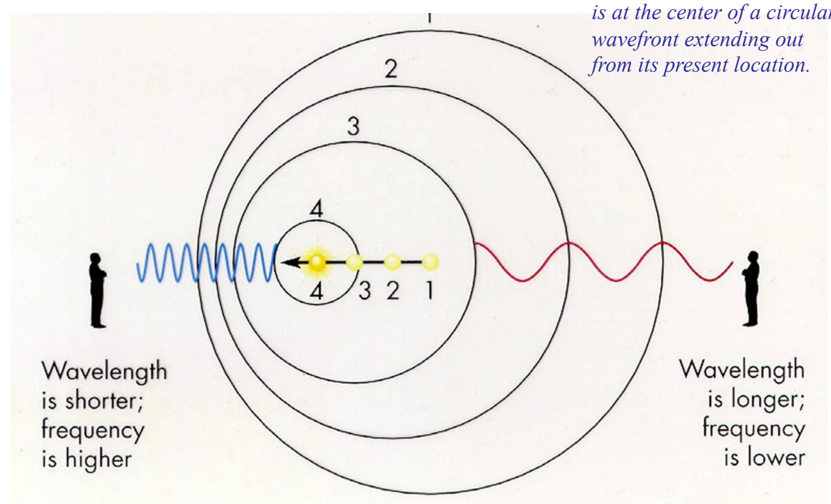
Spectroscopy, the Doppler Shift and Masses of Binary Stars

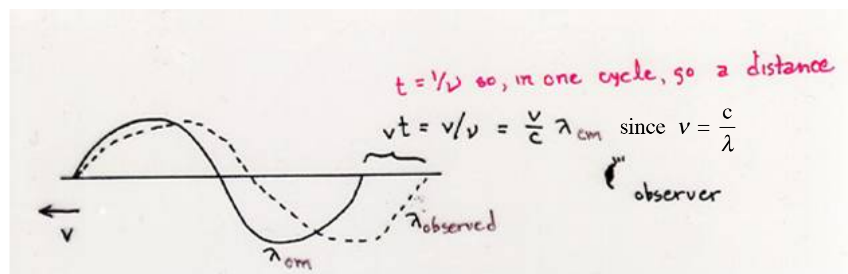
http://apod.nasa.gov/apod/astropix.html

Doppler Shift

At each point the emitter is at the center of a circular wavefront extending out from its present location.

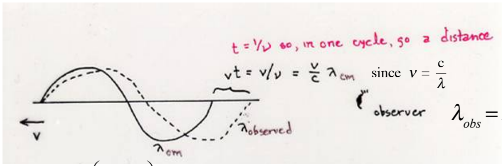


The Doppler Shift



$$\lambda_{obs} = \lambda_{emit} + \lambda_{emit}\left(\frac{v}{c}\right)$$

The Doppler Shift



$$\lambda_{obs} = \lambda_{emit} + \lambda_{emit}\left(\frac{v}{c}\right)$$

$\lambda_{obs} = \lambda_{emit}\left(1+\frac{v}{c}\right)$ if moving away from you with speed v

$\lambda_{obs} = \lambda_{emit}\left(1-\frac{v}{c}\right)$ if moving toward you with speed v

$$\Delta\lambda = \lambda_{obs} - \lambda_{emit}$$
$$= \lambda_{emit}\,(1 \pm \frac{v}{c} - 1)$$

$\boxed{\dfrac{\Delta\lambda}{\lambda_{emit}} = \pm\dfrac{v}{c}}$ if v is away from you $\Delta\lambda > 0$ red shift

if v is toward you $\Delta\lambda < 0$ blue shift

This formula can only be used when v << c

Otherwise, without proof,

$$\lambda_{obs} = \lambda_{emit}\left(\frac{1+v/c}{1-v/c}\right)^{1/2}$$

Astronomical Examples of Doppler Shift

- A star or (nearby) galaxy moves towards you or away from you (can't measure transverse motion)
- Motion of stars in a binary system
- Thermal motion in a hot gas
- Rotation of a star

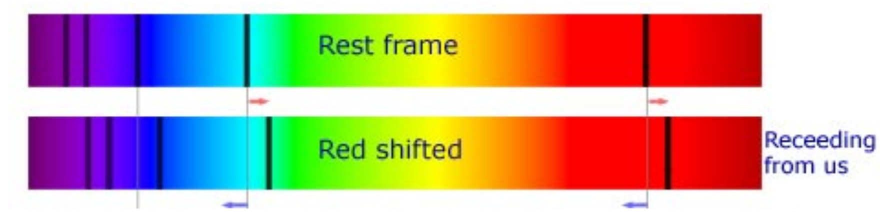
E.g. A H atom in a star is moving away from you at 3.0×10^7 cm s^{-1} = 0.001 times c.

At what wavelength will you see H_α?

$$\lambda_{obs} = 6562.8\,(1+0.001) = 6569.4\ \text{A}$$

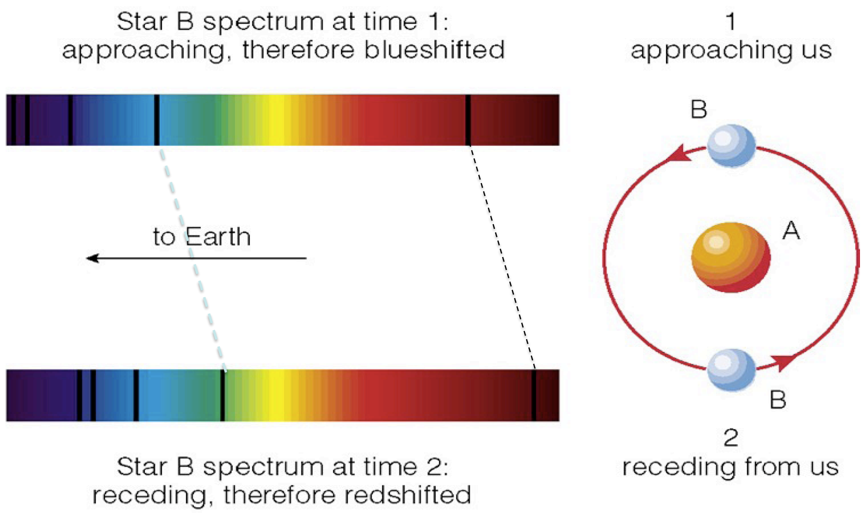
Note that the Doppler shift only measures that part of the velocity that is directed towards or away from you.

Doppler Shift:

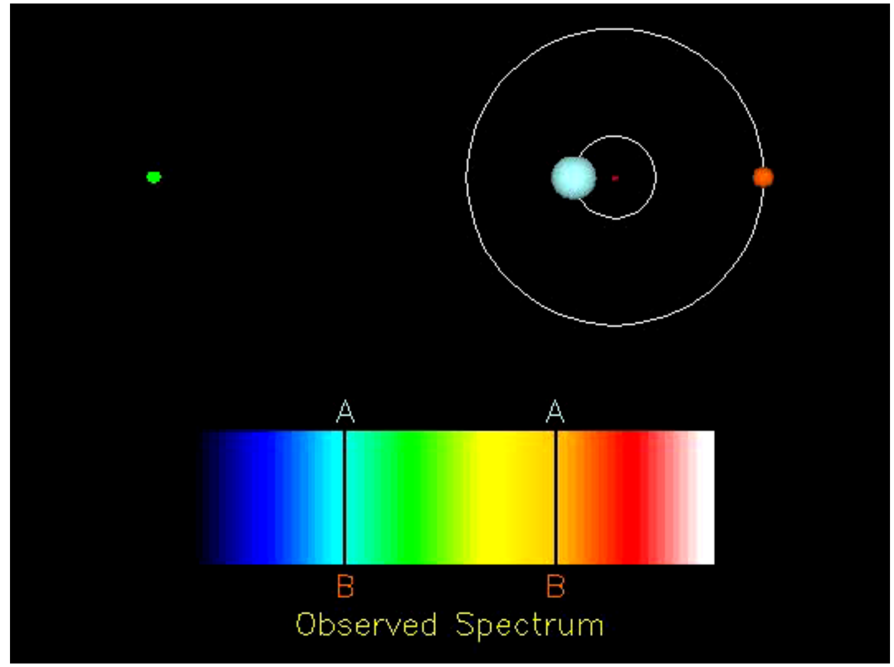


Note – different from a cosmological red shift!

A binary star pair



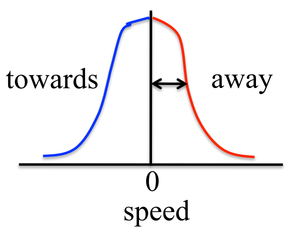
Again, only see a shift due to motion along our line of sight



Thermal Line Broadening

In a gas with some temperature T atoms will be moving around in random directions. Their average speed will depend upon the temperature. Recall that the definition of temperature, T, is

$$\frac{1}{2} m_{atom} \langle v^2 \rangle = \frac{3}{2} k\, T$$



$$\boxed{v_{average} = \sqrt{\frac{3kT}{m_{atom}}}}$$

where $k = 1.38 \times 10^{-16}$ erg K^{-1}

Here $\langle\rangle$ means "average". Some atoms will be moving faster than the average, others hardly at all. Some will be moving towards you, others away, still others across your line of sight.

Thermal Line Broadening

The full range of wavelengths, hence the width of the spectral line will be

$$\frac{\Delta\lambda}{\lambda} = 2\frac{v_{average}}{c} = \frac{2}{c}\sqrt{\frac{3kT}{m_{atom}}}$$

The mass of an atom is the mass of a neutron or proton (they are about the same) times the total number of both in the nucleus, this is an integer "A".

$$\frac{\Delta\lambda}{\lambda} = 2\left(\frac{(3)(1.38\times10^{-16})(T)}{(1.66\times10^{-24})(A)}\right)^{1/2}\left(\frac{1}{2.99\times10^{10}}\right)$$

A = 1 for hydrogen
4 for helium
12 for carbon
16 for oxygen
etc.

$$\boxed{\frac{\Delta\lambda}{\lambda} = 1.05\times10^{-6}\sqrt{\frac{T}{A}}}$$

where T is in K; *nb* depends on A

Thermal Line Broadening

$$\text{Full width} = \Delta\lambda = 1.05 \times 10^{-6} \sqrt{\frac{T(\text{in K})}{A}}\ \lambda$$

Eg. H_α at 5800 K (roughly the photospheric temperature of the sun)

A = 1 T= 5800 λ = 6563 A

$$\Delta\lambda = 1.05 \times 10^{-6}\left(\frac{5800}{1}\right)^{1/2}(6563) = 0.53\ A$$

This is (another) way of measuring a star's temperature

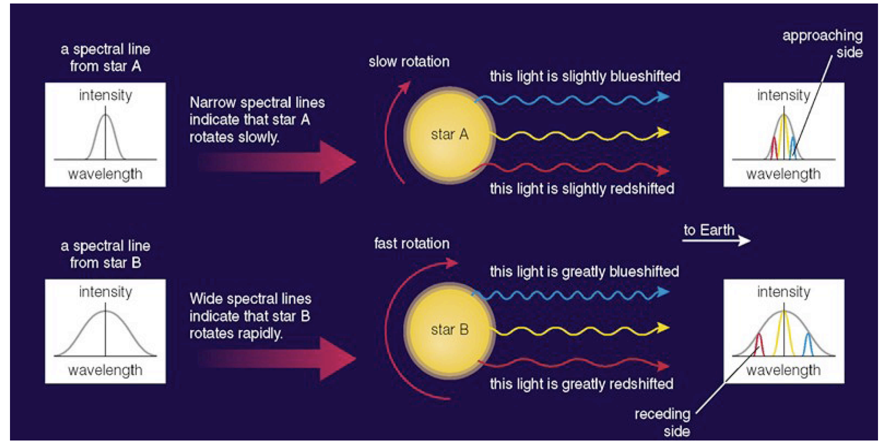
Wien's law (wavelength where most emission comes out)

Spectral class (O,B, F, G, K, M and subsets thereof)

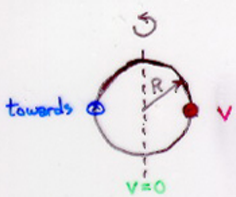
$$L = 4\pi R^2 \sigma T^4 \Rightarrow T = \left(\frac{L}{4\pi R^2 \sigma}\right)^{1/4}$$

Thermal line broadening

ROTATION
(as viewed in the plane of the equator)



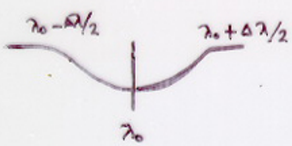
Rotation



If look at * in plane of equator:

v away $= V_{rot} = \frac{2\pi R}{P}$ P = period

$V_{rot} = -\frac{2\pi R}{P}$



For an absorption line:

$$\frac{\Delta\lambda}{\lambda} = 2\frac{V_{rot}}{c} = \frac{4\pi R}{P}$$

Can be used to get the rotational speed and period (if we know R)

Note: <u>Potential</u> <u>complications</u>:

1) Star may have both thermal and rotational broadening

2) May see the star at some other angle than in its equatorial plane.

Example: H_α in a star with equatorial rotational speed 100 km/s = 10^7 cm/s

$$\text{Full width} = \Delta\lambda = 2\left(\frac{v}{c}\right)\lambda$$

$$= (2)(6563)\left(\frac{10^7}{3 \times 10^{10}}\right) = 4.4\ A$$

Average rotational velocities (main sequence stars)

Stellar Class	$v_{equator}$ (km/s)
O5	190
B0	200
B5	210
A0	190
A5	160
F0	95
F5	25
G0	12

Stellar winds and magnetic torques are thought to be involved in slowing the rotation of stars of class G, K, and M.

Stars hotter than F5 have stable surfaces and perhaps low magnetic fields.

The sun, a G2 star rotates at 2 km/s at its equator

Red giant stars rotate very slowly. Single white dwarfs in hours to days. Neutron stars may rotate in milliseconds

<u>3 sources of spectral line broadening</u>

1) Pressure or "Stark" broadening (Pressure)

2) Thermal broadening (Temperature)

3) Rotational broadening (w, rotation rate)

One can distinguish rotational broadening from thermal broadening because rotation affects all ions equally, but in a hot gas the heavier ions move slower. Thermal width depends on $A^{-1/2}$.

SUMMARY

SPECTROSCOPY: WHAT WE CAN LEARN

1) Temperature

Ionization stages that are present

Thermal line broadening

Wien's Law ($\lambda_{max} \propto 1/T$)

2) Radius

Blackbody $L = 4\pi R^2 \sigma T^4$

3) Rotation rate

Spectral line widths

4) Composition

From a detailed analysis of what lines are present and their strengths

5) Surface pressure

Also from line broadening. Is the star a white dwarf or a red giant or a main sequence star

6) Velocity towards or away from us

Is the star or galaxy approaching us or receding?

7) Binary membership, period, and velocity planets?

From periodic Doppler shifts in spectral lines

8) Magnetic fields

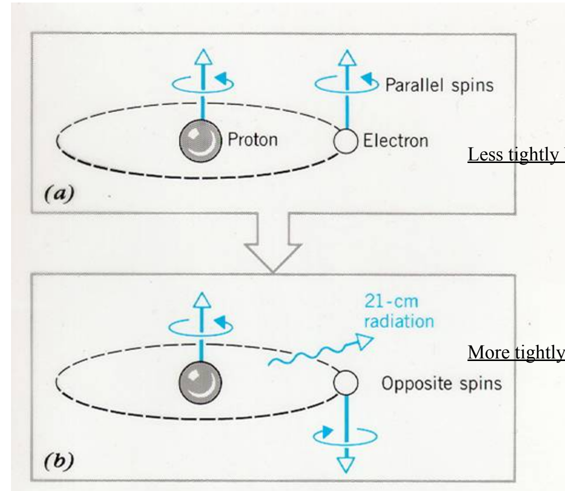
From Zeeman splitting

9) Expansion speeds in stellar winds and explosions

Supernovae, novae, planetary nebulae

10) From 21 cm - rotation rates of galaxies. Distribution of neutral hydrogen in galaxies. Sun's motion in the Milky Way.

Hyperfine Splitting   The 21 cm Line



The 21-cm emission from hydrogen atoms.

Less tightly bound

More tightly bound

21 cm (radio)

λ= 21 cm $\lambda = c\nu$

ν = 1.4 x 10^9 Hz

hν = (6.63 x 10^{-27})(1.4 x 10^9) = 9.5 x 10^{-18} erg

= 5.6 x 10^{-6} eV

Must have neutral H I

Emission collisionally excited

Lifetime of atom in excited state about 10^7 yr

Galaxy is transparent to 21 cm

Merits:

- Hydrogen is the most abundant element in the universe and a lot of it is in neutral atoms - H I
- It is not so difficult to build big radio telescopes
- The earth's atmosphere is transparent at 21 cm

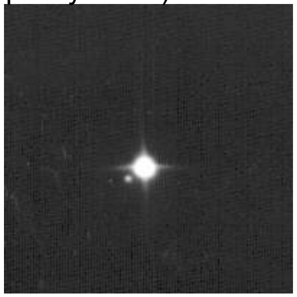

Aerecibo - 305 m radio telescope - Puerto Rico

Getting Masses in Binary Systems

Binary and Multiple Stars

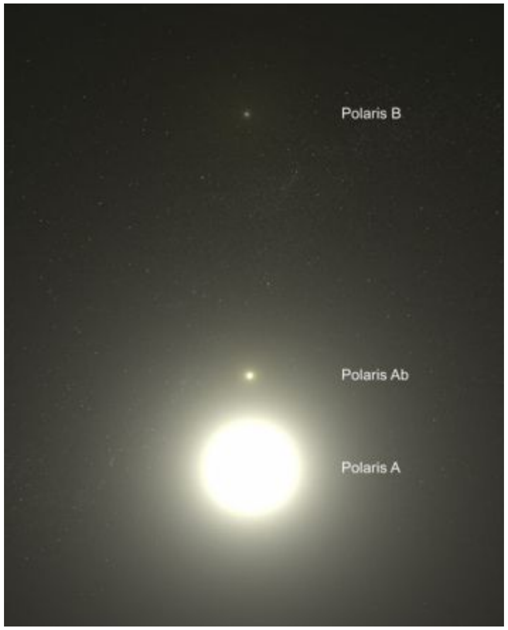
(about half of all stars are found in multiple systems)

Beta-Cygnus (also known as Alberio)
Separation 34.6". Magnitudes 3.0 and 5.3.
Yellow and blue. 380 ly away. Bound?
P > 75000 y. The brighter yellow component is also a (close) binary. P ~ 100 yr.



Alpha Ursa Minoris (Polaris)
Separation 18.3". Magnitudes 2.0 and 9.0. Now known to be a triple.
Separation ~2000 AU for distant pair.

When the star system was born it apparently had too much angular momentum to end up as a single star.



Polaris

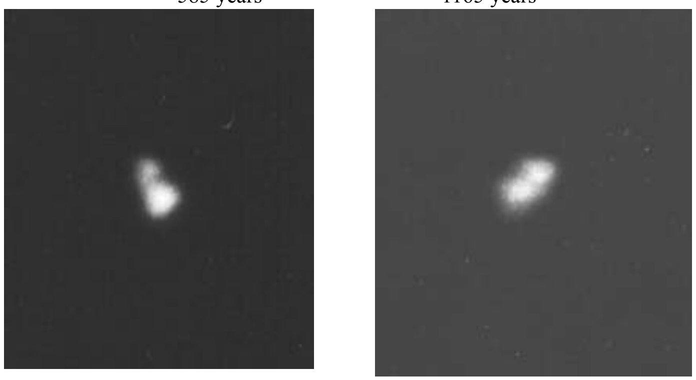
1.2 Msun Polaris Ab
Type F6 - V
4.5 Msun Polaris A
Cepheid

Period 30 yr

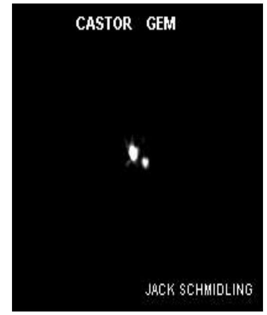
Polaris B is
F3 - V

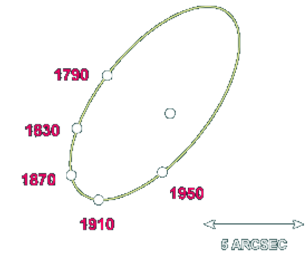
$> 10^5$ years

585 years 1165 years



Epsilon Lyra – a double double.
The stars on the left are separated by 2.3” about 140 AU; those on the right by 2.6”. The two pairs are separated by about 208” (13,000 AU separation, 0.16 ly between the two pairs, all about 162 ly distant). Each pair would be about as bright as the quarter moon viewed from the other.





Castor A and B complete an orbit every 400 years. In their elliptical orbits their separation varies from 1.8” to 6.5”. The mean separation is 8 billion miles. Each star is actually a double with period only a few days (not resolvable with a telescope). There is actually a “C” component that orbits A+B with a period of of about 10,000 years (distance 11,000 AU).

Castor C is also a binary. 6 stars in total

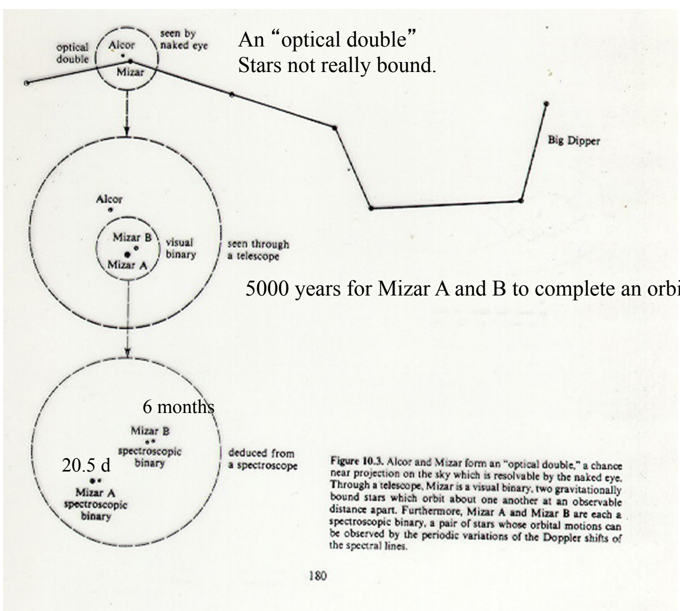
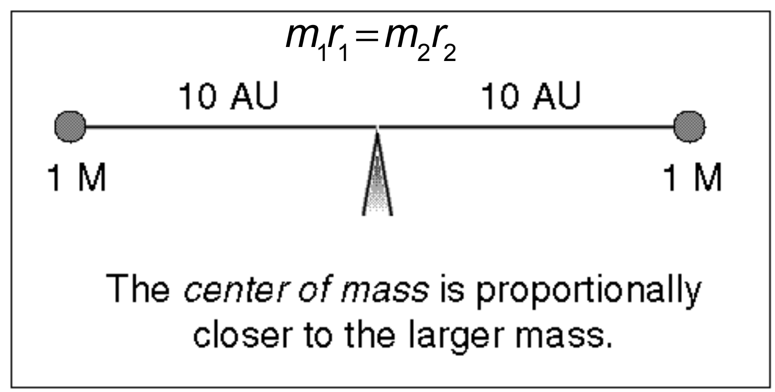


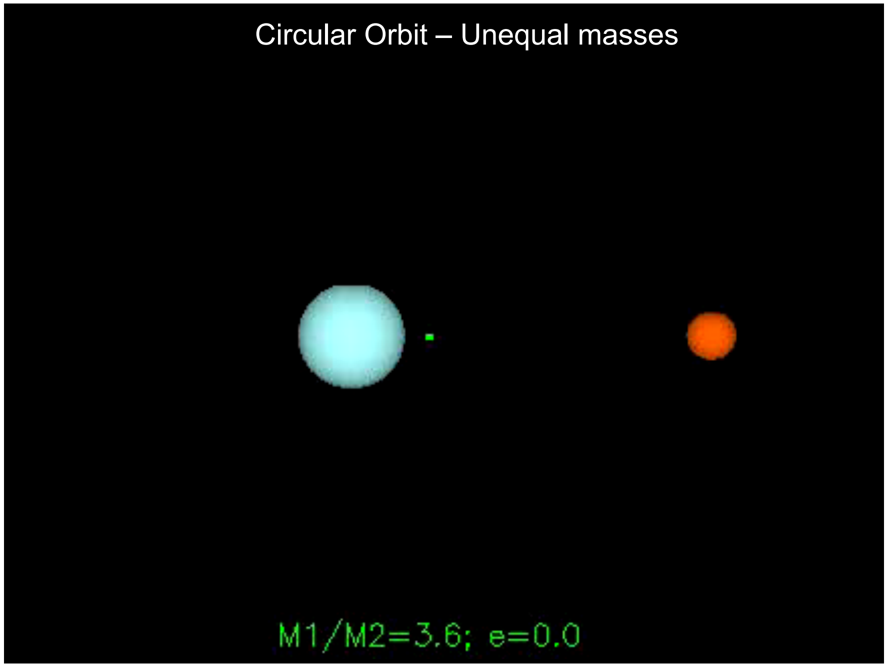
Figure 10.3. Alcor and Mizar form an "optical double," a chance near projection on the sky which is resolvable by the naked eye. Through a telescope, Mizar is a visual binary, two gravitationally bound stars which orbit about one another at an observable distance apart. Furthermore, Mizar A and Mizar B are each a spectroscopic binary, a pair of stars whose orbital motions can be observed by the periodic variations of the Doppler shifts of the spectral lines.

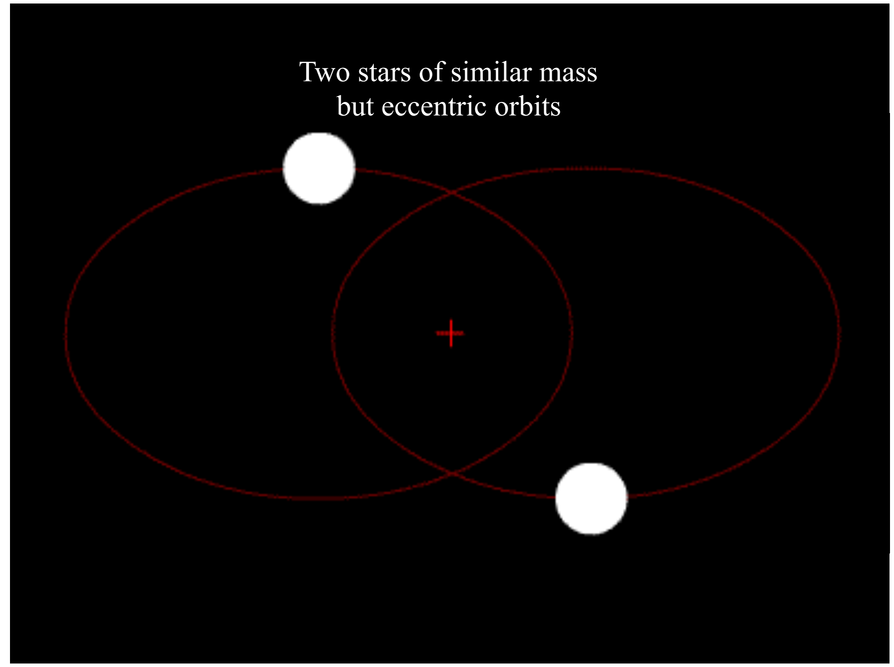
180

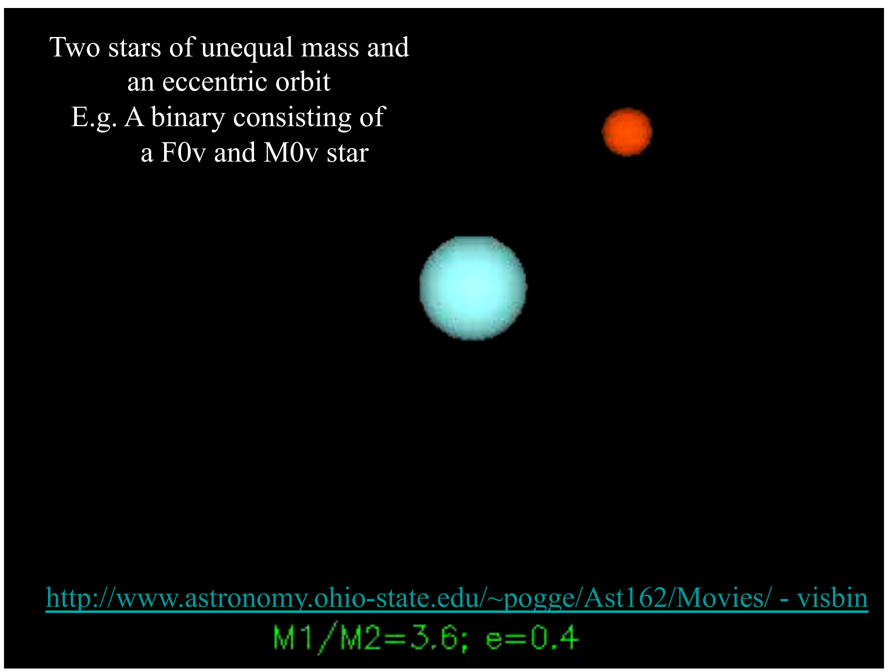
Center of Mass



For constant total separation, 20 AU, vary the masses







Aside

The actual separation between the stars is obviously not constant in the general case shown of non-circular orbits.

The separation at closest approach is the sum of the semi-major axes of the two elliptical orbits, $a = a_1+a_2$, times (1-e) where e is the eccentricity.

At the most distant point the separation is "a" times (1+e).

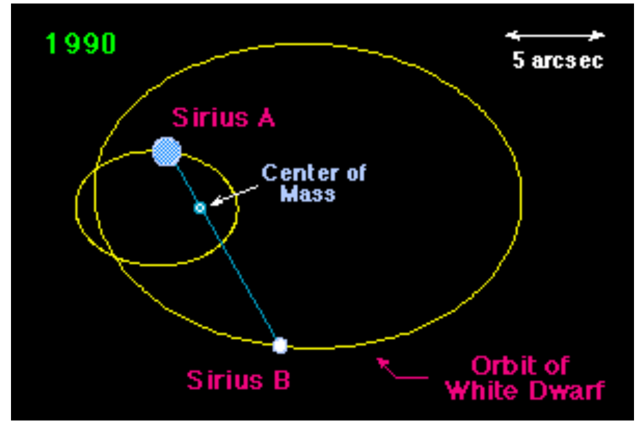
So if one measures the separation at closest approach and farthest separation, adds and divides by 2, one has $a_1 + a_2$

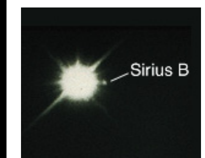
For circular orbits e = 0 and the separation is constant, r_1+r_2 .

$(a_1 + a_2)\ (1 + e)$ farthest separation

$+\ (a_1 + a_2)\ (1 - e)$ closest separation

$$2\,(a_1 + a_2) \quad /\,2 \quad = (a_1 + a_2)$$



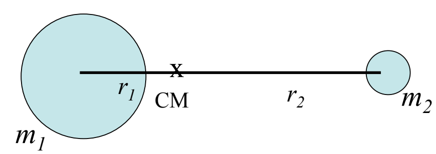


Period = 50.1 years
semi-major axis of A is 6.4 AU and B is 13.4 AU
(In Kepler's equation use the sum of the semimajor axes)

Some things to note:

- A binary star system has only one period. The time for star A to go round B is the same as for B to go round A
- A line connecting the centers of A and B always passes through the center of mass of the system
- The orbits of the two stars are similar ellipses with the center of mass at a focal point for both ellipses
- For the case of circular orbits, the distance from the center of mass to the star times the mass of each star is a constant. (next page)

ASSUME CIRCULAR ORBITS



both stars feel the same gravitational attraction and thus both have the same centrifugal force

$$\frac{m_1 v_1^2}{r_1} = \frac{G m_1 m_2}{(r_1 + r_2)^2} = \frac{m_2 v_2^2}{r_2}$$

$$\frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} = \text{Period}$$

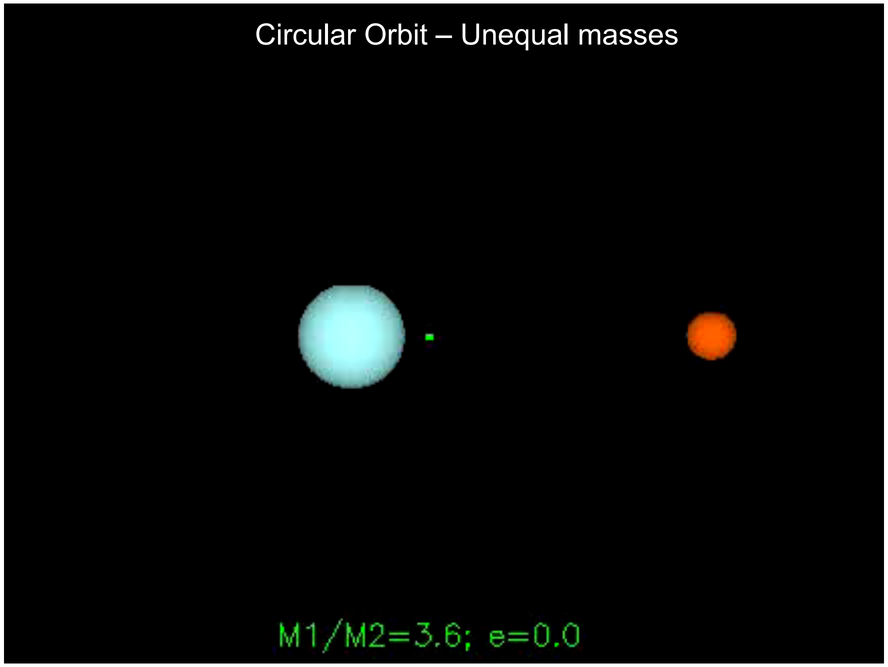
$$\therefore\ v_1 = \frac{r_1 v_2}{r_2}$$

$$\frac{m_1 r_1^{\cancel{2}} \cancel{v_2^2}}{\cancel{r_1} r_2^{\cancel{2}}} = \frac{m_2 \cancel{v_2^2}}{\cancel{r_2}}$$

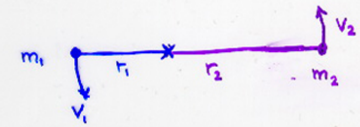
More massive star is closer to the center of mass and moves slower.

$$\boxed{m_1 r_1 = m_2 r_2}$$

$$\boxed{\frac{r_1}{r_2} = \frac{m_2}{m_1}}$$



For simplicity, assume circular motion



m_1 goes around "x" in period P

m_2 also goes around "x" in period P

$$\frac{2\pi r_1}{P} = v_1 \qquad \frac{2\pi r_2}{P} = v_2$$

$$P = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$$

$$\boxed{r_1 v_2 = r_2 v_1} \qquad \frac{r_1}{r_2} = \frac{v_1}{v_2}$$

$$\boxed{\frac{r_1}{r_2} = \frac{v_1}{v_2}}$$

So

since $\dfrac{r_1}{r_2} = \dfrac{m_2}{m_1}$

$$\boxed{\frac{m_2}{m_1} = \frac{v_1}{v_2}}$$

E.g.. Motion of the sun because of Jupiter;
Roughly the same as two stars in circular orbits

$$m_1 r_1 = m_2 r_2$$
$$M_\odot d_\odot = M_J\, d_J$$

$$d_\odot = \frac{M_J}{M_\odot}\, d_J$$
$$= (9.95 \text{ x } 10^{-4})(7.80 \text{ x } 10^{13})$$
$$= \boxed{7.45 \text{ x } 10^{10} \text{ cm}}$$

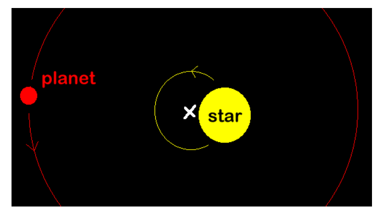
$d_\odot$ = radius of sun's orbit around center of mass

d_J = Jupiter's orbital radius
= 5.20 AU
= $7.80 \text{ x } 10^{13}$ cm

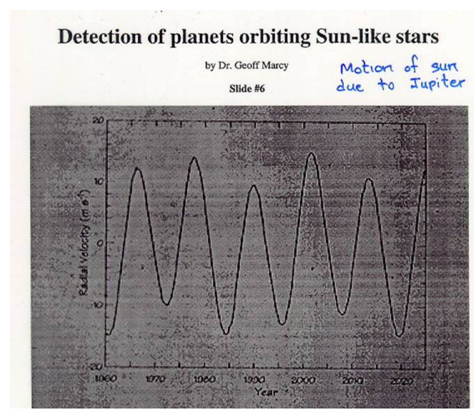
M_J = $1.90 \text{ x } 10^{30}$ gm
= $9.55 \text{ x } 10^{-4}$ $M_\odot$

Can ignore the influence of the other planets.

P = 11.86 years



Note: "wobble" of the star is bigger if the planet is bigger or closer to the star (hence has a shorter period).



12.5 m/s
11.86 years

Doppler shift

P_J = Period Jupiter $= 11.86\,y$
$= 3.75 \times 10^8\ s$

$$V_\odot = \frac{2\pi d_\odot}{P_J} = \frac{(2\pi)(7.45\times10^{10}\ cm)}{3.75\times10^8\ s}$$

$$= 1.25\times10^3\ cm/s$$

$$= 12.5\ m/s$$

About 40 mph

$$\frac{v}{c} = \frac{1.25\times10^3\ cm/s}{2.99\times10^{10}\ cm/s} = 4.18\times10^{-8}$$

$$= \frac{\Delta\lambda}{\lambda}$$

eg. $H\alpha$ $6563\ \AA$ $\qquad \underline{\underline{\Delta\lambda = 2.74\times10^{-4}\ \AA}}$

small compared to thermal + rotational broadening

As of today –2052 extra solar planets in 1300 stellar systems and the number is growing rapidly.

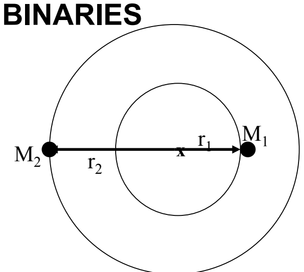
Many were detected by their Doppler shifts.
Many more by the "transits" they produce as they cross the stellar disk.

http://exoplanet.eu/catalog.php

KEPLER'S THIRD LAW FOR BINARIES

$$\frac{G\cancel{M_1}M_2}{(r_1+r_2)^2} = \frac{\cancel{M_1}v_1^2}{r_1}$$

$$+\quad \frac{GM_1\cancel{M_2}}{(r_1+r_2)^2} = \frac{\cancel{M_2}v_2^2}{r_2}$$



$$\frac{G(M_1+M_2)}{(r_1+r_2)^2} = \frac{v_1^2}{r_1} + \frac{v_2^2}{r_2} = \frac{4\pi^2 r_1^2}{P^2 r_1} + \frac{4\pi^2 r_2^2}{P^2 r_2} \qquad v = \frac{2\pi r}{P}$$

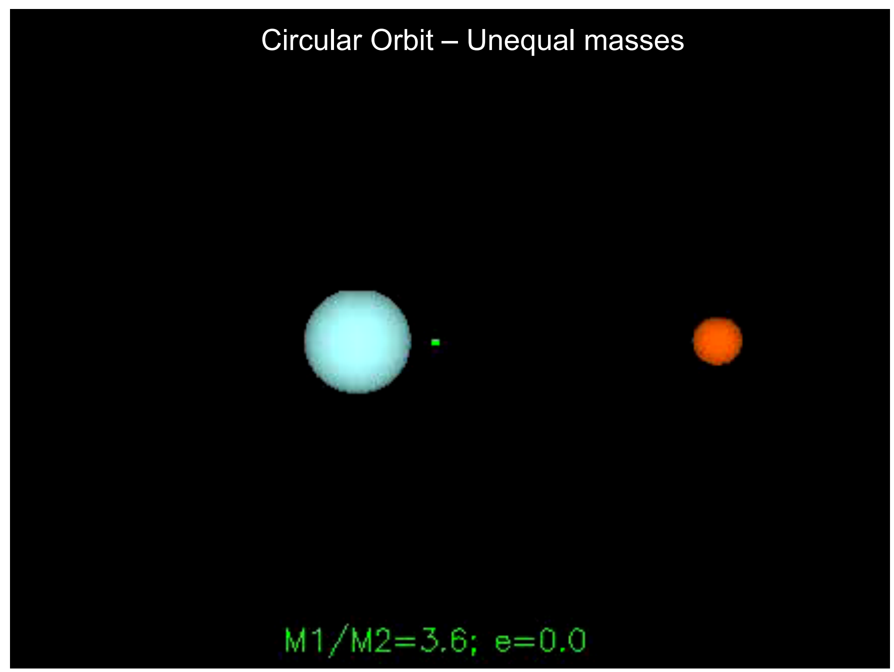
$$= \frac{4\pi^2}{P^2}(r_1+r_2)$$

$$\boxed{P^2 = K(r_1+r_2)^3} \qquad K = \frac{4\pi^2}{G(M_1+M_2)}$$

i.e., just like before but

$M \rightarrow M_1+M_2 \qquad R \rightarrow r_1+r_2$

In the general case $(r_1+r_2) \rightarrow (a_1+a_2)$ where a_1 and a_2 are the semimajor axes of the two stars in ellipical orbits around each other



$$(M_1 + M_2) = \frac{4\pi^2}{GP^2}(r_1 + r_2)^3$$

$$M_\odot = \frac{4\pi^2}{G(1\,yr)^2}(AU)^3$$ for the earth

Divide the two equations

$$\frac{M_1 + M_2}{M_\odot} = \left(\frac{(r_1 + r_2)^3_{AU}}{P^2_{yr}}\right)$$

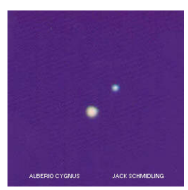
$$\frac{M_1}{M_2} = \frac{r_2}{r_1} \quad \text{or} \quad \frac{M_1}{M_2} = \frac{v_2}{v_1}$$

If you know r_1, r_2 *in AU*, or v_1, v_2, and P in years you can solve for the two masses.

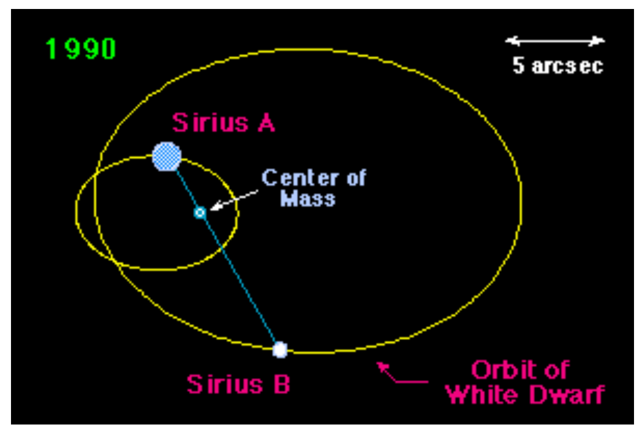
Getting Stellar Masses – Visual binaries

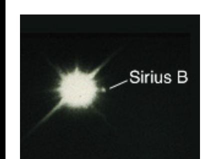
Measure:

- Period
- Separation (if circular orbit; sum of semi-major axes if elliptical; average of max and min separations)
- Ratio of speeds or separations from center of mass if circular (or ratio of semi-major axes if elliptical)



E.g., Sirius A and B; distance 2.67 pc)





Period = 50.1 years
semi-major axis of A is 6.4 AU and B is 13.4 AU
(In Kepler's equation use the sum of the semimajor axes)

Calculation

$$\frac{M_A + M_B}{M_\odot} P^2(yr) = A^3(AU)$$

$$A = A_A + A_B = 6.4 + 13.4 = 19.8\ AU \qquad P = 51\ yr$$

$$\frac{M_A + M_B}{M_\odot} = \frac{19.8^3}{51^2} = 2.99$$

So the sum of the masses is 2.98 solar masses.

The ratio of the masses is $A_A / A_B = 6.4/13.4 = 0.478 = M_B / M_A$

$$M_A + 0.478\, M_A = 2.99 \qquad M_A = 2.02\, M_\odot$$

$$M_B = 0.478\, M_A = 0.97\, M_\odot$$

and since we see the orbits face on these are the actual masses.

Sirius B only has a radius 0.0084 times that of the sun. What is it?

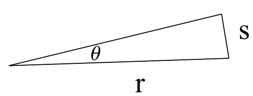
In the case of circular orbits seen face on the separation is a constant. As shown on the next page the separation in AU is just the distance in pc times the angular separation in arc seconds. This is a consequence of the way the pc is defined.

One can also measure, in the same way the separation between star 1 and the center of mass, r_1, and star 2 and the center of mass, r_2.

The total mass is then given by the usual equation

$$\frac{M_1 + M_2}{M_\odot} P^2(yr) = R^3(AU) = (r_1 + r_2)^3$$

and $$\frac{M_1}{M_2} = \frac{r_2}{r_1}$$



s (in pc) = r (in pc) θ (in radians)

s (in AU) = r (in AU) θ (in radians)

$$\text{r (in AU)} = \text{r (in pc)} \left(\frac{\text{number AU}}{1\ \text{pc}}\right)$$

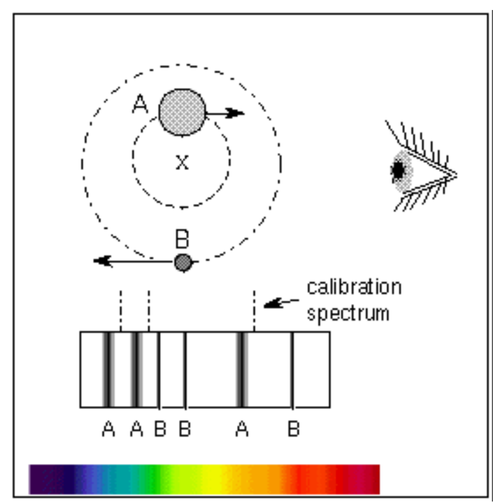
$$\theta \text{ in radians} = \theta \text{ (in arc sec)} \left(\frac{1\ \text{radian}}{\text{number arc sec}}\right)$$

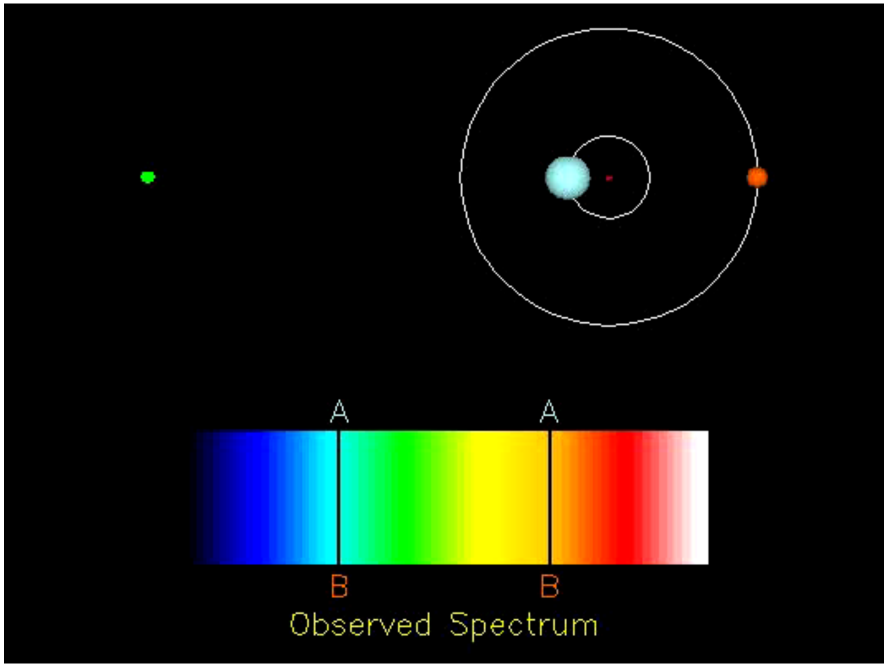
$$\text{s in AU} = \text{r (in pc)} \left(\cancel{\frac{\text{number AU}}{1\ \text{pc}}}\right) \theta \text{ (in arc sec)} \left(\cancel{\frac{1\ \text{radian}}{\text{number arc sec}}}\right)$$

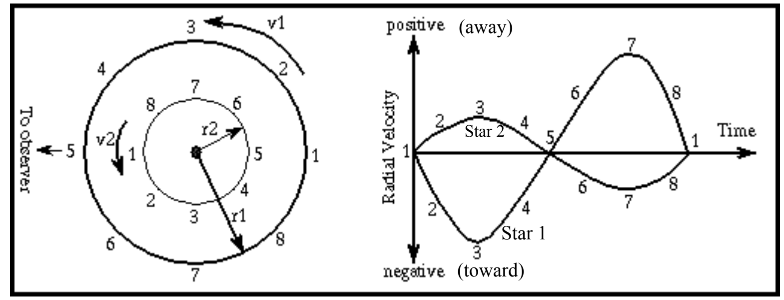
e.g. The separtion of Sirius A and B varies from 3.5 to 11.5 arc sec. The average is therefore 7.5 arc sec. Sirius is 2.67 pc away so

$a_1 + a_2 = (2.67)(7.5) = 20$ AU

<u>Spectroscopic Binaries</u>







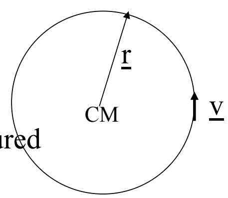
Complication:

The viewing angle

Getting Stellar Masses For Sprectroscopic Binaries

For spectroscopic binaries measure:

- Period
- Velocity of each star
- Inclination will be unknown so mass measured will be a lower bound (TBD)



CALCULATION

$$P = \frac{2\pi r}{v}$$

Assume circular orbits

First get r_1 and r_2 from v_1 and v_2

$$r_i = \frac{v_i P}{2\pi}$$

Example:

$v_1 = 75 \text{ km s}^{-1}$ $v_2 = 25 \text{ km s}^{-1}$

P= 17.5 days

$$P = \frac{2\pi r}{v} \Rightarrow r = \frac{Pv}{2\pi}$$

$$
\begin{aligned}
A &= r_1 + r_2 \\
&= \frac{P}{2\pi}(v_1 + v_2) \\
&= \left[\frac{17.5\ \cancel{day}}{(2)(3.14)}\right]\left[\frac{8.64\times 10^4\ \cancel{sec}}{1\ \cancel{day}}\right]\left[100\frac{\cancel{km}}{\cancel{sec}}\right] \\
&\quad \left[\frac{10^5\ \cancel{cm}}{\cancel{km}}\right]\left[\frac{AU}{1.50\times 10^{13}\ \cancel{cm}}\right] \\
&= 0.16\ \mathrm{AU}
\end{aligned}
$$

$$
\begin{aligned}
P &= 17.5\ \mathrm{d}\left(\frac{1\ \mathrm{yr}}{365.25\ \mathrm{d}}\right) \\
&= 0.0479\ \mathrm{yr}
\end{aligned}
$$

and can now solve as before

$$
\begin{aligned}
M_1 + M_2 &= \frac{(0.16)^3}{(0.0479)^2} = \frac{A^3}{P^2} \\
&= 1.8\ M_\odot
\end{aligned}
$$

and since $M_1/M_2 = v_2/v_1 = 1/3$, $M_1 = 0.45\ M_\odot$ and $M_2 = 1.35\ M_\odot$.

Note - the bigger the speeds measured for a given P the bigger the masses

Complication – The Inclination Angle

Let i be the angle of the observer relative to the rotation axis, i.e., i = 0 if we re along the axis.

Only if i = 90 degrees do we measure the full velocity.

Measure v Sin i which is a lower bound to v.

$$P^2 = \frac{4\pi^2}{G(M_1+M_2)}(r_1+r_2)^3$$

$$r_i = \frac{P v_i}{2\pi}$$

but measure $\tilde{v} = v\,\mathrm{Sin}\,i$, so we end up measuring

$\tilde{r} = r\,\mathrm{Sin}\,i$ and calculate

$$\tilde{M}_1 + \tilde{M}_2 = \frac{4\pi^2}{GP^2}\left(\frac{\tilde{v}_1+\tilde{v}_2}{2\pi}\right)^3 P^3 \qquad \text{measured}$$

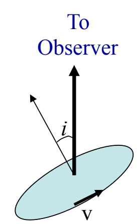
when the actual mass is

$$M_1 + M_2 = \frac{4\pi^2}{GP^2}\left(\frac{v_1+v_2}{2\pi}\right)^3 P^3 \qquad \text{actual}$$

hence the measurement gives a low bound on the actual mass

$(\tilde{M}_1 + \tilde{M}_2) = (M_1+M_2)\,\mathrm{Sin}^3\,i$

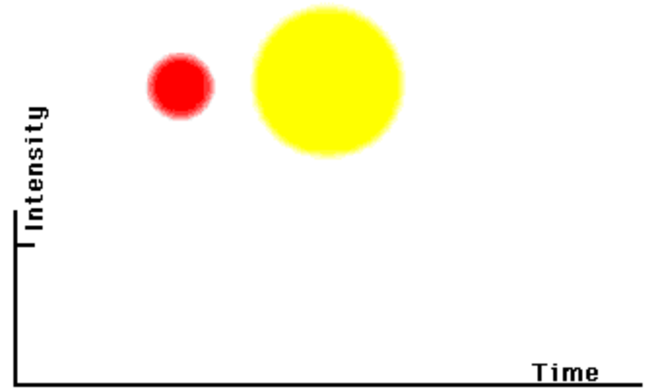
Since Sin i < 1, the measurement is a lower bound.



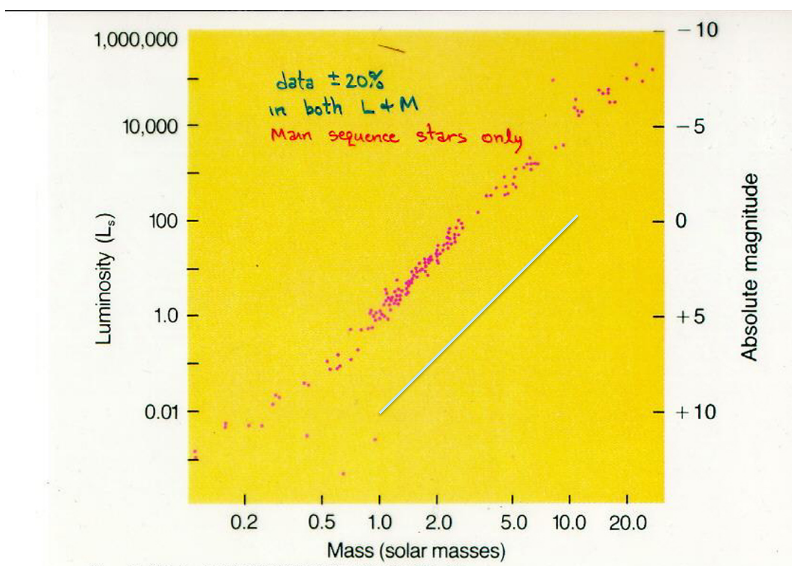
$\langle \mathrm{Sin}^3 i \rangle = 0.59$

But we tend to discover more edge on binaries so 2/3 is perhaps better

Eclipsing Binary



For an eclipsing binary you know you are viewing the system in the plane of the orbit. I.e., Sin i = 1



67. Abell/Morrison/Wolff: EXPLORATION OF THE UNIVERSE, 6/E

Limits of stellar mass:

Observed stars end up having masses between 0.08 $M_\odot$ and about 150 $M_\odot$.

The upper number is uncertain (130? 200?). The lower number will be derived later in class (minimum mass to ignite H burning before becoming degenerate).

STELLAR LIFETIMES

On the main sequence:

- Luminosity determined by mass - $L \propto M^n$ $n \approx 3$ to 4

- Say star has a total energy reservoir proportional to its mass (as in a certain fraction to be burned by nuclear reactions)

$$E_{tot} = fM$$

Then the lifetime on the main sequence will be shorter for stars of higher mass;

$$\tau_{MS} \propto \frac{fM}{M^n} \qquad n=3$$

$$\boxed{\tau_{MS} \approx 10^{10}\,\mathrm{yr}(M_\odot/M)^2}$$

This explains some important features of the HR-diagram.