II. HII Regions (Ionization State)

A. Motivations

- Theoretical: HII regions are intamitely linked with past, current and future star-forming regions in galaxies. To build theories of star-formation it is vital to have a working knowledge of HII regions.
- Observational: The spectra of emission-line galaxies are dominated by its HII regions. To derive basic properties about the galaxy (e.g. its dynamics, metallicity, age, etc.), one must understand the physical nature of HII regions

B. Ionization balance (Hydrogen)

$$e^- + H^+ \leftrightarrow H^0 + h\nu \tag{1}$$

- Balance $\Rightarrow$ Equilibrium
  - ◇ Photoionization rate equals the recombination rate
  - ◇ This balance holds *locally*, i.e. in every cm^{-3} of the gas
- Photionization of H
  - ◇ Photoionization rate per H atom :: Γ
  - ◇ Therefore, the photoionization rate per cm^{-3} is $n_{HI}\Gamma$
  - ◇ Define

$$n_{HI}\Gamma = n_{HI} \int \frac{4\pi J_\nu}{h\nu} \sigma_\nu^{ph} \, d\nu \tag{2}$$

  - ◇ J_ν :: Mean intensity of ionizing radiation (see radproc notes)
  - ◇ σ_ν^{ph} :: Cross-section of H to ionizing photons

$$\sigma_\nu^{ph} = \frac{2^9 \pi^2}{3} \alpha a_0^2 \left(\frac{I_H}{h\nu} \right)^4 f(\eta) \tag{3}$$

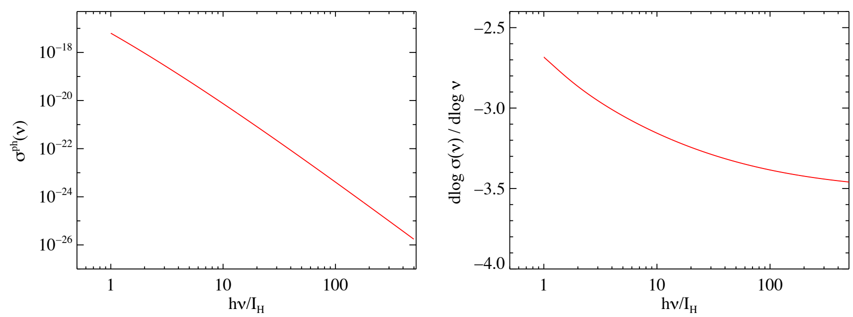
$$f(\eta) \equiv \frac{\exp\left[-4\eta \cot^{-1} \eta\right]}{1 - \exp\left[-2\pi\eta\right]} \tag{4}$$

$$\eta \equiv \left(\frac{I_H}{E_f} \right)^{\frac{1}{2}} = \left(\frac{I_H}{\hbar\omega - I_H} \right)^{\frac{1}{2}} \tag{5}$$

▲ See http://www.ucolick.org/∼xavier/AY204b/ay204b_phtrec.ps

▲ $I_H = h\nu_0$

$$\sigma_\nu^{ph} \propto \begin{cases} 0 & \hbar\omega < I_H \\ \nu^{-8/3} & \hbar\omega \approx I_H \\ \nu^{-3} & \hbar\omega \gtrsim I_H \\ \nu^{-7/2} & \hbar\omega \gg I_H \end{cases} \tag{6}$$



- ▲ $\sigma^{ph}(\nu_0) = 6 \times 10^{-18}\,\mathrm{cm}^{-2}$
- ▲ Overall,

$$\sigma^{ph}(\nu > \nu_0) \approx \sigma^{ph}(\nu_0) \left(\frac{\nu}{\nu_0}\right)^{-3} \tag{7}$$

- Recombination of H
  - ◇ Total recombination rate $n_e n_p \alpha(T)$
  - ◇ Total recombination rate per proton and per electron :: $\alpha(T)$

$$\alpha(T) = \sum_n \alpha_n(T) \quad \text{(n specifies the energy level)} \tag{8}$$

  - ◇ Recombination rate to a given energy level n :: α_n

$$\alpha_n(T) \equiv < \sigma_n^{rec}(v) v > \tag{9}$$

    - ▲ $\sigma_n^{rec}(v)$ is the cross-section for recombination to level n. It is convolved with the velocity distribution $f(v)$ of electrons.
    - ▲ The functional form of $f(v)$ is generally a Maxwellian described by the electron temperature T.
    - ▲ See the radproc notes on Milne's relation to derive:

$$\sigma_n^{rec}(v) = 2 \left(\frac{h\nu}{cm_e}\right)^2 \frac{n^2}{v^2} \sigma_n^{ph}(\nu) \tag{10}$$

C. Stromgren Radius (Simplified case)

- Crude estimate of the size for an HII region
- Concept
  - ◇ Globally balance the photoionizing rate by a star with the recombination rate of the gas surrounding it
  - ◇ Ignore radiative transfer, dust, etc.
  - ◇ Specific assumptions
    - ▲ Single O or B star
    - ▲ Embedded within a 'cloud' of HI gas (no dust)
    - ▲ Constant density

- Ionizing photon luminosity of a star
  - ◇ Total number of ionizing photons emitted per second ϕ

$$\phi = \int_{\nu_0}^{\infty} \frac{\pi B_\nu}{h\nu} 4\pi R_*^2 d\nu \quad \text{(photons/s)} \tag{11}$$

    - ▲ B_ν is the blackbody spectrum
    - ▲ $h\nu_0 = 1\,\text{Ryd} = 13.6\,\text{eV}$
  - ◇ Recall Astro 101

$$F = \int_0^{\infty} \pi B_\nu d\nu = \sigma T_*^4 \tag{12}$$

    - ▲ σ is the Stefan-Boltzman constant

$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67 \times 10^{-5} \quad (cgs) \tag{13}$$

    - ▲ Black body

$$B_\nu(T) \equiv \frac{2h\nu^3}{c^2} \frac{1}{(\mathrm{e}^{h\nu/kT} - 1)} \tag{14}$$

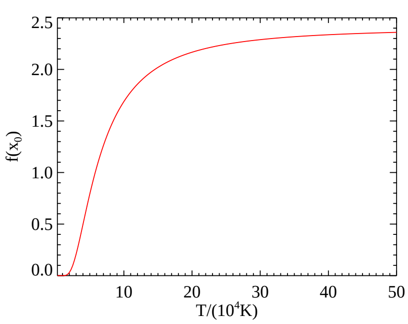
  - ◇ Similiarly

$$L = 4\pi R_*^2 F \tag{15}$$

  - ◇ For $kT_* > h\nu_0$,
    - ▲ Expect $\phi \approx L/h\nu_0$
    - ▲ Note $T_0 = h\nu_0/k \approx 150,000\,\text{K}$
  - ◇ Substituting $x \equiv h\nu/kT_*$

$$\phi = \frac{15}{\pi^4} \frac{L}{h\nu_0} \int_{x_0}^{\infty} \frac{x^2 dx}{(\mathrm{e}^x - 1)} \tag{16}$$

  - ◇ Plotting $f(x_0)$ equal to the x integral



- Recombination rate of the surrounding medium
  - ◇ Key: Ignrore recombinations to the ground-state because these will produce an ionizing photon (i.e. no net change)

◇ Recombination rate per cm^{-3}

$$n_e n_p < \sigma_{n\geq 2}\, v > \tag{17}$$

▲ Where we have defined

$$\sigma_{n\geq 2} \equiv \sum_{n=2}^{\infty} \sigma_n \tag{18}$$

▲ Note $n_p = n_e$ for pure Hydrogen

◇ Define Case B recombination coefficient

$$\alpha_B(T) \; \equiv < \sigma_{n\geq 2}(v)\, v > \tag{19}$$

▲ Integrated over a Maxwellian

▲ T is the temperature of the electrons

▲ For Hydrogen

$$\alpha_B(T) \approx \frac{2.60 \times 10^{-13}}{(T/10^4\text{K})^{0.8}} \frac{\text{cm}^3}{\text{s}} \tag{20}$$

◇ Recombination rate of a constant density cloud with radius R_S

$$n_e^2 \alpha_B(T) \frac{4}{3} \pi R_S^3 \tag{21}$$

- Stromgren Radius: R_S

◇ Adopt ionization equilibrium: Total number of recombinations equals the total number of ionizations

◇ Physics

$$\phi = \; n_e^2 \alpha_B(T) \frac{4}{3} \pi R_S^3 \qquad (\text{s}^{-1}) \tag{22}$$

◇ Rearrange

$$R_S = \left(\frac{3\phi}{4\pi n_e^2 \alpha_B(T)} \right)^{1/3} \tag{23}$$

◇ Consider real stars (TABLE)

Table 1: Stromgren Radii

Spectral type	$T_*/10^4$	$\log \phi$	$\log(R_S^3 n_e^2)$	R_S(pc) [$n_e = 10$]
B0	3.0	47.67	4.1	4.9
O9	3.2	48.24	4.6	7.3
O7	3.5	48.84	5.2	12
O5	4.8	49.67	6.1	23

- What happens at $r \gtrsim R_S$?

- ◇ Consider the mean free path of a photon λ

$$\lambda_{mfp} \sim \frac{1}{\sigma^{ph}(\nu_0)\; n_H} \tag{24}$$

- ◇ From equation 7 and with $\nu \approx \nu_0$

$$\lambda_{mfp} \sim \frac{1}{\sigma^{ph}(\nu_0) n_H} \sim 0.1 \left(\frac{n_H}{\mathrm{cm}^{-3}} \right)^{-1} \quad \mathrm{pc} \tag{25}$$

- ◇ Therefore, the transition from HII to HI is very sharp!
- ◇ For harder spectra, $\bar{\sigma} < \sigma(\nu_0)$ and λ_{mfp} is larger

D. Ionization structure:: "On the spot"

- Next simplest model is to approximate the radiative transfer and emissivity within the HII region
- Assumptions
  - ◇ Pure Hydrogen
  - ◇ Allow for partial ionization, radiative transfer
  - ◇ On-the-spot approximation
- Radiative transfer (see radproc notes)
- Ionization equilibrium
  - ◇ In steady state, ionizations balance recombinations

$$n_{HI} \int \frac{4\pi J_\nu}{h\nu} \sigma_\nu^{ph} \, d\nu = n_p n_e < \sigma_{n\geq 1}^{rec} v > \tag{26}$$

  - ◇ Define: Case A recombination coefficient

$$\alpha_A(T) = < \sigma_{n\geq 1}^{rec}(v) v >_{maxwellian} \tag{27}$$

- Equation of transfer (for ionizing photons $\nu \geq \nu_0$)

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu \tag{28}$$

  - ◇ Opacity: Ionization of Hydrogen

$$\kappa_\nu = n_{HI} \sigma_\nu^{ph} \tag{29}$$

  - ◇ Emissivity: Reemission of photons with $\nu \geq \nu_0$ due to recombinations to $n = 1$. We will refer to this as the 'diffuse' radiation field
- Express the intensity as two terms

$$I_\nu = I_{\nu *} + I_{\nu d} \tag{30}$$

- ◇ $I_{\nu *}$: Stellar radiation field
- ◇ $I_{\nu d}$: Diffuse radiation field

- Stellar radiation field

$$I_{\nu *}(r) = I_{\nu *}(R) \frac{R^2 e^{-\tau_\nu}}{r^2} \tag{31}$$

- ◇ $1/r^2$ term is standard dilution
- ◇ τ_ν is the optical depth due to the HI atoms in the HII region

$$\tau_\nu(r) = \int_0^r n_{HI}(r')\, \sigma_\nu^{ph} dr' \tag{32}$$

- Diffuse radiation field

$$\frac{dI_{\nu d}}{ds} = -n_{HI} \sigma_\nu^{ph} I_{\nu d} + j_\nu \tag{33}$$

- ◇ Recombinations give j_ν
- ◇ We will discuss an exact expression later
- ◇ Consider the total number of photons generated per cm^{-3}

$$4\pi \int_{\nu_0}^{\infty} \frac{j_\nu}{h\nu} d\nu = n_p n_e < \sigma_{n=1}\, v > \tag{34}$$

$$= n_p n_e \left[\, \alpha_A(T) - \alpha_B(T) \,\right] \tag{35}$$

- ◇ Text books frequently define

$$\alpha_1(T) \equiv \alpha_A - \alpha_B = < \sigma_{n=1}^{rec}(v) v >_{max} \tag{36}$$

- 'On-the-spot' (OTS) approximation
  - ◇ In an optically thick 'cloud' where no ionizing photon escapes, every diffuse photon generated is absorbed somewhere in the cloud

$$4\pi \int \frac{j_\nu}{h\nu} dV = 4\pi \int n_{HI} \frac{J_{\nu d}}{h\nu} \sigma_\nu^{ph} dV \tag{37}$$

  - ▲ $J_{\nu d}$ is the mean intensity of the diffuse radiation field
  - ▲ $J_{\nu d} n_{HI} \sigma_\nu^{ph}$ is the rate of absorption by HI atoms in the HII region
  - ◇ If this relation holds locally, we have the on-the-spot approximation

$$J_{\nu d} = \frac{j_\nu}{n_{HI} \sigma_\nu^{ph}} \tag{38}$$

  - ▲ This implies a very small mean free path
  - ▲ Only valid in regions with large n_{HI}
  - ▲ This is *not* the case for most of the HII region, yet the approximation still works well!

- Ionization structure assuming OTS Approx
  - ◇ Original equation

$$n_{HI}\int_{\nu_0}^{\infty}\frac{4\pi J_\nu}{h\nu}\sigma_\nu^{ph}d\nu = n_p n_e \alpha_A \tag{39}$$

  - ◇ Substitute

$$\begin{aligned} J_\nu &= J_{\nu *} + J_{\nu d} && (40)\\ &= J_{\nu *} + \frac{j_\nu}{n_{HI}\sigma_\nu^{ph}} && (41)\end{aligned}$$

  - ◇ Rearrange

$$n_{HI}\int_{\nu_0}^{\infty}\frac{4\pi J_{\nu *}}{h\nu}\sigma_\nu^{ph}d\nu = n_p n_e \alpha_B \tag{42}$$

  - ◇ Substituting for $J_{\nu *}$ from Equation 31

$$\frac{n_{HI}R^2}{r^2}\int_{\nu_0}^{\infty}\frac{4\pi J_{\nu *}(R)}{h\nu}\sigma_\nu^{ph}\mathrm{e}^{-\tau_\nu}d\nu = n_p n_e \alpha_B(T) \tag{43}$$

- Solving for the ionization structure
  - ◇ Adopt a density profile

$$n_H(r) = n_{HI}(r) + n_p(r) \tag{44}$$

  - ◇ Assume a temperature profile $T(r)$
  - ◇ Can integrate Equation 43 and the optical depth (Equation 32) to find $n_{HI}(r)$ and $n_p(r)$
- Rough calculation
  - ◇ Recall

$$\phi(R_*) = 4\pi\int_{\nu_0}^{\infty}\frac{J_{\nu *}(R)4\pi R^2}{h\nu}d\nu \tag{45}$$

  - ◇ Introduce the intensity weighted ionization cross-section

$$\bar{\sigma} \equiv \frac{\int_{\nu_0}^{\infty}\frac{J_{\nu *}\sigma_\nu^{ph}}{h\nu}d\nu}{\int_{\nu_0}^{\infty}\frac{J_{\nu *}}{h\nu}d\nu} \tag{46}$$

  - ◇ Using our on-the-spot approximation, we rewrite

$$n_{HI}\bar{\sigma}\frac{\phi}{4\pi r^2} = n_e n_p \alpha_B \tag{47}$$

◇ Ionization fraction x

$$x \equiv \frac{n_{HII}}{n_{HII} + n_{HI}} = \frac{n_{HII}}{n_H} \tag{48}$$

▲ $n_e = x n_H$ for Hydrogen gas

▲ $n_{HI} = (1-x) n_H$ for Hydrogen gas

◇ Ionization fraction in the on-the-spot approximation

$$\frac{x^2}{1-x} = \frac{\bar{\sigma}}{\alpha_B} \frac{\phi}{4\pi r^2} \frac{1}{n_H} \tag{49}$$

▲ At radii $r \ll R_S$, the gas is highly ionized: $x \approx 1$

$$\frac{1}{1-x} = \frac{\bar{\sigma}}{\alpha_B} \frac{\phi}{4\pi r^2} \frac{1}{n_H} \tag{50}$$

$$1 - x \approx \frac{\alpha_B}{\bar{\sigma}} \frac{4\pi r^2}{\phi} n_H \tag{51}$$

∘ Example: O7 star (see Table 1)

∘ Evaluate at $r = R_S/2 = 6\,\mathrm{pc}$

$$1 - x = \frac{2.6 \times 10^{-13}}{6 \times 10^{-18}} \frac{4\pi(6\mathrm{pc})^2}{10^{48.8}} \cdot 10 = 2.7 \times 10^{-4} \ll 1 \tag{52}$$

▲ HI density profile

$$n_{HI} = (1-x) n_H \propto r^2 \tag{53}$$

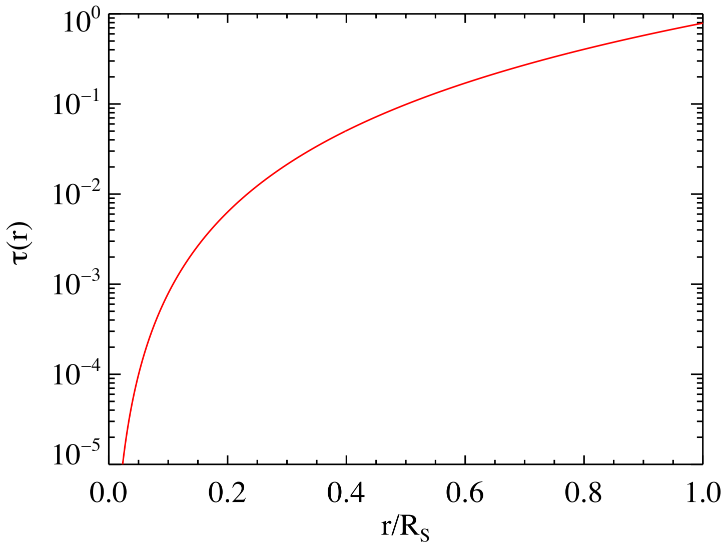
▲ If $x \approx 1$, $n_{HI} \ll n_H \Rightarrow$ small optical depth

$$\tau = \int_0^r n_{HI} \bar{\sigma} dr' \tag{54}$$

$$= n_H \int_0^r (1-x) \bar{\sigma} dr' \tag{55}$$

▲ Example: O7 star again

$$\tau = n_H \int_0^r \frac{r'^2}{(R_S/2)^2} 1.6 \times 10^{-21} dr' \tag{56}$$



▲ This plot breaks down at $r \gtrsim R_S$ where x is no longer very close to 1

▲ At $r < R_S/2$, the ionizing spectrum is largely unattenuated

$$J_{\nu *}(r) = R^2 \frac{J_{\nu *}}{r^2} \mathrm{e}^{-\tau_\nu} \approx \frac{R^2}{r^2} J_{\nu *}(R) \tag{57}$$

$$\phi(r) \approx \phi(R) \mathrm{e}^{-\tau} \approx \phi(R) \tag{58}$$

- Consider the mean free path at $R_S/2$

$$\lambda \approx \frac{1}{n_{HI}\bar{\sigma}} \approx \frac{1}{n_H(1-x)\bar{\sigma}} \sim 20\,\mathrm{pc} \tag{59}$$

  ◇ This is greater than R_S!!

  ◇ Ionizing radiation is <u>not</u> absorbed on-the-spot

  ◇ Need to include an extra term for diffuse $n \to 1$ ionizing radiation

  ◇ On the other hand, in most conditions $J_{\nu d} \ll J_{\nu *}$

- Interesting aside: Life cycle of an electron in an HII region

  ◇ Time scales

  (a) Recombination time

$$t_{rec} = \frac{1}{n_H \alpha_B} = \frac{10^5}{n_H}\ \mathrm{yrs} \tag{60}$$

  (b) Cascade time (spontaneous emission)

$$t_{cascade} = 1/A_{n'\ell' \to n=1} \approx 10^{-7}\ \mathrm{s} \tag{61}$$

  (c) Time in ground state

$$t_{n=1} = \frac{4\pi r^2}{\phi \bar{\sigma}} \approx 3\mathrm{yrs} \left(\frac{r}{R_S/2} \right)^2 \tag{62}$$

◇ Electrons spend the majority of their time as free particles

E. Ionization structure:: Helium, heavy elements and dust

- Helium
  - ◇ Physics
    - ▲ Extra opacity for $h\nu > 24.6\,\mathrm{eV}$
    - ▲ Extra source of electrons
  - ◇ He^+ region
    - ▲ Follows HII region for stars with $h\bar{\nu} > 24.6\,\mathrm{eV}$
    - ▲ Much smaller region for $h\bar{\nu} < 24.6\,\mathrm{eV}$
- Heavy elements
  - ◇ Mainly contribute electrons
  - ◇ Important for cooling in the HII region
  - ◇ Offer useful observational diagnostics of HII regions
- Dust
  - ◇ Optical depth

$$d\tau_d = -\kappa_d ds \tag{63}$$

  - ◇ Rough estimate

$$\tau_d = \kappa_d N_d \approx 2 \times 10^{-21}\,\mathrm{cm}^2 \cdot N_H \tag{64}$$

    - ▲ $N_H = n_H R_S$
    - ▲ Evaluating with Equation 23 for R_S

$$\tau_d = 0.19 \left(\frac{n_H}{\mathrm{cm}^{-3}} \right)^{1/3} T_4^{0.26} \left(\frac{\phi}{10^{48}} \right)^{1/3} \tag{65}$$

  - ◇ Therefore, dust can be very important for dense HII regions