Lecture 3

Basic Physics of Astrophysics - Force and Energy

http://apod.nasa.gov/apod/

Wed 5 Thur 4

ISB 165



Forces

Momentum is the product of mass and velocity - a vector

$$\vec{p} = m\vec{v}$$

(generally *m* is taken to be constant)

An unbalanced force is capable of producing a change in momentum

$$\vec{F} = \frac{d\vec{p}}{dt}$$

(n.b., a mass times an acceleration)

Units:

$$\vec{\mathrm{F}} = m \left(\frac{d\vec{v}}{dt} \right) = m \, \vec{a}$$

dyne = $\frac{\text{gm cm}}{\text{s}^2}$

1 dyne = 2.248×10^{-6} pounds force

Intuitively, force is the "push or pull" on an object.



Note that a force may also be required to balance another force even when nothing is moving. For example a block setting on a table is pulled downwards by gravity but supported by the table.

NEWTON'S THREE LAWS

1) In the absence of an external force, The product of mass and (vector) velocity is a constant. Also may be stated "Objects at rest tend to remain at rest; objects in motion tend to remain in motion in the same straight line". This defines *momentum*, p = m v.

2) When a force acts on a body it produces a change in the momentum in the direction of and proportional to the applied force.

3) For every action there is an equal but op- Act posite reaction.



Conservation of momentum in the absence of forces

Definition of force $\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$

Action-reaction



A force is required to change either the speed of an object *or the direction in which it is going*.

FORCES IN THE UNIVERSE

There are four fundamental interactions that exist, each of which is associated with a force.

• The weakest but most commonly experienced force is **GRAVITY**. This force is exerted by anything that has mass or energy on all other masses. The force is proportional to each of the two masses that interact and declines as $1/r^2$. It thus can be experienced, albeit weakly, over very large distances (infinite range force).

$$F = \frac{GMm}{r^2}$$
$$G = 6.673 \times 10^{-8} \frac{dynes \, cm^2}{gm^2}$$

• The next to strongest (skipping one for now) is the **ELECTRIC** or **ELECTROMAGNETIC** force, the most obvious example being the interaction between two charges. The force is mathematically very similar to gravity (though fundamentally different in physical nature). It is proportional to the two interacting charges and also declines a $1/r^2$



All of chemistry is due to the electric force. So are the strength of solids and all processes related to the emission and absorption of light. Two other forces have only been recognized during the last century. They only affect phenomena on the scale of nuclei and individual particles, i.e., they are *"short ranged"*.

The *strong* force is responsible for binding together the neutrons and protons in the atomic nucleus. On an even smaller scale, the strong force binds together quarks to make the neutron and proton and other particles on the sub-atomic scale. The typical range of the strong force is 10^{-13} cm.

The strong force is strong enough at short range to overcome the repulsion of electrically charged protons in the nucleus (as well as the "degeneracy" energy of the nucleus itself). But outside the nucleus it falls rapidly to zero. At very short range the nuclear force is actually repulsive.



Its chief effects are that it allows neutrinos to be produced and to interact with matter and it allows neutrons to change into protons and vice versa if energy conservation allows it.

A free neutron outside the nucleus will decay into a proton in 10.3 minutes by the weak interaction

(udd) (uud)

 $n \rightarrow p + e^- + \overline{V}_e$

Note conserved quantities - charge, baryon number, lepton number, energy, momentum



SUMMARY

FORCE	<u>STRENGTH</u>	RANGE	<u>EXAMPLE</u>
Strong	1	10 ⁻¹³ cm	nucleus
Electric	10-2	1/r ²	chemistry
Weak	10-6	$< 10^{-13} \text{ cm}$	$n \rightarrow p + e^- + \overline{v}_e$
Gravity	10-38	1/r ²	binds earth to sun

Figure from Nick Strobel's electronic text. See his website.

$F_{g} = \frac{GMm}{d^{2}}$

Gravity

For spherically symmetric objects, gravity acts as if all the mass were concentrated at the center of the sphere (more generally at the center of mass). [often we Use r instead of d for distance]

Tides

The calculations on the previous page would suggest that the sun is actually more influential on the Earth (tides, etc.) than the moon, *but the Earth is in free fall around the sun*. At the center of the earth the gravitational force of the sun and the centrifugal force due to the earth's orbit precisely cancel.

What matters for tides is the *differential force* because the Earth has finite size.

GRAVITY • Examples: Your weight (assume m = 70 kg): $g = 980.66 \frac{cm}{s^2}$ $GM_{Earth}(m)$ F = -= m g R^2_{Earti} $6.67 \times 10^{-8} \text{dyne cm}^2 (5.32 \times 10^{27} \text{ gm}) (7 \times 10^4 \text{ gm})$ ${ m gm^2(6.38 imes 10^8 \, cm)^2}$ $= -6.9 \times 10^7$ dyne 1 dyne = 2.248×10^{-6} pound fprce = 154 pound force The moon's force: $F = -\frac{GM_{Moon}rn}{F}$ R^2_{Moon} $= -233 \,\mathrm{dynes}$ The sun's force: $F=-\frac{GM_{\odot}m}{}$ AU^2 $= -4.1 \times 10^4$ dynes almost 200 times that of the moon Force from sun at $4.4 \text{ ly} = 5 \times 10^{-7} \text{ dyne}$ Force from Jupiter at nearest point =2.2 dynes

Tides

Force from another peson at 1 m = 0.033 dynes



The moon pulls on all parts of the Earth. It pulls strongest on the part that is closest, less on the center, and least of all on the far side. Subtracting the force at the center of mass from all components leads to a bulge in the oceans both on the near and far side of the Earth.



Tidal force without calculus:





With calculus

$$\frac{dF}{dr} = \frac{2GMm}{r^3} \qquad \text{i.e., } \frac{d}{dr} \left(-\frac{1}{r^2}\right) = \frac{2}{r^3}$$

For an object of diameter d, the difference in force from one side to the other is

$$\Delta F = \frac{dF}{dr}(d) = F\left(\frac{2d}{r}\right)$$

The distance (r in this equation) to the sun is 390 times that to the moon so the tidal forces from each are comparable

$$F_{tide}(moon) = \left(\frac{GM_{moon}m}{r_{moon}^2}\right) \left(\frac{2d_{earth}}{r_{moon}}\right)$$

$$F_{tide}(sun) = \left(\frac{GM_{sun}m}{r_{sun}^2}\right) \left(\frac{2d_{earth}}{r_{sun}}\right)$$

distance to sun
$$\left(\frac{F_{tide}(moon)}{F_{tide}(sun)}\right) = \left(\frac{F_{grav}(moon)}{F_{grav}(sun)}\right) \left(\frac{r_{sun}}{r_{moon}}\right) = \left(\frac{233}{4.1 \times 10^4}\right) \left(\frac{1.5 \times 10^{13}}{3.84 \times 10^{10}}\right)$$

$$= \left(\frac{390}{176}\right) = 2.2$$

distance to moon

Q: Are there tides at the earth's poles?

A: Yes. But the biggest tides are in the tropics and near the equator.

Remember that it is the sun and moon causing the tides. These are not generally in the celestial equator. The path of the sun is the ecliptic, which we discussed. The moon's orbit is inclined by about 5° to the ecliptic.

The biggest tides are when the moon and sun are in the same direction or in opposite directions, i.e. at new moon and full moon.

Because two bodies are responsible and their position with respect to the equator changes during the year there is no place on earth with no tides. There are also small forces due to the rotation of the earth.





Combining the definition of centrifugal force and Newton's equation for gravitational attraction we get



ORBITS KEPLER'S THREE LAWS

http://en.wikipedia.org/wiki/Kepler's laws of planetary_motion

Soulution of

 $\frac{d^2\mathbf{r}}{dt^2} = -\frac{GM}{r^3}\mathbf{r}$

general equation – not circular orbits

• The planets orbit the sun in elliptical orbits with the sun at one focus of the ellipse.

• A line connecting the planet and the sun sweets out equal areas in equal times

• $P^2 \propto r^3$ as we derived

In Ay 12 we shall presume dircular c. bits to be a good approximation most of the time

<u>Kepler' s First</u> Law



The orbit of a planet around the sun is an ellipse with the sun at one focus of the ellipse. a is called the "semi-major axis" of the ellipse. e is the "eccentricity" For earth e = 0.0167. For Mercury e = 0.206.



The equation for an ellipse The equation for a circle is $x^2 + y^2 = r^2$ *i.e.*, a=b

Kepler's Second Law



A line connecting the orbiting both with one focus of the ellipse (e.g., the sun) sweeps out equal areas in equal times.

Using Ratios: $\rho^{2} = \frac{4\pi^{2}}{cM} r^{3}$ Kepler's Third Law for Circular Orbits for the Earth $(1 \text{ yr })^{2} = \frac{4\pi^{2}}{GM_{\odot}} (Au)^{3}$ So for any object <u>orbiting</u> the <u>sun</u> $\frac{P^{2}}{(1 \text{ yr })^{2}} = \frac{\left(\frac{4\pi^{2}}{GM}\right)}{\left(\frac{4\pi^{2}}{GM_{\odot}}\right)}$ $\left(\frac{r}{1 \text{ AU}}\right)$ $\frac{p^2}{lyr^2} = r^3/(Au)^3$ $r(in Au) = \left[P(in yr) \right]^{2/3}$

This only works when comparing small masses, each of which orbits the same big mass, M (e.g., planets around the sun).

Kepler's Third Law

The squares of the periods of the planets are proportional to the cubes of their semi-major axes

Planet	P(yr)	a(AU)	P ²	a³
Mercury	0.24	0.39	0.058	0.058
Venus	0.62	0.72	0.38	0.38
Earth	1.0	1.0	1.0	1.0
Mars	1.88	1.52	3.52	3.52



THE ASTRONOMICAL UNIT (AU)

The astronomical unit (AU) is the distance from the earth to the sun. It is measured from the center of the earth to the center of the sun.

Actually the distance from the earth to the sun varies about 1.7% during the year since its orbit is elliptical. Prior to 1976 the AU was defined as the semi-major axis of the earth's orbit but in 1976 the IAU redefined it to be the *average* distance to the sun over a year.

Its standard (1996) value is 1.495978707 x 1013 cm

http://en.wikipedia.org/wiki/Astronomical unit



$$P^{2} = \frac{4\pi^{2}}{GM} r^{3}$$

Rearrange \Rightarrow $M = \frac{4\pi^{2}}{GP^{2}} r^{3}$

and the length of the year and AU are known

$$M_{\odot} = \frac{4\pi^{2}}{G(1 \text{ yr})^{2}} (AU)^{3}$$

= $\frac{(4)(3.14)^{2} (\text{gm})^{2}}{(6.67 \times 10^{-8})(\text{dyne cm}^{2})(1 \text{ yr})^{2}} (1 \text{ AU})^{3} \left(\frac{1.50 \times 10^{13} \text{ cm}}{1 \text{ AU}}\right)^{3} \left(\frac{\text{dyne s}^{2}}{\text{gm cm}}\right) \left(\frac{1 \text{ yr}}{3.16 \times 10^{7} \text{ s}}\right)^{2}$
= $\left(\frac{(4)(3.14)^{2}(1.50 \times 10^{13})^{3}}{(6.67 \times 10^{-8})(3.16 \times 10^{7})^{2}}\right) \text{ gm}$
= $2.00 \times 10^{33} \text{ gm}$

Later we shall see how to get the masses of other stars (and planets) if those stars are in binary systems



• The Earth

 $\begin{array}{l} {\rm The\ moon\ has\ a\ distance\ (radar)\ of\ 3.84\times10^{10}}\\ {\rm cm\ (semi-major\ axis).}\\ {\rm The\ month\ is\ 27.32\ days\ =\ 2.36\times10^6\ seconds.} \end{array}$

$$\begin{split} M_E &= \frac{4\pi^2}{G(1\ mo)^2} r_{M\ oon}^3 & \mathsf{M} = \left(\frac{4\pi^2}{\mathrm{GP}^2}\right)^{-1} \\ &= \frac{(4)(3.14)^2}{(6.67\times10^{-8})(2.36\times10^6)^2} \ (3.84\times10^{10})^3 \\ &= 6.0\ \times10^{27}\ \mathrm{gm} \\ &(5.977\ \mathrm{x}\ 10^{27}) \end{split}$$

$$\bullet \text{ Other planets that have satellites} \quad (\texttt{all but Mercury} + \mathrm{Venus}) \end{split}$$

• The moon

• Other stars that are in binary systems

• Lest we forget Kepler's orginal intent, if we lnow the period of other planets their semi-major axes follow immediately. E.g. Jupiter

Jupiter year = 11.9 Earth Years

Therefore its semi-major axis around the sun is $(11.9)^{2/3} = 5.2 \text{ AU}$

How massive is the Milky Way Galaxy? (and how many stars are in it?)



COBE (1990) - the galaxy as seen in far infrared 1.25, 2.2, and 3.5 microns (optical = 0.4 - 0.7 microns) stars are white; dust is reddish

THE MASS OF THE MILKY WAY GALAXY



Measurements of the parallax of numerous star forming regions using radio (Very Long Baseline Array) have given (2009) accurate measurements of the distance from the galactic center to the earth (28,000 ly) and the speed of the solar system in its orbit around the center (254 km/s; used to be 220 km/s prior to January, 2009)

http://www.nrao.edu/pr/2009/mwrotate/

[orbital speed of earth around sun is 29.8 km/s]

Treat Milky Way as a sphere (the bulge) plus a disk (but not a rigid disk – the galaxy rotates "differentially"). Gravitationally, the sun "sees" mostly the mass interior to its orbit and that mass acts as if it were all located at the center of mass, i.e., the center of the Galaxy. If the galaxy were a sphere this approximation would be exact. If we are far outside the edge of the disk it is also nearly exact

ttp://people.cs.nctu.edu.tw/~tsaiwn/sisc/runtime_error_200_div_by_0/www.merlyn.demon.co.uk/gravity1.htm - FiSSh

Milky Way interior to sun's orbit

The matter outside the sun's orbit exerts little net force on the sun. To reasonable accuracy can treat the disk mass as being concentrated at the center, especially at large distances

Sun







Figure 7. Disk Force vs. Force for Point Mass (L80 Mass Distribution)

1 = edge of disk

THE MASS OF THE MILKY WAY GALAXY



Where is the edge of the Milky Way, i.e. where does the amount of mass enclosed reach a constant?

Once M = constant then as r increases





KEPLER'S LAW for the orbital velocity of planets in the solar system, in which is north in 99 percent of the total mass resides in the sun, yields this plotted curve. Orbital velocity, her mass inversely as the square root of r, the planet's mean distance from 0⁺ scu. The c't ance is shown here in astronomical units; one A.U. equals the mean distance between the earth and the sym. Pluto, at 39.5 A.U., lies 100 times farther from the sun than Mercury, at 39.4 A.U., lies 100 times farther from the sun than Mercury, at 39.4 A.U., lies 100 times farther from the sun than Mercury, at 39.4 A.U., lies 100 times farther from the sun than Mercury, at 39.4 A.U., Mercury's factor of 10, or 4.7 kilometers per second; Pluto's velocity is accordingly slower by a factor of 10, or 4.7 kilometers in a spiral galaxy depart strongly from a Kepterian distribution.







If density were independent of r,

 $M \propto r^3$ (sphere), $M \propto r^2$ (disk)

so the density is indeed declining



That is the orbital velocity should decrease with increasing distance from the center of the filky Way.

- In fact the orbital velocity measured by radio observations does not fall off but tends to stay nearly constant outside the sun's orbit, maybe even increase
- But the optical luminosity of our galaxy outside the sun's orbit does decline dramatically. Therefore there is a lot of non-luminous mass. The mass interior to 75,000 ly is about 3 times that interior to the sun's orbit.
- Note that the Milky Way does not rotate as a "rigid" body. Stars in the disk closer to the center execute an orbit quicker than stars farther out. Note also that v stays about constant interior to the suns orbit.





http://arxiv.org/abs/0801.1232

May 27, 2008 - Sloan sky survey using a sample of over 2400 Population II (blue horizonal branch) stars infers a mass of slightly less than 10¹² solar masses

What is Dark Matter?

Anything with a large mass to light ratio

Baryonic dark matter(made of neutrons, protons and electrons)

- White dwarfs
- Black holes (large and small)
- Neutron stars
- Brown dwarfs and planets
- Gas either in small cold clouds or a hot inter-galactic cluster gas

How much matter is in the universe?

Big Bang nucleosynthesis limits the amount of baryonic mass.

Baryon (p and n) - Density

The Ratio between Protonand Neutron- Abundance at the Time of Nucleosynthesis (as it is today) depends on the Baryon-Density

The Abundance of light Elements (He, D, ³He) reflects the n/p-Ratio

Light Elements => Baryon-Density

The value inferred is less than that required to bind galaxies and especially clusters of galaxies together.



Predictions from Big Bang Nucleosynthesis

The initial universe consisted of

 $24 \pm 1\%$ by mass helium 2.8 $\pm 0.5 \times 10^{-5}$ deuterium a trace of lithium and the rest was hydrogen

These numbers agree with observations but suggest a baryon density of 0.021 ± 0.002 of the value needed to close the universe

http://www.astronomy.ohio-state.edu/~dhw/A5682/notes7.pdf

Recently, CMB anisotropies have provided an entirely independent way to constrain the baryon density, yielding $\Omega bh2 = 0.022 \pm 0.001$, so the theory is pretty tight.

But still, Big Bang nucleosynthesis implies several times more baryonic matter than we see in galaxies and stars. Where is the rest?

Probably in an ionized hot intergalactic medium.

But observations of the dynamics of galactic clusters suggest even more matter than that.

Dark Matter in the Universe

Hot Gas in Galaxy Clusters 'to much' kinetic Energy (measured by X-Ray Emission)

+ large Scale Motion of Galaxies at 'too high' Speed

> About 5 times more mass than BB nucleosynthesis suggests



Josef Jochun

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More Exotic

Anything with a large mass to light ratio

Non-baryonic dark matter:

- Massive neutrinos
- Axions (particle that might be needed to understand absence of CP violation in the strong interaction)
- Photinos, gravitinos,
- WIMPs
- unknown...

In general, particles that have mass but little else

Best indications are that dark matter is composed of two parts "baryonic dark matter" which is things made out of neutrons, protons and electrons, and non-baryonic dark matter, which is something else.

Of the *baryonic* matter, stars that we can see are at most about 50% and the rest (the dark stuff) is in some other form. Part of the baryonic matter may be in the hot intergalactic medium and in ionized halos around galaxies (50%?).

The non-baryonic matter has, in total, 5 times more mass than the total baryonic matter. I.e. baryonic matter is 1/6 of matter and non-baryonic matter is 5/6.

- The universe was not born recently (i.e., there is nothing special about the present
- We are not at the center of the solar system
- Our solar system is not at the center of the Milky Way Galaxy
- Our galaxy is not at the center of the universe (and there are many other galaxies)
- We are not made of the matter which comprises most of the universe (as thought <30 years ago)
- [Still to come matter is not the dominant constituent of the universe]