IV. Temperature of HII Regions

A. Motivations

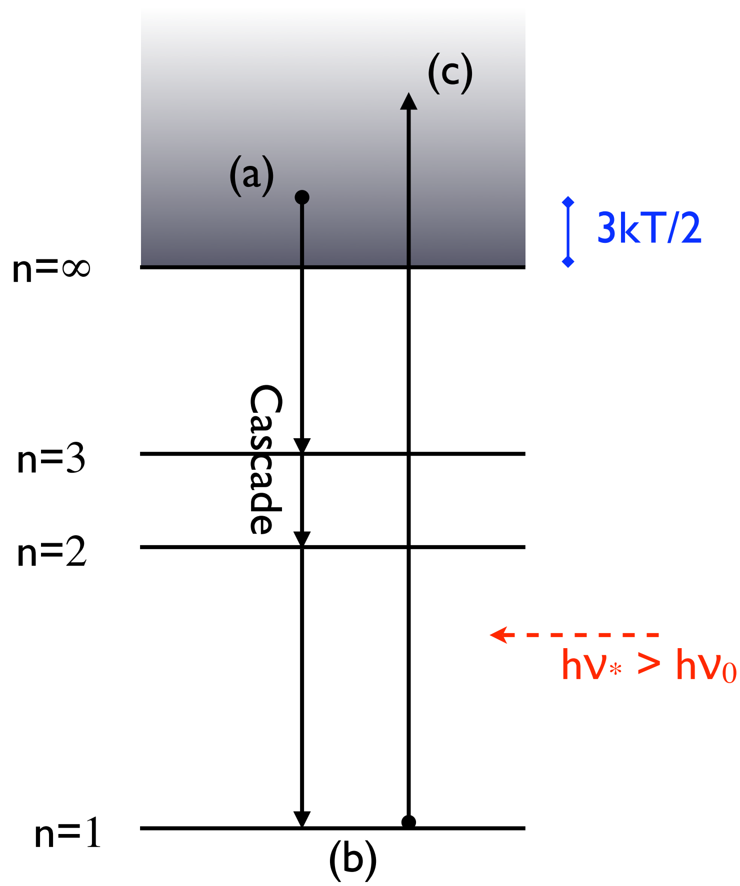
- In star-forming galaxies, most of the heating + cooling occurs within HII regions
- Heating occurs via the UV photons from O and B stars
- Cooling occurs via dust and line-emission

B. History

- 'Epochal' series of papers: Spitzer 1948, 1949ab, 1954
- Summary: Burbidge, Gould, & Pottasch 1963
- Ostriker, Chapter 3

C. Heating: Basics

- Heating occurs by photoionization of incident (stellar) radiation in the Lyman continuum
  - ◇ Photoionization only occurs if there is an H^0 atom in the region
  - ◇ Ultimately, we require recombination



- Figure

- ⋄ Because $\sigma_n^{rec}(v) \propto 1/v^2$, electrons with energy $f \cdot \frac{3}{2}kT$ are more likely to recombine ($f \approx 0.5$). This releases a cascade of photons.
- ⋄ The neutral HI atom is ionized by a hard photon $h\nu_* > h\nu_0$
- ⋄ The heating is "recombination driven" (a) − > (b) − > (c)
- ⋄ The gas is heated by

$$\Delta E = E_f - E_i = (h\nu_* - h\nu_0) - f \cdot \frac{3}{2}kT \tag{1}$$

- ⋄ If $h\nu_* < h\nu_0 + f \cdot \frac{3}{2}kT$, the gas is cooled!

- Are the electrons in kinematic equilibrium?
  - ⋄ Consider the e$^-$e$^-$ relaxation time
  - ⋄ Strong collisions occur when

$$\frac{e^2}{r} \sim \frac{1}{2}mv^2 \tag{2}$$

  - ⋄ The cross-section for such a collision is

$$\sigma_{ee} \sim \pi r^2 \sim \pi \left(\frac{2e^2}{mv^2} \right)^2 \tag{3}$$

  - ⋄ Mean free path

$$\lambda \sim \frac{1}{\sigma_{ee} n_e} \tag{4}$$

  - ⋄ Relaxation time

$$t_{ee} \sim \frac{\lambda_e}{v_e} \sim \frac{m_e^{\frac{1}{2}} (\, kT \,)^{\frac{3}{2}}}{n_e e^4} \sim 0.92 \frac{T^{\frac{3}{2}}}{n_e} \ \text{s} \tag{5}$$

  - ⋄ Therefore, the electrons form a Maxwellian distribution
- How about the protons?

$$t_{pp} \sim \left(\frac{m_p}{m_e} \right)^{\frac{1}{2}} t_{ee} \sim 43 t_{ee} \tag{6}$$

- Are the protons and electrons coupled?
  - ⋄ Equipartition time
  - ⋄ Only a fraction of energy (m_e/m_p) is exchanged during e$^-$p collisions

$$t_{ep} \sim 1836 t_{ee} \tag{7}$$

- All of these time-scales are short compared to $t_{rec} \sim 10^5$yrs

D. Heating of Hydrogen

- $G(T)$ :: Heating rate (erg cm^3/s)

- Start by ignoring the diffuse radiation

$$n_e n_p G(T) = n_e n_p \left\{ \sum_{n=1}^{\infty} < \sigma_n(v) v >_{Max} [< h\nu >_* - h\nu_0] - \sum_{n=1}^{\infty} < \sigma_n v \frac{1}{2} m v^2 >_{Max} \right\} \tag{8}$$

  - ◇ The latter term is related to $f \cdot \frac{3}{2} kT$
  - ◇ Define:

$$<E>_* = [< h\nu >_* - h\nu_0] \tag{9}$$

- Now consider the diffuse ionizing radiation
  - ◇ Let's try an "on-the-spot" assumption

$$n_e n_p G(T) = \sum_{n=2}^{\infty} < \sigma_n v >_{Max} <E>_* - \sum_{n=2}^{\infty} < \sigma_n v \frac{1}{2} m v^2 >_{Max} + \left\{ < \sigma_1 v >_{Max} <E>_d - < \sigma_1 v \frac{1}{2} m v^2 >_{Max} \right\}$$

    - ▲ But, the On-the-spot approximation implies $\{\} = 0$
    - ▲ And this ignores the fact that some electrons that cascade to the ground state are ionized by the diffuse radiation field where $< h\nu >_d \neq < h\nu >_*$!
    - ▲ We need a different approach

- A Self-consistent solution (or close to it):
  - ◇ Consider the photoionization point of view
    - ▲ Energy in:

$$n_{\rm HI} \Gamma_* <E>_* + n_{\rm HI} \Gamma_d <E>_d \tag{10}$$

    - ▲ Now we need to solve for Γ_* and Γ_d
  - ◇ Of course, we have (from our on-the-spot approx):

$$n_{\rm HI} \Gamma_* = n_e^2 \alpha_B(T) \quad \{\text{Stromgren}\} \tag{11}$$

$$n_{\rm HI} \Gamma_d = n_e^2 \alpha_1(T) \quad \{\text{on the spot}\} \tag{12}$$

  - ◇ Consider $\Gamma_* \equiv$ Photoionization by starlight/s

$$F_{\nu *} = \frac{\pi B_\nu 4\pi R_*^2}{4\pi r^2} \mathrm{e}^{-\tau_\nu} = \frac{L_\nu}{4\pi r^2} \mathrm{e}^{-\tau_\nu} \tag{13}$$

    - ▲ Assume $\tau_\nu \ll 1$
    - ▲ Local density of ionizing photons is

$$n_\gamma = \frac{F_\nu}{c h\nu} \tag{14}$$

    - ▲ Therefore

$$n_{\rm HI} \Gamma_* = \int_0^\infty n_{\rm HI} \frac{L_\nu}{4\pi r^2} \frac{1}{c} \frac{d\nu}{h\nu} \cdot c \cdot \sigma^{ph}(\nu) \tag{15}$$

- ▲ Finally, $L_\nu \rightarrow L_\nu e^{-\tau_\nu}$ with τ_ν defined as always

◇ But, we can actually estimate the heating rate without calculating Γ_* explicitly

- ▲ Replace Γ with the recombination rate

$$n_{\rm HI}\Gamma_* <E>_* + n_{\rm HI}\Gamma_d <E>_d = n_e^2\alpha_B <E>_* + n_e^2\alpha_1 <E>_d \tag{16}$$

- ▲ For a highly ionized gas $n_p = n_e$
- ▲ We infer the total heating rate

$$n_e^2 G(T) = n_e^2 \left\{ \alpha_B <E>_* + \alpha_1 <E>_d - < \sum_{n=1}^{\infty} \sigma_n(v) v \frac{1}{2} m v^2 >_{Max} \right\} \tag{17}$$

- ▲ We are left to evaluate $<E>_*, <E>_d, < \sum \sigma_n v \frac{1}{2} m v^2 >_{Max}$

• Define $<E>$

$$<E> = < h\nu - h\nu_0 > = \frac{\text{total K.E. of ejected electrons}}{\text{total number of photoionizations}} \tag{18}$$

$$<E> = \frac{n_{\rm HI} \int\limits_{\nu_0}^{\infty} (h\nu - h\nu_0) \frac{u_\nu d\nu}{h\nu} \sigma_\nu^{ph}}{n_{\rm HI} \int\limits_{\nu_0}^{\infty} \frac{u_\nu d\nu}{h\nu} \sigma_\nu^{ph}} \tag{19}$$

◇ Calculate $<E>_*$ for a star (blackbody)

$$u_\nu \propto B_\nu \propto \frac{\nu^3}{e^{h\nu/kT} - 1} \tag{20}$$

- ▲ Assume $\tau_\nu = 0$ (no attenuation of stellar radiation)
- ▲ Approximate

$$\sigma_\nu^{ph} = \sigma_0 \left(\frac{\nu}{\nu_0} \right)^{-3} \tag{21}$$

$$<E>_* = \frac{h \int\limits_{\nu_0}^{\infty} \left(1 - \frac{\nu_0}{\nu} \right) \frac{\nu^3}{e^{h\nu/kT_*} - 1} \frac{d\nu}{\nu^3}}{\int\limits_{\nu_0}^{\infty} \frac{\nu^3}{e^{h\nu/kT_*} - 1} \frac{1}{\nu} \frac{d\nu}{\nu^3}} \tag{22}$$

- ▲ Substitute variables

$$y \equiv \frac{h\nu}{kT_*} \quad \beta_* = \frac{h\nu_0}{kT_*} = \frac{158000\text{K}}{T_*} \tag{23}$$

- ▲ Express

$$<E>_* = kT_* \left\{ \frac{\int\limits_{\beta_*}^{\infty} \frac{dy}{e^y - 1}}{\int\limits_{\beta_*}^{\infty} \frac{dy}{y(e^y - 1)}} - \beta_* \right\} \equiv kT_* \psi(\beta_*) \tag{24}$$

- ▲ To evaluate the integrals:
  - ○ Expand the denominator

$$\frac{1}{e^y - 1} = \frac{e^{-y}}{1 - e^{-y}} = e^{-y}\left(1 + e^{-y} + e^{-2y} + \dots\right) \tag{25}$$

$$= \sum_{n=1}^{\infty} e^{-ny} \tag{26}$$

  - ○ Uniformly convergent series can be integrated term by term

$$\psi(\beta_*) = \frac{\sum\limits_{n=1}^{\infty}\int\limits_{\beta_*}^{\infty} e^{-ny}dy}{\sum\limits_{n=1}^{\infty}\int\limits_{\beta_*}^{\infty} e^{-ny}dy/y} - \beta_* \tag{27}$$

$$= \frac{\sum\limits_{n=1}^{\infty} \frac{1}{n}e^{-n\beta_*}}{\sum\limits_{n=1}^{\infty} E_1(n\beta_*)} - \beta_* \tag{28}$$

  - ○ Leading terms

$$\psi(\beta_*) \approx 1 - \frac{1}{\beta_*} + \dots \tag{29}$$

Table 1: Evaluation of $\psi(\beta_*)$

$T_*/10^4$	β_*	$\psi(\beta_*)$
10.5	1.5	0.686
5.27	3.0	0.808
3.16	5.0	0.868
0.8	20.0	0.95

◇ Now calculate $<E>_d$ for the diffuse heating

$$<E>_d = \frac{\int\limits_{\nu_0}^{\infty}(h\nu - h\nu_0) j^{ph}_{\nu d}\sigma^{ph}_{1\nu} d\nu}{\int\limits_{\nu_0}^{\infty} j^{ph}_{\nu d}\sigma^{ph}_{\nu 1} d\nu} \tag{30}$$

- ▲ Photon emissivity :: j^{ph}_d

$$j^{ph}_d \propto \frac{1}{\nu} e^{-h\nu/kT} \tag{31}$$

- ▲ σ^{ph}_1 is the photoionization cross-section to $n = 1$

▲ Substitute: $\beta \equiv h\nu_0/kT$

$$<E>_d = kT\beta \frac{E_3(\beta) - E_4(\beta)}{E_4(\beta)} \equiv kT\xi(\beta) \tag{32}$$

$$\xi(\beta) \approx 1 - \frac{4}{\beta} + \dots \quad (\beta > 1) \tag{33}$$

◇ Finally, the loss of energy by electron recombination

$$\sum_{n=1}^{\infty} < \sigma_n(v) v \frac{1}{2} m v^2 >_{Max} = \frac{2m_e L^{\frac{3}{2}}}{\sqrt{\pi}} \sum_{n=1} \int_0^{\infty} \sigma_n(v) v^5 \mathrm{e}^{-Lv^2} dv \tag{34}$$

▲ $L = m/2kT$

▲ $f(v) = (4/\sqrt{\pi}) L^{\frac{3}{2}} v^2 \mathrm{e}^{-Lv^2}$

▲ Define $\beta = h\nu_0/kT,\ A = (2^5/3^{3/2})\alpha^3 \pi a_B^2$

$$\sum_{n=1}^{\infty} < \sigma_n(v) v \frac{1}{2} m v^2 >_{Max} = \frac{mA}{\sqrt{\pi L^3}} \beta \sum_{n=1}^{\infty} \frac{\beta}{n^3} \left\{ 1 - \frac{\beta}{n^2} \mathrm{e}^{\beta/n^2} E_1\left(\frac{\beta}{n^2}\right) \right\} \tag{35}$$

▲ The sum is written as $\chi(\beta)$

▲ Using the E-M sum rule: (e.g. BGP63)

$$\chi(\beta) \approx \frac{1}{2}\left[0.735 + \ln\beta + \frac{1}{(3\beta)}\right] \tag{36}$$

Table 2: Evaluation of $\chi(\beta)$

$T/10^4$	β	$\chi(\beta)$
15.8	1	0.53
7.9	2	0.80
3.16	5	1.21
1.00	10.5	1.56
0.8	20	1.87
0.31	50	2.32

• Altogether now!

$$G(T) = \alpha_B <E>_* + \alpha_1 <E>_d - \sum_{n=1}^{\infty} < \sigma_n(v) v \frac{1}{2} m v^2 >_{Max} \quad (\mathrm{erg\ cm^3/s}) \tag{37}$$

$$= \alpha_B(T) kT_* \psi\left(\frac{h\nu_0}{kT_*}\right) + \alpha_1(T) kT \xi\left(\frac{h\nu_0}{kT}\right) - \frac{Am_e}{\sqrt{\pi L^3}} \beta \chi\left(\frac{h\nu_0}{kT}\right) \tag{38}$$

◇ Approximations of the recombination coefficients

$$\alpha_B \approx 2.6010^{-13} \left(\frac{T}{10^4}\right)^{-0.8} \quad \frac{\mathrm{cm}^3}{\mathrm{s}} \tag{39}$$

$$\alpha_1 \approx 1.5710^{-13} \left(\frac{T}{10^4}\right)^{-0.525} \quad \frac{\mathrm{cm}^3}{\mathrm{s}} \tag{40}$$

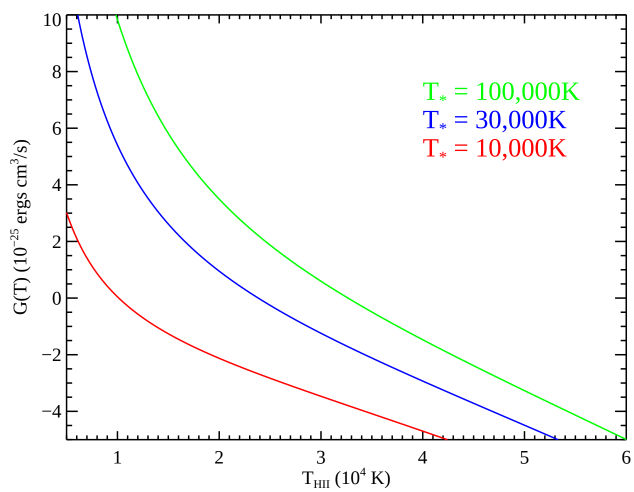
◇ And the integrals

$$\psi(\beta_*) \approx 1 - \frac{1}{\beta_*} \tag{41}$$

$$\xi(\beta) \approx 1 - 4/\beta \tag{42}$$

$$\chi(\beta) \approx \frac{1}{2}\left[0.735 \ln \beta - 1/3\beta\right] \tag{43}$$

◇ Figure



E. Cooling: Collisional Excitation of Hydrogen

- Key excitation process

$$1\,^2S \rightarrow 2\,^2P \tag{44}$$

  ◇ Followed by emission of Lyα photon with $\lambda = 1215.67$Å(10.2eV)

  ◇ Dominant coolant for gas with $T \sim 10^{4-5}$ K

- Also

$$1\,^2S \rightarrow 2\,^2S \tag{45}$$

  ◇ Emission of 2 photons

  ◇ $A_{2\,^1S,1\,^2S} = 8.23\ \mathrm{s}^{-1}$ (vs. $A_{Ly\alpha} \approx 10^9\ \mathrm{s}^{-1}$)

  ◇ $h\nu' + h\nu'' = 10.2\,\mathrm{eV}$

- Cross-section

- ◇ σ_{HI}^{ex} is not simply proportional to v^{-2}
- ◇ It is complicated, in particular, by 'wiggles' due to resonances
- ◇ Fig

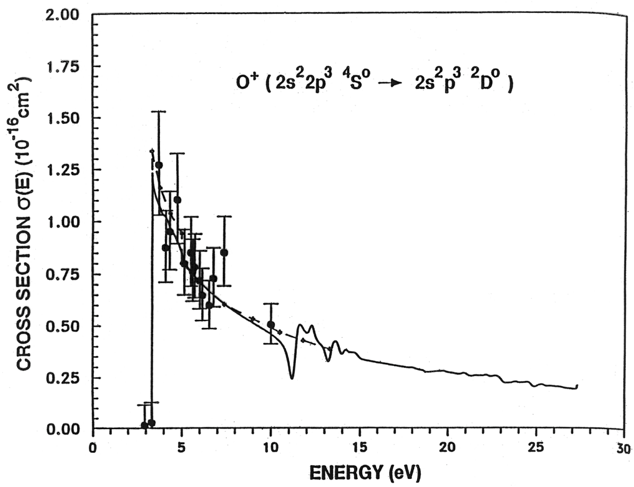


Figure 1. Cross sections for electron impact excitation of the forbidden $^4S^O \to 2s^22p^3\,{}^2D^O$ transition in O^+. ●) Merged-beams electron energy loss experiment[32]; —) 11-state R-matrix calculation[32]; ---) two-state close-coupling calculation.[178]

- Collision strengths: See Table 3.12 in Osterbrock
  - ◇ Amazingly, accurate cross-sections are <u>not</u> available for $n > 3$
- Line emissivity (see RadProc notes)

F. Cooling: Collisional Excitation of Heavy Elements

- Definitions
  - ◇ $n_k^{(m)}$ = Number density of element m in k^{th} ionization state
  - ◇ $f_{k,j}^{(m)}$ = Fraction of the ion in level j
- Cooling from element m in k^{th} ionization state from level $j \to i$

$$L_{k,ij}^{(m)} = n_e n_k^{(m)} \left[f_{ki}^{(m)} q_{ij} - f_{kj}^{(m)} q_{ji} \right] \tag{46}$$

- Total cooling rate

$$L_{line}(T) = \sum_m \sum_k \sum_{j>i} L_{k,ij}^{(m)} \tag{47}$$

- We will return to line emission from heavy elements as a means to probe the physical conditions within HII regions

G. Cooling: Brehmstrahlung (a.k.a. Free-free emission)

- For a (nearly) full quantum treatment, see
  - ◇ http://www.ucolick.org/∼xavier/AY204b/Lectures/ay204b_phtrec.ps

- ◇ A semi-classical, heuristic treatment is given in the RadProc notes

- Astrophysics
  - ◇ Dominant cooling process at high T (e.g. cluster gas)
  - ◇ Means of diagnosing the T in HII regions

- Quantum result

$$I = \frac{4}{\pi\sqrt{3}} \bar{g}_{ff}(\nu) \mathrm{e}^{-h\nu/kT} \tag{48}$$

  - ◇ $\bar{g}_{ff}$ is the Gaunt factor
  - ◇ $\bar{g}_{ff} = 1$ for most frequencies

$$\bar{g}_{ff} = \begin{cases} \sqrt{\frac{3}{\pi}} \left(\ln \frac{4kT}{h\nu} - 0.577 \right) & h\nu \ll kT \\ 1 & h\nu \gg kT \end{cases} \tag{49}$$

  - ◇ Finally,

$$j_\nu^{ff} = \sum_i n_i n_e \left(\frac{2m_e}{3\pi kT} \right)^{\frac{1}{2}} \left[\frac{32\pi^2 Z_i^2 e^6}{3m_e^2 c^3} \right] \bar{g_{ff}}(\nu) \mathrm{e}^{-h\nu/kT} \tag{50}$$

- Radio Frequency Brehmstrahlung (i.e. HII Regions)
  - ◇ The log term in j_ν introduces a weak, power-law frequency dependence

$$j_\nu = 6.51 \times 10^{-38} \left(\sum n_e n_i Z_i^2 \right) T^{-0.35} \nu^{-0.1} \tag{51}$$

    - ▲ For fully ionized H, He the sum is $\approx 1.4 n_e^2$
  - ◇ Contrast with X-ray Brehmstrahlung

$$j_\nu = 7.6 \times 10^{-31} n_e^2 T^{-\frac{1}{2}} \mathrm{e}^{-h\nu/kT} g_x \tag{52}$$

$$g_x = \begin{cases} (0.551 + 0.682x) \ln(2.25/x) & x \leq 1 \\ x^{-0.4} & x \geq 1 \end{cases} \tag{53}$$

$$x \equiv h\nu/kT \tag{54}$$

  - ◇ Total Bolometric free-free emission

$$\varepsilon_{ff} = 4\pi \int j_\nu^{ff} d\nu \tag{55}$$

$$= 2.4 \times 10^{-27} T^{\frac{1}{2}} n_e^2 \tag{56}$$

    - ▲ HII region with $T \approx 10^4$ K

$$\varepsilon_{ff} \approx 2 \times 10^{-25} n_e^2 \tag{57}$$

▲ Compare with H line emission (or even [OIII])

$$\varepsilon_H \approx 10^{-24} n_e^2 \gg \varepsilon_{ff} \tag{58}$$

- Diagnosing HII Regions
  - ◇ Specific flux from a thermal radio source at distance R

$$F_\nu = \frac{1}{4\pi R^2} \int 4\pi j_\nu dV \tag{59}$$

$$= \frac{2.5 \times 10^3}{R_{pc}^2 \nu_{GHz}^{0.1}} \int \frac{n_e^2 dV_{pc}}{T^{0.35}} \quad \text{Jy} \tag{60}$$

  - ◇ Observe the radio flux to infer:

    (a) $\int n_e^2 dV = <n_e^2>$ if T is known (or assumed)

    (b) Ionizing photon density

$$\phi = \int n_e^2 \alpha_B(T) dV \approx \alpha_B(T) \int n_e^2 dV \tag{61}$$

$$\propto \ \alpha_B(T) F_\nu R^2 T^{0.35} \tag{62}$$

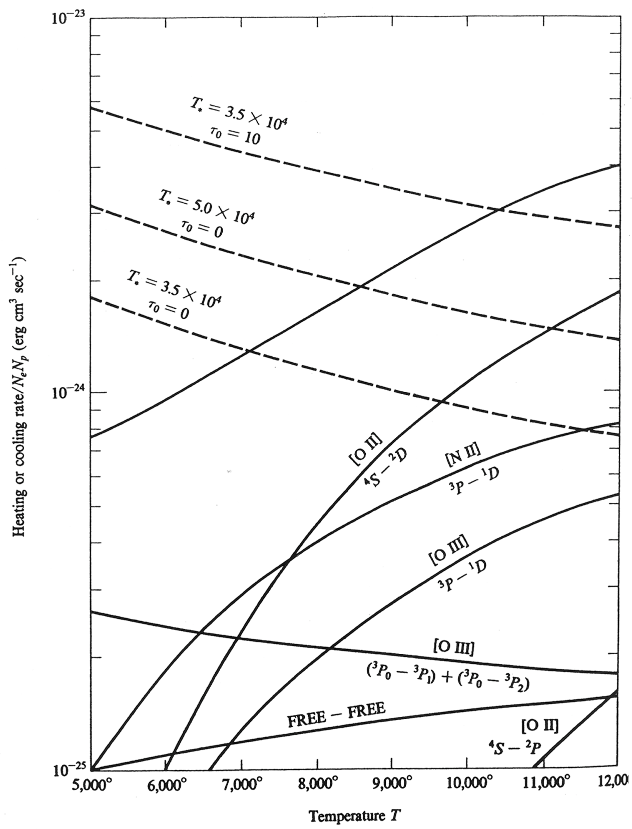
    (c) Interstellar reddening: Ratio of free-free flux to Hβ is independent of R^2 and $<n_e^2>$ and insensitive to T

H. Temperature Profile

- Equate heating with cooling

$$G(T) = L_R(T) + L_{line}(T) + L_{ff} \tag{63}$$

- Balance and solve for T
- Approximate value
  - ◇ x = 0.9, solar abundances
  - ◇ Figure
  - ◇ Heating curve is $G(T) - L_R(T)$
  - ◇ Free-free, collision excitation of HI is small
  - ◇ Line radiation
    - ▲ Peaks when $kT \approx \chi$
    - ▲ Decreases slowly for $kT > \chi$
  - ◇ Equilibrium T is where the combined curve (not labeled) crosses the heating curve



- Temperature profile
  - ◇ Need to iteratively solve for ionization balance and the T together
  - ◇ And perform Radiative Transfer!
  - ◇ CLOUDY: Gary Ferland and Associates
- CLOUDY Example
  - ◇ Input file

```
c hii_typical.in
title typical HII region
sphere
c 05 star with temperature 42000=10**4.6232 K radius 10**11.92cm
blackbody 4.6232, radius=11.92
c gas density in log of number density of all protons
hden = 2
c log of starting radius r_cm
radius 17
stop temperature 3
plot continuum range 0.1
iterations = 3
print last iteration
punch overview last file="hii_typical.ovr"
```

  - ◇ Solution

- ▲ Red curve: Solar metallicity
- ▲ Blue curve: Zero metallicity

- Discussion
  - ◇ We note $T \uparrow$ as $r \uparrow$
  - ◇ This is because $\sigma_\nu^{ph} \propto \nu^{-3}$
  - ◇ Therefore the radiation field becomes harder as it propogates out
  - ◇ This leads to more efficient heating (similiar to a hotter star)