

AY230 – Homework Set #5

Due December 4, 2008

- (1) Dust acceleration: Because of radiation pressure, a dust grain at a distance r_0 from a star with luminosity L_* will be accelerated to a terminal velocity that depends on the grain size a , ρ_s the specific density of the grain material, and Q_{pr} the radiation pressure efficiency.
 - (a) Derive an analytic expression for the terminal velocity.
 - (b) Calculate the terminal velocity for a grain radius of $0.1\mu\text{m}$, a specific density of 3 g cm^{-3} , $r_0 = 10\text{ pc}$, a radiation pressure efficiency of 1 and a luminosity of $10^4 L_\odot$.

- (2) Molecular estimate
Consider an electrically neutral medium of diatomic molecules (nuclear mass M_A and M_B) in thermal equilibrium with temperature T . [**Note:** Portions of this problem are discussed in the Solutions to Rybicky & Lightman. You are on your honor to ignore these solutions.]
 - (a) Derive an expression for the temperature of the gas where you would expect purely rotational emission lines from a substantial fraction of the molecules.
 - (b) What range of temperatures would be appropriate for the CO molecule which has rotational constant $B = 2.77\text{K}$?

- (3) CO rotational emission [Consult Tielens, Chapter 2]:
 - (a) Assuming optically thin emission in LTE, what is the expected intensity of the $J = 1 \rightarrow 0$ line for a column of 10^{15} cm^{-2} CO molecules at $T = 10\text{K}$?
 - (b) For a line width of 2 km/s , what is the corresponding peak brightness temperature?
 - (c) What is the expected optically thick peak brightness temperature for the line?
 - (d) The measured peak brightness temperatures for the two main isotopes of CO are $T_B(^{12}\text{C}^{16}\text{O}) = 12.4\text{K}$ and $T_B(^{13}\text{C}^{16}\text{O}) = 4.9\text{K}$. Assuming that $^{12}\text{C}^{16}\text{O}$ is optically thick and $^{13}\text{C}^{16}\text{O}$ is optically thin and adopting $^{12}\text{C}^{16}\text{O}/^{13}\text{C}^{16}\text{O} = 65$, what is the column density of $^{12}\text{C}^{16}\text{O}$?