AY230 – Homework Set #2

Due October 23, 2008

(1) Brehmstrahlung emission from HII regions [Stolen from Shu Vol I] Assume an HII region to have a uniform electron temperature T and density n_e which we would like to determine by observational means.

(a) Since the free-free emission associated with the thermal distribution of electrons occurs under conditions of LTE, show that our radiative transfer equation can be rewritten as:

$$I_\nu = I_\nu(0)\mathrm{e}^{-\tau_\nu} + B_\nu(T)\left(1 - \mathrm{e}^{-\tau_\nu}\right) \tag{1}$$

(b) For radio observations spanning $\lambda \sim$ 100cm to 1mm, show that $h\nu \ll kT$ for all likely values of T. In this case it represents a good approximation to replace $B_\nu(T)$ by its Rayleigh-Jeans limit:

$$B_\nu(T) = \frac{2\nu^2}{c^2}kT \tag{2}$$

(c) Using our definition for the brightness temperature, show that the radiative transfer equation becomes

$$T_b = T_b(0)\mathrm{e}^{-\tau_\nu} + T\left(1 - \mathrm{e}^{-\tau_\nu}\right) \tag{3}$$

(d) Focusing on free-free emission, the optical depth can be expressed as

$$\tau_\nu = \int \kappa_\nu^{ff}\, ds \tag{4}$$

where κ_ν^{ff} is the free-free opacity given by $j_\nu^{ff}/4\pi B_\nu$. Assuming a pure Hydrogen plasma, show that for small $h\nu/kT$, we have

$$\kappa_\nu^{ff} = C n_e^2 T^{-\frac{3}{2}} \nu^{-2} g_\nu^{ff} \tag{5}$$

where

$$C \equiv \left(\frac{2m_e}{3\pi k}\right)^{\frac{1}{2}} \left[\frac{4\pi e^6}{3m_e^2 ck}\right] \tag{6}$$

(e) An expression for the Gaunt factor (somewhat different from the one in the notes) is:

$$\bar{g}_\nu^{ff} = \frac{\sqrt{3}}{2\pi}\left[\ln\left(\frac{8k^3T^3}{\pi^2 e^4 m_e \nu^2}\right) - 5\gamma\right] \tag{7}$$

with $\gamma = 0.5772$. Compute $\bar{g}_\nu^{ff}$ for $\nu = 10^9$Hz and $T = 10^4$K and show that $\bar{g}_\nu^{ff}$ should not be approximated by unity here (unlike at optical wavelengths).

(f) Define the emission measure as the integral

$$\text{EM} \equiv \int n_e^2 \, ds \tag{8}$$

and show that τ_ν can be expressed as

$$\tau_\nu = (\text{EM}) C T^{-\frac{3}{2}} \nu^{-2} \bar{g}_\nu^{ff} \tag{9}$$

(g) At low frequencies $\tau_\nu \gg 1$, whereas at high frequencies $\tau_\nu \ll 1$. With no background source, show that this implies $T_b \approx T$ at low frequencies, while $T_b \approx T\tau_\nu$ at high ν.

(h) For a spherical HII region with radius R_S, show that the observed flux (measured in Janksys = 10^{-26} watts m^{-2} Hz^{-1})

$$F_\nu = \pi I_\nu \left(\frac{R_S}{r} \right)^2 \tag{10}$$

where r is the distance to the source.

(i) The size R_S can be determined if the source is angularly resolved and its distance is known. Show that F_ν is proportional to ν^2 at low frequencies and $\bar{g}_\nu^{ff}$ (i.e. $\nu^{-0.1}$) at high (radio) frequencies.

(j) Describe qualitatively how this information could be used to deduce T and EM if the spectrum on both sides of the turnover frequency ν_c (where $\tau_\nu = 1$) can be measured.

(k) Observations of an HII region in the Orion nebula [see Shu Figure P4.1] show F_ν =50Jy at $\nu = 10^2$MHz and 300Jy at $\nu = 10^4$MHz where $\nu = 10^3$MHz corresponds to the transition from low to high frequency. Compute approximate values for T and n_e assuming the region has size $R_S \approx 0.6$pc and is at a distance $r \approx 500$pc.

Quoted values in the literature are $T \approx 8000$K and $n_e \approx 2000\,\text{cm}^{-3}$. Check to see at what frequency $\tau_\nu = 1$ for your results. Surprisingly, your T value will not agree with the value of 8000 K because the low-frequency measurements have larger effective beam sizes than the high-frequency measurements. See the discussion in Osterbrock (p. 128-130) for more.

(l) Is the size $R_S = 0.6$pc consistent with our estimate of the Stromgren radius for an O5 star? Discuss.

(2) More FUN with CLOUDY: Temperature structure

(a) Repeat the calculation for the temperature profile of an HII region as done in the notes. This time consider these scenarios:

  i. Solar metallicity gas
  ii. Zero metallicity gas
  iii. H+O only (solar metallicity)
  iv. H+N only (solar metallicity)

v. H+N+O only (solar metallicity)

Plot T against r for all of these cases.

(b) Plot the ionization fraction of the various O ions as a function of radius for the Solar metallicity HII region.

(c) Using CLOUDY, determine the flux (or luminosity) of the [OIII] and [OII] forbidden lines for the Solar metallicity case.