Spectroscopy, the Doppler Shift and Masses of Binary Stars

http://apod.nasa.gov/apod/astropix.html

Doppler Shift

At each point the emitter is at the center of a circular wavefront extending out from its present location.

The Doppler Shift

\[ \lambda_{\text{obs}} = \lambda_{\text{emit}} + \lambda_{\text{emit}} \left( \frac{v}{c} \right) \]
The Doppler Shift

\[ \lambda_{\text{obs}} = \lambda_{\text{rest}} + \lambda_{\text{shift}} \left( \frac{v}{c} \right) \]

\[ \lambda_{\text{shift}} = \begin{cases} \frac{1}{1 - \frac{v}{c}} & \text{if moving toward you with speed } v \\ \frac{1}{1 + \frac{v}{c}} & \text{if moving away from you with speed } v \end{cases} \]

\[ \Delta \lambda = \lambda_{\text{obs}} - \lambda_{\text{rest}} \]

**Astronomical Examples of Doppler Shift**

- A star or (nearby) galaxy moves towards you or away from you (can’t measure transverse motion)
- Motion of stars in a binary system
- Thermal motion in a hot gas
- Rotation of a star

E.g. A H atom in a star is moving away from you at 3.0 \times 10^7 \text{ cm s}^{-1} = 0.001 \text{ times } c.

At what wavelength will you see H\( \alpha \)?

\[ \lambda_{\text{obs}} = 6562.8 \left( 1 + 0.001 \right) = 6569.4 \text{ Å} \]

Note that the Doppler shift only measures that part of the velocity that is directed towards or away from you.

**Doppler Shift:**

Note – different from a cosmological red shift!
A binary star pair

Again, only see a shift due to motion along our line of sight

Thermal Line Broadening

In a gas with some temperature $T$ atoms will be moving around in random directions. Their average speed will depend upon the temperature. Recall that the definition of temperature, $T$, is

$$\frac{1}{2} m_{\text{atom}} \langle v^2 \rangle = \frac{3}{2} kT$$

where $k = 1.38 \times 10^{-16}$ erg K$^{-1}$

Here $\langle \rangle$ means "average". Some atoms will be moving faster than the average, others hardly at all. Some will be moving towards you, others away, still others across your line of sight.

Thermal Line Broadening

The full range of wavelengths, hence the width of the spectral line will be

$$\frac{\Delta \lambda}{\lambda} = 2 \frac{v_{\text{average}}}{c} = 2 \frac{3kT}{c \sqrt{m_{\text{atom}}}}$$

The mass of an atom is the mass of a neutron or proton (they are about the same) times the total number of both in the nucleus, this is an integer "A".

$$\frac{\Delta \lambda}{\lambda} = 2 \left( \frac{3(1.38 \times 10^{-16})(T)}{(1.66 \times 10^{-24})(A)} \right)^{1/2} \left( \frac{1}{2.99 \times 10^{10}} \right)$$

$A = 1$ for hydrogen
4 for helium
12 for carbon
16 for oxygen
etc.

where $T$ is in K; $nb$ depends on $A$
Thermal Line Broadening

Full width = $\Delta \lambda = 1.05 \times 10^{-6} \sqrt{\frac{T(\text{in K})}{A}} \lambda$

Eg. $H_{\alpha}$ at 5800 K (roughly the photospheric temperature of the sun)

$A = 1 \quad T = 5800 \quad \lambda = 6563 \ \text{A}$

$\Delta \lambda = 1.05 \times 10^{-6} \left( \frac{5800}{1} \right)^{1/2} (6563) = 0.53 \ \text{A}$

This is (another) way of measuring a star’s temperature

Wien’s law (wavelength where most emission comes out)

Spectral class (O, B, F, G, K, M and subsets thereof)

$L = 4 \pi R^2 \sigma T^4 \Rightarrow T = \left( \frac{L}{4\pi R^2 \sigma} \right)^{1/4}$

Thermal line broadening

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**Rotation**

(as viewed in the plane of the equator)

Note: Potential complications:

1) Star may have both thermal and rotational broadening

2) May see the star at some other angle than in its equatorial plane.

Example: $H_{\alpha}$ in a star with equatorial rotational speed

$100 \ \text{km/s} = 10^7 \ \text{cm/s}$

Full width = $\Delta \lambda = 2 \left( \frac{v}{c} \right) \lambda$

$= 2(6563) \left( \frac{10^7}{3 \times 10^{10}} \right) = 4.4 \ \text{A}$
Average rotational velocities (main sequence stars)

<table>
<thead>
<tr>
<th>Stellar Class</th>
<th>$v_{equator}$ (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O5</td>
<td>190</td>
</tr>
<tr>
<td>B0</td>
<td>200</td>
</tr>
<tr>
<td>B5</td>
<td>210</td>
</tr>
<tr>
<td>A0</td>
<td>190</td>
</tr>
<tr>
<td>A5</td>
<td>160</td>
</tr>
<tr>
<td>F0</td>
<td>95</td>
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<tr>
<td>F5</td>
<td>25</td>
</tr>
<tr>
<td>G0</td>
<td>12</td>
</tr>
</tbody>
</table>

Stellar winds and magnetic torques are thought to be involved in slowing the rotation of stars of class G, K, and M. Stars hotter than F5 have stable surfaces and perhaps low magnetic fields.

The sun, a G2 star rotates at 2 km/s at its equator.

Red giant stars rotate very slowly. Single white dwarfs in hours to days. Neutron stars may rotate in milliseconds.

3 sources of spectral line broadening

1) Pressure or “Stark” broadening (Pressure)
2) Thermal broadening (Temperature)
3) Rotational broadening ($w$, rotation rate)

One can distinguish rotational broadening from thermal broadening because rotation affects all ions equally, but in a hot gas the heavier ions move slower. Thermal width depends on $\lambda^{-1/2}$.

SUMMARY

SPECTROSCOPY: WHAT WE CAN LEARN

1) Temperature
   - Ionization stages that are present
   - Thermal line broadening
     - Wien’s Law ($\lambda_{max} \propto 1/T$)
2) Radius
   - Blackbody $L = 4\pi R^2 \sigma T^4$
3) Rotation rate
   - Spectral line widths
4) Composition
   - From a detailed analysis of what lines are present and their strengths
5) Surface pressure
   - Also from line broadening. Is the star a white dwarf or a red giant or a main sequence star?
6) Velocity towards or away from us
   - Is the star or galaxy approaching us or receding?
7) Binary membership, period, and velocity
   - From periodic Doppler shifts in spectral lines

planets?
8) Magnetic fields

From Zeeman splitting

9) Expansion speeds in stellar winds and explosions

Supernovae, novae, planetary nebulae


21 cm (radio)

\[ \lambda = 21 \text{ cm} \quad \lambda = c \nu \]
\[ \nu = 1.4 \times 10^9 \text{ Hz} \]
\[ h\nu = (6.63 \times 10^{-27})(1.4 \times 10^9) = 9.5 \times 10^{-18} \text{ erg} \]
\[ = 5.6 \times 10^{-6} \text{ eV} \]

Must have neutral H I

Emission collisionally excited

Lifetime of atom in excited state about 10^7 yr

Galaxy is transparent to 21 cm

Merits:

- Hydrogen is the most abundant element in the universe and a lot of it is in neutral atoms - H I
- It is not so difficult to build big radio telescopes
- The earth's atmosphere is transparent at 21 cm

Arecibo - 305 m radio telescope - Puerto Rico
Getting Masses in Binary Systems

Beta-Cygnus (also known as Alberio) Separation 34.6", Magnitudes 3.0 and 5.3. Yellow and blue. 380 ly away. Bound? P > 75000 y. The brighter yellow component is also a (close) binary. P ~ 100 yr.

Alpha Ursae Minoris (Polaris) Separation 18.3". Magnitudes 2.0 and 9.0. Now known to be a triple. Separation ~2000 AU for distant pair.

When the star system was born it apparently had too much angular momentum to end up as a single star.

Polaris

1.2 Msun Polaris Ab
Type F6 - V
4.5 Msun Polaris A
Cepheid

Period 30 yr

Polaris B is
F3 - V
Epsilon Lyra – a double double.
The stars on the left are separated by 2.3" about 140 AU; those on the right by 2.6". The two pairs are separated by about 208" (13,000 AU separation, 0.16 ly between the two pairs, all about 162 ly distant). Each pair would be about as bright as the quarter moon viewed from the other.

Castor A and B complete an orbit every 400 years. In their elliptical orbits their separation varies from 1.8 to 6.5. The mean separation is 8 billion miles. Each star is actually a double with period only a few days (not resolvable with a telescope). There is actually a “C” component that orbits A+B with a period of of about 10,000 years (distance 11,000 AU).

Castor C is also a binary. 6 stars in total

Center of Mass

\[ m_1 r_1 = m_2 r_2 \]

The center of mass is proportionally closer to the larger mass.

For constant total separation, 20 AU, vary the masses
Circular Orbit – Unequal masses

Two stars of similar mass but eccentric orbits

Two stars of unequal mass and an eccentric orbit
E.g. A binary consisting of a F0v and M0v star

http://www.astronomy.ohio-state.edu/~pogge/Ast162/Movies/ - visbin
M1/M2 = 3.6; e = 0.0

Aside

The actual separation between the stars is obviously not constant in the general case shown of non-circular orbits.

The separation at closest approach is the sum of the semi-major axes of the two elliptical orbits, \( a = a_1 + a_2 \), times \( (1-e) \) where \( e \) is the eccentricity.

At the most distant point the separation is “\( a \)” times \( (1+e) \).

So if one measures the separation at closest approach and farthest separation, adds and divides by 2, one has \( a_1 + a_2 \).

For circular orbits \( e = 0 \) and the separation is constant, \( r_1 + r_2 \).
(a_1 + a_2) (1 + e) \quad \text{farthest separation}

(a_1 + a_2) (1 - e) \quad \text{closest separation}

\frac{2(a_1 + a_2)}{2} = (a_1 + a_2)

Some things to note:

• A binary star system has only one period. The time for star A to go round B is the same as for B to go round A.

• A line connecting the centers of A and B always passes through the center of mass of the system.

• The orbits of the two stars are similar ellipses with the center of mass at a focal point for both ellipses.

• For the case of circular orbits, the distance from the center of mass to the star times the mass of each star is a constant. (next page)

Period = 50.1 years
semi-major axis of A is 6.4 AU and B is 13.4 AU
(In Kepler’s equation use the sum of the semimajor axes)

\begin{align*}
\frac{m_1 v_1^2}{r_1} &= G m_1 m_2 \left( \frac{r_1 + r_2}{r_1 r_2} \right) \\
\frac{2 \pi r_1}{v_1} &= \frac{2 \pi r_2}{v_2} = \text{Period}
\end{align*}

\begin{align*}
\frac{m_1 v_1^2}{r_1^2} &= \frac{G m_1 m_2}{r_1^2} = \frac{G m_2 m_2}{r_2^2} \\
\therefore v_1 &= \frac{r_2 v_2}{r_1} \\
\frac{m_1 r_1^2 v_1^2}{r_1^2} &= \frac{m_2 r_2^2 v_2^2}{r_2^2} \\
\therefore m_1 r_1 &= m_2 r_2
\end{align*}

More massive star is closer to the center of mass and moves slower.
Circular Orbit – Unequal masses

$\frac{r_1}{r_2} = \frac{v_1}{v_2}$

So since $\frac{r_1}{r_2} = \frac{m_2}{m_1}$

$\frac{m_2}{m_1} = \frac{v_1}{v_2}$

E.g., Motion of the sun because of Jupiter;
Roughly the same as two stars in circular orbits

$m_1r_1 = m_2r_2$

$M_\odot d_\odot = M_J d_J$

$d_\odot = \frac{M_J}{M_\odot} d_J$

= \frac{9.95 \times 10^{-4}}{7.80 \times 10^{13}}$

= \frac{7.45 \times 10^{10}}{\text{cm}}$

Can ignore the influence of the other planets.

$P = 11.86 \text{ years}$

Roughly the same as two stars in circular orbits

$P = \frac{2\pi r_1}{V_1} = \frac{2\pi r_2}{V_2}$

since $\frac{r_1}{r_2} = \frac{m_2}{m_1}$

$m_1v_1 = m_2v_2$

$m_1 \times M_\odot = M_\odot \times m_1$

Note: “wobble” of the star is bigger if the planet is bigger or closer to the star (hence has a shorter period).
About 40 mph

As of today – 2052 extra solar planets in 1300 stellar systems and the number is growing rapidly.  Many were detected by their Doppler shifts. Many more by the “transits” they produce as they cross the stellar disk.

http://exoplanet.eu/catalog.php

KEPLER’S THIRD LAW FOR BINARIES

\[
\frac{GM_1M_2}{(r_1+r_2)^2} = \frac{M_1v_1^2}{r_1} \quad \text{and} \quad \frac{GM_1M_2}{(r_1+r_2)^2} = \frac{M_2v_2^2}{r_2}
\]

\[
\frac{G(M_1+M_2)}{(r_1+r_2)^2} = \frac{v_1^2}{r_1} + \frac{v_2^2}{r_2} = \frac{4\pi^2r_1^2}{P^2r_1} + \frac{4\pi^2r_2^2}{P^2r_2}
\]

\[
v = \frac{2\pi r}{P}
\]

\[
P^2 = K (r_1+r_2)^3
\]

\[
K = \frac{4\pi^2}{G(M_1+M_2)}
\]

i.e., just like before but

\[
M \rightarrow M_1+M_2 \quad R \rightarrow r_1+r_2
\]

In the general case \((r_1+r_2) \rightarrow (a_1 + a_2)\)

where \(a_1\) and \(a_2\) are the semimajor axes of the two stars in elliptical orbits around each other

Circular Orbit – Unequal masses

\[
M1/M2 = 3.6; e = 0.0
\]
\[ (M_1 + M_2) = \frac{4\pi^2}{GP^2} (r_1 + r_2)^3 \]

\[ M_\odot = \frac{4\pi^2}{G(1\,yr)^2} (AU)^3 \quad \text{for the earth} \]

Divide the two equations

\[ \frac{M_1 + M_2}{M_\odot} = \left( \frac{r_1 + r_2}{AU} \right)^3 \]

\[ \frac{M_1}{M_2} = \frac{r_2}{r_1} \quad \text{or} \quad \frac{M_1}{M_2} = \frac{v_2}{v_1} \]

If you know \( r_1, r_2 \) in AU, or \( v_1, v_2, \) and \( P \) in years you can solve for the two masses.

**Getting Stellar Masses – Visual binaries**

Measure:

- Period
- Separation (if circular orbit; sum of semi-major axes if elliptical; average of max and min separations)
- Ratio of speeds or separations from center of mass if circular (or ratio of semi-major axes if elliptical)

E.g., Sirius A and B; distance 2.67 pc)

- Period = 50.1 years
  - semi-major axis of A is 6.4 AU and B is 13.4 AU
  - (In Kepler’s equation use the sum of the semimajor axes)
Sirius B only has a radius $0.0084$ times that of the sun. What is it?

In the case of circular orbits seen face on the separation is a constant. As shown on the next page the separation in AU is just the distance in pc times the angular separation in arc seconds. This is a consequence of the way the pc is defined.

One can also measure, in the same way the separation between star 1 and the center of mass, $r_1$, and star 2 and the center of mass, $r_2$.

The total mass is then given by the usual equation

$$\frac{M_1 + M_2}{M_\odot} P^2 (\text{yr}) = R^3 (\text{AU}) = (r_1 + r_2)^3$$

and

$$\frac{M_1}{M_2} = \frac{r_2}{r_1}$$

Calculation

$$\frac{M_\odot + M_B}{M_\odot} P^2 (\text{yr}) = A^3 (\text{AU})$$

$$A = A_A + A_B = 6.4 + 13.4 = 19.8 \ \text{AU} \ \quad P = 51 \ \text{yr}$$

$$\frac{M_\odot + M_B}{M_\odot} = 19.8^3 = 2.99$$

So the sum of the masses is 2.98 solar masses.

The ratio of the masses is $A_A / A_B = 6.4 / 13.4 = 0.478 = M_B / M_A$

$$M_A + 0.478 M_A = 2.99 \quad M_A = 2.02 M_\odot$$

$$M_B = 0.478 M_A = 0.97 M_\odot$$

and since we see the orbits face on these are the actual masses.

Sirius B only has a radius 0.0084 times that of the sun. What is it?

\[ s \text{ (in pc)} = r \text{ (in pc)} \theta \text{ (in radians)} \]

\[ s \text{ (in AU)} = r \text{ (in AU)} \theta \text{ (in radians)} \]

\[ r \text{ (in AU)} = r \text{ (in pc)} \left( \frac{\text{number AU}}{1 \text{ pc}} \right) \]

\[ \theta \text{ in radians} = \theta \text{ (in arc sec)} \left( \frac{1 \text{ radian}}{\text{number arc sec}} \right) \]

\[ s \text{ in AU} = r \text{ (in pc)} \left( \frac{\text{number AU}}{1 \text{ pc}} \right) \theta \text{ (in arc sec)} \left( \frac{1 \text{ radian}}{\text{number arc sec}} \right) \]

e.g. The separation of Sirius A and B varies from 3.5 to 11.5 arc sec. The average is therefore 7.5 arc sec. Sirius is 2.67 pc away so $a_1 + a_2 = (2.67)(7.5) = 20$ AU
Complication:
The viewing angle

Getting Stellar Masses For Spectroscopic Binaries

For spectroscopic binaries measure:

- Period
- Velocity of each star
- Inclination will be unknown so mass measured will be a lower bound (TBD)

CALCULATION

\[ P = \frac{2\pi r}{v} \]

Assume circular orbits

First get \( r_1 \) and \( r_2 \) from \( v_1 \) and \( v_2 \)

\[ r_j = \frac{v_j P}{2\pi} \]

Example:

\[ v_1 = 75 \text{ km s}^{-1} \quad v_2 = 25 \text{ km s}^{-1} \]

\[ P = 17.5 \text{ days} \]

Note - the bigger the speeds measured for a given \( P \) the bigger the masses
Complication – The Inclination Angle

Let \( i \) be the angle of the observer relative to the rotation axis, i.e., \( i = 0 \) if we are along the axis.

Measure \( v \sin i \) which is a lower bound to \( v \).

\[
p' = \frac{4\pi}{G(M_1 + M_2)} (r_1 + r_2)
\]

\[
r = \frac{v'}{2\pi}
\]

but measure \( \dot{v} = v \sin i \), so we end up measuring \( \dot{v} = \frac{v'}{\sin i} \) and calculate

\[
M_1 + M_2 = \frac{4\pi}{G} \left( \frac{\dot{v}}{v} \left( \frac{r_1 + r_2}{2\pi} \right) \right)
\]

when the actual mass is

\[
M_1 + M_2 = \frac{4\pi}{G} \left( \frac{v + v}{2\pi} \right) \rho'
\]

hence the measurement gives a low bound on the actual mass

\( (M_1 + M_2) = (M_1 + M_2) \sin^i \)

Since \( \sin i < 1 \), the measurement is a lower bound.

Only if \( i = 90 \) degrees do we measure the full velocity.

But we tend to discover more edge-on binaries so \( 2/3 \) is perhaps better

For an eclipsing binary you know you are viewing the system in the plane of the orbit. i.e., \( \sin i = 1 \)

Limits of stellar mass:

Observed stars end up having masses between 0.08 \( M_\odot \) and about 150 \( M_\odot \).

The upper number is uncertain (130? 200?). The lower number will be derived later in class (minimum mass to ignite H burning before becoming degenerate).
STEellar lifetimes

On the main sequence:

- Luminosity determined by mass - \( L \propto M^n \), \( n \approx 3 \text{ to } 4 \)

- Say star has a total energy reservoir proportional to its mass (as in a certain fraction to be burned by nuclear reactions)

\[ E_{\text{tot}} = fM \]

Then the lifetime on the main sequence will be shorter for stars of higher mass;

\[ \tau_{MS} \propto \frac{\frac{fM}{M}}{M^n} \text{ with } n = 3 \]

\[ \tau_{MS} \approx 10^{10} \text{ yr} \left(\frac{M_0}{M}\right)^{3} \]

This explains some important features of the HR-diagram.