

V. Diagnostics of HII Regions

A. Motivations

- Primary means of studying the metallicity, SFR, density, extinction, temperature, etc. in star-forming galaxies
- For faint galaxies, emission lines may be the only aspects (beyond photometry) that are observable

B. Primer on Saha Equation and Collisional Strengths (see RadProc notes)

C. Hydrogenic Recombination Lines

- The relative strengths of the Balmer decrement ($H\alpha/H\beta/H\gamma$) can be calculated theoretically
 - ◇ The line ratios are nearly independent of T and n_e
 - ◇ Therefore, they are useful for inferring reddening because they are at widely separated wavelengths
 - ◇ Furthermore, the flux of $H\beta$ is useful for estimating other diagnostics of the HII Region
- Setup
 - ◇ Let $n_{n'\ell}$ equal the density of atoms in the $n'\ell'$ level
 - ◇ The statistical weights of these levels are

$$g_\ell = 2\ell + 1 \quad (1)$$

$$g_n = \sum_{\ell=0}^{n-1} (2\ell + 1) = n^2 \quad (2)$$

- ◇ Assume $\varepsilon_{n\ell}$ is independent of ℓ
 - ◇ Selection rule: $\Delta\ell = \pm 1$
- Line emissivity

$$j_{n'n} = \frac{1}{4\pi} \sum_{\ell'=0}^{n'-1} \sum_{\ell=\ell'\pm 1} n_{n'\ell'} A_{n'\ell'n\ell} h\nu_{n'n'} \quad (3)$$

- ◇ $A_{n'\ell'n\ell}$ is the spontaneous emission rate

$$A_{n'\ell'n\ell} = 4\pi^2 \nu_{n'n'}^3 \left[\frac{8\pi\alpha a_0^2}{Z^2 3c^2} \right] \frac{\max(\ell, \ell')}{2\ell' + 1} |P(n'\ell', n\ell)|^2 \quad (4)$$

$$P = \int_0^\infty R_{n'\ell'}(r) r R_{n\ell}(r) dr \quad (5)$$

- ◇ $R = r\Psi$ is the normalized radial function

- ◇ See Condon & Shortley or my AY204b notes for more
- Oscillator strength $f_{nl'n'l'}$

$$A_{n'l'n'l} = K_K \varepsilon_{nn'}^2 \frac{2\ell + 1}{2\ell' + 1} f_{nl'n'l} \quad (6)$$

$$\varepsilon_{nn'} = \frac{1}{n^2} - \frac{1}{n'^2} \quad (7)$$

$$K_K = \frac{1}{2} \frac{\alpha^5 m c^2}{\hbar} = 8.03 \times 10^9 \text{ s}^{-1} \quad (8)$$

- ◇ The f -values are often tabulated
- ◇ Or empirically determined
- ◇ A values are also tabulated in Wiese et al., NBS 1966
- Total decay rate from state $n\ell$

$$A_{n\ell} = \sum_{n''=n_0}^{n-1} \sum_{\ell''=\ell\pm 1} A_{n\ell n''\ell''} \quad (9)$$

- (a) Case A ($n_0 = 1$): This presumes all line transitions produced in the HII region escape. It is a good assumption for low mass HII regions but these are rare or, at the least, difficult to study.
- (b) Case B ($n_0 = 2$): Assume all Lyman lines higher than Ly α are trapped and ultimately converted to Ly α and other lines.
 - ◇ Reality generally lies between the two cases
 - ◇ Lyman line cross-section (center of the line; we will derive this later)

$$\sigma(\text{Lyn}) = \frac{3\lambda_{n1}^3}{8\pi} \left(\frac{m_H}{2\pi kT} \right)^{\frac{1}{2}} A_{nP,1S} \quad (10)$$

- ▲ At $T \approx 10000 \text{ K}$, $\tau(\text{Ly}\alpha) \approx 10^4$
- ▲ Similarly, $\tau(\text{Ly}\beta) \approx 10^3$, $\tau(L8) \approx 10^2$, $\tau(L18) \approx 10$
- ▲ The majority of interactions are scatterings
- ▲ Occasionally, however, the photon is converted to H α , H β , etc.
- Cascade Matrix

- ◇ Means of calculating the population of various levels
- ◇ Consider steady state conditions (i.e. detailed balancing)

- ▲ Ignoring g_ℓ

$$n_e n_p \alpha_{n\ell}(T) + \sum_{n'>n; \ell'=\ell\pm 1} n_{n'\ell'} A_{n'\ell'n\ell} = n_{n\ell} A_{n\ell} \quad (11)$$

- ▲ Averaging over ℓ

$$n_e n_p \alpha_n + \sum_{n'=n+1}^{\infty} n_{n'} A_{n'n} = n_n A_n \quad (12)$$

▲ Definitions

$$\alpha_n = \sum_{\ell} \alpha_{n\ell} \quad (13)$$

$$A_{n'n} = \sum_{\ell} \sum_{\ell'=\ell\pm 1} \frac{g_{\ell}}{(n')^2} A_{n'\ell'n\ell} \quad (14)$$

$$A_n = \sum_{n''=n_0}^{n-1} A_{nn''} \quad (15)$$

▲ This assumes the population of ℓ -levels is proportional to g_{ℓ}

▲ This is a good approximation for high n and ℓ where both e^- and p collisions give $n\ell \rightarrow n, \ell \pm 1$

- ◇ Define $P_{n'n}$ = probability that every entry into n' is followed by a direct transition to $n < n'$

$$P_{n'n} = \frac{A_{n'n}}{A_n} \quad (\text{branching ratio}) \quad (16)$$

- ◇ Define $C_{n'n}$ = probability that entry into n' is followed by a transition to $n < n'$ by any route.

$$C_{n,n} = 1 \quad (17)$$

$$C_{n+1,n} = P_{n+1,n} \quad (18)$$

$$C_{n+2,n} = P_{n+2,n+1}C_{n+1,n} + P_{n+2,n} \quad (19)$$

$$\vdots \quad (20)$$

- ◇ Altogether

$$C_{n'n} = \sum_{p=n'-1}^n P_{n'p} C_{pn} \quad (21)$$

- ◇ Our equation for detailed balance becomes

$$n_n A_n = n_e n_p \sum_{n'=n}^{\infty} \alpha_{n'} C_{n'n} \quad (22)$$

▲ One generally express the solution in terms of the ‘departure coefficient’

$$b_n \equiv \frac{n_n}{n_n^*} \quad (23)$$

- n_n^* is given by the Boltzman equation (i.e. thermodynamic equilibrium)
- Using the Boltzman-Saha equation

$$b_n = \left(\frac{2\pi m k T}{h^3} \right)^{\frac{3}{2}} \frac{e^{-I_n/kT}}{n^2 A_n} \sum_{n'=n}^{\infty} \alpha_{n'} C_{n'n} \quad (24)$$

- Note, there is no density dependence in the above equation
- ◊ An approximate solution to the Cascade matrix comes from Seaton
 - ▲ He noted $C_{gn} \rightarrow C_{\infty n}$ as $g \rightarrow \infty$
 - ▲ He replaced the higher order terms of the sum by an integral

$$\sum_{n'=n}^{\infty} \alpha_{n'} C_{n'n} = \sum_{n'=n}^{g-1} \alpha_{n'} C_{n'n} + \frac{1}{2} \alpha_g C_{gn} + \frac{1}{2} (C_{gn} + C_{\infty n}) \int_g^{\infty} \alpha_{n'} d_{n'} \quad (25)$$

- ◊ Given these results, the emissivity becomes

$$j_{n'n} = n_{n'} A_{n'n} h \nu_{n'n} / 4\pi \quad (26)$$

$$= b_{n'} n_{n'}^* A_{n'n} h \nu_{n'n} / 4\pi \quad (27)$$

- ◊ Table

Table 1: SEATON'S RESULTS (MODERN VALUES)

| $j_{n'n}$ | Case A | | Case B | |
|------------|------------|---------------------|------------|---------------------|
| | $T = 10^4$ | $T = 2 \times 10^4$ | $T = 10^4$ | $T = 2 \times 10^4$ |
| H α | 191 | 199 | 271 (286) | 279 |
| H β | 100 | 100 | 100 (100) | 100 |
| H γ | 58.9 | 56.9 | 50.6 (47) | 49.1 |
| H10 | 10.4 | 9.2 | 7.1 (5.4) | 6.3 |
| H20 | 1.6 | 1.4 | 1.0 (1.6) | 0.8 |

- ▲ Case B values of H α are much higher due to the trapping of Lyman lines
- ▲ Values in () are from Osterbrock (i.e. modern values)
- ▲ Note that all of the lines weaken together as $T \uparrow$ ($\alpha_n \propto T^{-1/2}$ to $T^{-3/2}$)
- More accurate Balmer decrements
 - ◊ Treat ℓ explicitly
 - ◊ Include collisional terms

$$\begin{aligned} n_e n_p \alpha_{n\ell}(T) + \sum_{n'=n+1}^{\infty} \sum_{\ell'=\ell\pm 1} A_{n'\ell'nl} n_{n'\ell'} + \sum_{\ell'=\ell\pm 1} n_e n_{n\ell'} q_{n\ell'nl} + \sum_{n'=n_0}^{\infty} \sum_{\ell'=\ell\pm 1} n_e n_{n'\ell'} q_{n'\ell'nl} \\ = n_{n\ell} \left[A_{n\ell} + \sum_{\ell'=\ell\pm 1} n_e q_{n\ell n\ell'} + \sum_{n=n_0}^{\infty} \sum_{\ell'=\ell\pm 1} n_e q_{n\ell n'\ell'} \right] \end{aligned}$$

- ▲ $\alpha_{n\ell}(T)$ are determined using Milne's relation
- ▲ Define

$$A_{n\ell} = \sum_{n''=n_0}^{n-1} \sum_{\ell''=\ell\pm 1} A_{n\ell n''\ell''} \quad (28)$$

- ▲ Collisions
 - $q_{n\ell \rightarrow n\ell \pm 1}$ collisions are due to impacts by ions (Pengalley & Seaton 1964)
 - $q_{n\ell \rightarrow n \pm 1 \ell \pm 1}$ collisions are due to electrons (Seraph 1964)
- ▲ See Osterbrock Tables 4.1 to 4.4

D. Effective Recombination Coefficient for $H\beta$: $\alpha_{H\beta}$

- Fig

- Define

$$\alpha_{H\beta} = \alpha_B P_{H\beta} \quad (29)$$

- ◇ $P_{H\beta}$ is the probability of $H\beta$ emission following recombination to higher excited levels
- ◇ Ignoring collisions

$$P_{H\beta} = \sum_{n'=4}^{\infty} \sum_{\ell'=0}^{n'-1} \alpha_{n'\ell'} \sum_{\ell'=\ell \pm 1} C_{n'\ell'n\ell} \sum_{\ell''=\ell \pm 1} \frac{A_{4\ell, 2\ell''}}{A_{4\ell}} \quad (30)$$

$$\approx 0.11 \quad (31)$$

- ◇ From Brocklehurst ($\varepsilon_{H\beta} \equiv \alpha_{H\beta} h\nu_{H\beta}$):

Table 2: $H\beta$ RECOMBINATION COEFFICIENT

| T | $\alpha_{H\beta}$ (cm^3/s) | $\varepsilon_{H\beta}$ ($\text{erg cm}^3 / \text{s}$) |
|-------------------|--|---|
| 0.5×10^4 | 5.4×10^{-14} | 2.2×10^{-25} |
| 1.0×10^4 | 3.0×10^{-14} | 1.24×10^{-25} |
| 2.0×10^4 | 1.6×10^{-14} | 6.5×10^{-26} |

- ◇ Functional fit

$$\alpha_{H\beta} h\nu_{H\beta} \approx 1.24 \times 10^{-25} \left(\frac{T}{10^4 \text{K}} \right)^{-0.8795} \quad (\text{erg cm}^3/\text{s}) \quad (32)$$

- Diagnostic of n_e or M_{HII}

- ◊ Measure $F_{H\beta}$

$$L_{H\beta} \approx F_{H\beta} 4\pi d^2 = \varepsilon_{H\beta} \frac{4}{3} \pi R^3 n_e^2 \quad (33)$$

- ◊ If R/d is known, n_e can be estimated

- ◊ If n_e is known (e.g. [OII]), then M_{HII} is inferred

$$M_{HII} \approx \frac{F_{H\beta} 4\pi d^2 m_p}{\varepsilon_{H\beta} n_e} \quad (34)$$

- Zanstra's Method for finding T_* of the ionizing star (or the ionizing luminosity, ϕ) in an HII region

(a) Observe the size of the HII region r_{HII} . Global ionization equilibrium (e.g. the Stromgren radius) implies:

$$\phi = \int_0^{r_{HII}} \alpha_B n_e^2 4\pi r^2 dr \quad (35)$$

- ◊ For an ionization bounded HII region, $r_{HII} = R_S$

- ◊ For $r_{HII} < R_S$, this is a density bounded region and ϕ is greater than the RHS of Equation 35.

(b) Observe $F_{H\beta}$

$$F_{H\beta} = \frac{1}{4\pi d^2} \left\{ h\nu_{H\beta} \int_0^{r_{HII}} \alpha_{H\beta} n_e^2 4\pi r^2 dr \right\} \quad (36)$$

- ◊ If the nebula is nearly isothermal, then $\alpha_{H\beta} = \text{constant}$ and $\alpha_B = \text{constant}$ with radius

- ◊ In this case, the integral is trivial

(c) Rearranging

$$\phi \geq \frac{4\pi d^2 F_{H\beta}}{h\nu_{H\beta}} \left(\frac{\alpha_B}{\alpha_{H\beta}} \right) \quad (37)$$

(d) For a star (blackbody)

$$\phi(T_*) = \frac{15}{\pi^4} \frac{L}{h\nu_0} f \left(\frac{h\nu_0}{kT_*} \right) \text{ s}^{-1} \quad (38)$$

- ◊ $L = 4\pi R_*^2 \sigma T_*^4$

- ◊ Note, ϕ is a double-valued function of T_*

(e) If the visible (V-band) flux is also known

$$F_V = \frac{4\pi R_*^2}{4\pi d^2} \int_0^\infty \pi B_\nu(T_*) V_\nu d\nu \quad (39)$$

$$= \frac{L}{\sigma T_*^4} \frac{1}{4\pi d^2} \int_0^\infty \pi B_\nu V_\nu d\nu \quad (40)$$

◇ V_ν is the V -band filter

(f) Rearrange the 3rd step

$$F_{H\beta} \leq \frac{\alpha_{H\beta}}{\alpha_B} \frac{h\nu_{H\beta}\phi}{4\pi d^2} = \frac{\alpha_{H\beta}}{\alpha_B} \frac{h\nu_{H\beta}}{4\pi d^2} \frac{15}{\pi^4} \frac{L}{h\nu_0} f \quad (41)$$

(g) Divide step 5 by 6

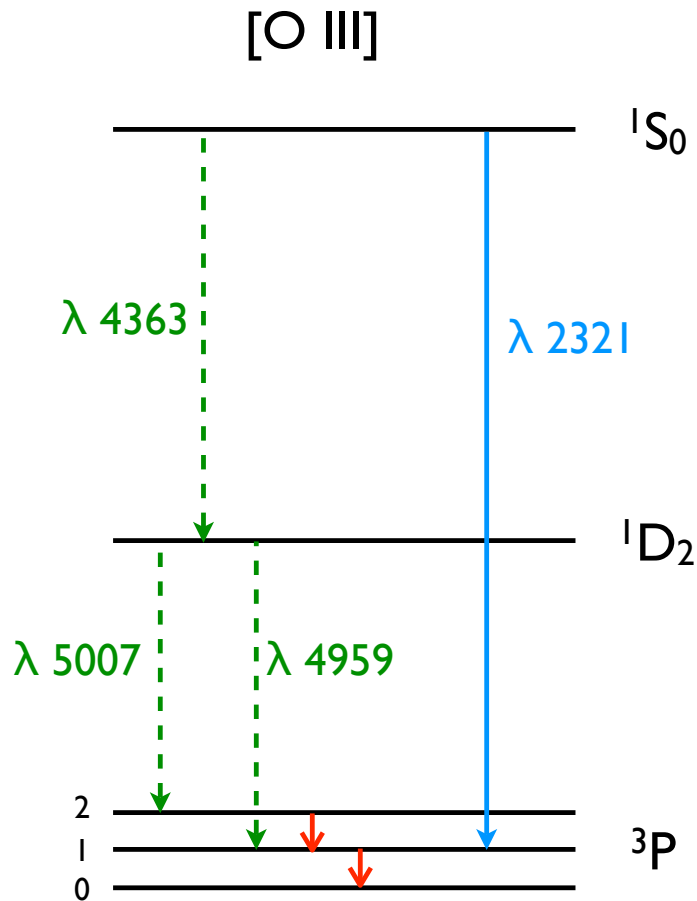
$$\frac{F_{H\beta}}{F_V} \leq \frac{\alpha_{H\beta}}{\alpha_B} h\nu_{H\beta} \frac{15}{\pi^4} \frac{f}{h\nu_0} \frac{\sigma T_*^4}{\int_0^\infty \pi B_\nu(T_*) V_\nu d\nu} \quad (42)$$

◇ Gives a known function of T and T_*

◇ Solve for T_* , independent of L and d

E. Forbidden Lines (Density, Temperature)

- These states are generally excited by collisions
 - ◇ Gas is optically thin to forbidden lines (no radiative transfer)
 - ◇ Excited levels of forbidden lines are populated by collisions \Rightarrow sensitive to n_e or T
 - ◇ Important diagnostics of density and temperature
 - ◇ e.g. [OIII], [OII]
- [OIII]
 - ◇ Configuration: $1s^2 2s^2 2p^2$
 - ◇ Two equivalent p electrons
 - ▲ Allowed states: $^1S_0; ^3P_{0,1,2}; ^1D_2$
 - ▲ Other states are excluded by Pauli
 - ◇ Relative energies: Hund's Rules (See RadProc notes)
 - (a) Higher $S \Rightarrow$ Lower energy
 3P_J has lowest energy
 - (b) Higher $L \Rightarrow$ Lower energy
 1D_2 has lower energy than 1S_0
 - (c) Higher $J \Rightarrow$ Higher energy for less than half filled
 3P_0 has lower energy than 3P_1 , etc.
 - ◇ Energy level diagram



- ◇ Wavelengths
 - $^1S_0 \rightarrow ^1D_2$ $\lambda = 4363\text{\AA}$
 - $^1D_2 \rightarrow ^3P_2$ $\lambda = 5007\text{\AA}$
 - $^1D_2 \rightarrow ^3P_1$ $\lambda = 4959\text{\AA}$
 - $^1D_2 \rightarrow ^3P_0$ $\lambda = 4931\text{\AA}$
- ◇ Transitions between any two of these states is forbidden
 - ▲ No change in parity
 - ▲ Parity of two p electrons is always even
- [OIII]: Temperature diagnostic
 - ◇ Low density limit ($n_e < 10^5 \text{ cm}^{-3}$)
 - ◇ The low lying 3P states have very nearly the same energy and are populated according to their statistical weights $g_J = (2J + 1)$
 - ◇ Keys:
 - ▲ Relative excitation of the upper levels is a function of T and not n_e
 - ▲ Every excitation is followed by spontaneous emission of a photon
 - ◇ Excitation of 1D level
 - ▲ Followed by emission of λ_{5007} or λ_{4959} photon
 - ▲ Relative probabilities $\rightarrow A_{1D,3P_2} : A_{1D,3P_1} \approx 3$

- ◇ Excitation of 1S level
 - ▲ Emission of $\lambda_{4363}(^1D_2)$ or $\lambda_{2321}(^3P_1)$ photon
 - ▲ Relative probabilities $\rightarrow A_{1S,1D} : A_{1S,3P_1} \approx 10$
 - ▲ Emission of λ_{4363} is followed by λ_{5007} or λ_{4959} photon (small contribution)
- ◇ Emission line strength (energy/s/volume)

- ▲ Consider $^1S \rightarrow ^1D$

$$j_{4363} = [\text{rate } ^1S \text{ is populated}] \times [h\nu(4363)] \times [\text{rate } ^1S \rightarrow ^1D] \quad (43)$$

- Collisional excitation rate

$$n_{3P} q_{3P,1S} = \frac{n_{3P} 8.629 \times 10^{-6} \Omega(^3P, ^1S) e^{-h\nu(^1S, ^3P)/kT}}{T^{\frac{1}{2}} g_{3P}} \quad (44)$$

- Rate $^1S \rightarrow ^1D_2$: Ratio of spontaneous emission coefficient

$$\frac{A_{1S,1D}}{A_{1S,1D} + A_{1S,3P_1}} \quad (45)$$

- Altogether

$$j_{4363} \propto \frac{n_{3P} \Omega(^3P, ^1S) e^{-h\nu(^1S, ^3P)/kT}}{T^{\frac{1}{2}} g_{3P}} \frac{\nu(^1S, ^1D) A_{1S,1D}}{A_{1S,1D} + A_{1S,3P_1}} \quad (46)$$

- ▲ Consider $^1D \rightarrow ^3P$

- Convenient to consider the two channels together

$$j_{4959} + j_{5007} \propto \frac{n_{3P} \Omega(^3P, ^1D) e^{-h\nu(^1D, ^3P)/kT}}{T^{\frac{1}{2}} g_{3P}} \bar{\nu}(^1D, ^3P) \quad (47)$$

- where

$$\bar{\nu}(^1D, ^3P) = \frac{\nu(^1D, ^3P_2) A_{1D,3P_2} + \nu(^1D, ^3P_1) A_{1D,3P_1}}{A_{1D,3P_2} + A_{1D,3P_1}} \quad (48)$$

- For higher accuracy, we should include the contribution of de-excitations from the $^1S \rightarrow ^1D$ transition

- But, these are small for $T < 30000K$

- ◇ Emission-line ratio

$$\frac{j_{4959} + j_{5007}}{j_{4363}} = \frac{\Omega(^3P, ^1D)}{\Omega(^3P, ^1S)} \frac{A_{1S,1D} + A_{1S,3P}}{A_{1S,1D}} \frac{\bar{\nu}(^3P, ^1D)}{\nu(^1S, ^1D)} e^{\Delta E/kT} \quad (49)$$

- ▲ $\Delta E = h\nu(^1S, ^1D)$
- ▲ The only physical dependence of the line ratio is temperature
- ▲ Excellent T diagnostic for low density regimes
- ▲ At higher n_e , collisional de-excitation is important and 1D is preferentially weakened.

- ▲ Can correct equation (49) by a factor

$$f = [\text{See AGN2}] \quad (50)$$

- ◇ Useful expression (not quite exact)

$$\frac{j_{4959} + j_{5007}}{j_{4363}} = \frac{6.91 \times \exp[(2.5 \times 10^4)/T]}{1 + 2.5 \times 10^{-3}(n_e/T^{1/2})} \quad (51)$$

- ◇ A similar relation holds for [NII] lines, but these lines are less luminous than oxygen
- ◇ Since [OIII] is a major coolant of HII regions, we expect higher temperatures when the O abundance is lower.
 - ▲ Indeed, T_e is observed to increase with the distance from the center of the Galaxy
 - ▲ Fig (Shaver et al. 1983)

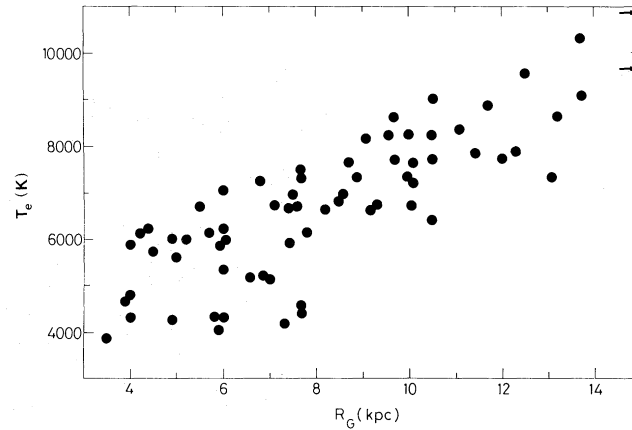


Figure 16. Electron temperatures plotted against galactocentric radius. The horizontal arrows at the upper right represent N 66 and 30 Doradus in the Magellanic Clouds.

- [OII] Configuration

- ◇ Ground state: $1s^2 2s^2 2p^3$
- ◇ Three equivalent p electrons
 - ▲ Add 1 equivalent p electron to the $2p^2$ states
 - ▲ Allowed states: $^4S_{3/2}; ^2P_{1/2, 3/2}; ^2D_{3/2, 5/2}$
- ◇ Relative energies: Hund's Rules
 - (a) Higher $S \Rightarrow$ Lower energy
 $^4S_{3/2}$ has lowest energy
 - (b) Higher $L \Rightarrow$ Lower energy
 $^2D_{3/2, 5/2}$ are second
 - (c) Half full \Rightarrow All bets are off $^2P_{1/2}$ is higher than $^3P_{3/2}$
 $^2D_{3/2}$ is higher than $^2D_{5/2}$

- ◇ Wavelengths
 - ${}^2D_{\frac{3}{2}} \rightarrow {}^4S_{\frac{3}{2}} \quad \lambda = 3726\text{\AA}$
 - ${}^2D_{\frac{5}{2}} \rightarrow {}^4S_{\frac{3}{2}} \quad \lambda = 3729\text{\AA}$
- ◇ Energy level diagram

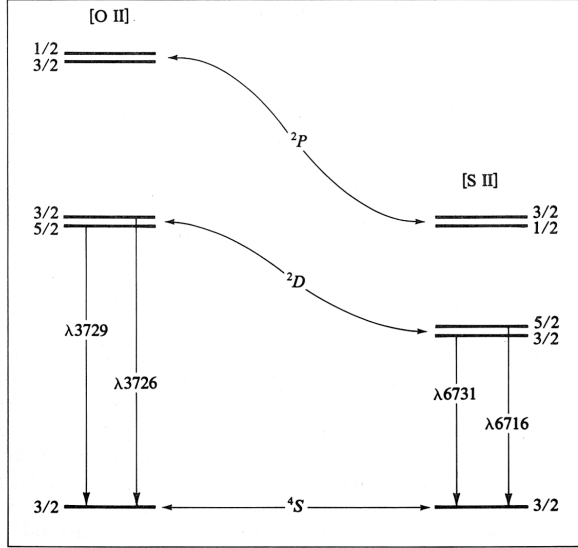


FIGURE 5.2
Energy-level diagrams of the $2p^3$ ground configuration of [O II] and $3p^3$ ground configuration of [S II].

- ◇ Transitions between any two of these states is forbidden
 - ▲ No change in parity
 - ▲ Parity of three p electrons is always odd
- [OII]: Density diagnostic
 - ◇ Pair of emission lines with very nearly the same energy
 - ▲ $\Rightarrow e^{\Delta E/kT} \approx 1$
 - ▲ No significant T dependence, only n_e
 - ◇ Low density limit: Every collisional excitation is followed by a spontaneous emission

$$\frac{j_{3729}}{j_{3726}} \propto \frac{\Omega({}^4S, {}^2D_{\frac{5}{2}}) \nu({}^4S, {}^2D_{\frac{5}{2}})}{\Omega({}^4S, {}^2D_{\frac{3}{2}}) \nu({}^4S, {}^2D_{\frac{3}{2}})} e^{-\Delta E/kT} \quad (52)$$

- ▲ $\Delta E \ll kT$ for reasonable $T \Rightarrow e^{-\Delta E/kT} \approx 1$
- ▲ $\nu({}^4S, {}^2D_{\frac{5}{2}})/\nu({}^4S, {}^2D_{\frac{3}{2}}) \approx 1$
- ▲ Useful relation:

$$\Omega(S'L'J', SLJ) = \frac{(2J+1)}{(2S+1)(2L+1)} \Omega(S'L', SL) \quad (53)$$

- ▲ Therefore

$$\Omega({}^4S, {}^2D_{\frac{5}{2}}) = \frac{3}{2} \Omega({}^4S, {}^2D_{\frac{3}{2}}) \quad (54)$$

▲ And

$$\frac{j_{3729}}{j_{3726}} \propto 1.5 \quad (n_e \ll 10^3 \text{ cm}^{-3}) \quad (55)$$

◇ High density limit: Levels are Boltzmann populated

$$\frac{j_{3729}}{j_{3726}} = \frac{g_{3729} A_{3729}}{g_{3726} A_{3726}} = 0.3 \quad (56)$$

◇ Transition between these two regimes occurs at the critical density

$$n_e = \frac{A}{q} \approx \begin{cases} 3 \times 10^3 \text{ cm}^{-3} & {}^2D_{\frac{5}{2}} \\ 1.6 \times 10^4 \text{ cm}^{-3} & {}^2D_{\frac{3}{2}} \end{cases} \quad (57)$$

◇ AGN² Fig 5.8

• Real World

◇ Often the observations of n_e and T from different forbidden lines or other methods do not always agree

▲ Could be due to incorrect assumptions in the relative abundances

▲ More likely, the problem is that there are variations in the temperature that are not considered in 'one-zone' models

◇ Piembert fluctuation method

F. Metallicity

• Definition and Notation

◇ Fraction of a gas/star/galaxy (by number or mass) that is made of metals

▲ Often assessed with a single element (e.g. O, Fe, C)

▲ And scaled against the Sun

◇ HII formalism: $12 + \log(\text{O}/\text{H})$

▲ O/H is the number density of the particles

▲ $(\text{O}/\text{H})_{\odot} \approx 10^{-3.4}$, but this keeps changing!

◇ 'Square-bracket' notation: $[\text{O}/\text{H}] = \log(\text{O}/\text{H}) - \log(\text{O}/\text{H})_{\odot}$

▲ Stellar dominated

▲ IGM, ISM

• Observations

◇ Local HII regions: Wealth of diagnostics we have been discussing and more

◇ Extragalactic: Often limited to the strongest lines observed at optical wavelengths

▲ Sensitivity of detectors and sky lines limit IR observations (improving)

▲ Things are progressing to the IR, however

◇ Lines

- ▲ Strong: [OII], H β , [OIII] λ 4959, 5007, H α , [NII]
 - ▲ Weak: [OIII] λ 4363, [SII]
- ◇ Key systematic observational error: Reddening
 - ▲ Often use H α /H β ratio and Case B assumption to estimate the reddening
 - ▲ One generally has to assume an extinction law
- T_e -Method
 - ◇ Often considered the ‘gold-standard’ of abundance estimates
 - ◇ Literature (Lots)
 - ▲ Shaver et al. 1983, MNRAS, 204, 53
 - ▲ Skillman 1998
<http://nedwww.ipac.caltech.edu/level5/Skillman/frames.html>
 - ▲ Dopita et al. 2006, ApJS, 167, 177
 - ◇ Basic procedure
 - (a) Use specific lines to estimate T_e and n_e for the HII region
 - (b) Construct an HII model based on these parameters
 - (c) Solve for the metallicity based on the line fluxes of other lines
 - (d) Iterate steps (a-c) as necessary
 - ◇ Challenges
 - ▲ Difficult to estimate T_e : [OIII] λ 4363 is very weak, especially in higher metallicity systems
 - ▲ One must still make many simplifying assumptions about the HII region(s)
 - ▲ How does one model a full galaxy of HII regions?!
 - ◇ More advanced efforts
 - ▲ Include extinction
 - ▲ Include variable T_e
 - ▲ Include a range of stellar temperatures or a synthesis of multiple HII regions
 - ▲ Fit for all lines together
- Empirical techniques
 - ◇ Basic approach
 - ▲ Measure O/H for a set of HII regions using the T_e -method
 - ▲ Search for correlations between O/H and strong emission lines
 - ▲ Derive fitting formulas and proceed
 - ◇ R_{23} method :: Historical favorite
 - ▲ Pagel et al. 1979, MNRAS, 189, 95
 - ▲ Definition

$$R_{23} = \frac{I_{[\text{OII}]\lambda 3727+3729} + I_{[\text{OIII}]\lambda 4259+\lambda 5009}}{I_{\text{H}\beta}} \quad (58)$$

▲ Empirical result (Pilyugin 2000, A& A, 362, 325)

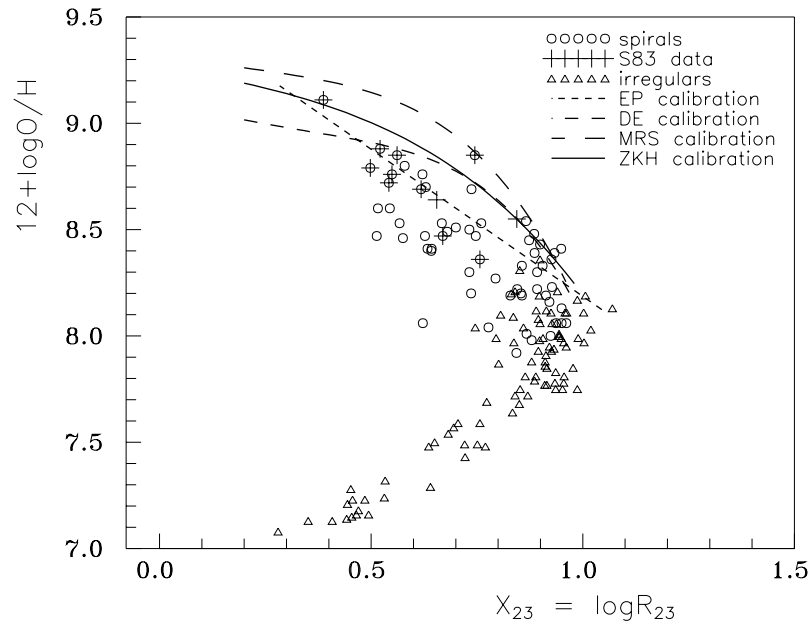


Fig. 1. The OH – $\log R_{23}$ diagram. The positions of H II regions in spiral galaxies (circles) (the H II regions in our Galaxy from Shaver et al. 1983 (S83) are indicated by pluses) and in irregular galaxies (triangles) are shown together with OH – R_{23} calibrations of different authors: Edmunds & Pagel 1984 (EP), McCall et al. 1985 (MRS), Dopita & Evans 1986 (DE), and Zaritsky et al. 1994 (ZKH).

○ Note: The results are double-valued!!!

○ One must have additional knowledge to break the degeneracy

▲ Functional form for low metallicity

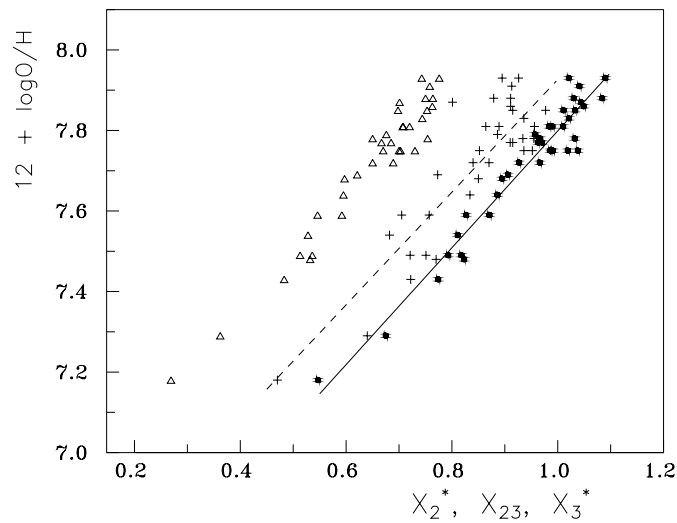


Fig. 4. The oxygen abundances $(O/H)_{T_e}$ versus observed X_{23} and

$$12 + \log(O/H)_{R23} = 6.53 + 1.40 \log R_{23} \quad (59)$$

▲ Functional form for high metallicity

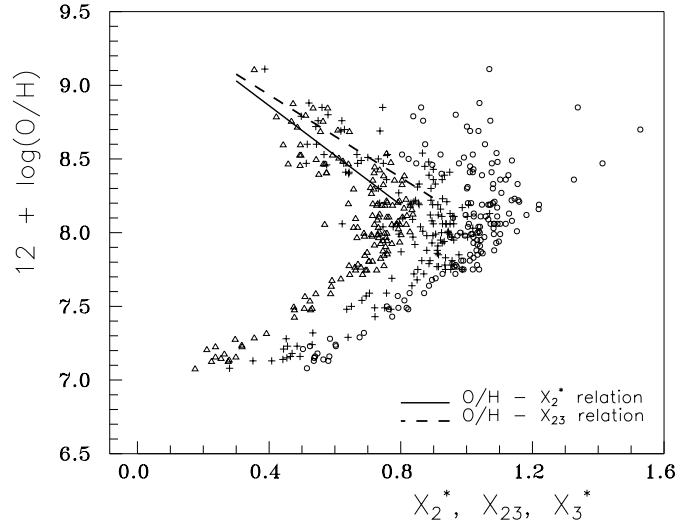


Fig. 8. The oxygen abundances $(O/H)_{T_e}$ versus observed X_{23} (plusses)

$$12 + \log(O/H)_{R23} = 9.50 - 1.40 \log R_{23} \quad (60)$$

◇ [NII], $H\alpha$, [OIII] methods

▲ Literature

- Alloin et al. 1979, A& A 78, 200
- Pettini & Pagel 2004, MNRAS, 348, L59

▲ Motivations

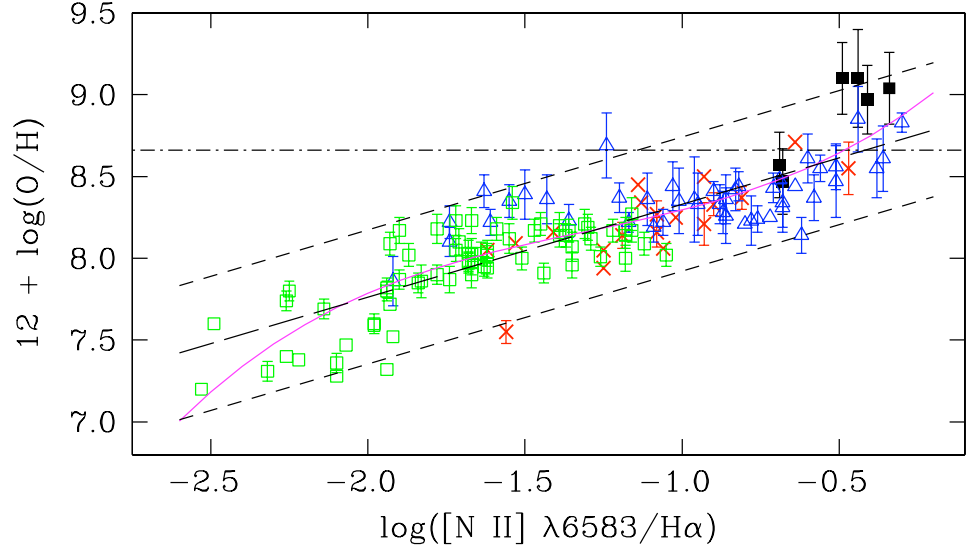
- Break the R_{23} degeneracy
- Still simpler than the T_e method
- Push to higher z

▲ N2 index

- Definition

$$N2 = \log([\text{NII}]\lambda 6583 / H\alpha) \quad (61)$$

- Empirical



○ Relation

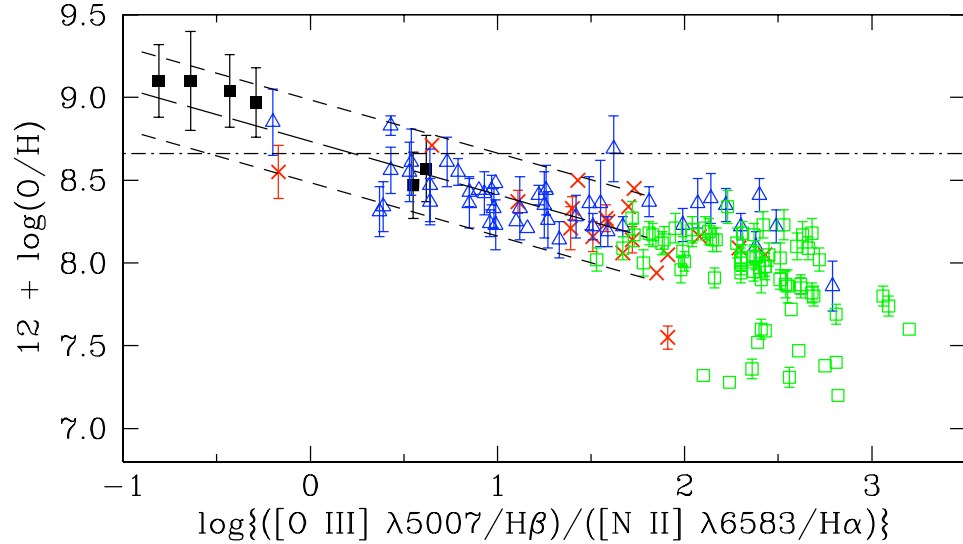
$$12 + \log(\text{O}/\text{H}) = 8.90 + 0.57 \times N2 \quad (62)$$

▲ O3N2 index

○ Definition

$$O3N2 = \log \left\{ \frac{([\text{O III}] \lambda 5007 / \text{H}\beta)}{([\text{N II}] \lambda 6583 / \text{H}\alpha)} \right\} \quad (63)$$

○ Empirical



○ Relation

$$12 + \log(\text{O}/\text{H}) = 8.73 - 0.32 \times O3N2 \quad (64)$$

G. Star Formation Rate (SFR)

- Because HII emission lines are produced by short-lived, massive stars, their integrated intensity should scale with the star-formation rate in a galaxy

- I will leave a proper treatment/discussion of this topic to other classes.
- Allow me to point you to Kennicutt 1998, ARA& A, 36, 189 as a starter