Star Formation and Pressure

http://apod.nasa.gov/apod/astropix.html

THE THREE COMPONENT INTERSTELLAR MEDIUM

Component	Fractional volume	Scale Height (pc)	Temperature	Density	State of Hydrogen	Observational Technique
Cold dense Molecular Clouds	< 1% but ~40% of mass	70 - 300	10 - 100	10^2 - 10^6	H_2	Radio and infrared (molecules)
Warm Neutral Medium (WNM)	30-70% volume about 50% of mass	300 - 1000	100-10000	0.2 – 50	H I	21 cm
Coronal Gas (Hot Ionized Medium)	30 – 70% but <5% of mass	1000 - 3000	10^6 - 10^7	10^{-4} - 10^{-2}	H II metals also ionized	x-ray ultraviolet ionized metals recombination

In which of these components can star formation take place?

A necessary condition is a region of gas that has greater gravitational binding energy than internal energy. (The force pulling the region together must be greater than the pressure pushing it apart.)

Since internal energy increases with the amount of mass that is present while binding energy increases as M^2, there is a critical mass that is bound.

The Jean's Mass

$$\boxed{\Omega \approx \mathrm{KE}}$$

Ignore factor of 2 in the Virial Theorem. The clouds we are envisioning have not reached equilibrium.

$$\Omega \approx \frac{3}{5}\frac{GM^2}{R} \approx (\text{Number of particles})\left(\frac{3}{2}kT\right)$$

$$\approx \frac{M}{m_H}\frac{3}{2}kT \quad \text{(if made of pure hydrogen)}$$

$$= \mathrm{N_A}\, M\frac{3}{2}kT \quad (\mathrm{N_A} \text{ is Avogadro's Number, } 6.02\times10^{23})$$

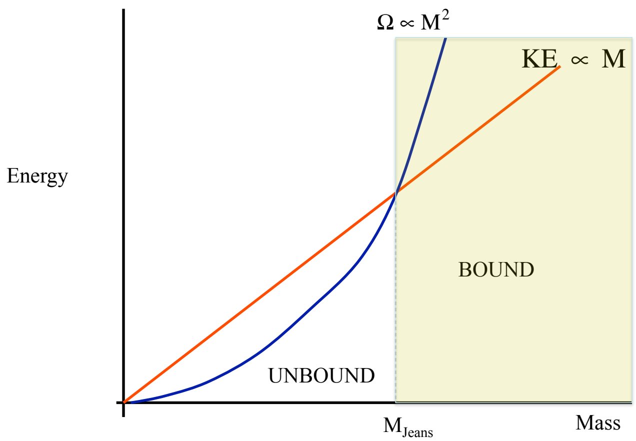
This can be solved for the "Jean's Mass", M_J

$$\frac{3}{5}\frac{GM_J^2}{R} = \frac{3}{2}N_A M_J kT$$

$$\boxed{\mathrm{M_J} = \frac{5N_A kTR}{2G}}$$

Clouds of gas with radius R and temperature T that have a mass bigger than this are unstable to gravitational collapse

For masses larger than the Jean's Mass gravitational binding energy exceeds internal energy



It is easier to measure densities and temperatures rather than radii, so the equation on the previous page can be transformed using

$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$ assume sphere, constant density $M = \frac{4}{3}\pi R^3 \rho$

previous page $$M_J = \frac{5N_A kTR}{2G} = \frac{5N_A kT}{2G}\left(\frac{3M_J}{4\pi\rho}\right)^{1/3}$$

$$M_J^{2/3} = \frac{5N_A k}{2G}\left(\frac{3}{4\pi}\right)^{1/3}\left(\frac{T^3}{\rho}\right)^{1/3}$$

$$M_J = \left(\frac{5N_A k}{2G}\right)^{3/2}\left(\frac{3}{4\pi}\right)^{1/2}\left(\frac{T^3}{\rho}\right)^{1/2}$$

$$= 8.5\times10^{22}\ \text{gm}\left(\frac{T^{3/2}}{\rho^{1/2}}\right) = 4.2\times10^{-11}\left(\frac{T^{3/2}}{\rho^{1/2}}\right)M_\odot$$

It is more frequent that one finds the density in this context expressed as atoms/cm^3 rather than gm/cm^3.

If $n = \rho N_A$ (strictly true only for H I), then

$$M_J = 8.5\times10^{22}\frac{T^{3/2}N_A^{1/2}}{n^{1/2}}\ \text{gm}$$

$$\boxed{M_J = 34\frac{T^{3/2}}{n^{1/2}}M_\odot}$$

where n is the density in atoms cm^{-3}.

By this criterion, only molecular clouds and possibly portions of the coldest neutral medium (depending on mass) are unstable to collapse.

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Example: Molecular cloud; T = 20 K, n = 10^4 atoms cm^{-3}

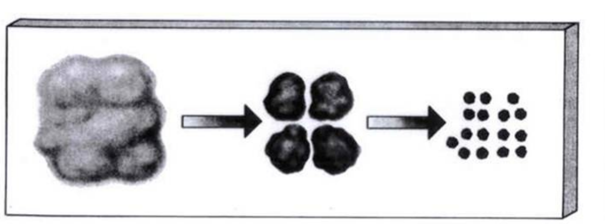
$$M_J = 34\,\frac{T^{3/2}}{n^{1/2}}$$

$$= 34\,\frac{(20)^{3/2}}{(10^4)^{1/2}} = 34\,\frac{89.4}{100}$$

$$= 30\ \mathrm{M}_\odot$$

Any cloud with this temperature and density and a mass over 30 solar masses is unstable to collapse

Fragmentation



Complications:

Rotation

Magnetic fields

How long does the collapse take?

$$v_{esc} = \sqrt{\frac{2GM}{R}} \qquad \tau_{ff} \approx \frac{R}{v_{esc}} = \sqrt{\frac{R^3}{2GM}}$$

but, ρ, the density, is given by

$$\rho = \frac{3\mathrm{M}}{4\pi \mathrm{R}^3} \Rightarrow \frac{R^3}{M} = \frac{3}{4\pi\rho}$$

so,

$$\boxed{\tau_{ff} \approx \sqrt{\frac{3}{8\pi G\rho}} \approx 1300\,\text{seconds}/\sqrt{\rho}}$$

Denser regions collapse faster

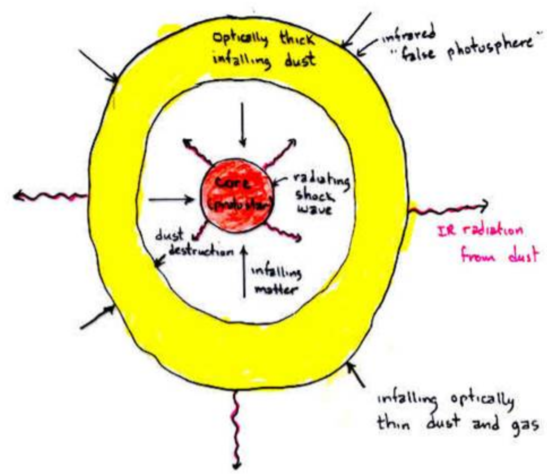
but $\rho \approx n/\mathrm{N_A}$, so

$$\tau_{ff} \approx 30 \text{ million years}/\sqrt{n}$$

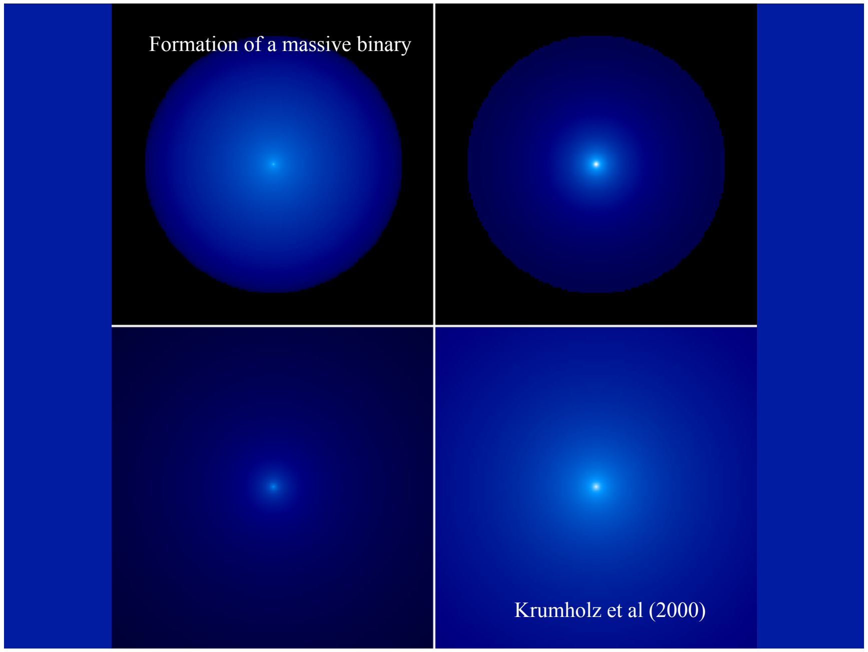
where n is the number of atoms per cubic cm.

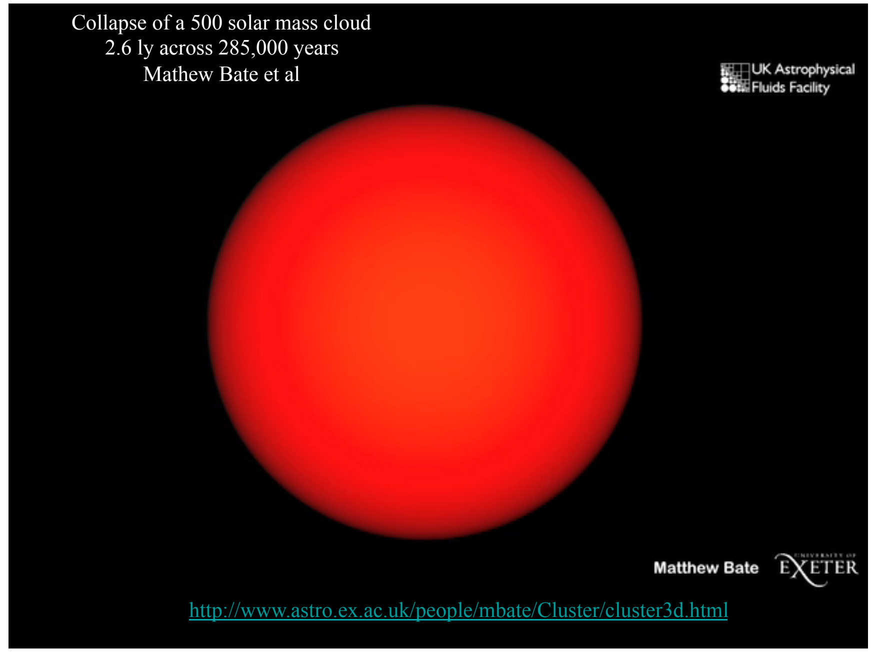
3 million years if n = 10^2 atoms/cm^3

Three million years is also the lifetime of the shortest lived stars

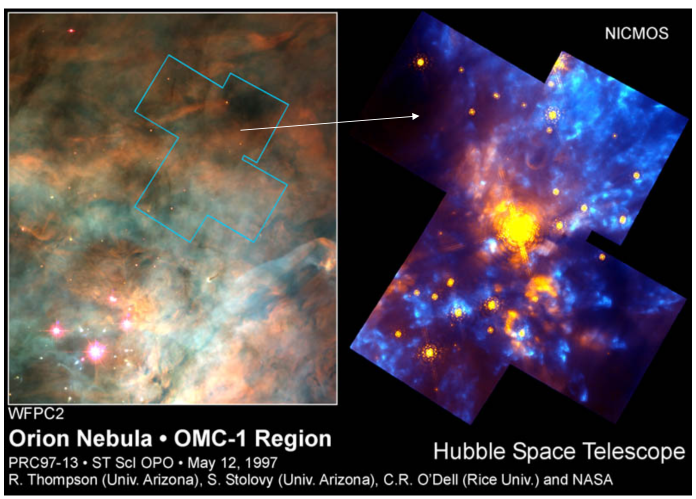


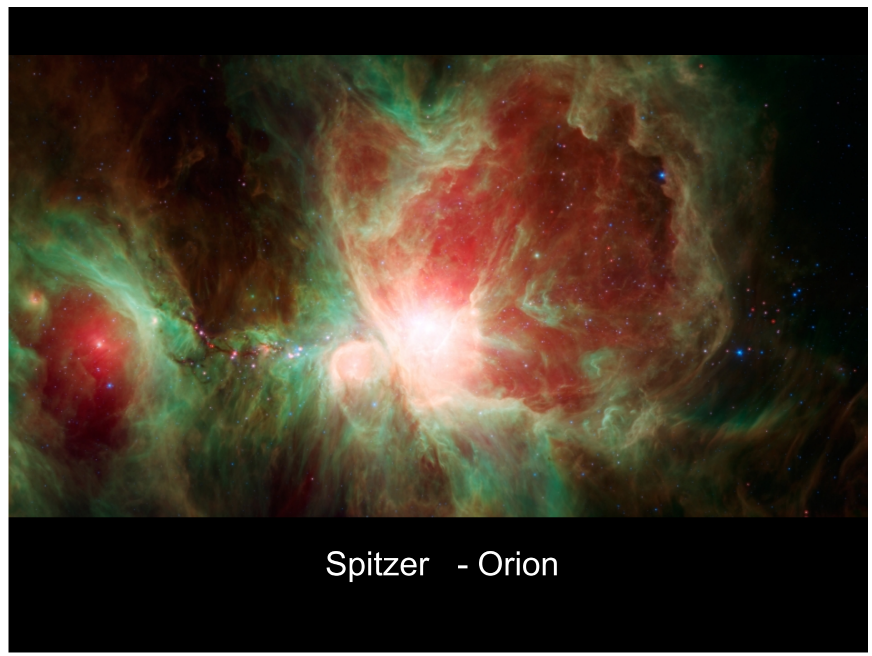
Star formation is inefficient. Even of the collapsing gas only 10 – 20 % of the gas ends up in the star, and overall an even smaller fraction of the cloud collapses to protostars.





Power of observing in the infrared

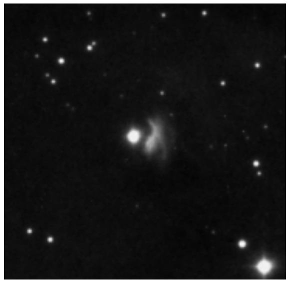




LH 95
A Stellar
Nursery in the
Large Magellanic
Cloud (HST)

The star formation region N11B in the LMC taken by WFPC2 on the NASA/ESA Hubble Space Telescope

T-Tauri Stars

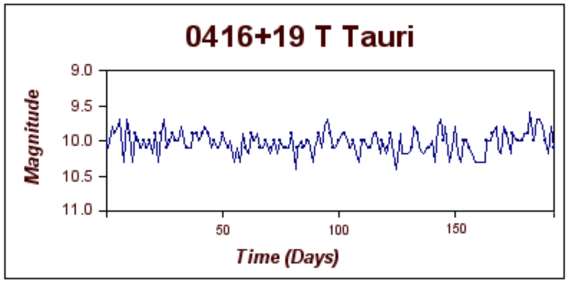


T-Tauri – in Taurus close to the Pleaides

T-Tauri discovered by John Hind in 1852 as a 10th magnitude star.
A faint nebula was subsequently discovered nearby ("Hind's nebula").
Both the star and nebula had variable brightness. The nebula was a "reflection" nebula, shining from the reflected light of T-Tauri.

By 1861 the nebula disappeared from view and by 1890 T-Tauri itself had faded to 14th magnitude, about the limit of telescopes then.
A faint nebula at the site of T-Tauri itself was observed at that time,

Over the next 10 – 20 years, T-Tauri brightened back to 10th magnitude and its local nebula became invisible against the glare. T-Tauri has remained at about 10th magnitude since (but varies).

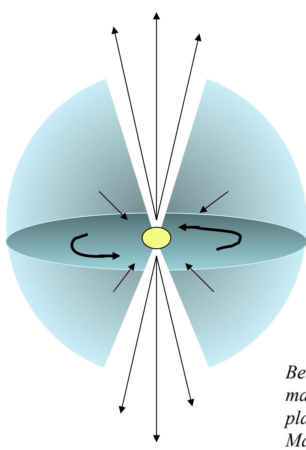


T-Tauri - about 400 ly away at the edge of a molecular cloud.
FOV here is 4 ly at the distance of T-Tauri http://apod.nasa.gov/apod/ap071213.htm

T-Tauri Stars

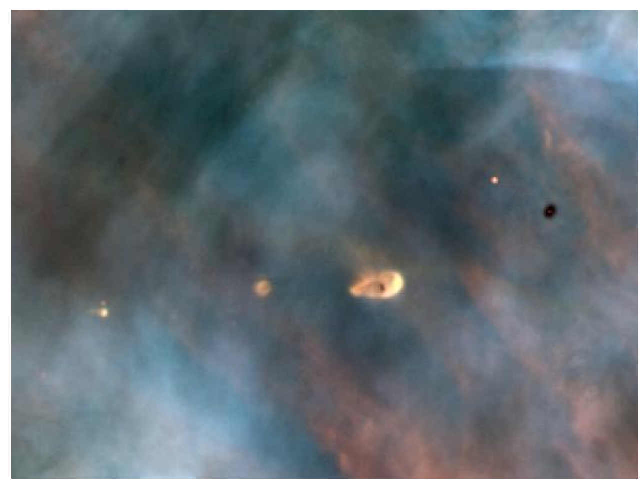
- Short lived phase in life of stars under 2 solar masses. Heavier stars evolve quicker and start burning by the time the star is visible. Above 2 solar masses the objects evolve rapidly and are rarely seen - "Herbig Ae and Be stars".
- Accretion disks and jets are common features
- Emission and absorption lines. High sunspot and magnetic activity
- Powered by gravitational contraction, not nuclear burning. In a Kelvin-Helmholtz phase
- May be forming planetary systems
- High lithium abundance
- Embedded in dense, dusty regions
- Can be highly variable. Larger luminosity than main sequence stars of same temperature implies larger radii

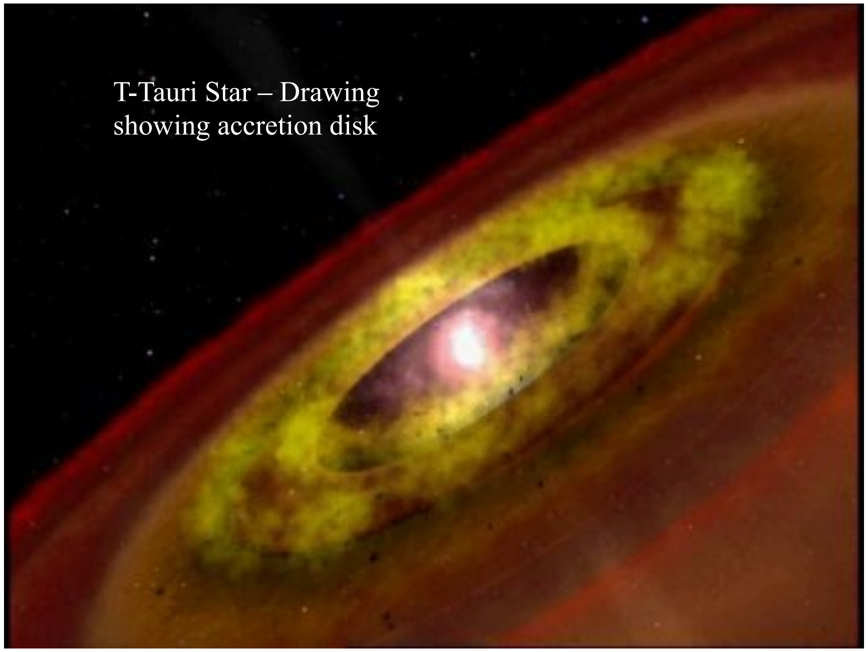
When the star first becomes visible it may still be surrounded by the gas and dust from which it formed. Often jets are seen.

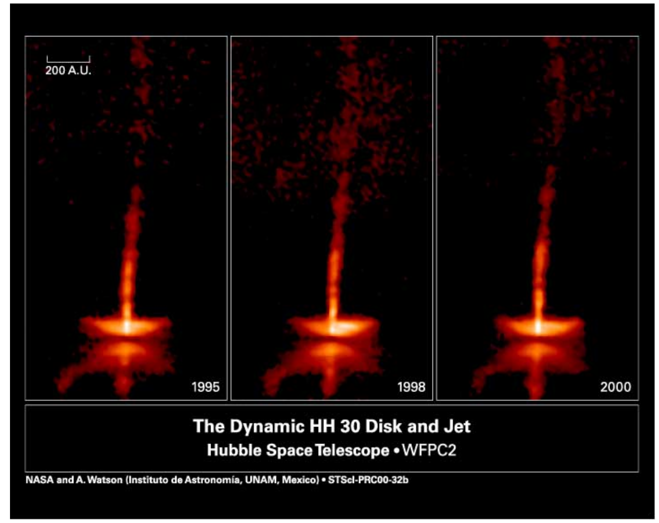


Because of rotational support matter hangs up in the equatorial plane forming an "accretion disk". Matter first rains down on the poles, but then later reverses direction in a strong collimated outflow called a "jet".

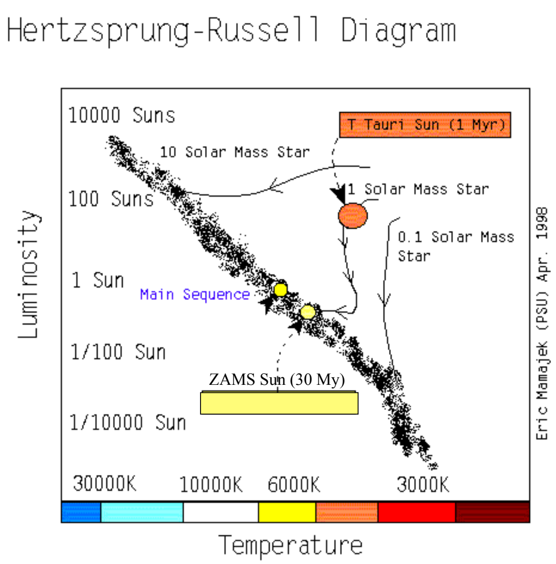
Protoplanetary disks orbit over half of T-Tauri stars.
This shows 5 such stars in the constellation Orion.
Picture using HST - field is about 0.14 ly across
http://en.wikipedia.org/wiki/T_Tauri_star





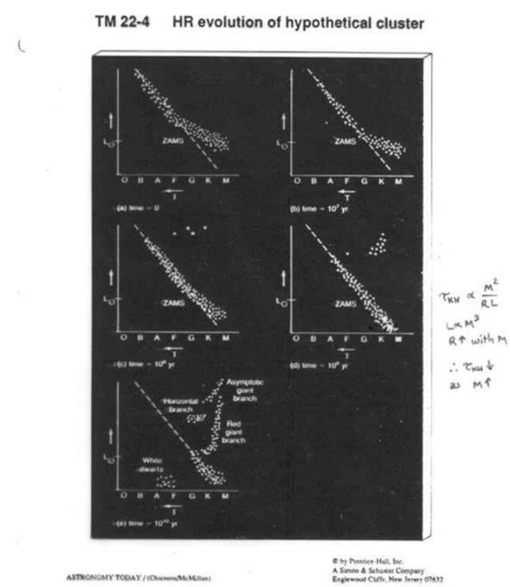


30” west of the brightest point in Hind’s nebula is a disk-jet system, Herbig-Haro 30. At the center of this is probably another T-Tauri like star.



Protostars start off with very large radii because they begin as contracting clouds of gas. They additionally have high luminosities because they are fully convective (more about this later) and able to transport the energy released by gravitational contraction efficiently to their surface.

Most of the time is spent close to the main sequence.

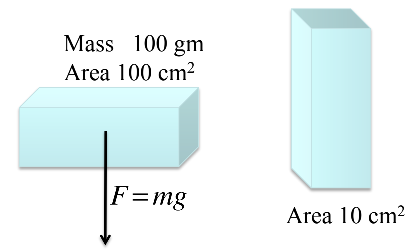


Stellar Interiors - Kinds of Pressure

Pressure is force *per unit area*

$$\text{Pressure} = \frac{\textit{Force}}{\textit{Area}}$$

$$g = \frac{GM_{earth}}{R^2_{earth}} = 980 \text{ cm s}^{-2}$$



$$P = \frac{mg}{A} = \frac{(100)(980)}{100} \qquad = \frac{(100)(980)}{10}$$

980 dyne cm^{-2} $\qquad$ $9800 \text{ dyne cm}^{-2}$

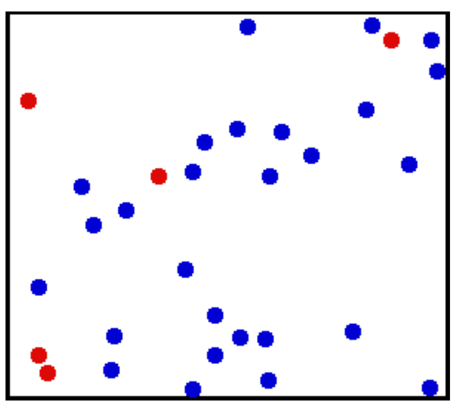
Gas Pressure

More particles -> more pressure

Faster particles -> more pressure

Heavier particles at the same speed -> more pressure

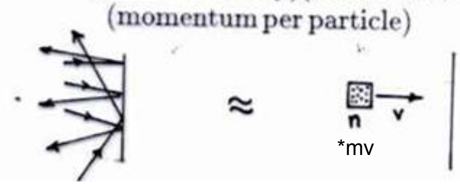
Particles exert pressure on on another, not just on the walls



http://intro.chem.okstate.edu/1314f00/laboratory/glp.htm

Qualitatively

$$P \sim (\text{number density})(\text{mean velocity})(\text{momentum per particle})$$



Each particle delivers a "kick" = 2 $m\ \Delta v_x$ *where* Δv_x is the change in x-velocity

Approximate this with a group of particles n in one cubic cm all moving to the right with $v_x = v$. The particle flux then = n times v and each particle imparts momentum of roughly mv

$$P \approx 2(mv)(nv) = 2nmv^2$$

For a gas, pressure is defined as

$$P = \frac{1}{3} \int \frac{dn}{dp} v\, p\, dp$$

where (dn/dp) * dp is the number density (per cm^3) of particles having momentum between p and $p+dp$, and v is their speed. The 1/3 is from an integral over angles.

Pressure thus has units

$$\frac{1}{cm^3}\,\frac{cm}{s}\,\frac{gm\ cm}{s} = \frac{gm\,cm}{cm^2\,s^2} = \frac{dyne}{cm^2}$$

Approximation:

suppose momentum p (and therefore v) is constant

Then

$$\frac{1}{3}\int \frac{dn}{dp} v\, p\, dp \approx \frac{1}{3} v\, p \int \frac{dn}{dp} dp = \frac{1}{3} v\, p\, \mathrm{n}$$

where n is the total number density of particles per cubic cm. When one integrates over a distribution of momenta, the 1/3 out front may change.

IDEAL GAS PRESSURE

- Due to the thermal motion of particles such as electrons, ions, molecules, etc. Particles only interact during their collisions. Particles moving slower than "c" and not "degenerate"

$$P \approx \frac{1}{3} n m v^2 \approx \frac{1}{3} n(3kT) = nkT$$

but $\frac{1}{2} m \left\langle v^2_{random} \right\rangle = \frac{3}{2} k\, T$

So

$$\boxed{P = n\, k\, T}$$

IDEAL GAS PRESSURE

But what is n? The number of particles per cm^3

For a given density, n depends upon the composition. E.g. for pure neutral atomic hydrogen, H I, the number of atoms in 1 gram is Advogadro's number,

N_A = 6.02 x 10^{23} atoms per mole.

Note that N_A= 1/m_H where m_H is the mass of the hydrogen atom.

For H I then

$$P_{ideal}(H\,I) = \rho\, N_A kT = 8.31\times10^7\ \rho T\ \ dyne\ cm^{-2}$$

but what if the hydrogen were ionized? Then there would be one electron for every proton. The electron, though lighter, would move faster and also contribute $n_e kT$ to the pressure. The total pressure would then be twice as great

$$P_{ideal}(H\,II) = 2\rho\, N_A kT = 1.66\times10^8\ \rho T\ \ dyne\ cm^{-2}$$

But what if the gas were fully ionized, 75% H II and 25% HeIII like the interior of a recently born star?

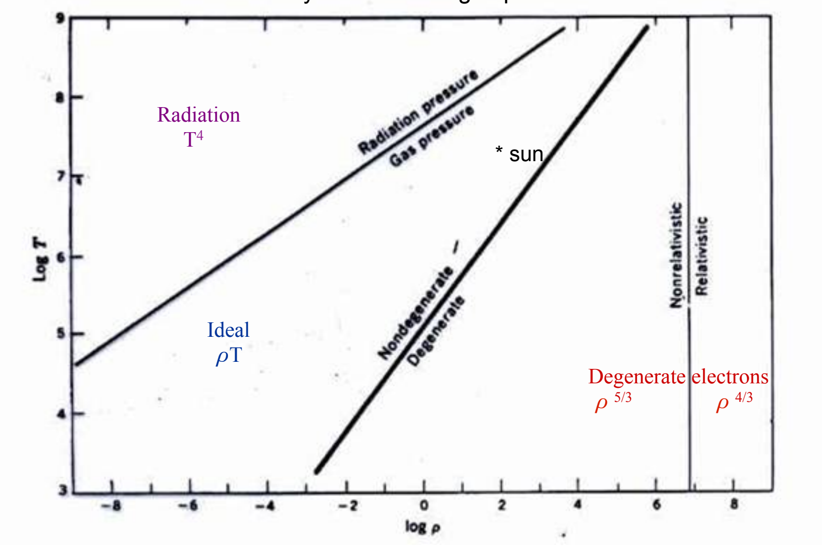
In general

$$P_{ideal} = F\rho\, N_A kT = 8.31\times10^7\, F\rho T\ dyne\ cm^{-2}$$

It is not difficult to evaluate F but it can be tedious so here is F for various compositions you may encounter

Composition	F	
H I	1	
H II	2	
ionized 75% H, 25% He	1.69	"star stuff"
ionized carbon	0.583	
ionized oxygen	0.563	
50% C; 50% O	0.573	
ionized 35%H; 65% He	1.19	

Most main sequence stars have pressures that are dominantly due to ideal gas pressure



DEGENERACY PRESSURE

Pressure due entirely to quantum mechanics and the wavelike nature of the electron.

Suppose one packs as many electrons with momentum p into a volume, V, as are quantum mechanically allowed by the wavelength of the electron

Each pair of electrons occupies a cell of size ~ $(\Delta x)^3$, but $\Delta x \sim \lambda = h/p$

$$\Delta x \cdot p = h$$

$$\text{Number of cells in volume V} = \frac{V}{(\Delta x)^3} = \frac{V}{\lambda^3} = \frac{Vp^3}{h^3}$$

$$\text{Number of electrons, N, in volume V} = \frac{2Vp^3}{h^3}$$

$$\text{Number of electrons per unit volume } n_e = \frac{N}{V} = \frac{2p^3}{h^3}$$

$$\text{So, } p_F \sim \left(\frac{n_e h^3}{2}\right)^{1/3}$$

This is commonly called the "Fermi Momentum"

http://en.wikipedia.org/wiki/Electron_degeneracy_pressure
http://en.wikipedia.org/wiki/Pauli_exclusion_principle

DEGENERACY PRESSURE

Now the pressure

$$P \sim \frac{1}{3} n_e\, p_F\, v = \frac{1}{3} n_e p_F \frac{mv}{m} = \frac{n_e\, p_F{}^2}{3m}$$

$$\sim \frac{n_e}{3m}\left(\frac{n_e h^3}{2}\right)^{2/3}$$

$$\boxed{P_{\deg} \sim \frac{h^2\, n_e^{5/3}}{3 \bullet 2^{2/3}\, m_e}}$$

The contribution of electrons, when present, is much larger than from neutrons or protons because of the 1/m

As n_e goes up the speed of each electron rises

$$p_F = m_e v \approx \left(\frac{n_e h^3}{2}\right)^{1/3} \qquad \text{more accurately } \left(\frac{3}{8\pi} n_e h^3\right)^{1/3}$$

$$v = \frac{1}{m_e}\left(n_e\left(\frac{3h^3}{8\pi}\right)\right)^{1/3} \qquad n_e \approx \frac{1}{2}\rho N_A \text{ for elements other than H}$$

$$v = \left(\frac{3\rho N_A h^3}{16\pi m_e^3}\right)^{1/3} \approx 2\ x\ 10^{10}\left(\frac{\rho}{10^6 \text{ gm cm}^{-3}}\right)^{1/3} \text{cm s}^{-1}$$

At around 10^7 gm cm^{-3} the electrons will move close to the speed of light.

$$\text{"}non-relativistic\text{" degeneracy pressure} = P_{NRD}$$

$$P_{NRD} \sim \frac{n_e p_F^2}{3m_e} = \frac{n_e\left(\frac{n_e h^3}{2}\right)^{2/3}}{3m}$$

$$= \frac{h^2}{3 \bullet 2^{2/3} m_e} n_e^{5/3} = 0.210 \frac{h^2}{m_e} n_e^{5/3}$$

A more accurate calculation gives

$$\boxed{P_{NRD} = \frac{1}{20}\left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{m_e}\, n_e^{5/3}} = 0.0485\, \frac{h^2}{m_e}\, n_e^{5/3}$$

http://scienceworld.wolfram.com/physics/ElectronDegeneracyPressure.html

For charge neutrality, number of electrons = number of protons and for pure hydrogen, $n_e = N_A \rho$.

For other compositions, $n_e = N_A \rho Y_e$ where $(Y_e)^{-1}$ is the number of electrons per atomic mass unit in the neutral atom. E.g., $Y_e = 1$ for hydrogen, 0.5 for ^{4}He, ^{12}C, etc, and 0.88 for 75% H and 25% He.

usually where P_{deg} is important $Y_e = 0.5$

Then

$$\boxed{P_{deg}^{NR} = 1.00 \times 10^{13} (\rho Y_e)^{5/3} \text{ dyne cm}^{-2}} \qquad \rho \lesssim 10^7 \text{ g/cm}^3$$

Note that the degeneracy pressure depends only on the density and not on the temperature

RELATIVISTIC DEGENERACY PRESSURE

The above remains true only so long as v of the electrons remains << c. As v approaches c

$$\boxed{P_{deg} \sim \tfrac{1}{3}(n_e)(c)(p) \sim n_e^{4/3}} \qquad (p \propto n_e^{1/3})$$

and in fact

$$\boxed{P_{deg}^{R} = 1.24 \times 10^{15}(\rho Y_e)^{4/3}\ \text{dyne cm}^{-2}} \qquad \rho \gtrsim 10^7\ \text{g/cm}^3$$

Once the electrons move near the speed of light, the pressure does not increase as rapidly with density as before.

RADIATION PRESSURE

Because electromagnetic radiation (light) carries energy, it also carries momentum. In general, for non-relativistic motion ($v << c$), momentum = (2)(kinetic energy)/(velocity), e.g., $2(1/2mv^2)/v = mv = p$. For photons the relation is a little different.

$$\boxed{p = E/c = h\nu/c}$$

So to get momentum flux just divide energy flux by c.

THE “PRESSURE” OF SUNLIGHT

From the sun, at the earth’s orbit (1AU), we receive a flux of radiation

$$\phi = \frac{L}{4\pi d^2} = \frac{L_\odot}{4\pi (AU)^2}$$

$$= 1.37 \times 10^6 \text{ erg cm}^{-2}\text{ s}^{-1}$$

This corresponds to a momentum flux, or pressure of

$$P = \frac{\phi}{c} = \frac{(1.37\times10^6)}{(3.00\times10^{10})}\ \frac{\text{erg}\qquad(\text{s})}{\text{cm}^2\text{ s}\ \ (\text{cm})}$$

$$= 4.57\times10^{-5}\ \frac{\text{dyne}}{\text{cm}^2} \qquad \text{since (dyne)(cm)=erg}$$

(1 square meter (10^4 cm^2) would be accelerated 0.46 cm/s^2 if it weighed 1 gm; would reach c in about 1000 years)

1997 Comet Hale Bop

The pressure is then

$$P \sim \text{(number flux)(momentum per photon)}$$
$$= \left(\frac{\text{E flux}}{\text{energy per photon}}\right)\text{(momentum per photon)}$$
$$= \left(\frac{\sigma T^4}{h\nu}\right)\left(\frac{h\nu}{c}\right)$$
$$= \frac{\sigma}{c} T^4$$

In fact the correct expression is

$$\boxed{P_{rad} = \frac{4\sigma}{3c}T^4 = \frac{1}{3}aT^4}$$

where $a = 7.56 \times 10^{-15}$ dyne cm^{-2} (K)$^{-4}$. $= \frac{4\sigma}{c}$

IN SUMMARY

There are 3 kinds of pressure:

- Ideal gas pressure ($\propto \rho$ and T)
- Radiation pressure ($\propto T^4$)
- Degeneracy pressure ($\propto \rho^n$) $4/3 < n < 5/3$

The *total* pressure is given by

$$P_{tot} = P_{ion} + P_{rad} + P_e$$

Except in neutron stars, P_{ion} is ideal. P_e can be quite complex (semidegenerate, semirelativistic) which can lead to some difficult math (Fermi integrals) which we will not consider.

Most main sequence stars have pressures that are dominantly ideal gas pressure