VI. 21cm Radiation

A. History

- van der Hulst 1941
  - Collq in a bunker!
  - Ground state of HI: $\ell = 0, s = 1/2, j = 1/2$
  - Degeneracy of the G.S. is removed by the interaction of the magnetic moments of the electron and proton

B. Physics

- Proton has a spin and therefore a magnetic moment
  \[ \vec{\mu}_p = \frac{g_p e}{2m_p c} \vec{I} \] (1)
  - $\vec{I}$ is the nuclear spin
  - $g_p \approx 5.5$
- Hyperfine interaction with the electron
  \[ H = -\vec{\mu}_p \cdot \vec{B} \] (2)
  - Spin aligned has highest energy
  - To estimate $\vec{B}$, model the electron orbit as a magnetized sphere
    \[ \vec{B} = \frac{8\pi}{3} \vec{M} = \frac{8\pi}{3} \vec{\mu}_e \Psi_0^2 \] (3)
  - Substituting
    \[ H = \frac{16\pi}{3\hbar^2} \left( \frac{e\hbar}{2m_e c} \right)^2 \frac{m_e}{m_p} g_p \Psi_0^2 \vec{I} \cdot \vec{s} \] (4)
- Introduce $\vec{F} = \vec{I} + \vec{s}$
  - Ala spin-orbit coupling
    \[ \vec{I} \cdot \vec{s} = \frac{1}{2} (F^2 - I^2 - S^2) \] (5)
  - $I = \frac{1}{2} = s$
  - $F = 0, 1$ for spins aligned, anti-aligned
  - For a 1s state
    \[ \Psi_0^2 = \frac{1}{\pi a_0^3} \] (6)
- Transition Energy
\( \diamond \) Energy

\[
\hbar \nu = U_+ - U_-
\]

\[
= \frac{8\pi}{3} \left( \frac{e \hbar}{2m_e c} \right)^2 \frac{m_e g_p}{m_p} \frac{2I + 1}{\pi a_0^3}
\]

\[
= \frac{8}{3} \alpha^2 g_p \frac{m_e}{m_p} \text{Ryd}
\]

\( \diamond \) Frequency: \( \nu_{21\text{cm}} = 1420.403 \text{ MHz} \)
\( \diamond \) Wavelength: \( \lambda = 21.1 \text{ cm} \)
\( \diamond \) Approx energy: \( \Delta E = 10^{-5} \text{eV} \)

- **Transition probability**
  - Spontaneous emission coefficient
    
    \[
    A_{10} = 2.85 \times 10^{-15} \text{ s}^{-1}
    \]

- **Lifetime**
    
    \[
    \tau_{21\text{cm}} \approx 10^7 \text{ yr}
    \]

- **Excited state is primarily de-excited by collisions**
    
    \[
    \langle \sigma \nu \rangle \sim 10^{-10} \text{ cm}^3 \text{ s}^{-1} \text{ for } T \sim 100\text{K}
    \]
    
    \[
    \tau_{\text{coll}} \approx 3 \times 10^2 \text{ yr}
    \]

**C. Spin Temperature**

- **\( T_S \):** Quantifies the level populations of the two hyperfine levels of H

- **Level population**
  - Boltzmann
    
    \[
    \frac{n_1}{n_0} = \frac{g_1}{g_0} \exp \left( -\frac{h\nu}{kT_S} \right)
    \]
    
    \[
    = 3 \exp \left( -\frac{T_*}{T_S} \right)
    \]
    
    \[
    \approx 3 \left[ 1 - \frac{T_*}{T_S} + \mathcal{O} \left( \frac{T_*}{T_S} \right)^2 \right]
    \]

- **\( T_* = 0.068\text{K} \)**
  - This generally implies \( T_* \ll T_S \)
  - The excitation temperature \( T_S \) is referred to as the Spin Temperature

- **G. Field 1959, ApJ, 129, 551:** \( T_S = T_{\text{kin}} \) even if the gas is NOT in thermal equilibrium

- **Majority of H gas will have 3/4 of the atoms in the excited state and only 1/4 in the ground state**
HI gas is always emitting 21cm radiation
Not a dominant coolant, but a very important diagnostic
Furthermore, one cannot neglect stimulated emission!!

D. Brightness Temperature

- Transfer equation
\[
\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu
\] (17)

- Optical depth
\[
d\tau_\nu = -\kappa_\nu ds
\] (18)

- Consider radiation from a single cloud with optical depth \( \tau_{\nu r} \) and incident radiation \( I_\nu(0) \)
\[
I_\nu = I_\nu(0)e^{-\tau_{\nu r}} + \int_{0}^{\tau_{\nu r}} \frac{j_\nu}{\kappa_\nu}e^{-\tau_\nu}d\tau_\nu
\] (19)

- If one has strict thermodynamic equilibrium, Kirchhoff’s law applies
\[
\frac{j_\nu}{\kappa_\nu} = B_\nu(T) = \frac{2\hbar \nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}
\] (20)

- In the low frequency regime, \( h\nu \ll kT \)
\[
B_\nu(T) = \frac{2\nu^2 kT}{c^2}
\] (21)

▲ Define ”brightness temperature”, \( T_b \) is the temperature where \( I_\nu = B_\nu(T) \)
▲ Radiation transfer equation becomes
\[
T_b = T_b\text{e}^{-\tau_{\nu r}} + T \left( 1 - \text{e}^{-\tau_{\nu r}} \right)
\] (22)

E. Optical Depth

- Opacity of HI gas to 21cm radiation
\[
\kappa_\nu = \tilde{\kappa}_\nu \left( 1 - \text{e}^{-T_*/T} \right)
\] (23)

- 2nd term is due to stimulated emission
- Without this term, we could not probe the Galaxy
- Again, \( T_* \ll T \)

\[
\kappa_\nu \approx \tilde{\kappa}_\nu \frac{T_*}{T}
\] (24)
\[
= \frac{h\nu_{10} B_{01} n_0 \phi_\nu}{4\pi} \frac{T_*}{T}
\] (25)
\[
= \frac{h\nu_{10} g_1}{4\pi g_0} \left( \frac{A_{10} c^2}{2h\nu_{10}^2} \right) \frac{1}{4} n_H \frac{T_*}{T} \phi_\nu
\] (26)
Hot regions are less opaque

\( \phi_v \) is the line profile (we will return to this when discussing Ly\( \alpha \) absorption)

- Optical depth
  - Frequency
    \[ \tau_\nu = \int_0^L \kappa_\nu ds \]  
    \[ = \frac{\text{(const)}}{T} \int_0^L n_H ds \]  
  - Velocity space
    \[ \delta \nu = \nu_{10} \frac{v}{c} \]  
    \[ \tau_v \propto N_{HI} \frac{\phi(v)}{T} \]

\( \phi(v) \) is in km/s
\( N_{HI} \) is in cm\(^{-2}\)

- For an isolated cloud with a Gaussian velocity distribution
  \[ \phi(v) = e^{-v^2/2\sigma^2} \]  

- Peak optical depth
  \[ \tau_0 = \frac{N_{HI}}{1.825 \times 10^{18} T^{1/2} \sigma \sqrt{2\pi}} \]

- For a single cloud with \( T \sim 80K, \sigma \sim 3\text{km/s}, \) and \( N_{HI} \sim 10^{20} \text{cm}^{-2} \), we find \( \tau_0 \sim 0.1 \)

F. Temperatures of HI Gas

- Observe a cloud in 21cm emission
  - Characterize the cloud by \( T_S \) and \( \tau_\nu \)
  - Radiative transfer (low frequency limit)
    \[ T_b = T_S \left( 1 - e^{-\tau_\nu} \right) \]

- Observe toward the Galactic center
\( \tau_0 \gg 1 \) because of the lack of differential rotation

- Line saturates at \( T_b \approx T_S \)
- \( T_S \approx 80 \text{K} \), the Harmonic mean of all gas
- \( T_b \)
• Observe off the Galactic center (emission)
  ◦ Cartoon

![Cartoon of the Sun with rays indicating observation](image)

◊ Now, $\tau_\nu \ll 1$

\[ T_b = T_S \tau_\nu \quad (34) \]

▲ It appears we have sensitivity to $T_S$

▲ But recall $\tau_\nu \approx 1/T_S \Rightarrow T_b$ is independent of $T_S$

◊ Thin limit ($\kappa_\nu$ negligible):

\[ I_\nu = \int j_\nu \, ds \quad (35) \]

\[ \propto A_{10} \int n_1 \, ds \quad (36) \]

▲ Observation is independent of $T_S$

▲ Can accurately measure $N_{\text{HI}}$

◊ Fig
- Require absorption spectra to determine $T_S$

![Diagram of Sun, Cloud, and Quasar]

- Absorption-line analysis of a continuum source (e.g. quasar) with intensity $T_C$

(a) Radiative transfer

$$T_b = T_S \left( 1 - e^{-\tau_\nu} \right) + T_C e^{-\tau_\nu}$$  \hfill (37)

(b) Determine the unattenuated brightness of the QSO

- Observe the quasar near but not at the 21cm frequency ("off-line")
- $\tau_\nu \ll 1$
- $T_{b_{\text{off}}} = T_C$ (by definition)

(c) Measure the brightness $T_{b_{\text{on}}}$ at the 21cm frequency ("on-line") [Eq. 37]

(d) Take the difference spectrum

$$\Delta T_b = T_{b_{\text{on}}} - T_{b_{\text{off}}}$$  \hfill (38)

$$= T_S \left( 1 - e^{-\tau_\nu} \right) + T_C e^{-\tau_\nu} - T_C$$  \hfill (39)

$$= (T_S - T_C) \left( 1 - e^{-\tau_\nu} \right)$$  \hfill (40)
(e) Subtle complication: The telescope has a finite beam size
- Beam size
  \[ \Theta_{\text{beam}} = \frac{\lambda}{D} \quad (\sim 3' \text{ for Arecibo}) \] (41)
- Source (quasar) size \( \Theta_S \ll \Theta_{\text{beam}} \)
- True intensity is related to the apparent intensity by
  \[ I_{\text{true}} = \frac{\Omega_{\text{beam}} I_{\nu}}{\Omega_S} = \left( \frac{\Theta_{\text{beam}}}{\Theta_S} \right)^2 I_{\nu} \] (42) (43)
- Therefore,
  \[ T_{b\text{true}} = \left( \frac{\Theta_{\text{beam}}}{\Theta_S} \right)^2 T_C \equiv T_A \] (44)
- Updating our expression
  \[ \Delta T_b = (T_S - T_A) \left( 1 - e^{-\tau_{\nu}} \right) \] (45)
- Given \( T_S \) or \( \tau_{\nu} \), one can infer the other

(f) Measuring the 21cm absorption (\( \tau_{\nu} \))
- Observe the cloud slightly off the quasar
- Assume physical properties of the cloud vary slowly
- Observe
  \[ T_{b\text{off}} = T_S \left( 1 - e^{-\tau_{\nu}} \right) \] (46)

(g) Solve for \( \tau_{\nu} \)
\[ e^{-\tau_{\nu}} = \frac{T_A - T_{b\text{off}} - \Delta T_b}{T_A} \] (47)
- Solving for \( T_S \) follows trivially

- Observations through the plane
  - There is gas detected in emission that is not seen in absorption
  - Fig

- Furthermore, the absorption components are more narrow than those in emission
Implication: Some gas detected in emission has low $\tau_\nu$ due to high $T_S$ (warm clouds)

- WNM: Warm Neutral Medium
  - Surveys of the Milky Way
    - Many areas with emission detected without absorption
    - Large filling factor of warm gas
    - $\sim 40\%$ of gas detected in emission is WNM
  - Low density
    - Large filling factor implies low density
    - This means the cooling time is long: $\Lambda \sim n^2$
  - Overall characteristics
    - Wide velocity components ($\sigma \sim 5 - 17\text{ km/s}$)
    - Large filling factor
    - Scale height of the gas (in our Galaxy)
      \[
      n(z) = n_0 \exp(-|z|/h) \quad (48)
      
      h \sim 250\text{ pc} \quad (49)
      \]
    - Surface density (vs. stars which have $\Sigma \sim 70M_\odot \text{pc}^{-2}$)
      \[
      \Sigma_{WMN} = \int \rho \, dz = 3.8M_\odot \text{pc}^{-2} \quad (50)
      \]

- CNM: Cold Neutral Medium
  - Dense clouds $\Rightarrow \Lambda_{CNM} \gg \Lambda_{WNM}$
  - Low filling factor: $\approx 1/3$ of radio sources do NOT show 21cm absorption
  - Dispersion: $\sigma_{CNM} \approx 2 - 3\text{ km/s}$
    - $\sigma_{CNM} < \sigma_{WNM}$
    - Dispersion is due to macroscopic velocities within the cloud, not thermal broadening
  - Cloud-cloud dispersion: $\sigma_{cc} \approx 7\text{ km/s}$
  - Surface density
    \[
    \Sigma_{CNM} = 4.5M_\odot \text{pc}^{-2} \quad (51)
    \]

- Actual temperature observations: Harmonic mean of the WNM and CNM
  - Consider two clouds, one warm and one cold
    - Temperatures $T_{S_1}, T_{S_2}$
    - Optical depths $\tau_1, \tau_2$
    - Cloud 1 is in front of cloud 2
  - Off-source temperature
    \[
    T_{b}^{off} = T_{S_1} \left( 1 - e^{-\tau_1} \right) + T_{S_2} \left( 1 - e^{-\tau_2} \right) e^{-\tau_1} \quad (52)
    \]
If we assigned an average temperature to the clouds
\[ T_{naive} = \frac{T_{S1} \left( 1 - e^{-\tau_1} \right) + T_{S2} \left( 1 - e^{-\tau_2} \right) e^{-\tau_1}}{1 - e^{-(\tau_1+\tau_2)}} \] (53)

Consider several limiting cases
(a) \( \tau_1 \gg 1, \tau_2 \ll 1 \)
\[ T_{naive} = T_{S1} \] (54)
(b) \( \tau_1 \ll 1, \tau_2 \gg 1 \)
\[ T_{naive} = T_{S1} \tau_1 + T_{S2} \] (55)
(c) \( \tau_1, \tau_2 \ll 1 \)
\[ T_{naive} = \frac{T_{S1} \tau_1 + T_{S2} \tau_2}{\tau_1 + \tau_2} \] (56)

Tend to overestimate \( T_S \) of the CNM and underestimate \( T_S \) of the WNM
Careful treatment

\[ 5000 \text{ K} < T_{WNM} < 10000 \text{ K} \] (57)
\[ 20 \text{ K} < T_{CNM} < 250 \text{ K} \] (58)
G. Galactic rotation curve

- Diagram

\[ \Theta_0 = \omega_0 R_0 = \text{rotational velocity at our position} \]
\[ \Theta = \omega R = \text{rotational velocity at radius } R \]
\[ r \text{ is the distance to a given 'cloud'} \]
\[ \ell \text{ is angle off the center of the galaxy} \]

- Observationally, we measure the velocity projected along the sightline

\[ v(r) = \Theta \cos \left[ \frac{\pi}{2} - (\ell + \theta) \right] - \Theta_0 \sin \ell \]  
\[ = \omega R [\sin \theta \cos \ell + \sin \ell \cos \theta] - \Theta_0 \cos \ell \]  

\[ R \sin \theta = r \sin \ell \]  
\[ R \cos \theta + r \cos \ell = R_0 \]
\( v(r) = \omega R_0 \left[ \frac{R}{R_0} \sin \theta \cos \ell + \frac{R}{R_0} \sin \ell \cos \theta \right] - \Theta_0 \sin \ell \) 
\( = \omega R_0 \sin \ell - \Theta_0 \sin \ell \) 
\( = R_0 \sin \ell [\omega(R) - \omega_0] \) 

If the Galaxy were rotating as a solid body (i.e. \( \omega(R) = \omega_0 \)), then every parcel of gas would have zero velocity on the los.

The optical depth would be extremely large.

We would be unable to see through the Galaxy!

• Observations of \( v(r) \)
  
  - Velocity peaks when \( \vec{R} \perp \vec{r} \)
  - Define this \( R = R_c \)
  - Define this velocity as \( v_T \)

\[ v_T = R_c \omega(R) - R_0 \omega_0 \sin \ell \] 

Finally,

\[ R_c = R_0 \sin \ell \] 
\[ \Theta_c = \omega(R_c)R_c = v_T + \Theta_0 \sin \ell \]
• Knowing \( R_0 \) and \( \Theta_0 \) (these are the hardest parts!), we can observe at a range of \( \ell \) to map out the rotation curve

\[
\begin{array}{c}
\text{Solid body in the interior} \\
\text{Flat at } R > 2 \text{ kpc}
\end{array}
\]

**H. Electron Density in HI Regions of the ISM**

• General: Radio observations but not 21cm
  ◦ Examine attenuation of Pulsar radiation by electrons
  ◦ Measure ‘Dispersion Measure’ and distance to determine \( n_e \)

• Faraday rotation
  ◦ Consider the e\(^-\) response to a plane wave
  ◦ Treat the electrons as a fluid
  ◦ EM radiation drives a current which modifies the incident radiation field
  ◦ Ignoring a magnetic field, the frequency of the plane wave becomes

\[
\begin{align*}
\omega^2 &= \omega_p^2 + c^2 k^2 \\
\omega_p^2 &= \frac{4\pi n_e e^2}{m_e} \quad (70)
\end{align*}
\]

◦ Index of refraction: \( m \)

\[
m \equiv \frac{c}{v_p} = \left[ 1 - \frac{\omega_p^2}{\omega^2} \right]^{\frac{1}{2}} \quad (71)
\]

◦ Can measure radiation over a frequency range to determine \( \omega_p \) and then \( n_e \)

• Observe pulsars
(a) $\omega < \omega_p$
  ◦ $\text{Re}(m) = 0$
  ◦ No propagation of the radiation!
  ◦ For $n_e = 0.03 - 0.1 \text{ cm}^{-3}$, $\omega_p = 10^4 \text{Hz}$
  ◦ Why doesn’t the wave propagate?
    ▲ Debye Radius

$$4\pi n_e e^2 R_D^2 \sim kT$$  \(72\)

$$\omega_p^2 m_e R_D^2 \sim kT$$  \(73\)

$$\omega_p^2 R_D^2 \sim \frac{kT}{m_e}$$  \(74\)

▲ Let $v_T^2 = kT/m_e$

$$\frac{1}{\omega_p} \sim \frac{R_D}{v_T}$$  \(75\)

▲ If $\omega < \omega_p$, then $e^-$ can travel fast enough to ‘cancel’ out the propagating wave!

(b) $\omega > \omega_p$
  ◦ Each pulse has a wide spectrum of frequencies
  ◦ Pulse delays will occur as a function of $\omega$
  ◦ Group velocity

$$v_g = \frac{d\omega}{dk} = c \left[ 1 - \frac{\omega_p^2}{\omega^2} \right]^{\frac{1}{2}}$$  \(76\)

$$\approx c \left( 1 - \frac{1}{2} \frac{\omega_p^2}{\omega^2} \right)$$  \(77\)

▲ Pulse arrival time

▲ Lower $\omega \Rightarrow$ longer time

$$\mathcal{T}(\omega) = \int_0^L \frac{ds}{v_g(\omega)} \approx \frac{L}{c} + \frac{1}{2c} \int_0^L ds \frac{\omega_p^2}{\omega^2}$$  \(78\)

▲ Take measurements at a wide range of $\omega$

▲ Evaluate

$$\frac{\mathcal{T}(\omega)}{d(1/\omega^2)} = \frac{1}{2} \int_0^L ds \frac{\omega_p^2}{c} = \frac{1}{2} \left( \frac{4\pi e^2}{m_e c} \right) \int_0^L n_e ds$$  \(79\)

▲ Define DM, the Dispersion measure

$$\text{DM} = \int n_e ds \quad \text{(pc cm}^{-3})$$  \(80\)

$$\frac{\mathcal{T}(\omega)}{d(1/\omega^2)} = 1.63 \times 10^{17} \text{DM}$$  \(81\)
Difficult part

- Measure $L$ for the pulsar
- Use 21cm + spiral arms

Answer

$$<n_e> = 0.03 \text{ cm}^{-3}$$ (82)

- Compare with Spitzer’s estimate
- Assumed all e⁻’s came from C

$$n_e = n_C = \frac{C}{H} n_H$$ (83)

$$= 10^{-4} n_H = 0.0001 \text{ cm}^{-3}$$ (84)

Puzzle

- What leads to the extra source of electrons?
- Is it related to ‘missing’ heating?

I. Extragalactic Rotation Curves

- As gas falls into a potential well (e.g. dark matter halo), it may dissipate and settle into a rotating disk
  - Assume non-negligible angular momentum
  - For galaxies, this angular momentum is believed to result from ‘tidal torques’

- HI gas – Excellent tracer of the potential well
  - Generally extends far beyond the detection limit of stars
  - 21cm telescope allow for excellent spectral resolution (kinematics)

Challenge: Spatial resolution

- Either study nearby galaxies
- Or build bigger telescopes
- Or use interferometric techniques
- Or use optical/molecular lines

First studies

Fig. 1. Rotation curves for the galaxies M 31, M 101, and M 81 are shown as solid lines. The rotation curve for our galaxy is included for comparative purposes only. It is shown as a dashed line.

Table 1. Properties of galaxies

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>Type</th>
<th>Distance (Mpc)</th>
<th>Incl. (°)</th>
<th>Semi major axis*(kpc)</th>
<th>Rot. curve limiting radius (kpc)</th>
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<tbody>
<tr>
<td>M 31</td>
<td>Sb</td>
<td>0.69</td>
<td>77</td>
<td>19.8</td>
<td>29.0</td>
</tr>
<tr>
<td>M 81</td>
<td>Sab</td>
<td>3.25</td>
<td>55</td>
<td>16.5</td>
<td>31.0</td>
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<tr>
<td>M 101</td>
<td>Scd</td>
<td>6.9</td>
<td>22</td>
<td>28.1</td>
<td>26.5</td>
</tr>
<tr>
<td>Galaxy</td>
<td>(Sbc)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>


Results
(a) The HI gas is extended far beyond the Holberg Radius
(b) The rotation curve flattens at large radii
(c) The surface density of gas decreases with radii
   (power-law or even exponential)

Implications
(a) HI gas exists beyond where stars form
(b) Dark matter!

- Current observational research
  - Large 21cm surveys: HIPASS, ALFALA, THINGS
  - Example (Walter et al. 2008)

![NGC 628](image)

Fig. 8.— NGC 628. Top left: integrated HI map (moment 0). Greyscale range from 0–218 Jy km s$^{-1}$. Top right: Optical image from the digitized sky survey (DSS). Bottom left: Velocity field (moment 1). Black contours (lighter greyscale) indicate approaching emission, white contours (darker greyscale) receding emission. The thick black contour is the systemic velocity ($v_{\text{sys}}=659.1$ km s$^{-1}$), the iso–velocity contours are spaced by $\Delta v=12.5$ km s$^{-1}$. Bottom right: Velocity dispersion map (moment 2). Contours are plotted at 5, 10 and 20 km s$^{-1}$. 
Fig. 55.— ISO and NFW rotation curve fits for NGC 7793 using the entire observed extent. Lines and symbols as in Fig. 25.

- Open questions
  - How does (not) star-formation trace HI gas?
  - How does the rotation curve behave in the very inner regions of low surface brightness galaxies?

J. HI Mass Function

- Concept
  - Mass function of galaxies in HI gas
- Number of objects per solar mass interval per comoving Mpc$^3$
- Akin to a luminosity function for galaxies

- Observations
  - Large sky surveys in 21cm
  - Aracebo, Parkes telescopes
  - Measure position, total flux, and velocity

- Analysis
  - Convert velocity to distance ($v = H_0 r$)
  - Convert 21cm flux to HI mass
  - Assess the survey volume and calculate number densities

- Example: Zwaan et al. 2005 (HIPASS)

Figure 2. Top: H$^1$ masses of HICAT detections as a function of their redshift. Points above the solid line are integrated flux-limited at $S_{int} = 9.4$ Jy km s$^{-1}$ (Zwaan et al. 2004). Only points that are used in the HIMF calculation ($C > 0.5$) are shown. Bottom: redshift distribution of HICAT galaxies. The histogram shows the measured distribution and the dashed line is the predicted distribution based on the selection function calculated from the maximum likelihood method.
K. Reionization

- Definition: Epoch when the ionizing radiation from quasars+galaxies ionized the majority of baryons in the Universe
  - Research area of strong, current interest
  - What are the exact sources?
  - How are they distributed?

- Challenges
  - Very few (if any) emitters to study
  - Very distant universe $\Rightarrow$ radiation is highly redshifted + dimmed
  - Very simple universe prior to reionization
    - Neutral hydrogen
    - Neutral helium
    - Not much else
  - A solution: Probe the hydrogen gas via 21cm emission

- Spin temperature at Reionization

\[ \alpha = -1.37 \]
\[ \log M_{\text{HI}}^* = 9.80 \]
\[ \theta^* = 0.0060 \]
Three processes will determine $T_S$
(a) Absorption of CMB photons (including stimulated emission)
(b) Spin-exchange collisions with other hydrogen atoms, free electrons and protons
(c) Scattering of ambient Ly$\alpha$ photons (the “Wouthuysen-Field” effect)

The third process may be neglected at very early times, when there is no source of Ly$\alpha$ photons.

- Level populations ($0=$ground, $1=$excited)

\[
\frac{dn_0}{dt} = -n_0 (C_{01} + B_{01}I_{\nu}^{\text{CMB}}) + n_1 (C_{10} + A_{10} + B_{10}I_{\nu}^{\text{CMB}}) \tag{85}
\]

- Definitions

$A_{10}$ is the spontaneous emission coefficient
$I_{\nu}^{\text{CMB}} = 2kT_{\text{CMB}}/\lambda^2_{10}$ is the intensity of the CMB at 21cm
$B_{10}, B_{01}$ are the Einstein coefficients

\[
B_{10} = \frac{\lambda^3_{10}}{2\hbar c}A_{10} \tag{86}
\]
\[
B_{01} = \frac{g_1}{g_0}B_{10} = 3B_{10} \tag{87}
\]

$C_{10}$ is the de-excitation rate (per atom) from collisions

- Rewriting

\[
B_{10}I_{\nu}^{\text{CMB}} = \frac{A_{10}T_{\text{CMB}}}{T_s} \tag{88}
\]

- Detailed balance in thermal equilibrium tells us

\[
n_0C_{01} = n_1C_{10} \tag{89}
\]
\[
\frac{C_{01}}{C_{10}} = \frac{n_1}{n_0} = \frac{g_1}{g_0} \exp (-T_s/T_K) \approx 3 \left(1 - T_s/T_K\right) \tag{90}
\]

$T_K$ is the gas kinetic temperature

- In steady state, it is set by adiabatic expansion

In steady state, $dn_0/dt = 0$, giving

\[
\frac{n_1}{n_0} = \frac{3C_{10} \left(1 - T_s/T_K\right) + 3A_{10}T_{\text{CMB}}/T_s}{C_{10} + A_{10} \left(1 + T_{\text{CMB}}/T_s\right)} \tag{91}
\]

- Solving for $T_S$

Solving for $T_S$ with $n_1/n_0 \approx 3(1 - T_s/T_S)$

\[
T_S = \frac{T_{\text{CMB}} + yT_K + T_s}{1 + y} \tag{92}
\]

$T_S$ is the weighted mean between $T_K$ and $T_{\text{CMB}}$
The collisional coupling coefficient $y$ is the sum of three terms,

$$y \equiv \frac{T_s}{A_{10}T_K} C_{10} = \frac{T_s}{A_{10}T_K} (C_H + C_e + C_p),$$  \hspace{1cm} (93)

where $C_H$, $C_e$, and $C_p$ are the de-excitation rates of the triplet due to collisions with neutral atoms, electrons, and protons.

The H-H collision term can be written as

$$C_H = n_H \kappa_{10}, \quad \kappa_{10} = 3.1 \times 10^{-11} T_K^{0.357} \exp(-32/T_K) \text{ cm}^3 \text{ s}^{-1}$$  \hspace{1cm} (94)

This is valid in the range $10 < T_K < 10^3 K$

**Diagnosing the gas during Reionization with 21cm**

- Key:
  - The CMB drives the temperature of the gas toward $T_{CMB}$
  - Need the gas to have $yT_K > T_{CMB}$ or lots of collisions to have $T_S \neq T_{CMB}$

$$C_H > A_{10} \frac{T_{CMB}}{T_s}$$  \hspace{1cm} (95)

- Consider the universe at $z = 100$
  - Characteristics of the gas
    - $T_K = 168K$
    - $n_H = 0.19 \text{ cm}^{-3}$
    - $yT_K = 730K \gg T_{CMB} = 275K$

Therefore, before the Reionization and the appearance of the first emitting objects, when the IGM is cooling adiabatically faster than the CMB, spin-exchange collisions between hydrogen atoms couple $T_S$ to the kinetic temperature $T_K$ of the cold gas.

Cosmic hydrogen can be observed in absorption (since $T_K < T_{CMB}$) against the CMB!

**Current+Future experiments**

- LOFAR (www.lofar.org)
- MWS
- Simple dipole experiments
Figure 1: (a) IGM temperature evolution if only adiabatic cooling and Compton heating are involved. The spin temperature $T_s$ includes only collisional coupling. (b) Differential brightness temperature against the CMB for $T_s$ shown in panel a. From Furlanetto, Oh, & Briggs 2006, PhR, 433, 181.