# LECTURE 15:

# WHITE DWARFS AND THE ADVANCED EVOLUTION OF MASSIVE STARS

$$M \geq 8\, M_{\odot}$$

http://apod.nasa.gov/apod/astropix.html

# White Dwarfs

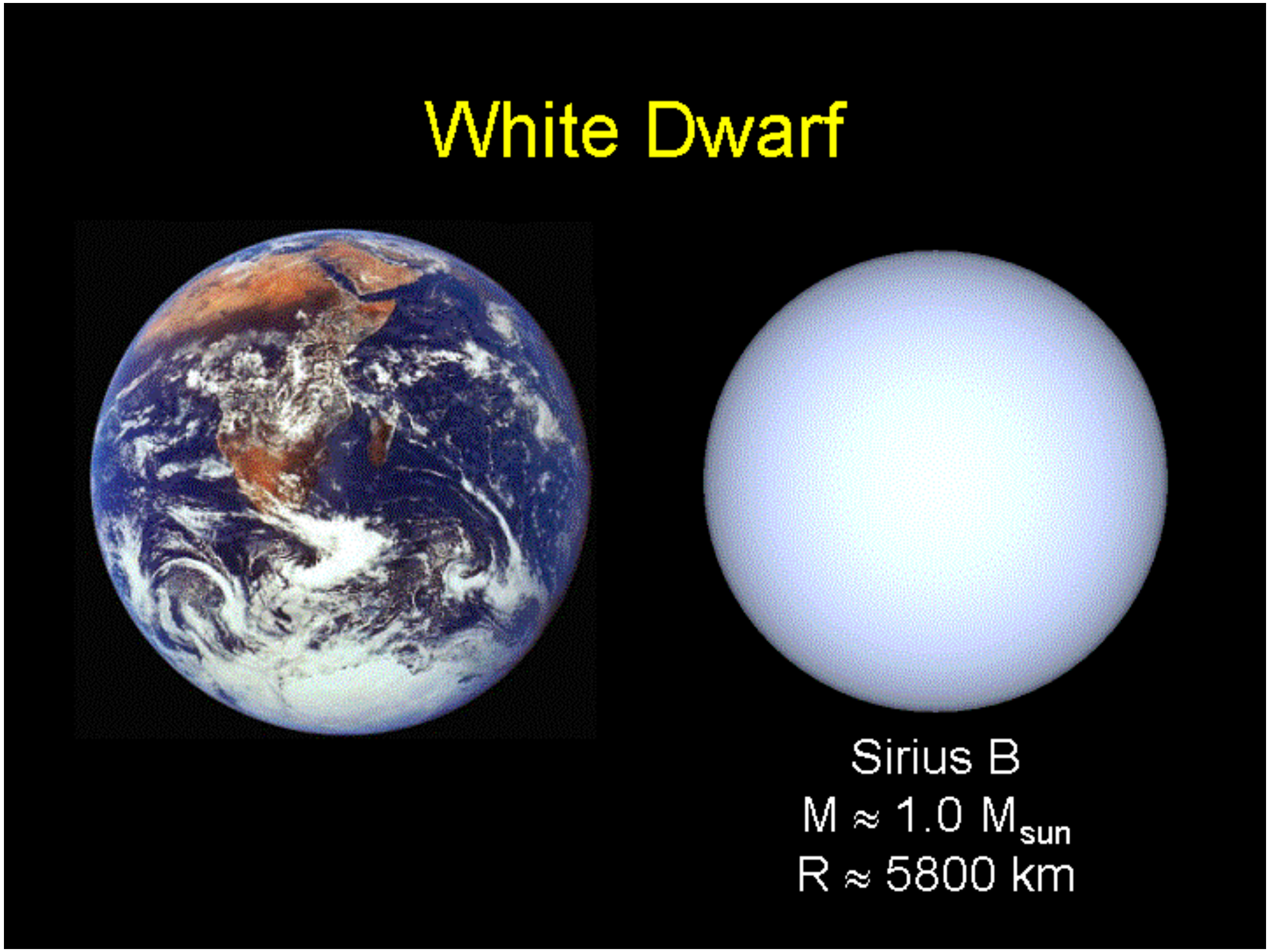
- Low mass stars are unable to reach high enough temperatures to ignite elements heavier than hydrogen in their core and become white dwarfs. Planetary nebulae reveal them. Can be He or CO. Rarely NeOMg.

- Hot exposed core of an evolved low mass star.

- Supported by electron degeneracy pressure.

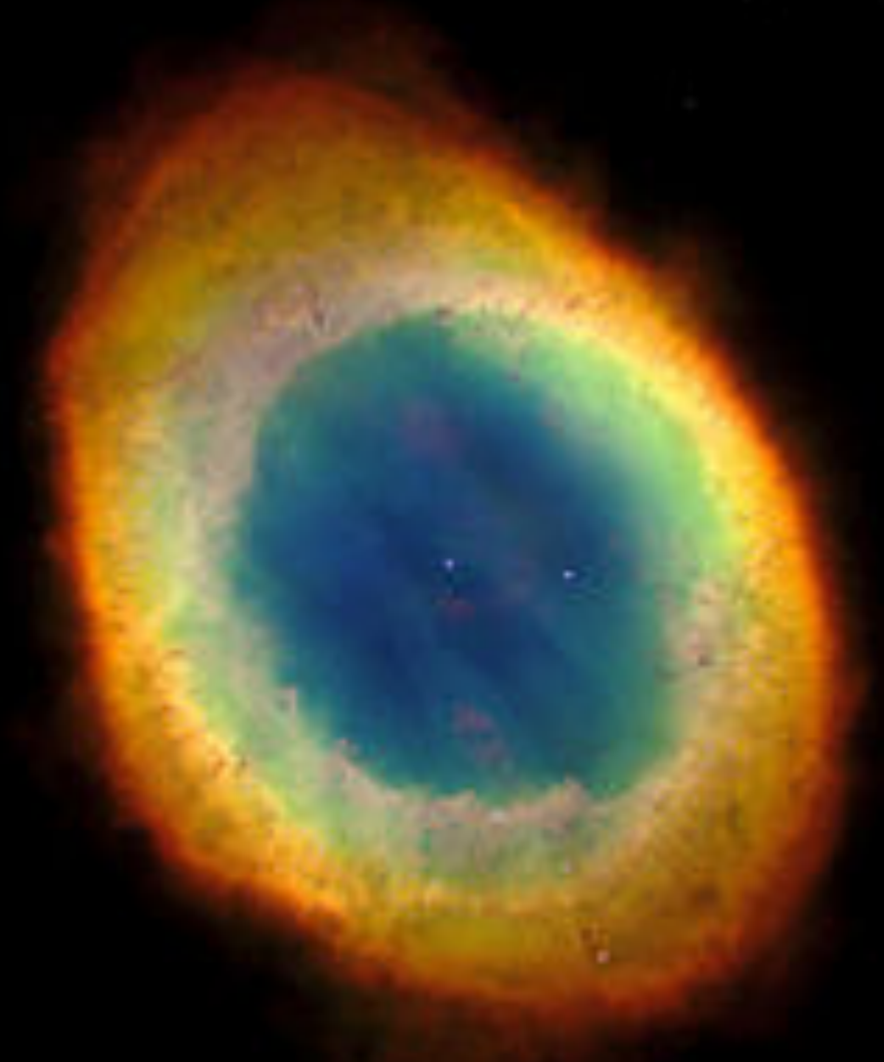
- Unable to produce energy by contracting or igniting nuclear reactions

- As white dwarfs radiate energy, they become cooler and less luminous gradually fading into oblivion, but it can take a long time….



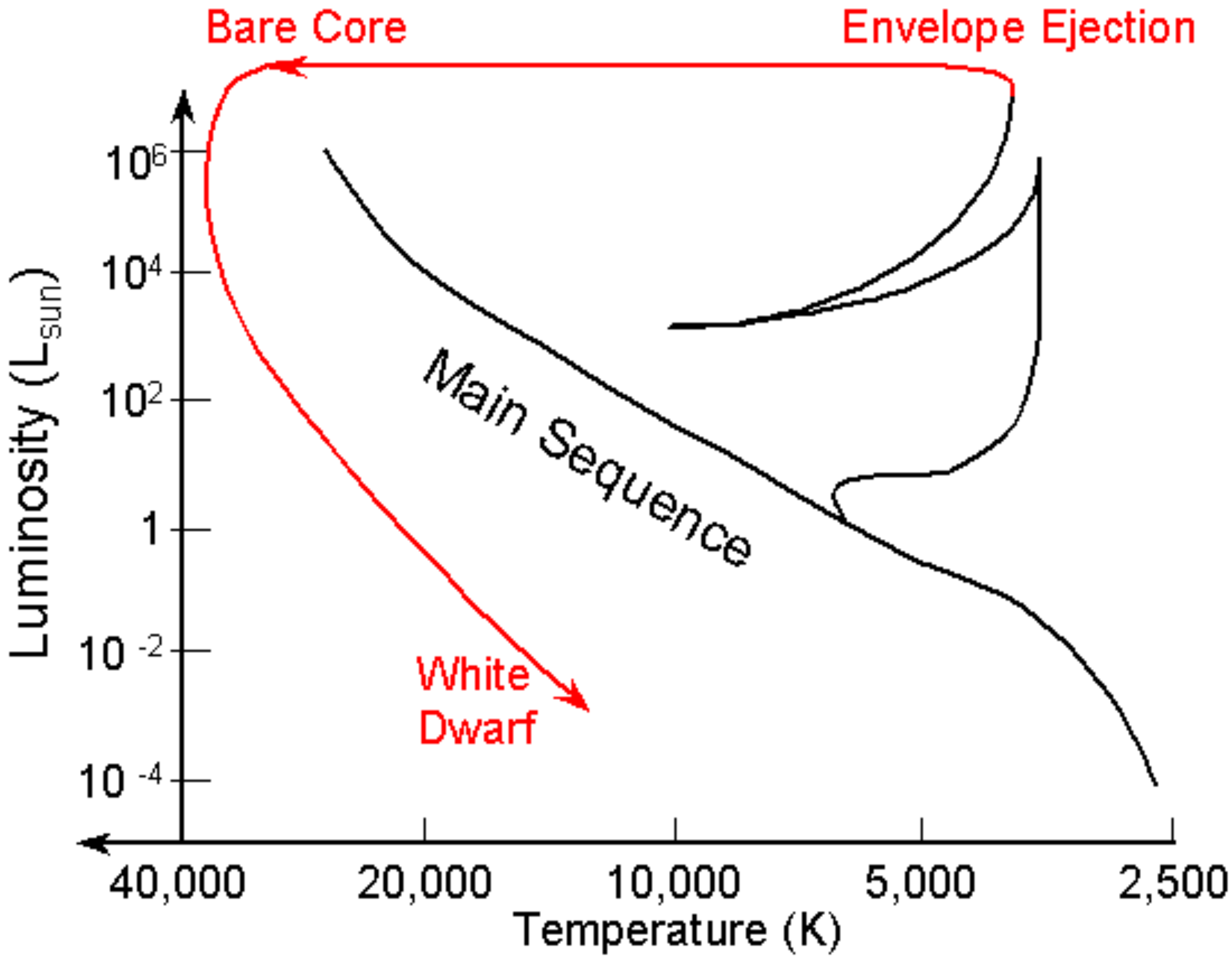

*A white dwarf is the remnant of stellar evolution for stars between 0.08 and 8 solar masses (below 0.08 one can have brown dwarfs). They can be made out of helium, or more commonly carbon and oxygen (rarely NeOMg).*

http://en.wikipedia.org/wiki/Ring_Nebula



The Ring Negula in Lyra (M57)
700 pc; magnitute 8.8

# Planetary Nebula Phase



# MASS RADIUS RELATION FOR WHITE DWARFS

$$P_c = \frac{GM\rho}{2R} = 1.00\times 10^{13}\ (\rho Y_e)^{5/3}$$

*if supported by non-relativistic electron degeneracy pressure*

$$\frac{GM\rho}{2R} \approx \frac{GM\left(\frac{3M}{4\pi R^3}\right)}{2R} = 1.00\times 10^{13}\left(\frac{3M}{4\pi R^3}\right)^{5/3}\left(\frac{1}{2}\right)^{5/3}$$

$$\left(\frac{3G}{8\pi}\right)\frac{M^2}{R^4} = 1.00\times 10^{13}\left(\frac{3}{8\pi}\right)^{5/3}\frac{M^{5/3}}{R^5}$$

$$M^{1/3} = 1.00\times 10^{13}\left(\frac{3}{8\pi}\right)^{2/3}\frac{1}{GR}$$

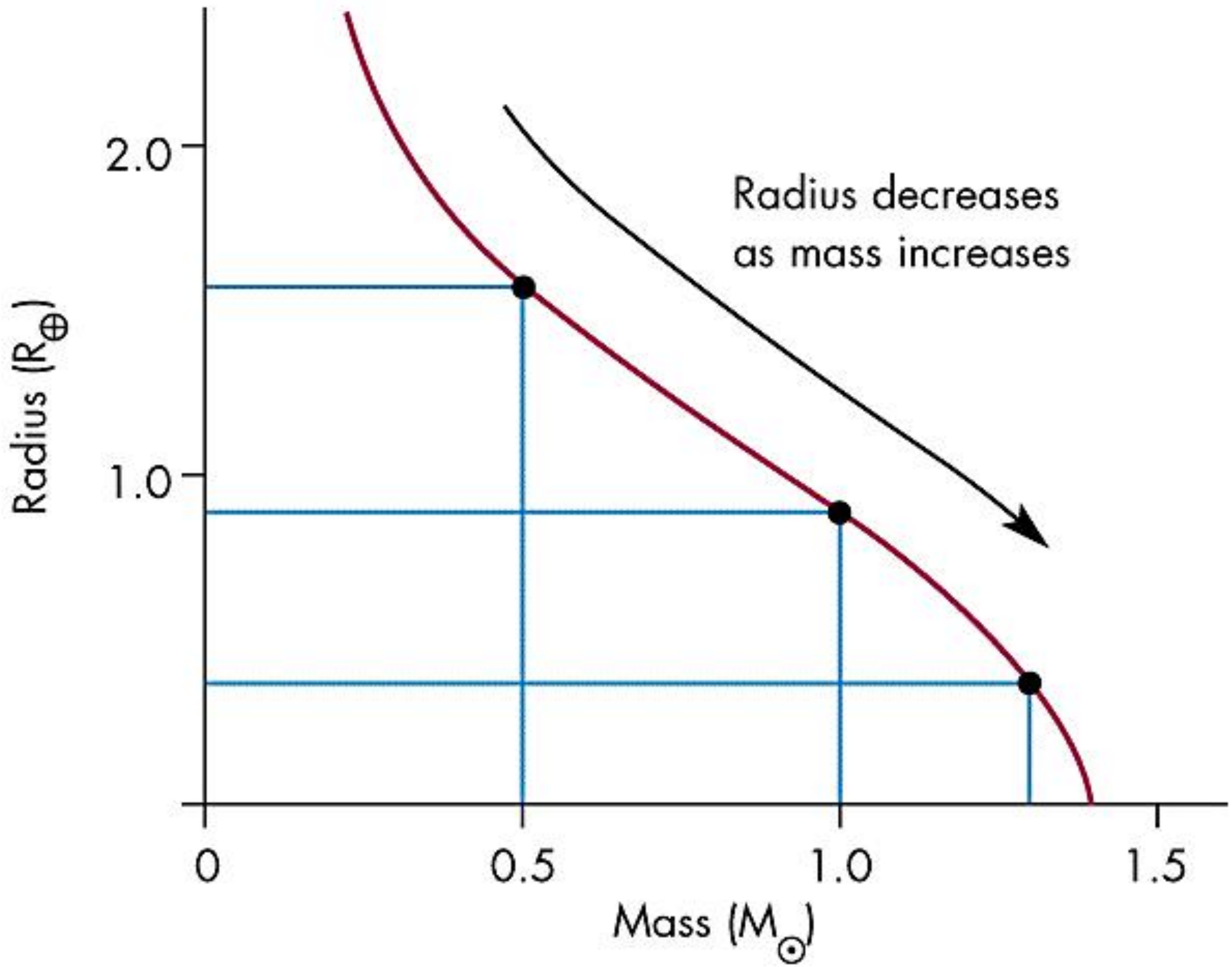
$$R = 3.63\times 10^{19}\ /\ M^{1/3}$$

$$R = 2.9\times 10^{8}\left(\frac{M_\odot}{M}\right)^{1/3}\ \text{cm}$$

Earth R = 6370 km

More massive white dwarfs are smaller

# Mass versus radius relation



# More massive white dwarf stars are denser

Actually $5 \times 10^8 \, (M_\odot/M)^{1/3}$ cm is more accurate.

for the radius

$$\rho = \frac{3M}{4\pi R^3}$$

$$= \frac{(3)(1.99 \times 10^{33})}{(4\pi)(5 \times 10^8)^3} \left(\frac{M}{M_\odot}\right)^2$$

$$= 4 \times 10^6 \left(\frac{M}{M_\odot}\right)^2 \text{ g cm}^{-3}$$

- Note implication: As M goes up, R gets smaller and $\rho$ gets larger. How far can this go?

# APPEARANCE

- $L \approx 0.01\ L_\odot$ (most are fainter)

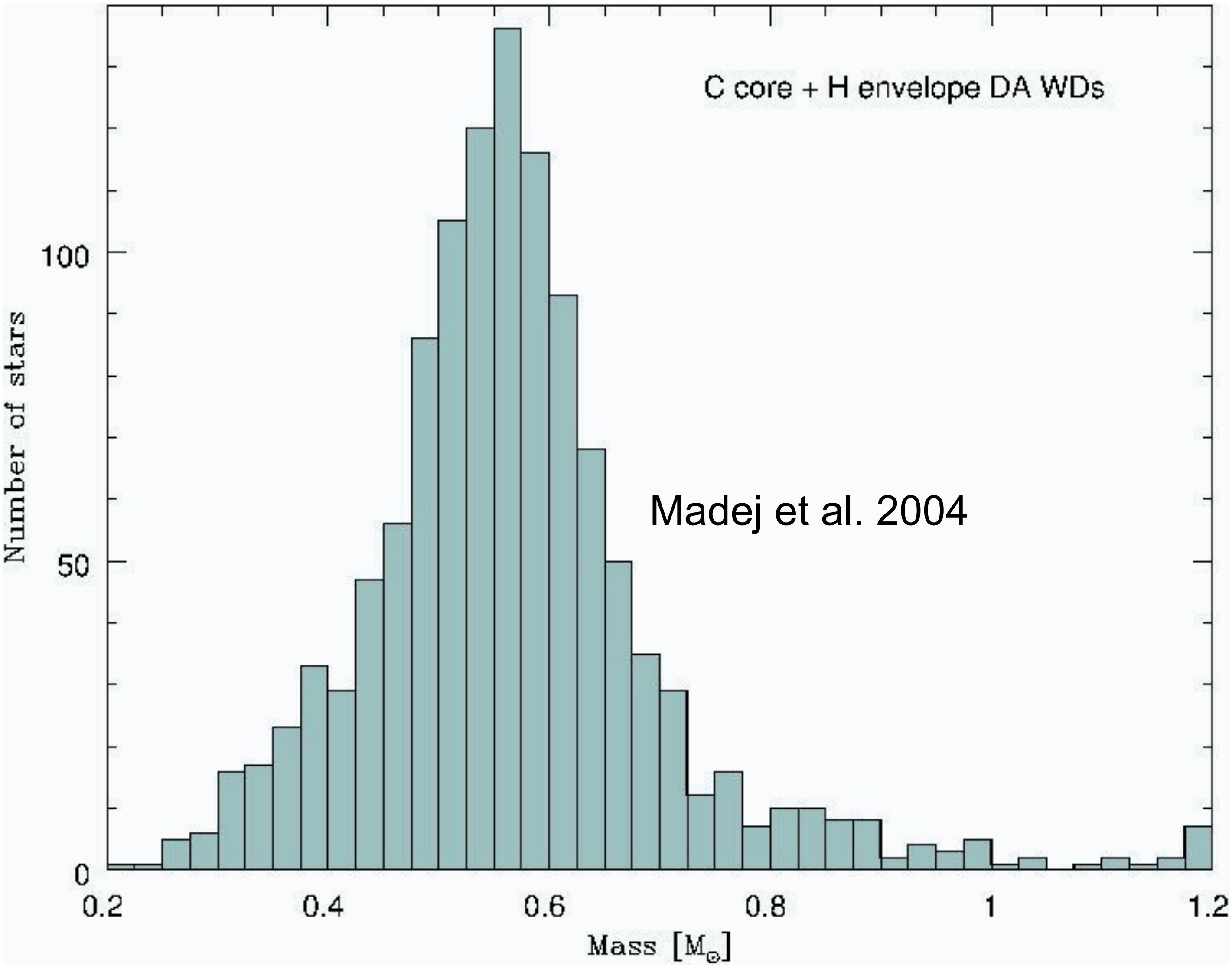
White dwarfs are known with temperatures ranging from 4000 K to 200,000 K

$$T_e = \left(\frac{L}{4\pi\sigma R^2}\right)^{1/4}$$
$$\approx \left[\frac{4 \times 10^{31}}{(4\pi)(5 \times 10^8)^2(5.6 \times 10^{-5})}\right]^{1/4}$$
$$= 20,000\ \mathrm{K}$$

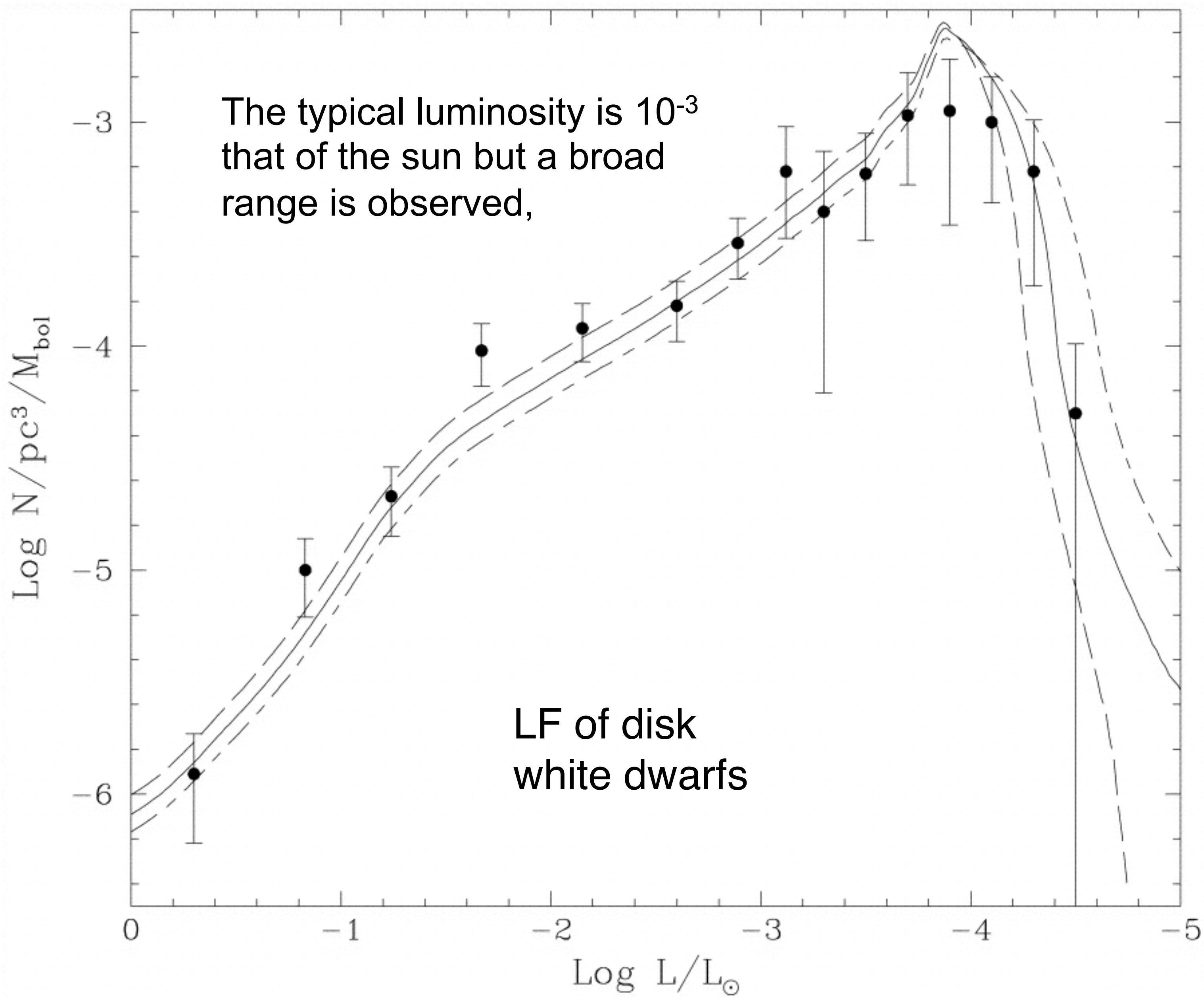
$$\lambda_{max} = \frac{2.89 \times 10^7\ \mathrm{A}}{T_{eff}}$$
$$= 1400\ \mathrm{A}$$

# Mass distribution

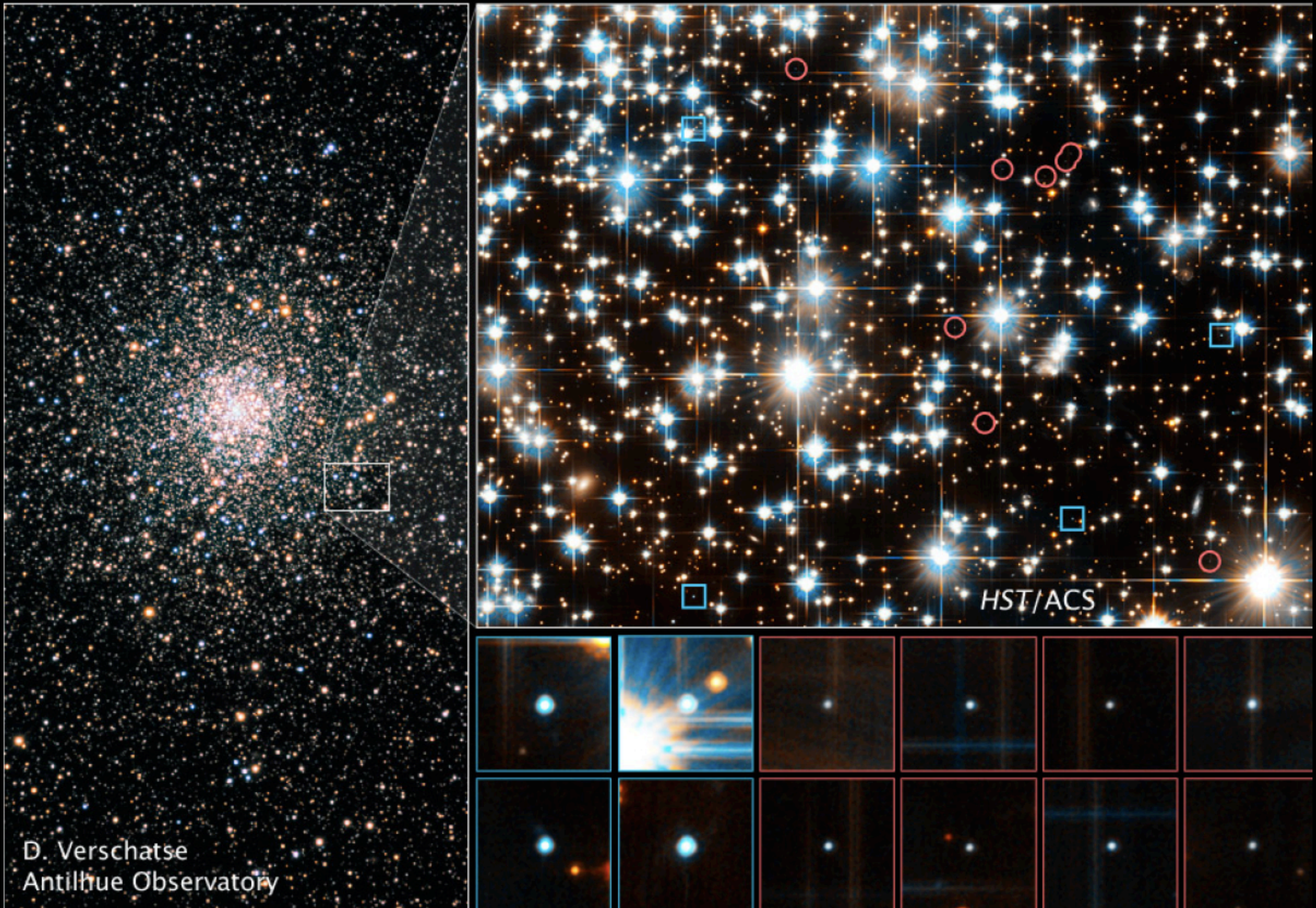
Most WDs cluster around 0.6 $M_\odot$. Narrow mass distribution



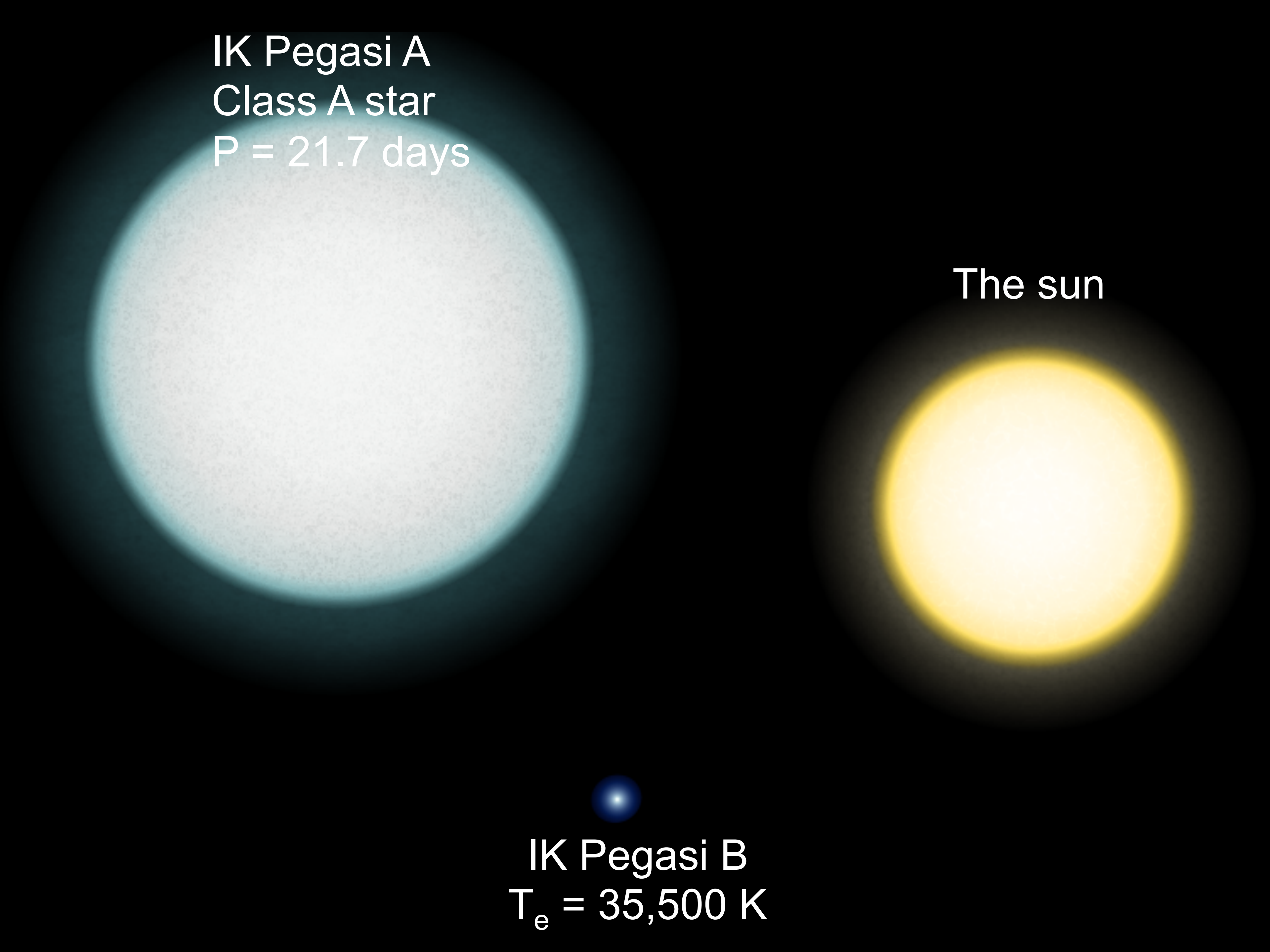
# Luminosity function



# White Dwarf Stars in Globular Cluster NGC 6397 ▪ *Hubble Space Telescope* ACS/WFC



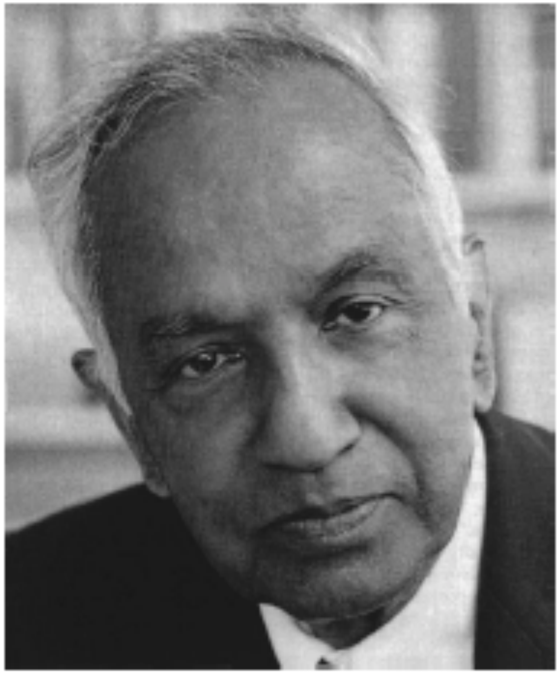

NASA, ESA, and H. Richer (University of British Columbia) ▪ STScI-PRC07-42


IK Pegasi A
Class A star
P = 21.7 days
The sun
IK Pegasi B
$T_e$ = 35,500 K

# Maximum white dwarf mass

- As mass increases, electron speed approaches *c*. Pressure becomes due relativistic electrons. Proportional to $\rho^{4/3}$

- Electron degeneracy cannot support a white dwarf heavier than 1.4 solar masses, the "Chandrasekhar limit".



$$M \approx \left( \frac{\hbar^3 c^3}{G^3 m_p^4} \right)^{1/2} Y_e^2$$

$Y_e = 0.50$ for He, C, O

# THE CHANDRASEKHAR MASS

As M gets larger and the radius decreases, the density rises

Eventually at $\rho$ greater than about $10^7$ g cm$^{-3}$ electrons in the *central* part of the white dwarf start to move close to the speed of light. As the mass continues to grow, a larger fraction of the star is supported by relativistic electron degeneracy pressure.

Consider the limit:

$$P_{\deg}^{R} = 1.24 \times 10^{15} \left(\rho Y_e\right)^{4/3} = \frac{GM\rho}{2R}$$

$$\mathrm{P}_{\mathrm{deg}}^{R} = 1.24 \times 10^{15} \left(\rho Y_e\right)^{4/3} = \frac{GM\rho}{2R}$$

As usual examine the constant density case for guidance

$$\rho \approx \left(\frac{3M}{4\pi R^3}\right)$$

$$1.24 \times 10^{15}\ \not{\rho}\, Y_e^{4/3} \left(\frac{3M}{4\pi \not{R}^3}\right)^{1/3} = \frac{GM\not{\rho}}{2\not{R}} = P_{central}$$

*Nb. R drops out*

$$M^{2/3} = 1.24 \times 10^{15}\ Y_e^{4/3} \left(\frac{3}{4\pi}\right)^{1/3} \frac{2}{G}$$

$$M^{2/3} = 2.3 \times 10^{22}\ Y_e^{4/3}$$

$$M = 3.5 \times 10^{33} Y_e^2 \ \text{gm} = 1.75\ \mathrm{Y}_e^2\ \mathrm{M}_\odot$$

Actually $\boxed{\mathrm{M} = 5.7\ \mathrm{Y}_e^2\ \mathrm{M}_\odot = 1.4\ \mathrm{M}_\odot}$ if $\mathrm{Y}_e = 0.5$

The fact that the radius drops out means that it can have any value – from zero to infinity. Taking the limit as the exponent approaches 4/3 in this case shows that infinity is the answer

Aside:

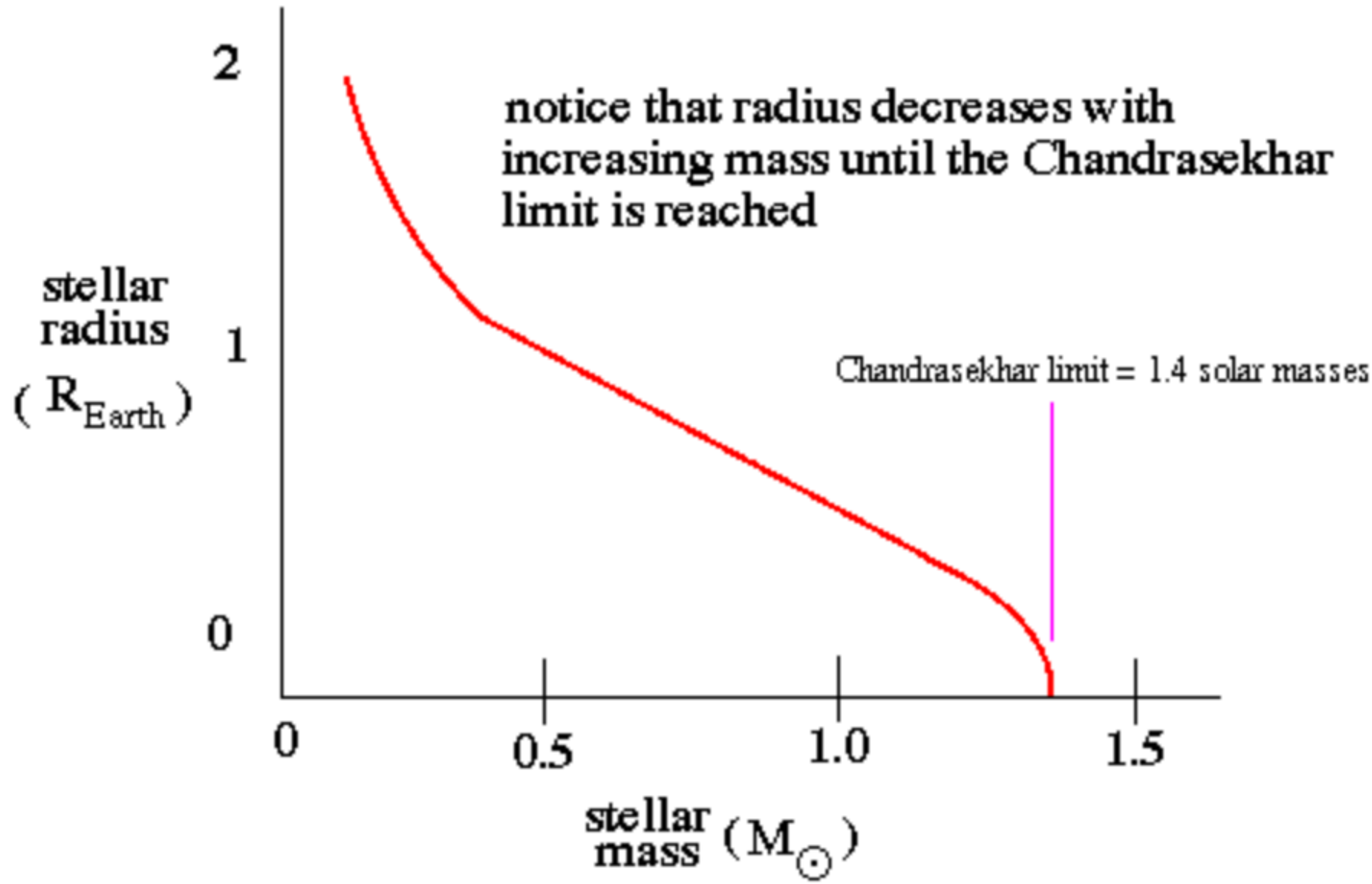
This result extends beyond white dwarfs.

There can be no stable star whose pressure depends on its density to the 4/3 power

**Table 8.5** Central Densities, Total Mass, and Radius of Different White Dwarf Models, Taking $\mu_e = 2$ (Negligible Hydrogen Concentration)*

| $\log \rho_c$ | $M/M_\odot$ | $\log R/R_\odot$ |
|---|---|---|
| 5.39 | 0.22 | −1.70 |
| 6.03 | 0.40 | −1.81 |
| 6.29 | 0.50 | −1.86 |
| 6.56 | 0.61 | −1.91 |
| 6.85 | 0.74 | −1.96 |
| 7.20 | 0.88 | −2.03 |
| 7.72 ↓ relativistic $e^-$ | 1.08 | −2.15 |
| 8.21 | 1.22 | −2.26 |
| 8.83 | 1.33 | −2.41 |
| 9.29 | 1.38 | −2.53 |
| $\infty$ | 1.44 | $-\infty$ ⇒ $R = 0$ |

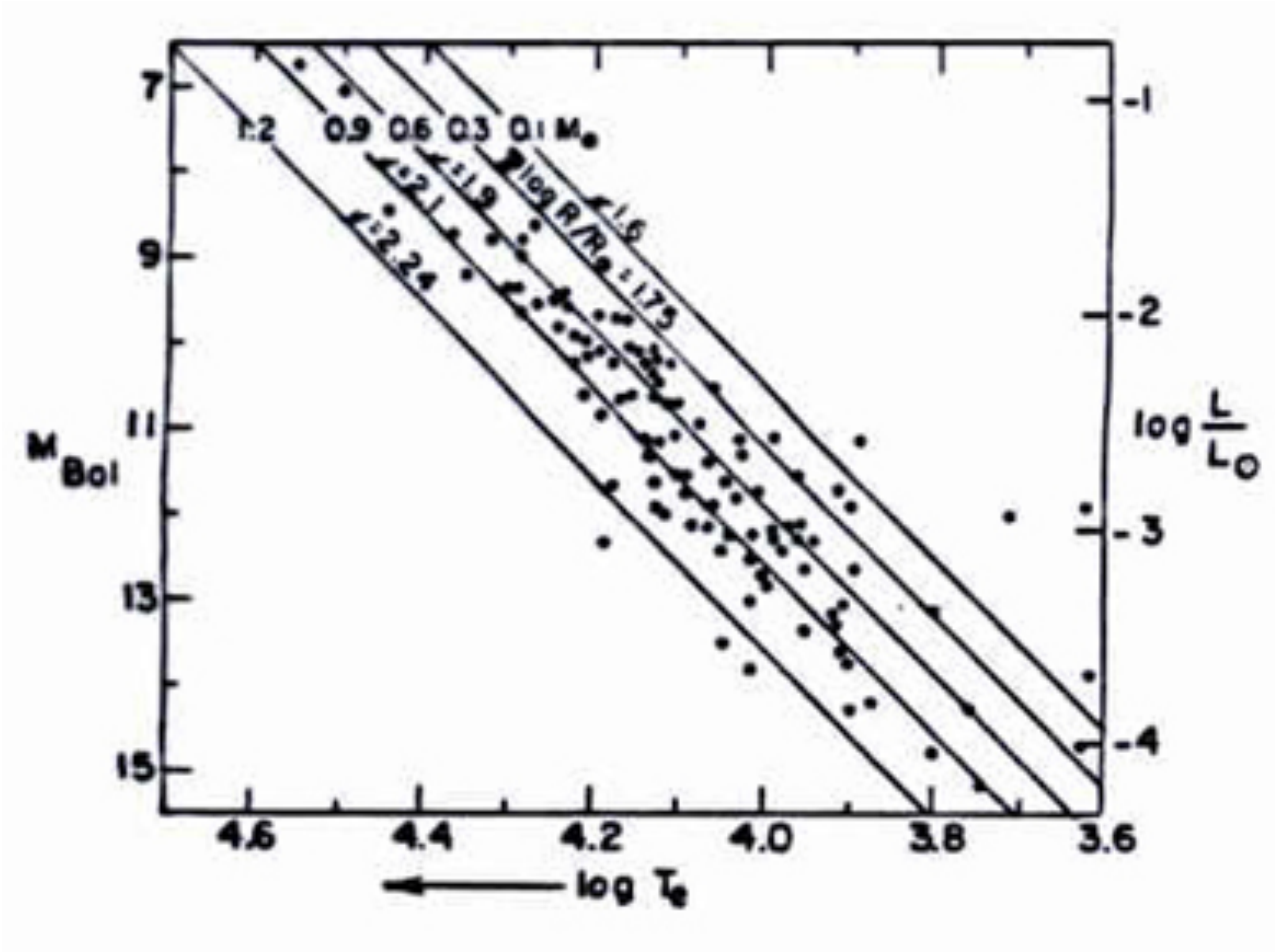
*See text for comments. (After M. Schwarzschild Sc58b.) From *Structure and Evolution of the Stars* (copyright © 1958 by Princeton University Press) p. 232.


notice that radius decreases with increasing mass until the Chandrasekhar limit is reached
Chandrasekhar limit = 1.4 solar masses
stellar radius ( $R_{Earth}$ )
2
1
0
0
0.5
1.0
1.5
stellar mass ( $M_{\odot}$ )

# What happens to a white dwarf more massive than 1.4 solar masses?

1. There aren’t any
2. They shrink to zero size
3. They explode
4. They become something else

# EVOLUTION OF WHITE DWARF STARS

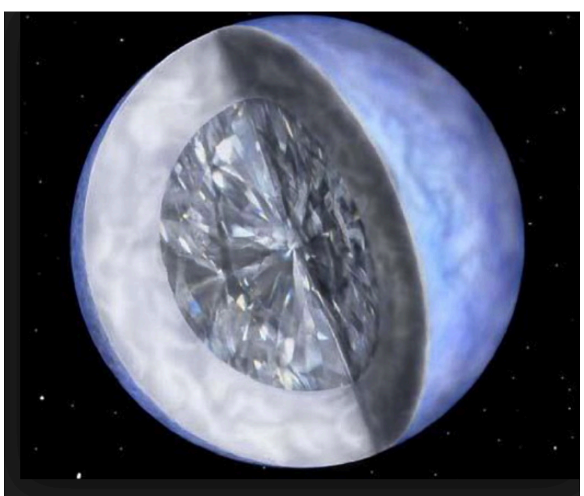



For a WD of constant mass, R = constant

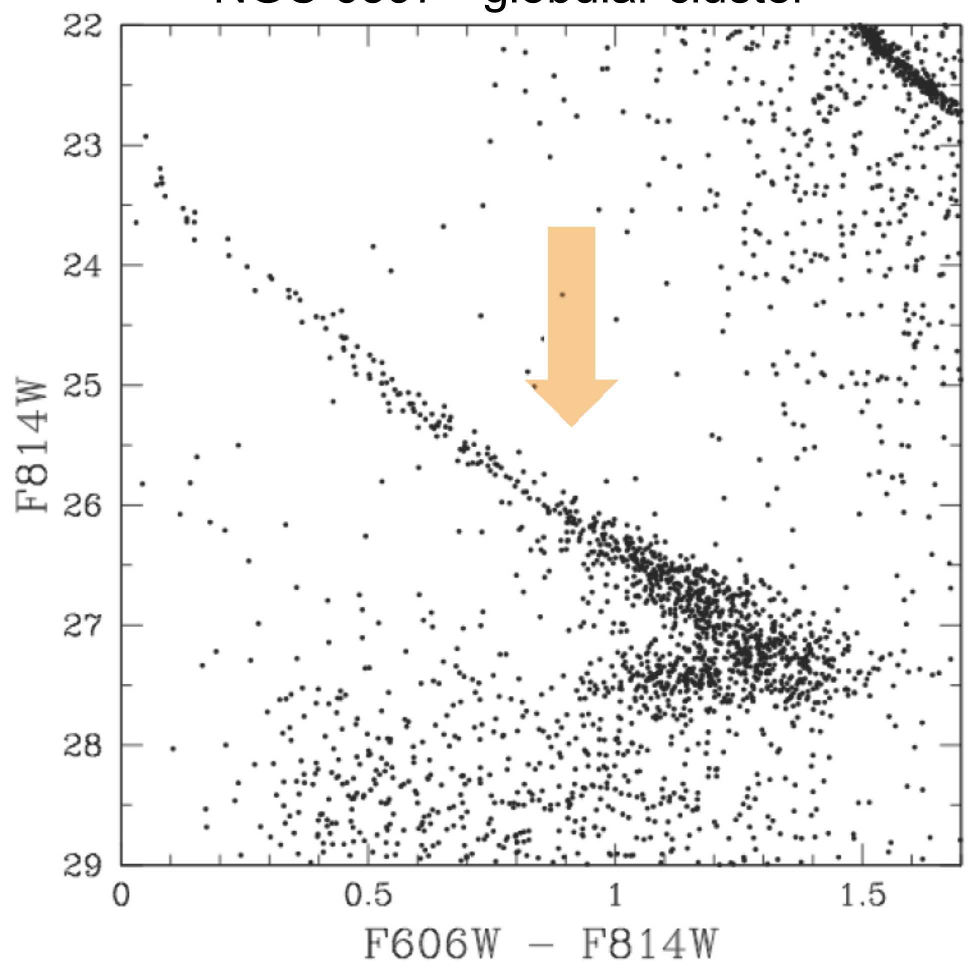
# Crystallization in white dwarfs

When the interior temperature declines to ~5000 K, the carbon and oxygen start to crystallize into a lattice. This crystallization releases energy and provides a source of luminosity that slows the cooling.

The number counts pile up.



Hansen et al (2007)
NGC 6397 - globular cluster




Lucy in the sky …

http://en.wikipedia.org/wiki/White_dwarf

The coolest, faintest white dwarfs still have a surface temperature of ~4000 K. The universe is not old enough for “black dwarfs” to have formed yet.

E.g., 0.59 solar mass WD - like the sun will make - takes about 1.5 billion years to cool to 7140 K and another 1.8 billion years to cool to 5550 K.

# Critical Masses

$0.08\ M_\odot$

*Contracting protostars below this mass do not ignite hydrogen burning on the main sequence. They become brown dwarfs or planets.*

$0.50\ M_\odot$

*Stars below this mass are completely convective on the main sequence*
*"   "   "   "   do not ignite helium burning*

$2.0\ M_\odot$

*Stars below this mass (and above .5) experience the helium core flash*
*Stars above this mass are powered by the CNO cycle (below by the pp-cycles)*
*Stars above this mass have convective cores on the main sequence (and radiative surfaces)*

$8\ M_\odot$

*Stars below this mass do not ignite carbon burning. They end their lives as planetary nebulae and white dwarfs. Stars above this mass make supernovae.*

$\sim 150\ M_\odot$

*Population I stars much above this mass pulse apart on the main sequence. No heavier stars exist.*


The Evolution and Explosion
of Massive Stars

# MAXIMUM MASS STAR

Because of the increasing dominance of radiation pressure, stars much above 150 solar masses become *pulsationally unstable* and experience episodes of *violent mass ejection* (not Cepheids nor supernovae or planetary nebulae, but a lot of fast mass loss).

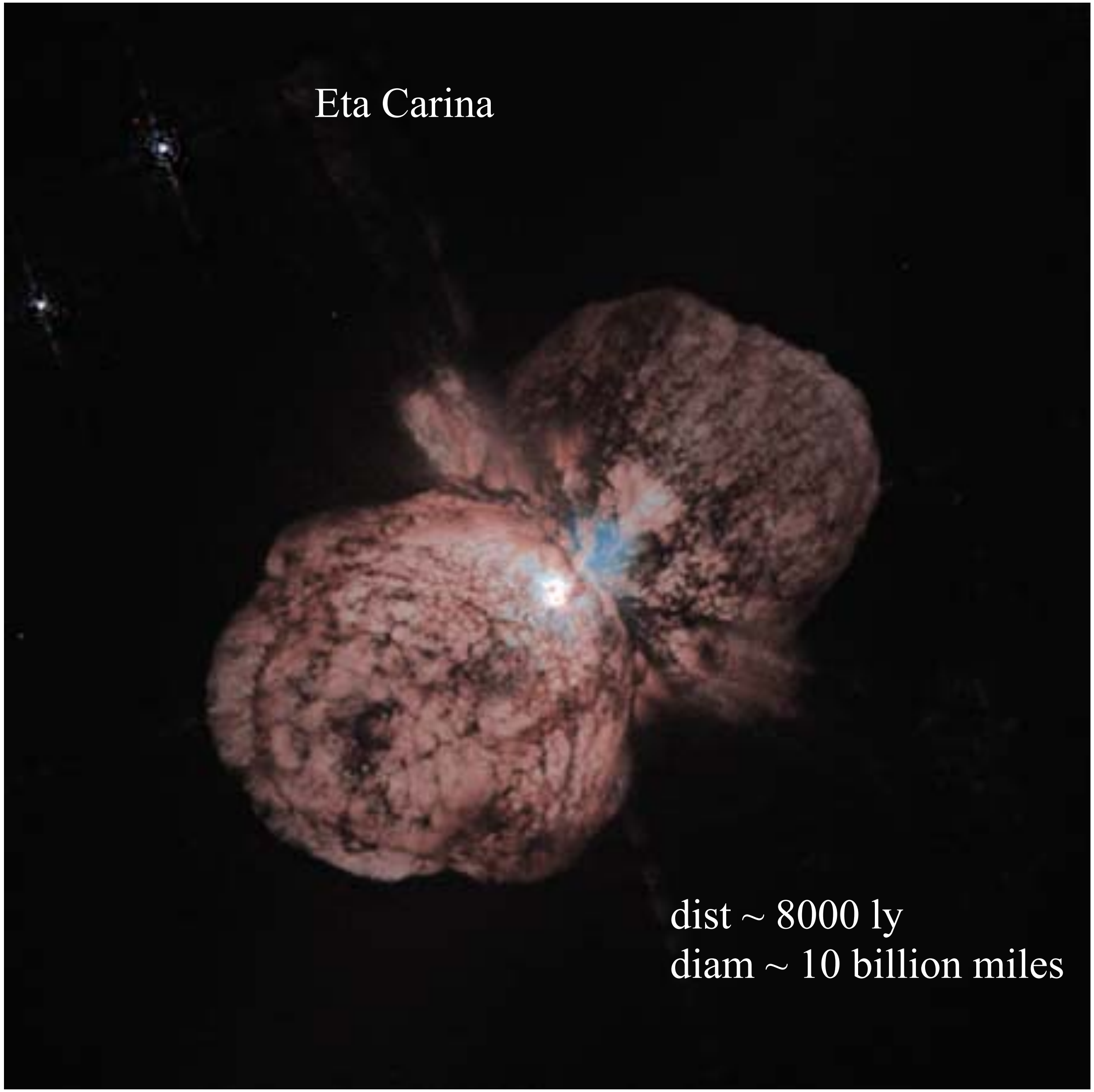
No star can be supported by 100% radiation pressure:

$$P_c = \frac{GM\rho}{2R} \approx \frac{1}{3} aT^4 \quad \text{if supported by } P_{radiation}$$

$$\text{but} \quad \rho \sim \left(\frac{3M}{4\pi R^3}\right) \Rightarrow \frac{3GM^2}{8\pi R^4} \sim \frac{1}{3} aT^4$$

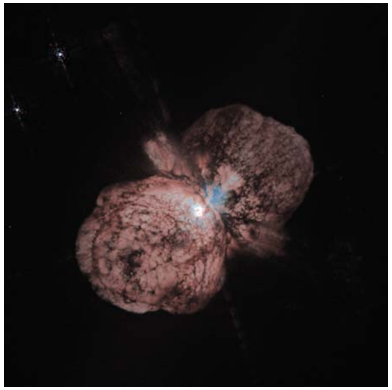
$$\text{so for a fixed M,} \quad T^4 \propto \left(\frac{1}{R}\right)^4 \propto \rho^{4/3}$$

$$P \propto \rho^{4/3}$$ which is known to have no stable solution




Most luminous star in our galaxy (that we can study well), several million times more luminous than the sun, bigger than the solar system.

# Peculiar star *Eta Carina* in Carina



1677 – discovered Edmond Halley – 4th magnitude star
1730 – brightness had reached 2nd magnitude
1801 – brightened again then faded back to 4th magnitude by 1811
1820 – began to brighten again
1822 – reached 2nd magnitude
1827 – reached 1st magnitude began to fade back to 2nd magnitude for about 5 years, then rose to magnitude 0 faded slightly then rose again
1843, April – magnitude -0.8 second brightest star in sky after Sirius, then faded continuously
1868 – became invisible
1900 – had faded to 8th magnitude, stayed there til 1941, then began to brighten again
1953 – 7th magnitude
early 1990's – 6th magnitude
1998-99 – brightened by a factor of 2

Eta Carina is about 8,000 light years away and one of the most massive stars in the sky (120 to 150 times the mass of the sun).99% of its luminosity is in the infrared.
Probably a supernova in the next 100,000 years, maybe sooner.

Observations suggest a cutoff around 150 Msun.
Controversial claims of heaver stars come and go. .

| Star Name | Mass (solar masses) |
|---|---|
| R136a1 | 265? |
| WR101e | 150 - 160 |
| HD 269810 | 150 |
| Peony Nebula Star | 150 |
| LBV 1806 - 20 | 130 |

July 2010. R136a1  265 Msun - controversial

# Overview of Evolution (150 > M > 8 Solar Masses)

- Above 8 solar masses, stars ignite carbon burning stably after helium depletion. They avoid becoming degenerate in their centers and go on to burn heavier fuels culminating in the production of an iron core. In a narrow range aounnd 8 – 9 Msun, carbon burns but oxygen does not

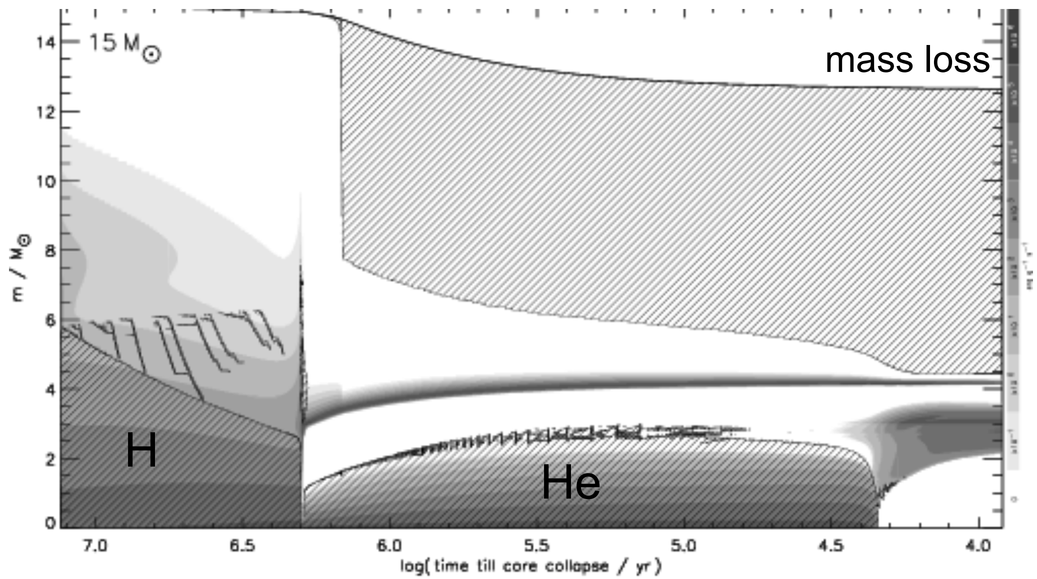
.

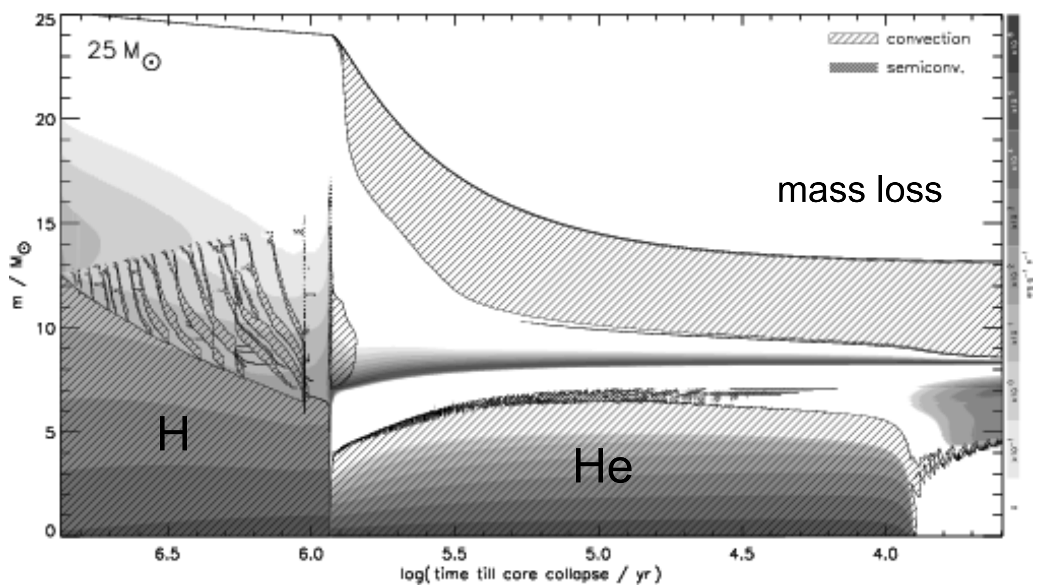
- Such massive stars have very high luminosities and short lives. They are all (presently) of Population I.

- During the red giant stage the very high luminosities of these stars (and their large radii) imply that the surface layers are very loosely bound. Extensive mass loss occurs due to "stellar winds"

- For stars above about 35 solar masses the entire hydrogen envelope is lost during helium burning. The star becomes a Wolf-Rayet star and even then mass loss continues at a rapid pace.

# Convective history 15 $M_\odot$ and 25 $M_\odot$ stars

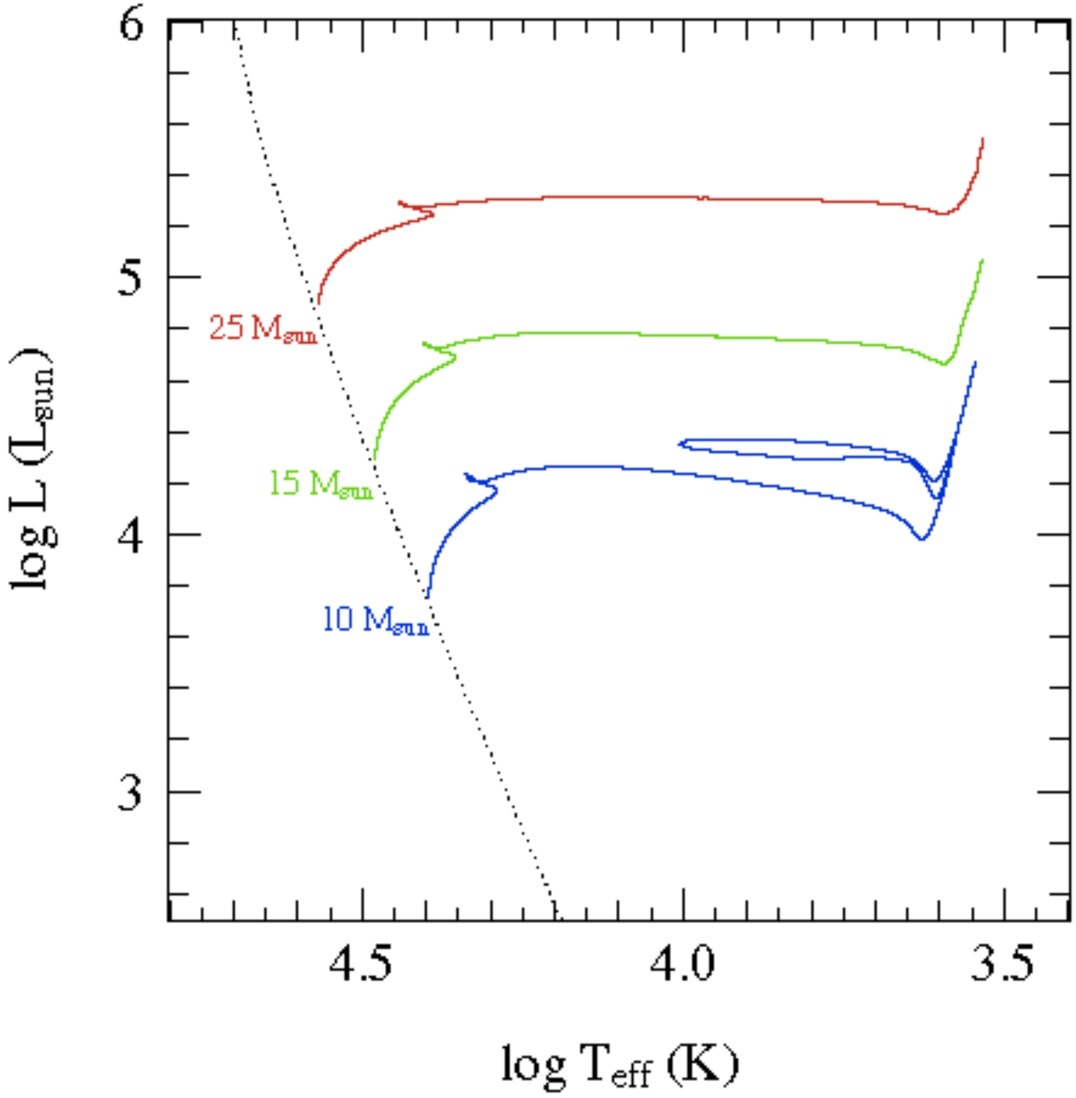



note: radiate surfaces and convective centers on the main sequence. Time axis is log time until death as a supernova.

*Above about 40 solar masses, everything outside the helium core is lost. This makes a Wolf-Rayet star.*

In the HR diagram, massive stars evolve at nearly constant luminosity off the main sequence and eventually explode as red or blue supergiants



## Overview of Evolution (150 > M > 8 Solar Masses)

- On the main sequence such massive stars have convective cores and are powered by the CNO cycle. Their surfaces are not convective. After burning hydrogen they ignite helium burning non-degenerately (no “helium flash”)

- Evolution beyond helium burning is greatly accelerated by thermal neutrino losses, especially from electron-positron pair annihilation (TBD).

- The massive stars that keep part of their hydrogen envelope become Type II supernovae. Those that lose their envelope (either in binaries or single stars above 40 solar masses) become Type Ib or Ic supernovae

# Post-Helium Burning Evolution

SUMMARY

# Advanced Nuclear Burning Stages

(e.g., 20 solar masses)

| Fuel | Main Product | Secondary Products | Temp ($10^9$ K) | Time (yr) |
|---|---|---|---|---|
| H | He | $^{14}N$ | 0.02 | $10^7$ |
| He | C,O | $^{18}O$, $^{22}Ne$ s- process | 0.2 | $10^6$ |
| C | Ne, Mg | Na | 0.8 | $10^3$ |
| Ne | O, Mg | Al, P | 1.5 | 3 |
| O | Si, S | Cl, Ar K, Ca | 2.0 | 0.8 |
| Si | Fe | Ti, V, Cr Mn, Co, Ni | 3.5 | 1 week |

*Massive stars are the ultimate "recyclers". They use the ashes of the previous stage as fuel for the next.*

# Why the big speed up?
# Pair Neutrino Losses

After helium burning the core contracts and the temperature rises. The most abundant fuel with the lowest charge is carbon ($^{12}C$). In order to get two carbons to fuse, a temperature of almost a billion K is required (actually 0.8 billion).

At such high temperatures, a new energy loss mechanism comes into play.

$$\text{Gamma rays } (\gamma) \rightleftarrows e^+ + e^-$$

$$\text{Very rarely though} \quad e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$$

For T ~ $10^9$ K, kT = 86 keV

$m_e c^2 = 511$ keV

number $e^+$ ~ number $e^-$ ~ $T^4$

Because the number of electron-positron pairs is very sensitive to the temperature, the energy loss rate due to neutrino losses also depends on a high power of the temperature.

For a temperatures over about $2 \times 10^9$ K

$$\varepsilon_{\nu,\mathrm{pair}} \approx -\frac{2\times10^{15}}{\rho}\left(\frac{T}{10^9\,K}\right)^9 \ \mathrm{erg}\ \ \mathrm{g}^{-1}\ \mathrm{s}^{-1}$$

For carbon burning and other later burning stages, these losses greatly exceed those due to radiative diffusion and convection.

Because the amount of energy released by each stage is roughly constant, the lifetime at each stage goes down very roughly as $1/T^9$. A higher T is required to burn each fuel.

# CARBON BURNING

At a temperature $T \approx 8 \times 10^8$ K and a density $\rho \approx 10^5$ g cm$^{-3}$, carbon fusion provides energy at a rate that balances losses due to neutrinos.
A little bit of extra energy powers convection and keeps the core hot. Simply carbon $\rightarrow$ neon and magnesium but in greater detail, the chief reaction is the fusion of two $^{12}C$ nuclei to produce isotopes of neon, sodium and magnesium

$$^{12}C + ^{12}C \rightarrow ^{23}Na + p + 2.24 \text{ MeV}$$

$$^{12}C + ^{12}C \rightarrow ^{20}Ne + \alpha + 4.62 \text{ MeV} \qquad (\alpha \equiv ^4He)$$

$$^{12}C + ^{12}C \rightarrow ^{23}Mg + n - 2.63 \text{ MeV} \qquad \text{(rarely)}$$

# CARBON BURNING

The neutrons, protons and alpha-particles (helium nuclei) react with other species that are there so that following the composition becomes complicated (but calculable)

$^{23}\mathrm{Na} + \mathrm{p} \rightarrow {}^{24}\mathrm{Mg} + \gamma$ $\qquad$ $^{23}\mathrm{Na} + \alpha \rightarrow {}^{27}\mathrm{Al} + \gamma$

$^{20}\mathrm{Ne} + \alpha \rightarrow {}^{24}\mathrm{Mg} + \gamma$ $\qquad$ $^{23}\mathrm{Mg} + \mathrm{n} \rightarrow {}^{24}\mathrm{Mg} + \gamma$

$^{24}\mathrm{Mg} + \mathrm{n} \rightarrow {}^{25}\mathrm{Mg} + \gamma$ $\qquad$ *etc.*

The net result is that $4 \times 10^{17}\ \Delta X_{12}$ erg $\mathrm{g}^{-1}$ are released and the most abundant isotopes of neon, sodium, magnesium and aluminum are created. Oxygen also survives with a slightly increased abundance. $\Delta X_{12} \approx 0.2$, so this is significantly less energy than helium burning gave

Note the gradual decrease in energy yield from $6 \times 10^{18}$ erg $\mathrm{g}^{-1}$ for hydrogen burning to about $1 \times 10^{18}$ erg $\mathrm{g}^{-1}$ for helium burning to about $10^{17}$ erg $\mathrm{g}^{-1}$ for carbon burning.

# OXYGEN BURNING

- Similar to carbon burning; at T ~ 2.0 x $10^9$K, $\rho$ ~ $10^6$ g cm$^{-3}$

$$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{28}\mathrm{Si} + \alpha$$

$$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{31}\mathrm{P} + \mathrm{p}$$

$$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{31}\mathrm{S} + \mathrm{n}$$

and a host of secondary reactions

- The net result is

$^{16}$O, $^{20}$Ne, $^{24}$Mg $\rightarrow$ abundant isotopes of silicon, sulfur, chlorine, argon potassium and calcium. Most abundant ashes - $^{28}$Si and $^{32}$S

$$q_{\mathrm{nuc}} \approx 5.0 \times 10^{17} \ \Delta X_{16} \ \ \mathrm{erg\ g}^{-1}$$

$$\varepsilon_{nuc} \propto \mathrm{T}^{33}$$

# SILICON BURNING

- $T \approx 3.5 \times 10^9$ K, $\rho \approx 10^7$ g cm$^{-3}$.

- At the end of oxygen burning the lighest element is silicon.

- Nuclear reactions are complicated, but in the end

$$[Si, S, Cl, Ar, K, Ca] \rightarrow [Ti, V, Cr, Mn, Fe, Co, Ni]$$

- The most abundant nucleus produced is $^{56}$Fe

$$q_{nuc} = 2 \times 10^{17} \text{ erg g}^{-1}$$

$$\epsilon_{nuc} \propto T^{47}$$

SUMMARY

# Advanced Nuclear Burning Stages

(e.g., 20 solar masses)

| Fuel | Main Product | Secondary Products | Temp ($10^9$ K) | Time (yr) |
|---|---|---|---|---|
| H | He | $^{14}N$ | 0.02 | $10^7$ |
| He | C,O | $^{18}O$,$^{22}Ne$ s- process | 0.2 | $10^6$ |
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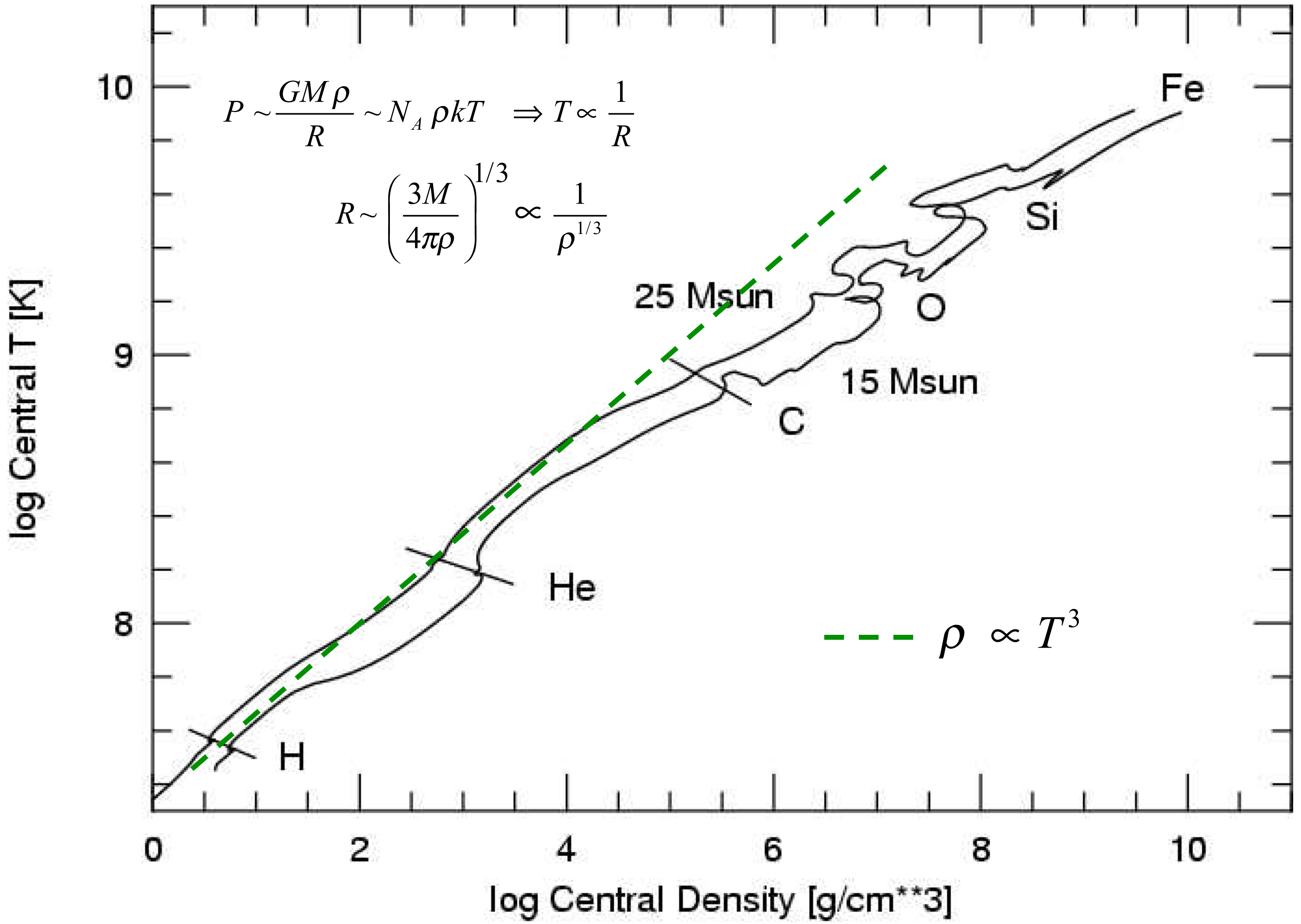
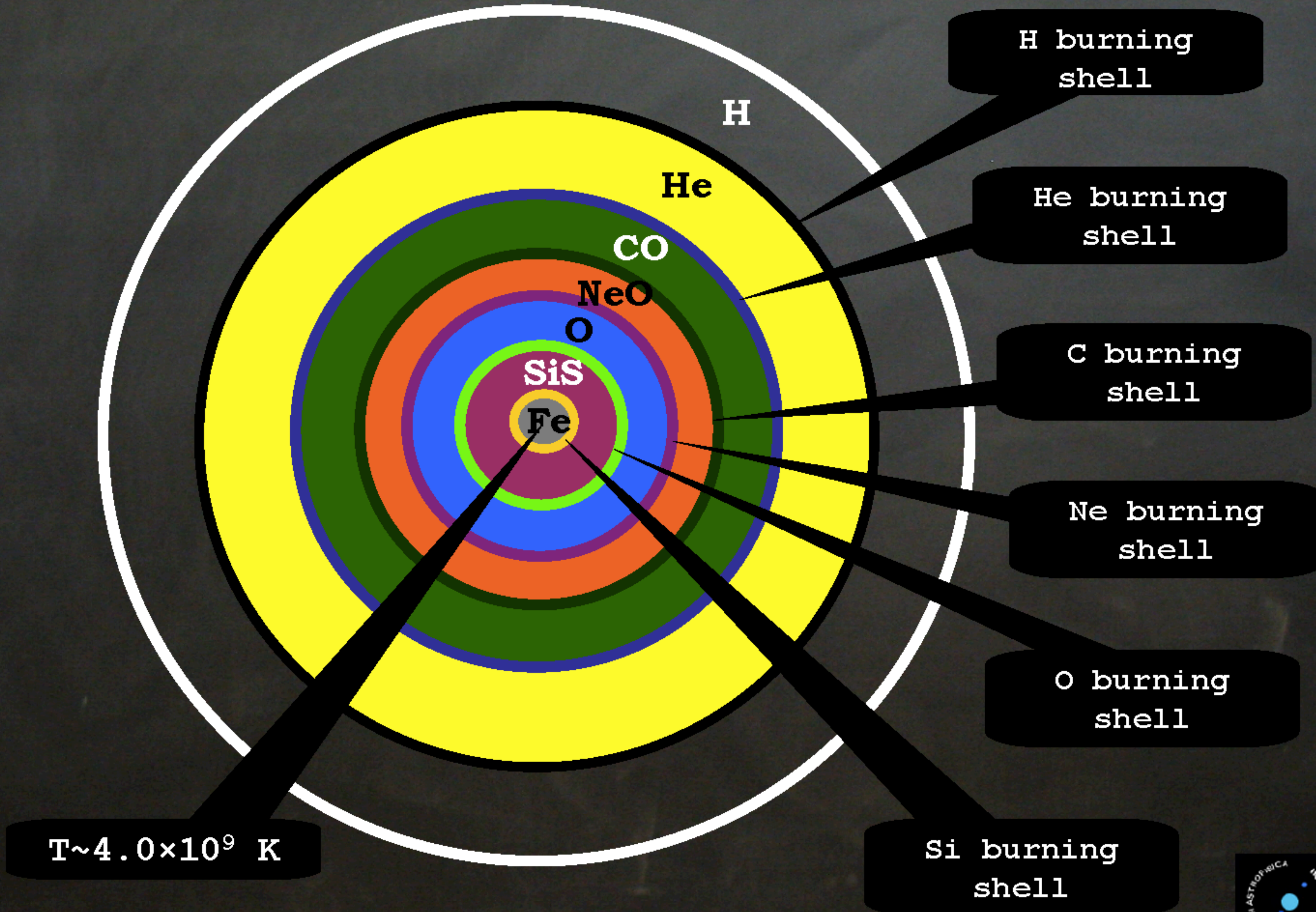
| O | Si, S | Cl, Ar K, Ca | 2.0 | 0.8 |
| Si | Fe | Ti, V, Cr Mn, Co, Ni | 3.5 | 1 week |

After each burning stage the core contracts, heats up and ignites another fuel

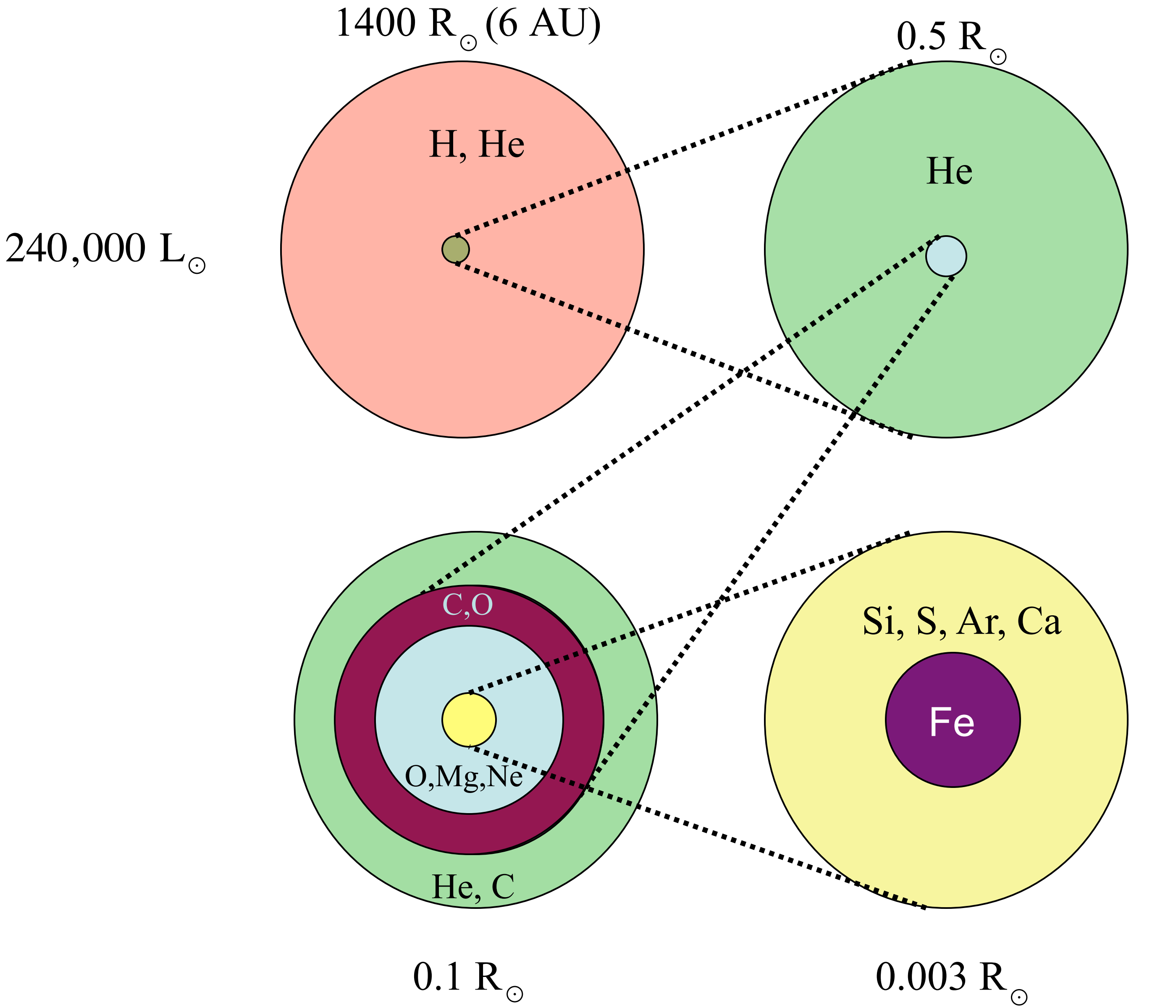


# Pre-SuperNova Stage

NOT TO SCALE



# 25 $M_\odot$ Presupernova Star (typical for 9 - 130 $M_\odot$)

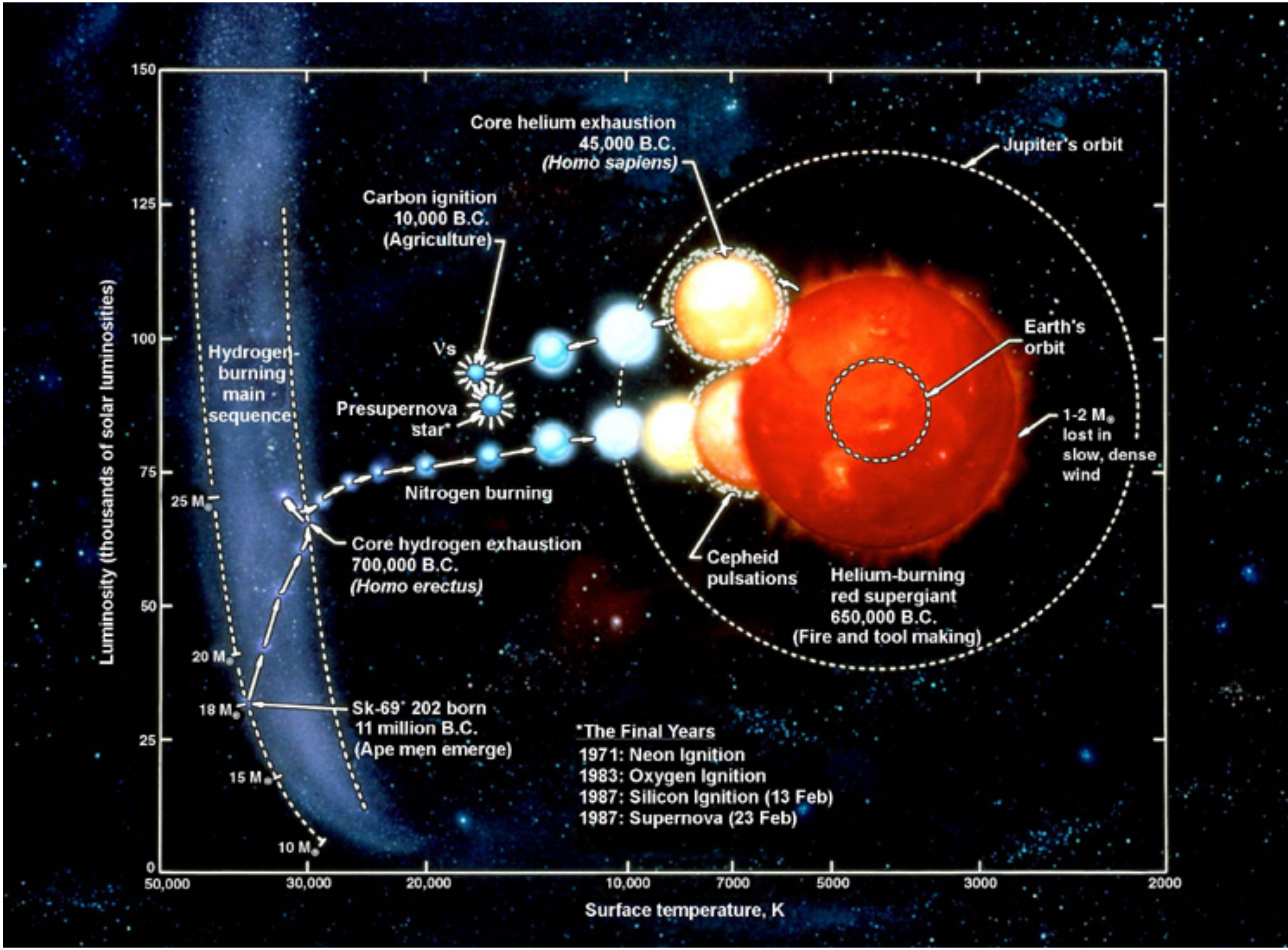



**Actual – to scale**

Neutrino emission dominates the energy budget after helium depletion in the center of the star…

**Table 1** Burning stages in the evolution of a 20-$M_\odot$ star

| Fuel | $\rho_c$ (g cm$^{-3}$) | $T_c$ ($10^9$ K) | $\tau$ (yr) | $L_{phot}$ (erg s$^{-1}$) | $L_\nu$ (erg s$^{-1}$) |
|---|---|---|---|---|---|
| Hydrogen | 5.6(0) | 0.040 | 1.0(7) | 2.7(38) | — |
| Helium | 9.4(2) | 0.19 | 9.5(5) | 5.3(38) | <1.0(36) |
| Carbon | 2.7(5) | 0.81 | 3.0(2) | 4.3(38) | 7.4(39) |
| Neon | 4.0(6) | 1.7 | 3.8(−1) | 4.4(38) | 1.2(43) |
| Oxygen | 6.0(6) | 2.1 | 5.0(−1) | 4.4(38) | 7.4(43) |
| Silicon | 4.9(7) | 3.7 | 2 days | 4.4(38) | 3.1(45) |




Most massive stars die as red supergiants. This one made a transition back to the blue just before dying