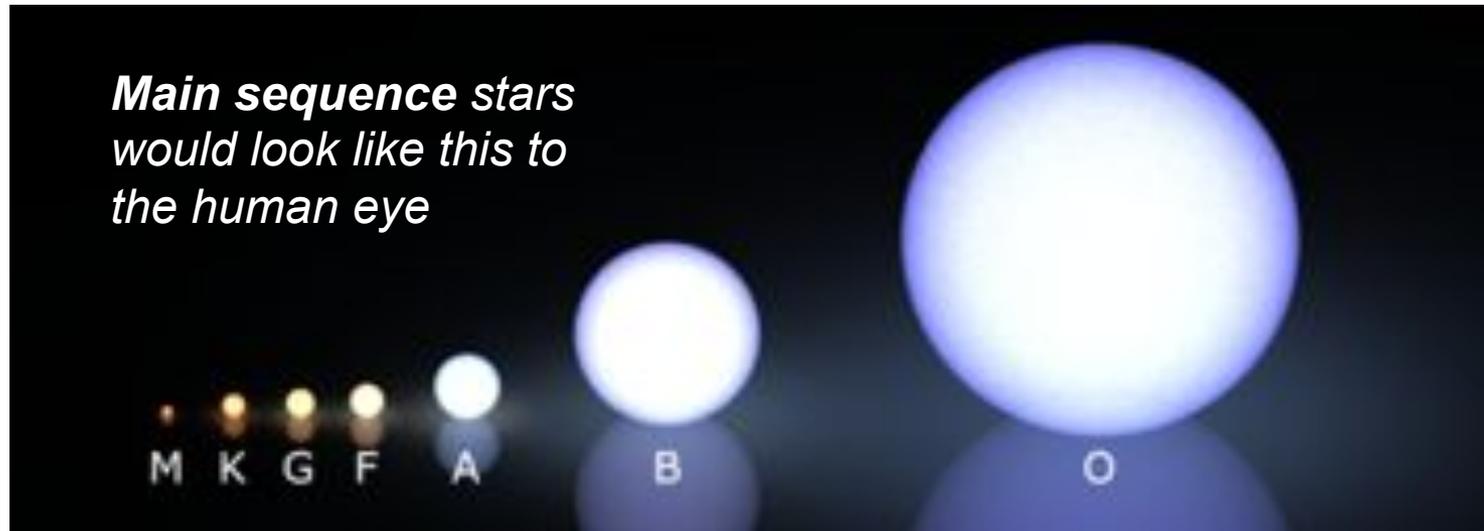
Bluest

Reddest

Spectral Sequence is a Temperature Sequence

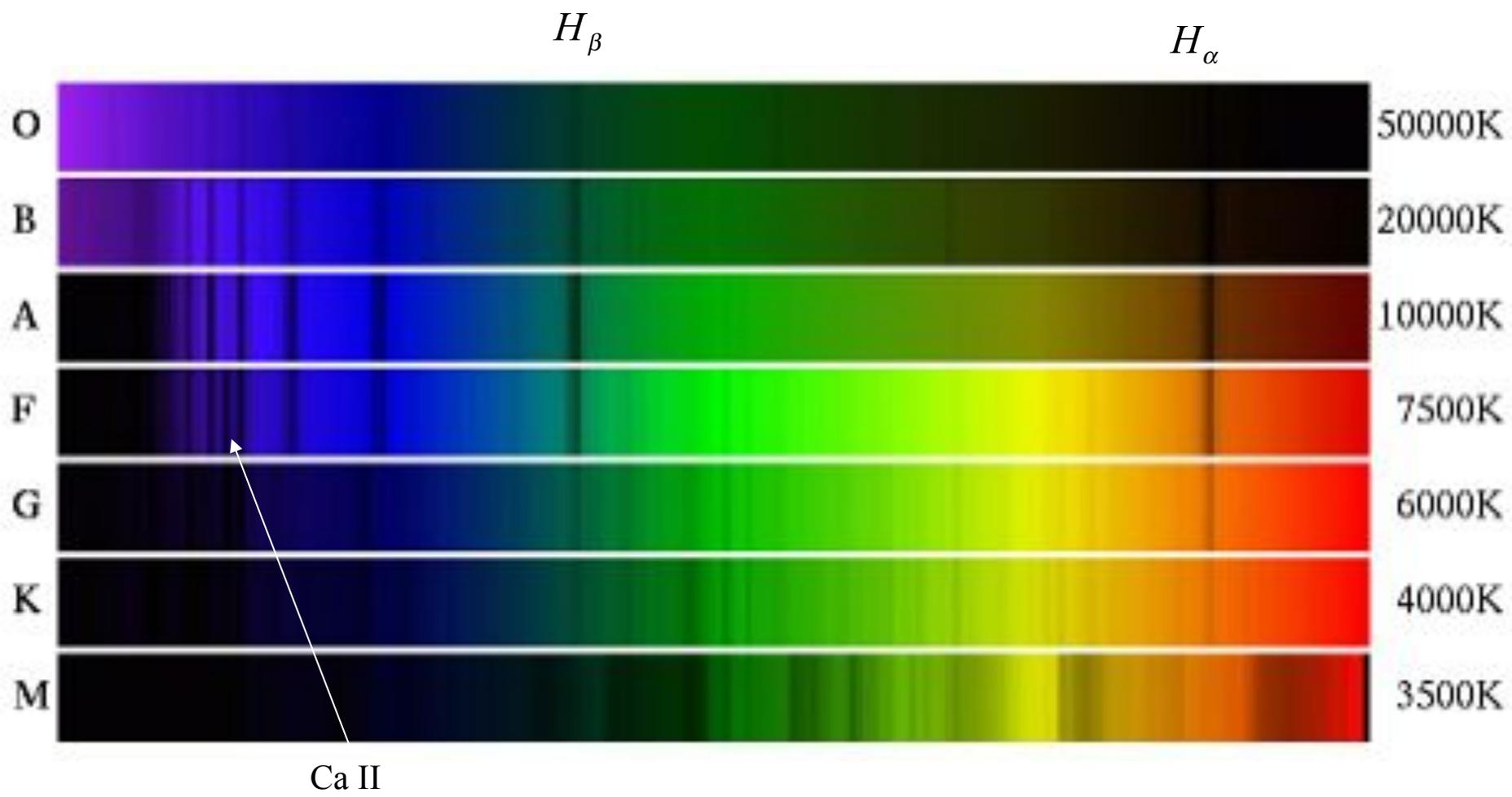
Our sun's spectral class is G2-V

			Fraction MS stars solar neighborhood
O	> 25,000 K	Delta Orionis	1/3,000,000
B	11,000 – 25,000	Pleiades brightest	1/800
A	7500 – 11,000	Sirius	1/160
F	6000 – 7500	Canopus	1/133
G	5000 – 6000	Sun	1/13
K	3500 – 5000	Arcturus	1/8
M	< 3500	Proxima Centauri	3/4



[http://en.wikipedia.org/wiki/Stellar\\_classification](http://en.wikipedia.org/wiki/Stellar_classification)

# Spectral Sequence



- Cannon further refined the spectral classification system by dividing the classes into numbered subclasses:
- For example, A was divided into  
**A0 A1 A2 A3 ... A9**
- Between 1911 and 1924, she classified about 220,000 stars, published as the Henry Draper Catalog.

O6.5

B0

B6

A1

A5

F0

F5

G0

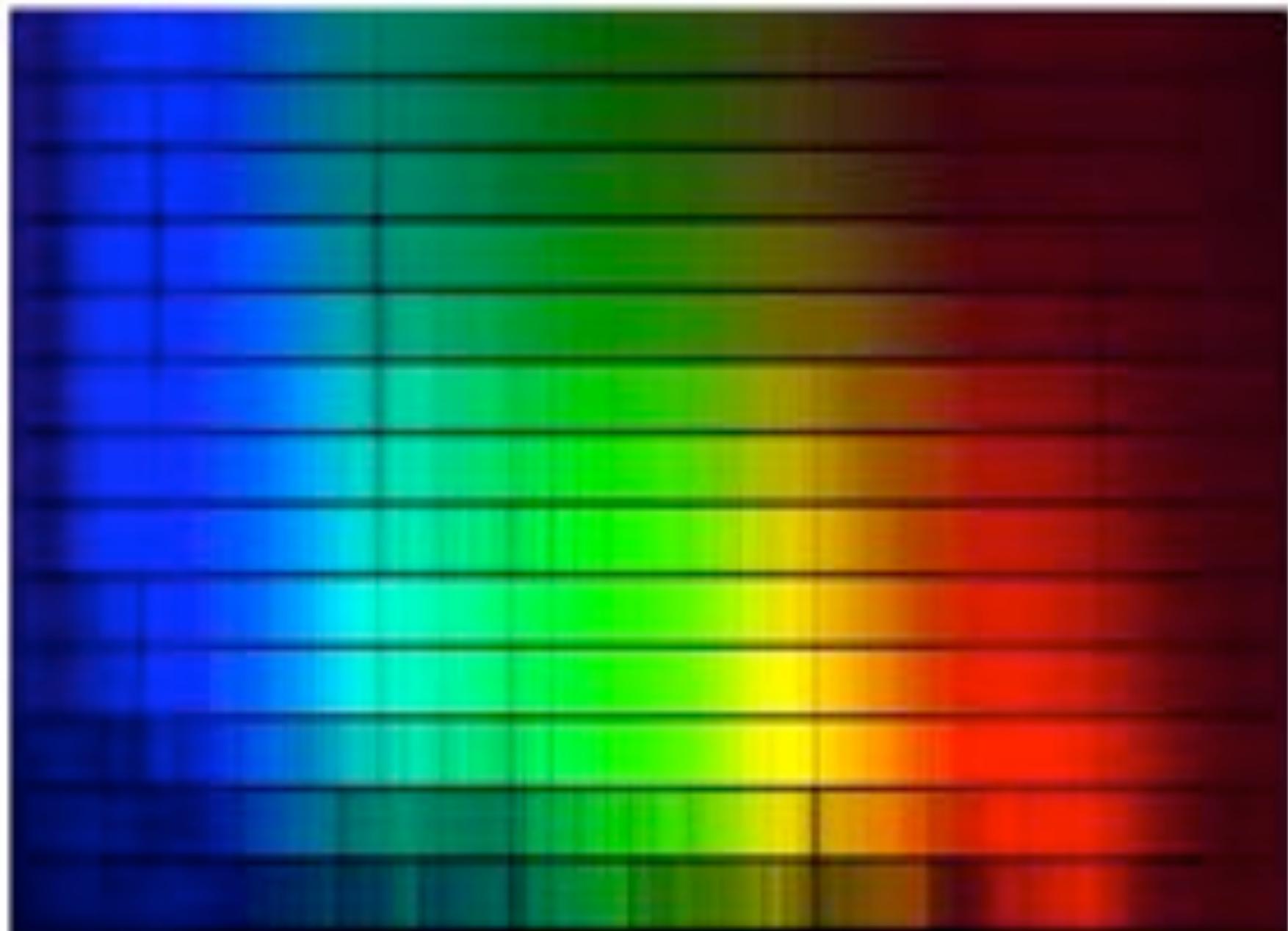
G5

K0

K5

M0

M5



## The Henry Draper Spectral Sequence

Spectral Type	Principal Characteristics	Spectral Criteria
O	Hottest blue stars Relatively few lines He II dominates	Strong He II lines—in absorption, sometimes emission. He I lines weak, but increasing in strength from O5 to O9. Hydrogen Balmer lines prominent, but weak compared to later types. Lines of Si IV, O III, N III, and C III.
B	Hot blue stars More lines He I dominates	He I lines dominate, with maximum strength at B2; He II lines virtually absent. Hydrogen lines strengthening from B0 to B9. Also Mg II and Si II lines.
A	Blue stars Ionized metal lines Hydrogen dominates	The hydrogen lines reach maximum strength at A0. Lines of ionized metals (Fe II, Si II, Mg II) at maximum strength near A5. Ca II lines strengthening. The lines of neutral metals are appearing weakly.

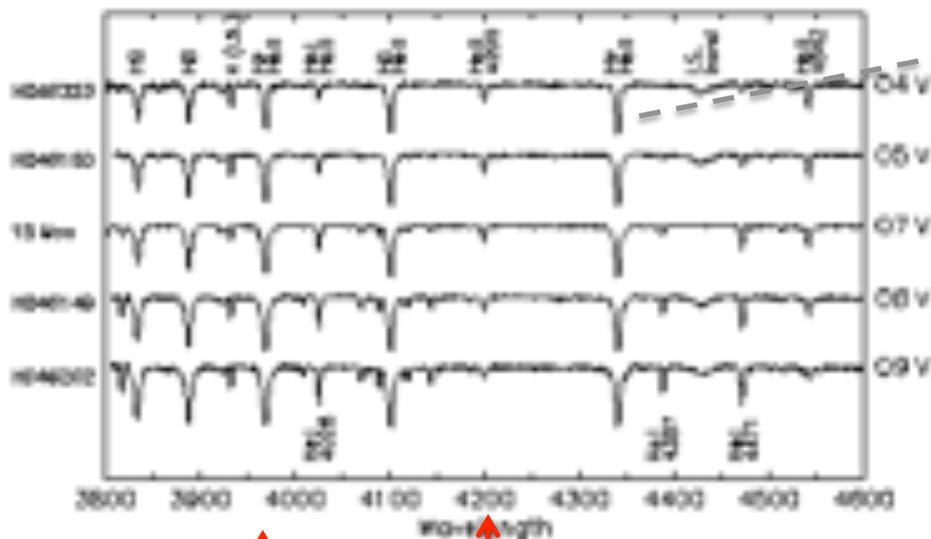
F	White stars Hydrogen lines declining Neutral metal lines increasing	The hydrogen lines are weakening rapidly, while the H and K lines of Ca II strengthen. Neutral metal (Fe I and Cr I) lines gaining on ionized metal lines by late F.
G	Yellow stars Many metal lines Ca II lines dominate	The hydrogen lines are very weak. The Ca II H and K lines reach maximum strength near G2. Neutral metal (Fe I, Mn I, Ca I) lines strengthening, while ionized metal lines diminish. The molecular G-band of CH becomes strong.
K	Reddish stars Molecular bands appear Neutral metal lines dominate	The hydrogen lines are almost gone. The Ca lines are strong. Neutral metal lines are very prominent. By late K the molecular bands of TiO begin to appear.
M	Coollest red stars Neutral metal lines strong Molecular bands dominate	The neutral metal lines are very strong. Molecular bands are prominent, with the TiO bands dominating the spectrum by M5. Vanadium oxide (VO) bands appear.

# Summary of spectroscopic types

Class	Temperature <sup>(K)</sup> (Kelvins)	Conventional color	Apparent color <sup>(V=0.11)</sup>	Mass <sup>(M<sub>☉</sub>)</sup> (solar masses)	Radius <sup>(R<sub>☉</sub>)</sup> (solar radii)	Luminosity <sup>(L<sub>☉</sub>)</sup> (bolometric)	Hydrogen lines	Fraction of all main sequence stars <sup>(%)</sup>
O	≥ 33,000 K	blue	blue	≥ 16 M <sub>☉</sub>	≥ 6.5 R <sub>☉</sub>	≥ 30,000 L <sub>☉</sub>	Weak	~0.00007%
B	10,000–33,000 K	blue to blue white	blue white	2.1–16 M <sub>☉</sub>	1.8–6.5 R <sub>☉</sub>	25–30,000 L <sub>☉</sub>	Medium	0.13%
A	7,500–10,000 K	white	white to blue white	1.4–2.1 M <sub>☉</sub>	1.4–1.8 R <sub>☉</sub>	5–25 L <sub>☉</sub>	Strong	0.6%
F	6,000–7,500 K	yellowish white	white	1.04–1.4 M <sub>☉</sub>	1.15–1.4 R <sub>☉</sub>	1.5–5 L <sub>☉</sub>	Medium	3%
G	5,200–6,000 K	yellow	yellowish white	0.8–1.04 M <sub>☉</sub>	0.96–1.15 R <sub>☉</sub>	0.6–1.5 L <sub>☉</sub>	Weak	7.6%
K	3,700–5,200 K	orange	yellow orange	0.45–0.8 M <sub>☉</sub>	0.7–0.96 R <sub>☉</sub>	0.08–0.6 L <sub>☉</sub>	Very weak	12.1%
M	≤ 3,700 K	red	orange red	≤ 0.45 M <sub>☉</sub>	≤ 0.7 R <sub>☉</sub>	≤ 0.08 L <sub>☉</sub>	Very weak	76.4%

[http://en.wikipedia.org/wiki/Stellar\\_classification](http://en.wikipedia.org/wiki/Stellar_classification)

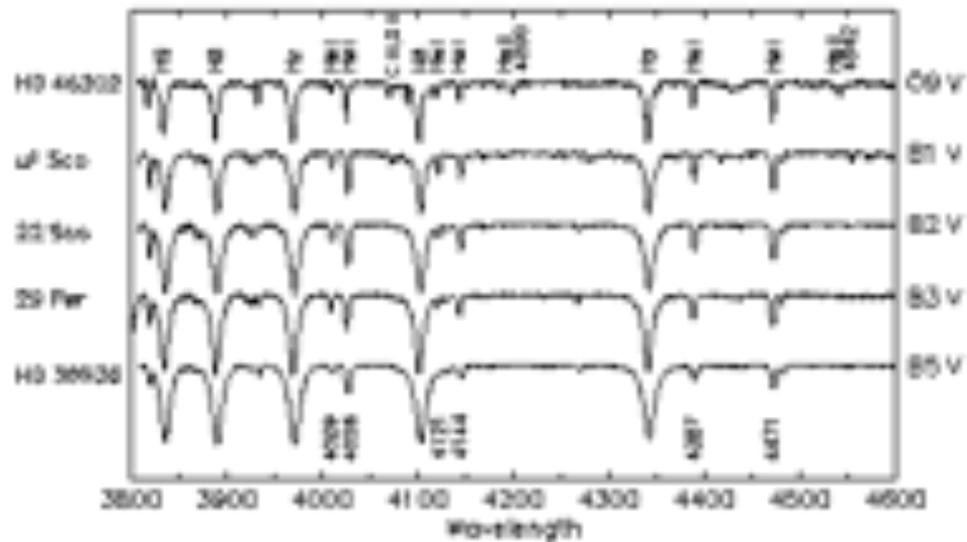
Main Sequence O4 - O9



10 → 4 in He II  
has same wavelength as  
5 → 2 in H I

He II strong,  
He I increasing  
from O4 to O9  
H prominent

Main Sequence O9 - B5

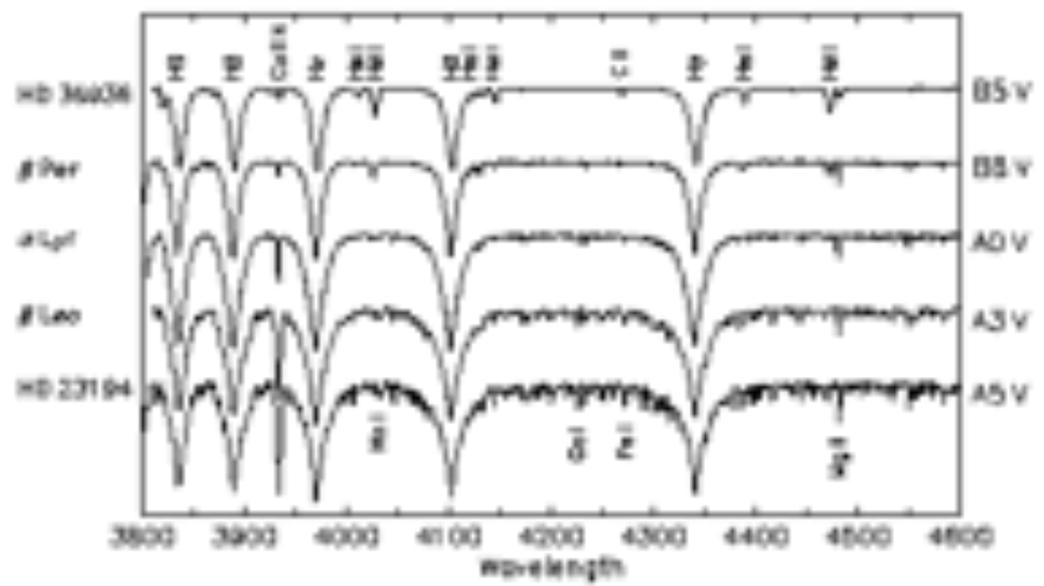


He I lines dominate  
H increasing in strength

# Balmer Series

Transition	3 -> 2	4 -> 2	5 -> 2	6 -> 2	7-> 2
Name	H $\alpha$	H $\beta$	H $\gamma$	H $\delta$	H $\epsilon$
Wavelength	6563	4861	4341	4102	3970
Color	Red	Blue-green	Violet	Violet	Ultra-violet

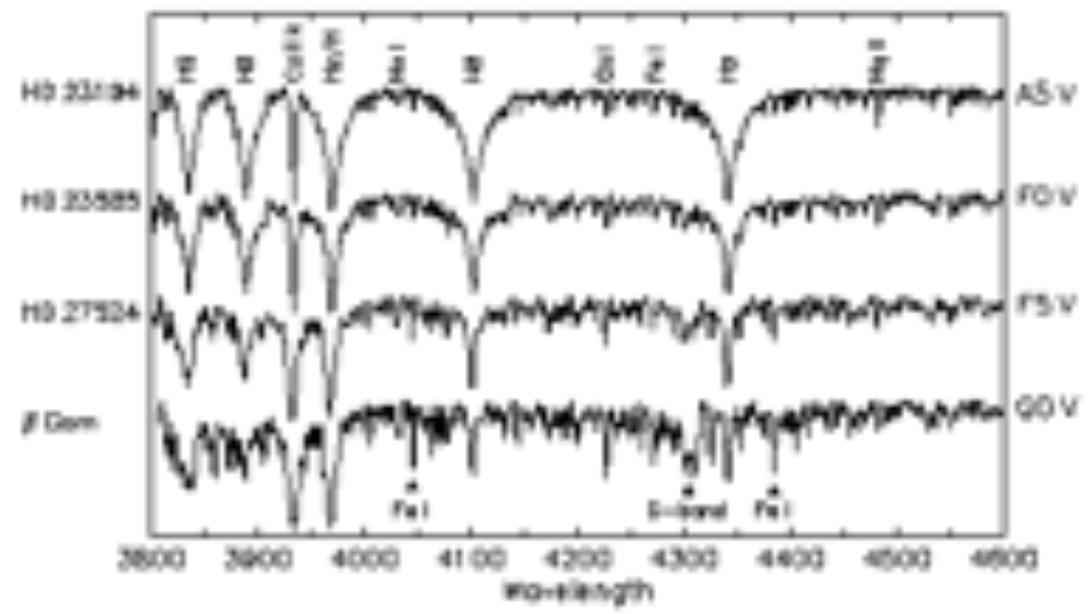
Main Sequence B5 – A5



H lines reach maximum strength. Ca II growing. Fe II, Si II, Mg II reach

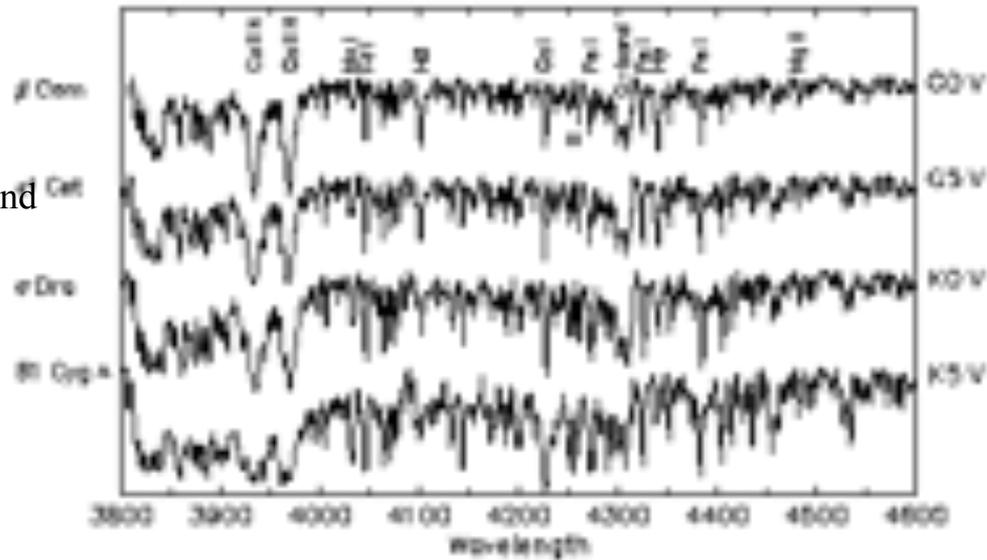
H $\gamma$  = 4341 Å  
H $\delta$  = 4102 Å

Main Sequence A5 – G0



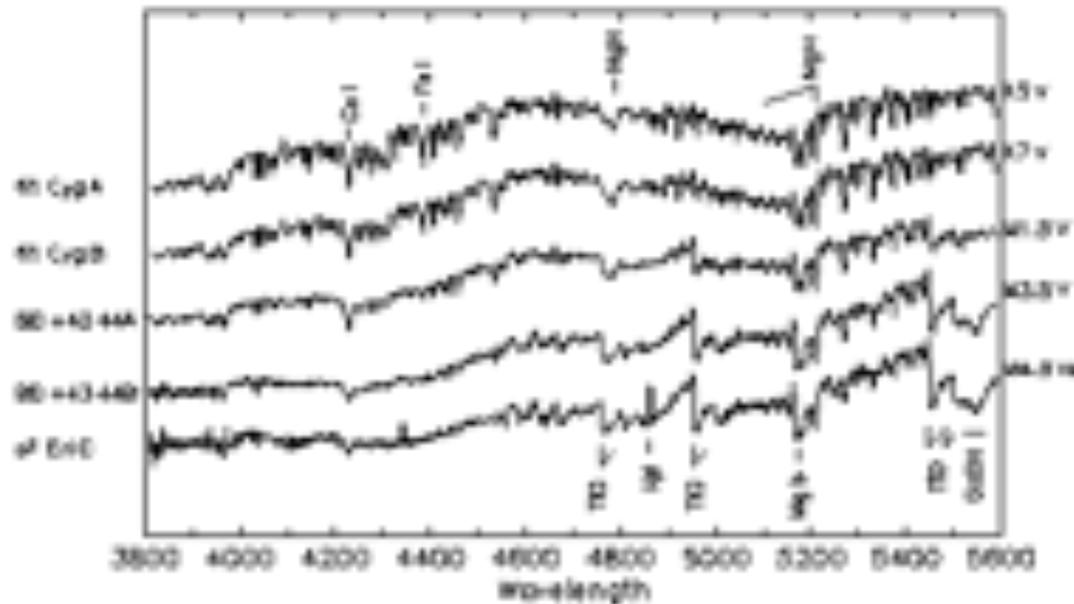
H lines start to decrease in strength. Ca II strong. Fe I growing in strength. Mg II decreasing.

Main Sequence G0 – K5



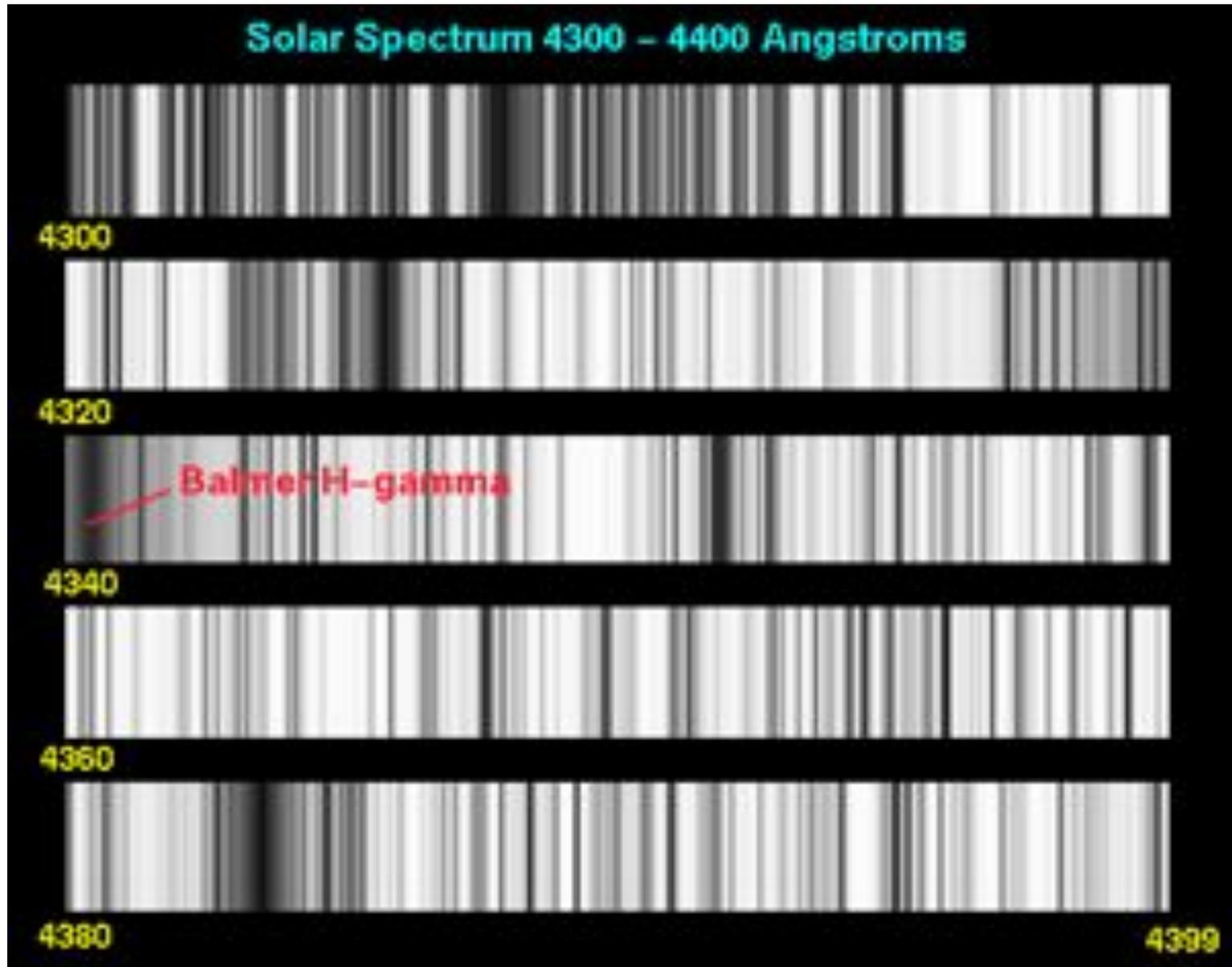
Ca II lines strongest,  
H lines weak, neutral  
metal lines strong. G-band  
of CH is strong.

Main Sequence K5 – M4.5  
Normalized Flux



H lines weak. Lines  
of neutral metals present  
but weakening. Major  
characteristic is bands  
from molecules like TiO  
and MgH

(Part of) the solar spectrum



# DISTINGUISHING MAIN SEQUENCE STARS

The surface gravity

$$g = \frac{GM}{R^2}$$

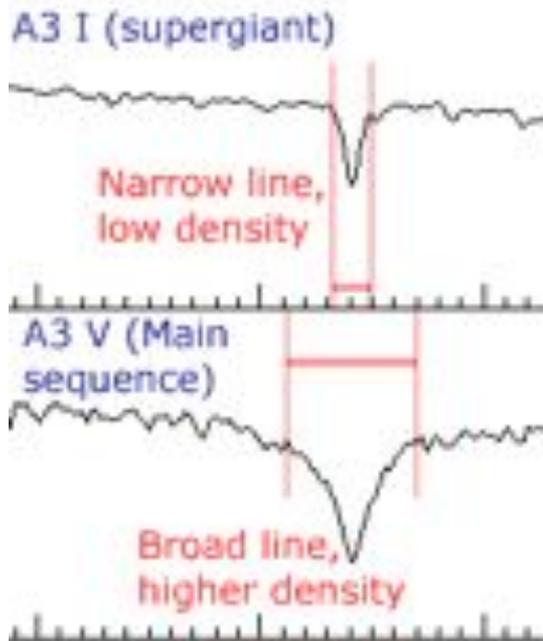
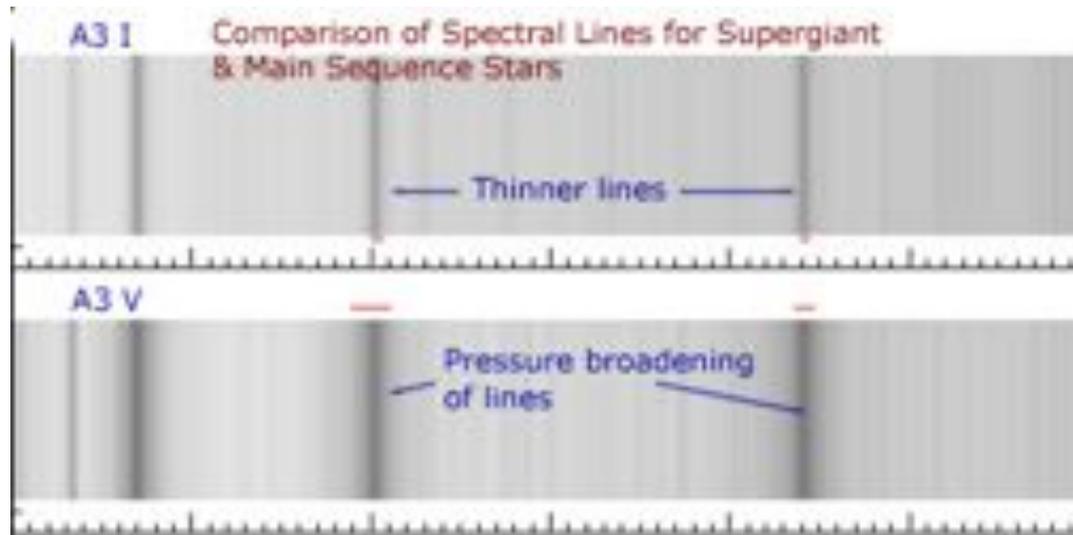
of a star is clearly larger for a smaller radius (if  $M$  is constant)

To support itself against this higher gravity, a the stellar photosphere must have a larger pressure. As we shall see later for an ideal gas

$$P = n k T$$

where  $n$  is the number density and  $T$  is the temperature. If two stars have the same temperature,  $T$ , the one with the higher pressure (smaller radius) will have the larger  $n$ , i.e., its atoms will be more closely crowded together. This has two effects:

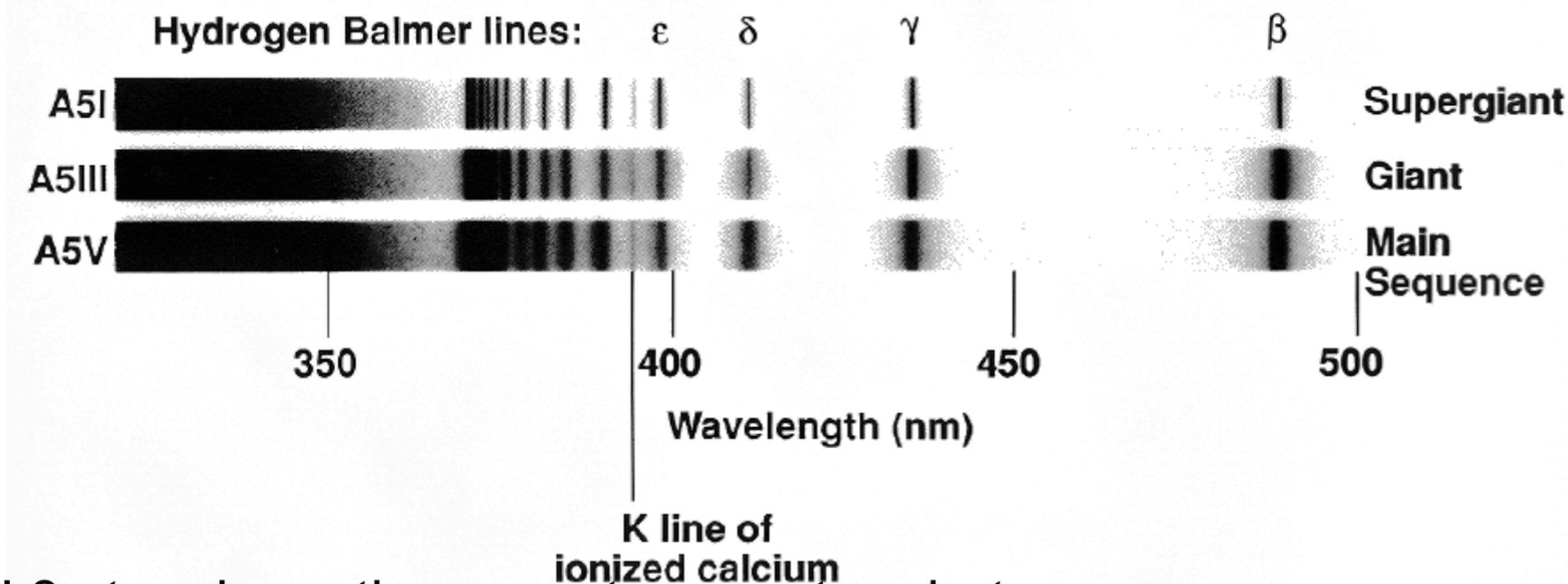
- 1) At a greater density (and the same  $T$ ) a gas is less ionized
- 2) If the density is high, the electrons in one atom “feel” the presence of other nearby nuclei. This makes their binding energy less certain. This spreading of the energy level is called “Stark broadening”



Note: Surface gravity on the main sequence is higher for *lower* mass stars

$$R \propto M^{0.65}$$

$$\frac{GM}{R^2} \text{ decreases with increasing } M$$



All 3 stars have the same temperature but,

- The supergiants have the narrowest absorption lines
- Small Main-Sequence stars have the broadest lines
- Giants are intermediate in line width and radius

- In 1943, Morgan & Keenan added the *Luminosity Class* as a second classification parameter:
  - Ia = Bright Supergiants
  - Ib = Supergiants
  - II = Bright Giants
  - III = Giants
  - IV = Subgiants
  - V = Main sequence

# Luminosity Classes

