# XI. Hot Diffuse Gas

## A. Motivations

- Hot diffuse gas pervades dark matter halos
- Dominant reservoir of baryons in the universe?
- Key to studying groups and galaxy clusters
- Critical to the SZ effect

## B. Ionization Equilibrium: Hydrogen

- For $T \gg 10^4$K, collisional ionization dominates

$$e^- + H^0 \rightarrow H^+ + 2e^- \tag{1}$$

  - ◇ Two body process
  - ◇ Rate for Collisional Ionization (CI)

$$n_e n_{\rm HI} < \sigma_{CI}({\rm HI}) v > \tag{2}$$

  - ◇ Express the latter term as

$$\zeta_{CI} =< \sigma_{CI}({\rm HI}) v > \tag{3}$$

$$= \int_{I_{\rm HI}}^{\infty} \sigma_{CI}({\rm HI}) v f(v) dv \tag{4}$$

  - ◇ Cross-sections are determined from theoretical calculations and fitted to experiment
    - ▲ Younger derived this 'convenient' expression

$$\sigma_{CI}(E) = \frac{1}{uI^2}\left[ A\left(1 - \frac{1}{u}\right) + B\left(1 + \frac{1}{u}\right) + C\ln u + D\frac{\ln u}{u}\right] \quad {\rm cm}^2 \tag{5}$$

$$u \equiv \frac{E}{I} \tag{6}$$

      - ○ This formulation is readily integrable with a Maxwellian
      - ○ And provides enough parameteric freedom to give good fits to experimental data
  - ◇ Integrating over a Maxwellian, we have

$$\zeta_{CI}(T) = \frac{6.69 \times 10^{-7}}{kT^{3/2}} \frac{e^{-x}}{x} \Phi(x) \tag{7}$$

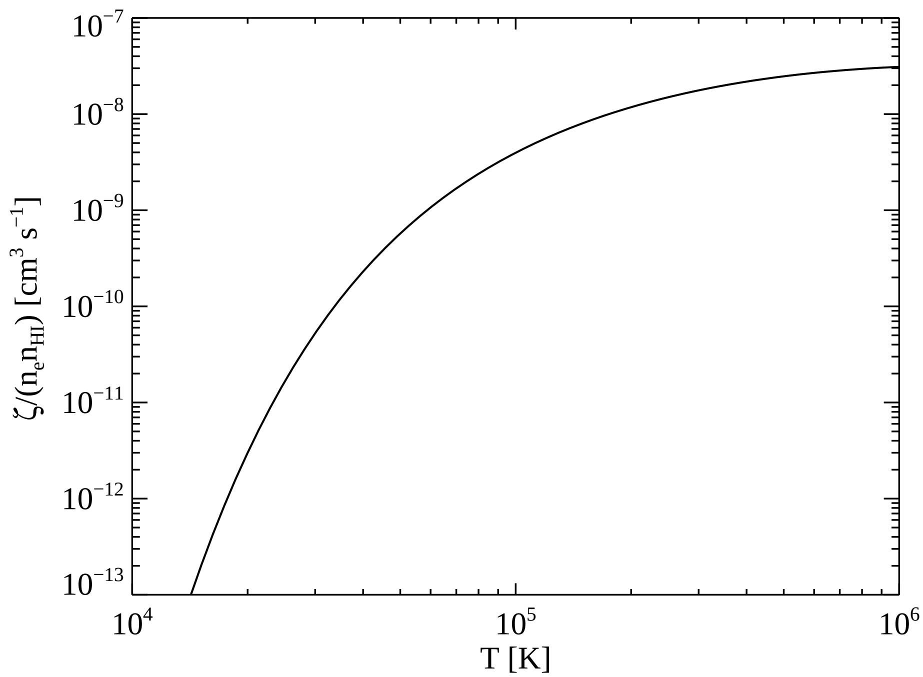
$$x = I/kT \tag{8}$$

$$\Phi(x) = [\, A + B(1+x)\,] - \left[\, C + Ax - B(2x - x^2)\,\right] e^x E_1(x) + D e^x E_2(x) \tag{9}$$

- ▲ $E_1(x)$ and $E_2(x)$ are the exponential integrals
- ◇ Verner has provided 4-parameter analytic fits to the rate of the form (good for $x < 80$)

$$\zeta_{CI} = \frac{b\,(\,1 + a\sqrt{x}\,)}{c + x} x^d \exp(-x) \tag{10}$$

- ▲ For Hydrogen, $a = 0, b = 2.91 \times 10^{-8}, c = 0.232, d = 0.39$
- ▲ Here is a plot




- Recombination
  - ◇ Also a two-body process
  - ◇ In general, the gas is sufficiently diffuse that the total recombination rate is considered

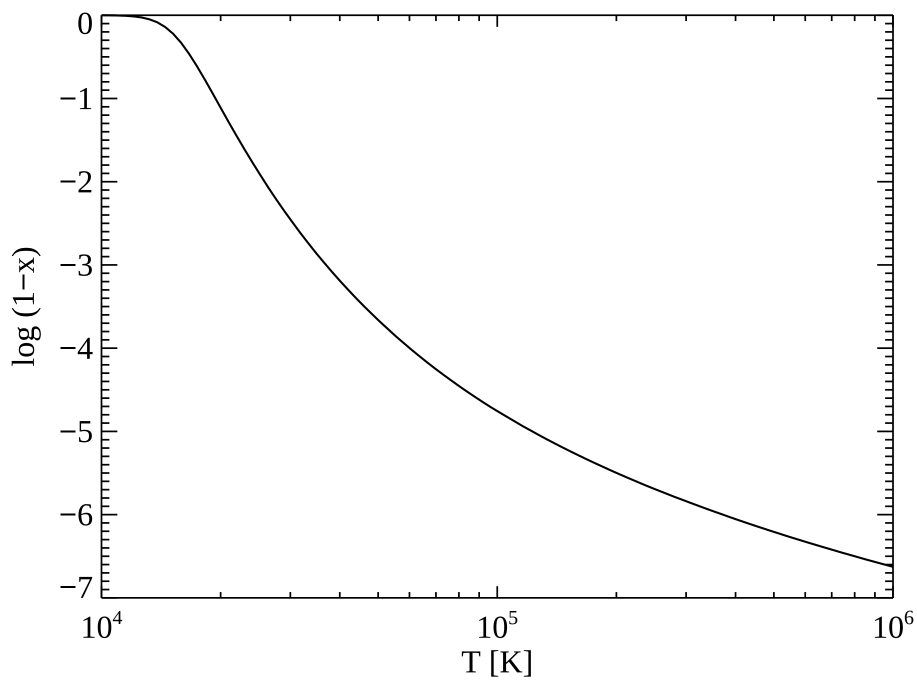
$$n_e n_{\mathrm{H}^+} \alpha(T) \tag{11}$$

- Balance the two processes
  - ◇ Both are proportional to $n_e n_{\mathrm{H}}$
  - ◇ Therefore the density dependence drops out!

$$\frac{n_{\mathrm{HI}}}{n_{\mathrm{H}^+}} = \frac{1-x}{x} = \frac{\zeta_{CI}(T)}{\alpha(T)} = f(T) \tag{12}$$

  - ◇ In a collisionally ionized gas, the ionization fraction is solely a function of temperature

◇ Here is a plot for the neutral fraction of Hydrogen $(1 - x)$




## C. Ionization Equilibrium: Metals

- Same concept, of course

$$e^- + X^i \rightarrow 2e^- + X^{i+1} \tag{13}$$

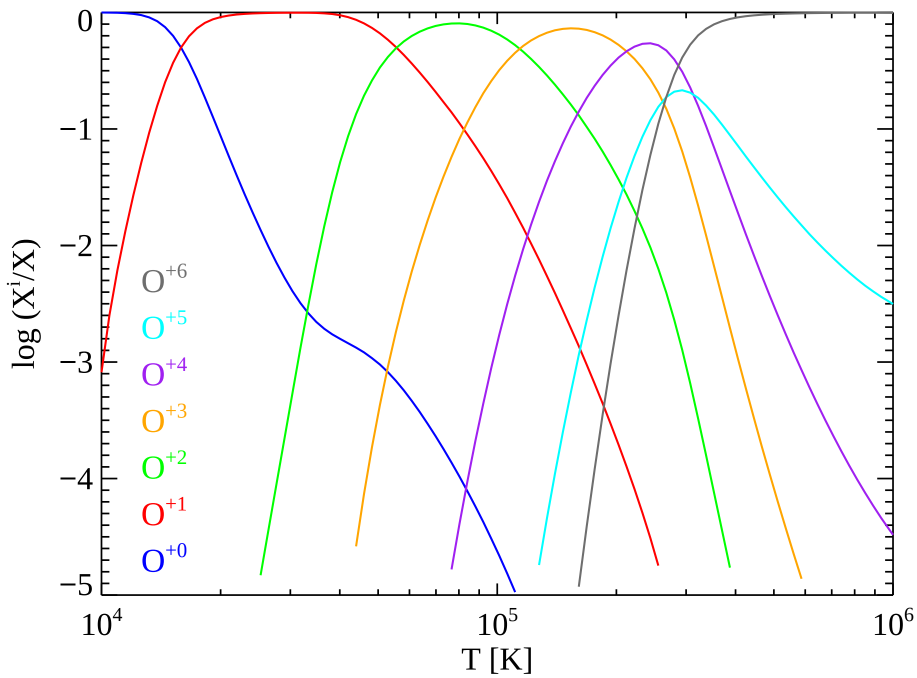
- For a pair of ions, the density dependence drops out

$$\frac{X^{i+1}}{X^i} = g(T) \tag{14}$$

- It is then straightforward to sum over all ionization states for a given element to determine the fractional dependence of any ion
  - ◇ Total

$$X = \sum_i X^i \tag{15}$$

  - ◇ Each ion tends to peak when $kT \approx I_i$
- Here are the curves for Oxygen (from Gnat & S 2007)




### D. Non-Equilibrium Conditions

### E. Heating of Diffuse Hot Gas

- Photoionizations are rare
  - ◇ For $T \gg 10^4$K, there are very few targets to hit
  - ◇ Heating via photoionization is a weak process
- Dust is probably not existent
- Heating is often external and of a kinetic form
  - ◇ Virial shocks
  - ◇ Galactic winds
  - ◇ AGN feedback (jets, etc.)
  - ◇ Bubbles!!
- Cosmic rays may play a role too

### F. Cooling: Recombinations and Collisions

- Recombination cascades yield a series of photons

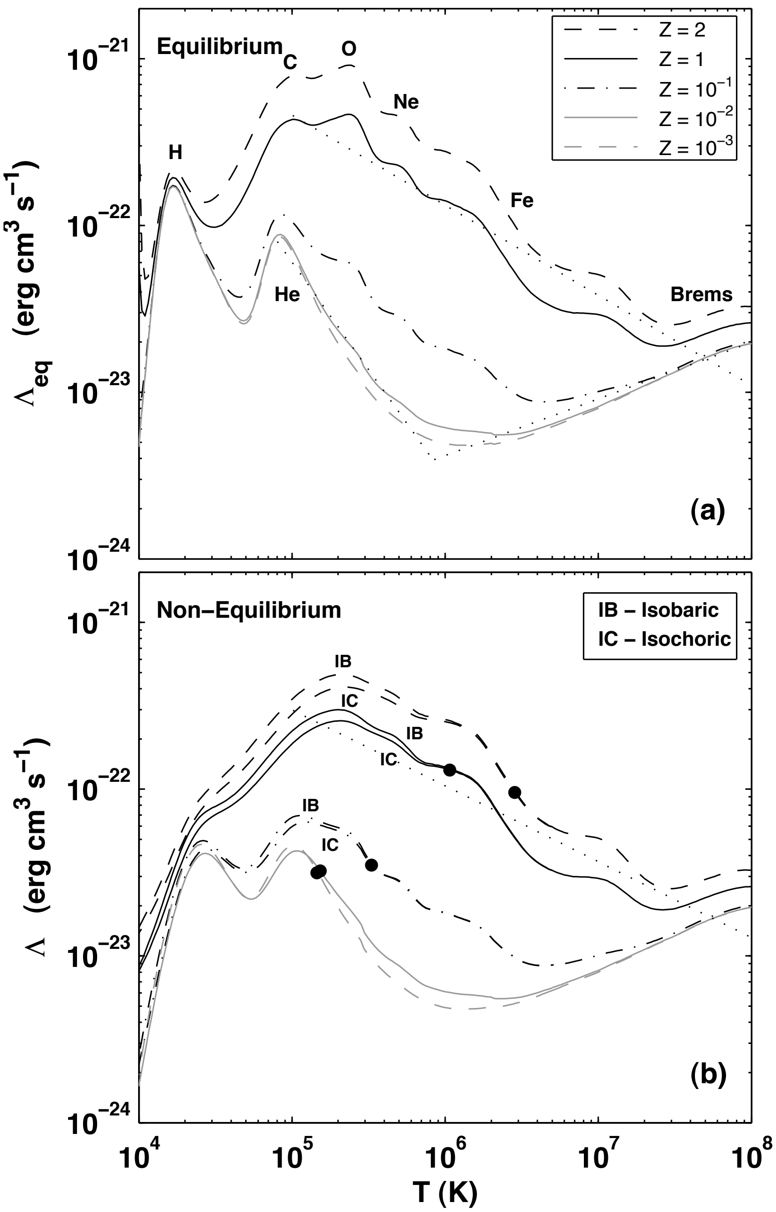
$$\mathrm{e}^- + X^{i+1} \rightarrow X^i + \gamma \tag{16}$$

  - ◇ This is generally a slow process in a hot, diffuse medium
  - ◇ Basically ignoreable

- More frequently an electron excites an ion

$$e^- + X^{i+1} \rightarrow e^- + X^{i+1,*} \rightarrow e^- + X^{i+1} + \gamma \tag{17}$$

  - ◇ A portion of the electron K.E. is transferred to the ion
  - ◇ It may then emit a photon which cools the gas
  - ◇ In the HI regions, recall cooling via 158$\mu$m emission
- Dominant cooling lines
  - ◇ UV resonance transitions from abundant ions at the given temperature
  - ◇ For $T = 10^5 - 10^7$K metal-lines dominate (e.g. CIV, OVI, NeVIII)
  - ◇ Therefore, there is a strong metallicity dependence on the cooling curve
  - ◇ The following is from Gnat & S 2007 (Fig 8)



- Approximate power-law expressions (good to 30%)
  - ◇ Solar metallicty ($Z = 1$; $T = 10^5 - 10^8$K)

$$\Lambda_{eq}^{Z=1} = 2.3 \times 10^{-19} T^{-0.54} \ \text{erg cm}^3 \text{s}^{-1} \tag{18}$$

  - ◇ 'Zero'-metallicity ($Z < 0.01$) – This includes Brehmstrahlung

$$\Lambda_{eq}^{Z=10^{-3}} = 1.7 \times 10^{-16} T^{-1.29} \quad [T = 8 \times 10^{4-5}\,\text{K}] \tag{19}$$

$$= 3.8 \times 10^{-26} T^{0.34} \quad [T = 8 \times 10^5 - 10^8\,\text{K}] \tag{20}$$

## G. Cooling: Brehmstrahlung

- At $T > 10^7$K, there are few abundant elements with a bound electron
  - ◇ Cooling via collisional excitation is severely reduced
  - ◇ Recombination cooling remains weak
  - ◇ At these temperatures, Brehmstrahlung emission dominates
- Semi-Classical treatment [following Shu]
  - ◇ Classical $\Rightarrow$ $v/c \ll Z\alpha$

$$\text{e}^- + \text{ion} \to \text{e}^- + \text{ion} + \gamma \tag{21}$$

  - ◇ Treat ion as a fixed particle
  - ◇ Stick to the small-angle scattering regime
  - ◇ Dipole moment

$$\vec{d} = -e\vec{r} \tag{22}$$

$$\ddot{\vec{d}} = -e\ddot{\vec{r}} \tag{23}$$

  - ◇ Non-relativistic equation of motion

$$|\ddot{\vec{r}}| = \frac{Ze^2}{m(b^2 + v^2t^2)} \tag{24}$$

  - ◇ Electric field (dipole approximation)

$$|\vec{E}| = \frac{e|\ddot{\vec{r}}|}{c^2 r} \sin\theta = \frac{Ze^3 \sin\theta}{mc^2 r(b^2 + v^2t^2)} \tag{25}$$

    - ▲ $\theta$ is the angle between the observer and $\ddot{\vec{r}}$
  - ◇ Fourier analysis (to gain frequency dependence)

$$|\vec{E}| = \int_{-\infty}^{\infty} \epsilon_\nu \text{e}^{-i2\pi\nu t} d\nu \tag{26}$$

$$= \int_{0}^{\infty} \left( \epsilon_\nu \text{e}^{-i2\pi\nu t} + \epsilon_\nu^* \text{e}^{i2\pi\nu t} \right) d\nu \tag{27}$$

- ⋄ $\epsilon_\nu$ is just the transform of $|\vec{E}|$

$$\epsilon_\nu = \frac{Ze^3 \sin\theta}{mc^2 r} \int_{-\infty}^{\infty} \frac{\mathrm{e}^{i2\pi\nu t}}{b^2 + v^2 t^2} dt \tag{28}$$

$$= \frac{Ze^3 \sin\theta}{mc^2 r} \left( \frac{\pi}{bv} \right) \mathrm{e}^{-2\pi\nu b/v} \tag{29}$$

- ⋄ Dipole Radiation

$$\frac{dP}{d\Omega} = \frac{|e\ddot{\vec{r}}|^2}{4\pi c^2} \sin^2\theta \tag{30}$$

$$= \frac{c|E|^2}{4\pi} \tag{31}$$

- ⋄ Derive the frequency dependence
  - ▲ Parseval's theorem of Fourier transform

$$\int_{-\infty}^{\infty} |E|^2 dt = \int_{-\infty}^{\infty} \epsilon_\nu \epsilon_\nu^* d\nu = 2 \int_{0}^{\infty} \epsilon_\nu \epsilon_\nu^* d\nu \tag{32}$$

  - ▲ $P_\nu$: The integrated flux in the frequency interval $\nu, \nu + d\nu$ for 1 electron with velocity $v$ at impact parameter $b$
  - ▲ From Equation 30

$$\frac{dP_\nu}{d\Omega} = \epsilon_\nu \epsilon_\nu^* \left( \frac{c}{2\pi} \right) \tag{33}$$

  - ▲ Therefore, integrating over $d\Omega$

$$P_\nu = \int_{0}^{\pi} \frac{c}{2\pi} \epsilon_\nu \epsilon_\nu^* 2\pi r^2 d\cos\theta \tag{34}$$

$$= \frac{4}{3} \left( \frac{Z^2 e^6}{m^2 c^3} \right) \frac{\pi^2}{(bv)^2} \mathrm{e}^{-4\pi\nu b/v} \tag{35}$$

- • Thermal Brehmstrahlung
  - ⋄ Total emission rate for all electrons in a thermal (i.e. Maxwellian distribution)

$$j_\nu^{ff} = n_i \int_{v_{min}}^{\infty} v \left[ n_e f(v) dv \right] \int_{b_{min}}^{\infty} P_\nu 2\pi b db \tag{36}$$

    - ▲ $v_{min}, b_{min}$ are cutoff values
    - ▲ i.e., thresholds that must be satisfied for Brehmstrahlung to occur

◇ Integrating

$$j_\nu^{ff} = n_i n_e \left(\frac{2m}{\pi kT}\right)^{\frac{1}{2}} \left(\frac{8\pi^3 Z_i^2 e^6}{3m^2c^3}\right) I \tag{37}$$

$$I \equiv \int_{x_{min}} dx \mathrm{e}^{-x} \int_{\xi_{min}}^{\infty} \frac{\mathrm{e}^{-\xi}}{\xi} d\xi \tag{38}$$

▲ $x_{min} \equiv m v_{min}^2 / 2kT$

▲ $\xi_{min} \equiv 4\pi\nu b_{min}/v$

▲ The second integral is our old friend $E_1(\xi_{min})$

◇ Evaluate $x_{min}, \xi_{min}$

▲ For the emission of a photon with frequency $\nu$, we have

$$x_{min} = \frac{h\nu}{kT} \tag{39}$$

▲ For $b_{min}$, we have two possibilities

(a) To insure small angle scattering, set

$$\frac{Ze^2}{b_{min}} = m_e v^2 \tag{40}$$

(b) Quantize angular momentum (or uncertainty principle)

$$b_{min} m v = \hbar \tag{41}$$

∘ These two conditions are satisfied when

$$v = \frac{Z_i e^2}{\hbar} \equiv v_c \tag{42}$$

∘ Let's impose two conditions

$$b_{min} = \begin{cases} Z_i e^2 / m v^2 & v \leq v_c \\ \hbar / m v & v \geq v_c \end{cases} \tag{43}$$

▲ Therefore

$$\xi_{min} = \begin{cases} (h\nu/\chi_i)(x/x_i)^{-\frac{3}{2}} & x \leq x_i \\ (h\nu/kT) x^{-1} & x \geq x_i \end{cases} \tag{44}$$

$$x \equiv \frac{m_e v^2}{2kT} \tag{45}$$

$$x_i \equiv \frac{\chi_i}{kT} \tag{46}$$

$$\chi_i \equiv \frac{m}{2}\left(\frac{Z_i e^2}{\hbar}\right)^2 \tag{47}$$

◇ Altogether now

$$I = \int_{h\nu/kT}^{\infty} \mathrm{e}^{-x} E_1(\xi_{min}) dx \tag{48}$$

◇ Low Frequency

▲ If $h\nu \ll \chi_i$ (or $kT$), we have $x \le x_i$

▲ Approximating $E_1(z) = \ln(\gamma/z)$ with $\gamma = 0.5772$ (Euler's constant)

▲ Therefore, for $h\nu/kT \ll 1$

$$I \approx \ln\left[\gamma \left( \chi_i/h\nu \right) x_i^{-\frac{3}{2}} \right] - \frac{3}{2}\gamma \tag{49}$$

▲ The Brehmstrahlung spectrum is a logarithmic function of $\nu$ (i.e. flat)

▲ Physics:

- Interaction between the e$^-$ and ion is very short compared to $\tau = 1/\nu$
- Approximately an impulse of radiation
- Fourier transform of an impulse (i.e. delta function) is flat

◇ High frequencies: $h\nu \gg \chi_i$ (or $kT$)

▲ $\xi_{min}$ is of order unity for small $x$

▲ Evaluate $I$ with $\xi_{min} = 1$

$$I = E_1(1)\mathrm{e}^{-h\nu/kT} \tag{50}$$

▲ The expected cutoff is simply due to the absence of e$^-$'s at wavelengths capable of producing photons with $h\nu$

- Quantum result

$$I = \frac{4}{\pi\sqrt{3}} \bar{g}_{ff}(\nu)\mathrm{e}^{-h\nu/kT} \tag{51}$$

◇ $\bar{g}_{ff}$ is the Gaunt factor

◇ $\bar{g}_{ff} = 1$ for most frequencies

$$\bar{g}_{ff} = \begin{cases} \sqrt{\frac{3}{\pi}} \left( \ln \frac{4kT}{h\nu} - 0.577 \right) & h\nu \ll kT \\ 1 & h\nu \gg kT \end{cases} \tag{52}$$

◇ Finally,

$$j_\nu^{ff} = \sum_i n_i n_e \left( \frac{2m_e}{3\pi kT} \right)^{\frac{1}{2}} \left[ \frac{32\pi^2 Z_i^2 e^6}{3m_e^2 c^3} \right] \bar{g_{ff}}(\nu)\mathrm{e}^{-h\nu/kT} \tag{53}$$

### H. Cooling Time

- Definition

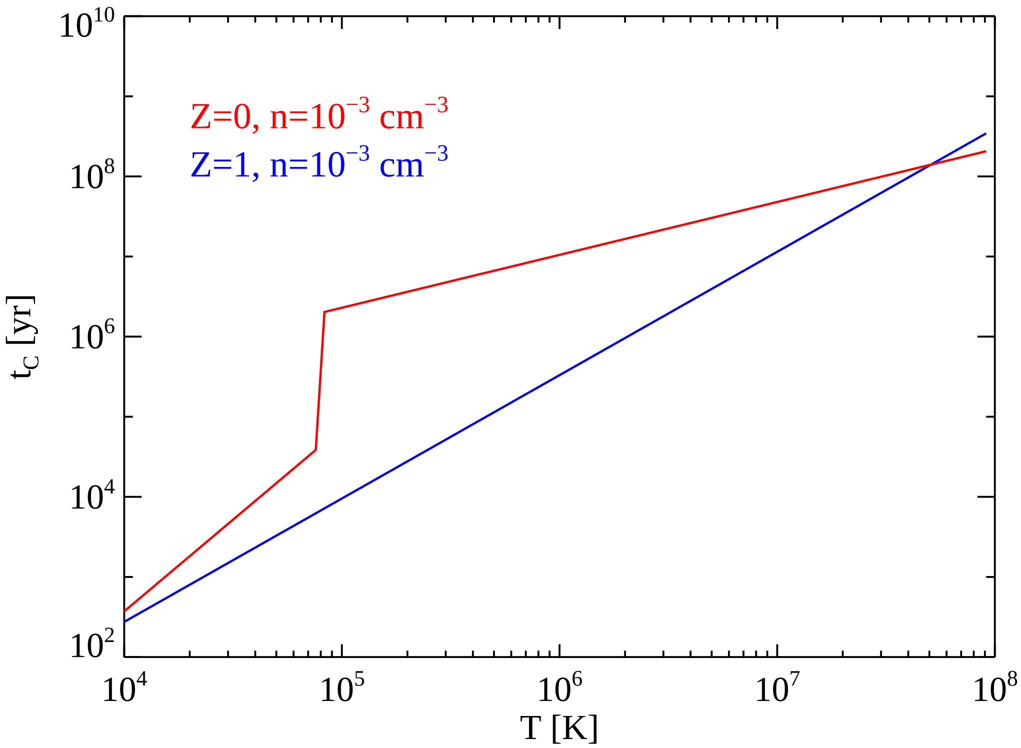
$$t_c \equiv \frac{T}{|dT/dt|} \tag{54}$$

◇ We can estimate this by comparing the thermal energy of the gas with the cooling rate

◇ Following Gnat & S 2007 for isobaric cooling,

$$t_c \approx \frac{10.8kT}{n\Lambda} \tag{55}$$

- Here is a plot for solar and primordial gas with $n = 10^{-3}\,\mathrm{cm}^{-3}$




◇ Gas at $T = 10^5$K cools very rapidly, especially with metals

◇ For very hot ($T > 10^6$K) and diffuse ($n \ll 10^{-3}\,\mathrm{cm}^{-3}$ gas, the cooling time may exceed the Hubble time

## I. Applications: Intracluster Medium (ICM)

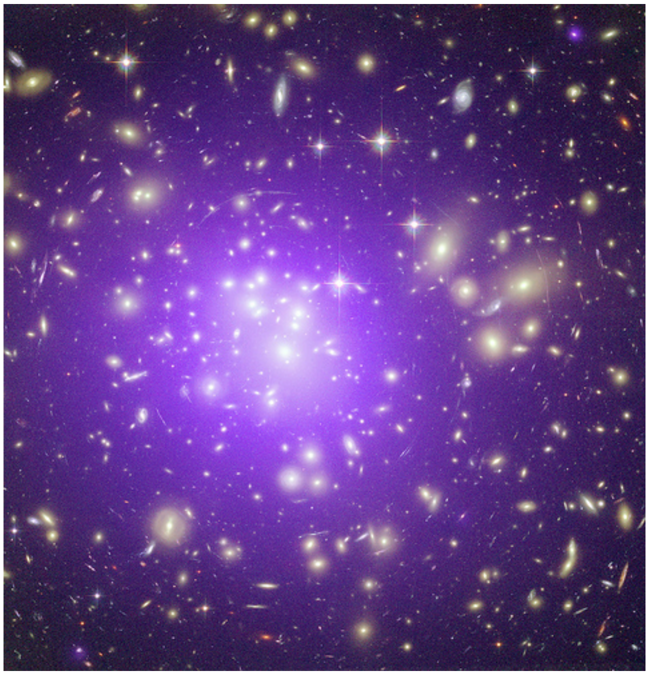
- Galaxy clusters have dark matter halo masses exceeding $10^{14} M_\odot$

◇ If these halos contain baryons, they may be shock heated to the virial temperature

◇ Estimate of $T_{\rm vir}$, assuming a fully ionized primordial gas

$$T_{\rm vir} \approx 4 \times 10^7 \left( \frac{M}{10^{14} M_\odot} \right)^{2/3} \;\mathrm{K} \tag{56}$$

◇ Gas at these temperatures will emit Brehmstrahlung X-ray radiation



- X-ray Observations
  - ◇ Detect point sources in cluster galaxies (X-ray binaries, AGN)
  - ◇ Detect diffuse, extended X-ray emission (ICM)
- ICM
  - ◇ Diffuse baryons filling the space between galaxy clusters
  - ◇ $T_{\rm ICM}$
    - ▲ Spectral analysis of the X-ray emission
    - ▲ Fit to Brehmstrahlung spectrum $(\exp(-h\nu/kT)$
      - ∘ Need to observe at energies $h\nu \approx kT$
      - ∘ $E \approx 1(T/10^7\,{\rm K}){\rm keV}$
      - ∘ X-rays
    - ▲ Presumably $T_{\rm ICM}$ is roughly the Virial temperature, at least for where the gas is too diffuse to cool efficiently
  - ◇ $n_e$
    - ▲ The ICM is optically thin to Brehmstrahlung emission
      - ∘ Therefore,

$$L \sim n_e n_p j_\nu^{ff}(T) V \tag{57}$$

      - ∘ V is the volume of the cluster
      - ∘ Measure $L(r)$ to estimate $n_e(r)$
    - ▲ The X-ray emission is often fitted by this profile

$$n_e(r) = n_0 \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{3\beta/2} \tag{58}$$

- ∘ See Reiprich & Bohringer 2002, ApJ, 567, 716
- ∘ $\beta \approx 0.6$
- ∘ $n_0 = 10^{-1}$ to $10^{-2}\,\mathrm{cm}^{-3}$ at $r_c = 300\,\mathrm{kpc}$

- Cooling flows
  - ◇ Near the center of the cluster, the cooling time is short
    - ▲ $t_c(r = 100\,\mathrm{kpc}) \approx 10^6\mathrm{yr}$
    - ▲ Expect to observe cooler gas flowing in
  - ◇ Initial X-ray analyses suggeseted cooling flows
    - ▲ Chandra (higher spatial resolution) refutes these
  - ◇ Outstanding problem: What prevents cooling flows in clusters?
    - ▲ AGN?
    - ▲ Bubbles?
    - ▲ Cosmic rays?
- SZ Effect
  - ◇ CMB photons travelling through a cluster will upscatter on the electrons
  - ◇ This alters the observed CMB spectrum
    - ▲ Signal is independent of redshift
    - ▲ But sensitive to cluster mass
  - ◇ Major campaigns underway to measure the SZ effect
    - ▲ Expect to discover thousands of high $z$ clusters
    - ▲ And estimate their masses
    - ▲ Cosmology!

## J. Applications: WHIM

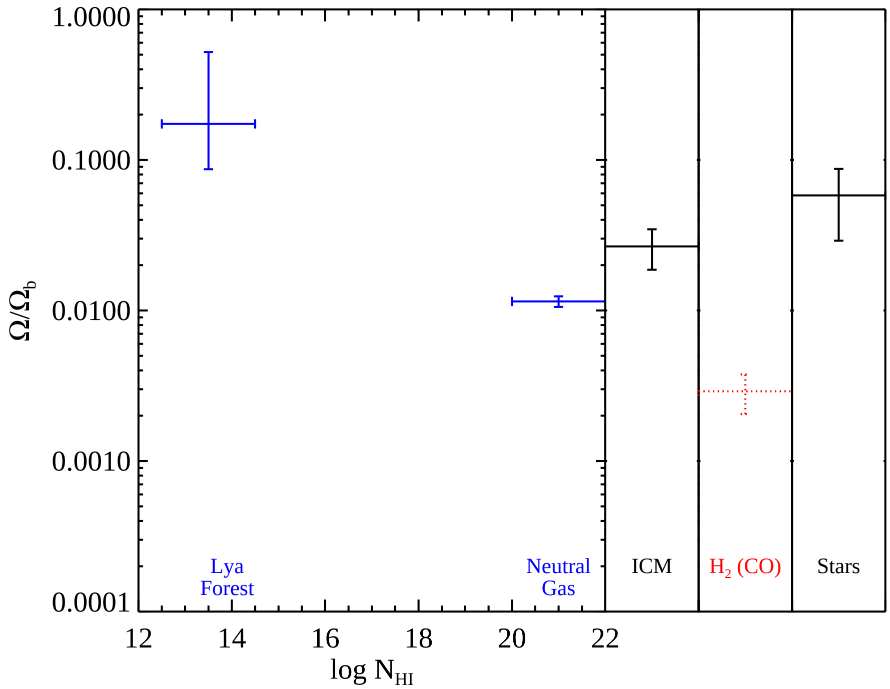
- D/H+BBN and CMB analysis yield a precise estimate of the baryon mass density

$$\Omega_b = 0.04 \tag{59}$$

- Consider a local census of baryons (e.g. Prochaska & Tumlinson 2009)

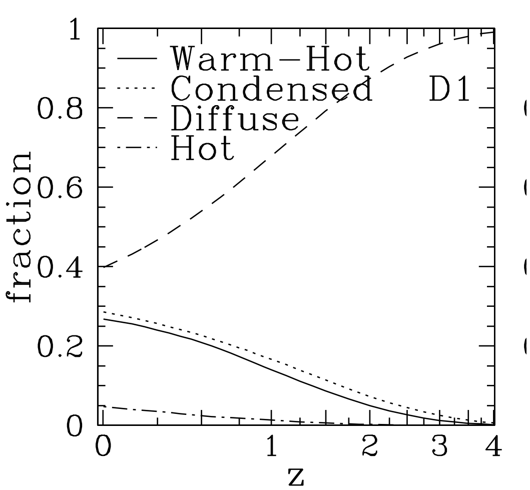
Table 1: $z \sim 0$ BARYON CENSUS

| Phase | Technique | Value ($\Omega/\Omega_b$) |
|---|---|---|
| Stars | $\phi(L)$, $M/L$ | $0.06 \pm 0.03$ |
| $H_2$ | $\phi(L)$, CO/$L$, $X$ | $0.003 \pm 0.001$ |
| HI | 21cm surveys | $0.011 \pm 0.001$ |
| ICM | $n_{DM}, f_{gas}$ | $0.027 \pm 0.001$ |
| IGM | Ly$\alpha$, EUVB | $0.2 - 0.5$ |




- ◇ The sum of these components is less than unity!

- Where are the missing baryons?
  - ◇ Black holes?? Not enough quasars
  - ◇ Hotter gas?? What would heat it?
  - ◇ Warm-Hot Intergalactic Medium (WHIM)
- Cosmological simulations of the WHIM (Davé et al., Cen & Ostriker)
  - ◇ Over the past several Gyr, large-scale (filamentary) structures have collapsed
  - ◇ In the process, the gas shocks and heats to $T \approx 10^6$ to $10^7$K
  - ◇ It has densities of several times the mean, e.g., $n \approx 10^{-5}\,\mathrm{cm}^{-3}$
  - ◇ Does the WHIM exist beyond theory-land?



- Empirical (observational) searches
  - ◇ Hydrogen in absorption
    - ▲ Standard Ly$\alpha$ absorption-line observations
    - ▲ But, the high $T$ implies very broad lines

$$b_{HI}^{WHIM} = 130\sqrt{\frac{T}{10^6\,\mathrm{K}}}\;\mathrm{km\,s^{-1}} \tag{60}$$

    - ▲ This implies a low optical depth,

$$\tau_0^{WHIM} = 0.05\frac{N_{\rm HI}}{10^{13}\,\mathrm{cm^{-2}}}\frac{130\,\mathrm{km\,s^{-1}}}{b} \tag{61}$$

    - ▲ To detect such a broad Ly$\alpha$ (BLA) line requires high S/N and a well-fluxed spectrum

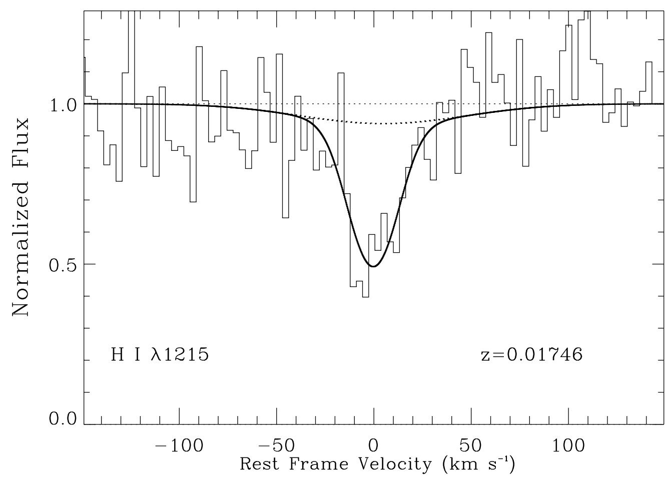



FIG. 16.—Broad component fit to the Ly$\alpha$ line at $z = 0.01746$. The solid line is a two-component fit with $b = 58\ \mathrm{km\,s^{-1}}$ and $\log N(\mathrm{H\,I}) = 12.71$ for the broad component. The dotted line shows the broad component profile only.

    - ▲ The BLAs may comprise > 20% of the baryons (Lehner et al. 2006,2007)
  - ◇ Metals in absorption
    - ▲ If the WHIM has metals, these will be in high ionization states (see our ion fractions of Oxygen)
    - ▲ $O^{+6}$, $O^{+7}$ are dominant but these require X-ray spectroscopy ($\lambda \approx 20$Å)

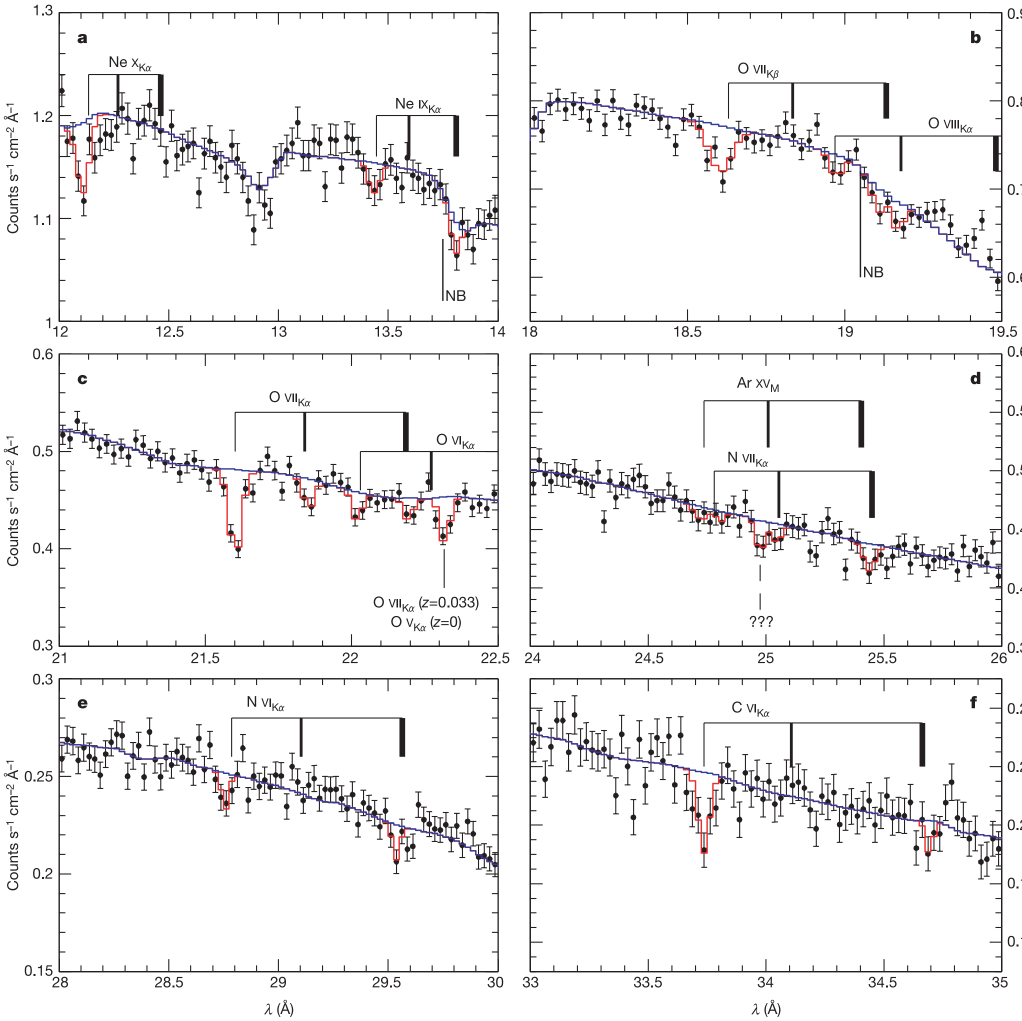



**Figure 1** The WHIM absorption in the Chandra LETG spectrum of Mkn 421. Six portions of the LETG spectrum of MKn 421 along with its best-fitting continuum (solid blue line) plus narrow absorption model (solid red line), centred around the rest wavelengths of the Ne IX–Ne X $_{K\alpha}$ (**a**), O VIII $_{K\alpha}$ (**b**), O VII $_{K\alpha}$ (**c**), N VII $_{K\alpha}$ (**d**), N VI $_{K\alpha}$ (**e**) and C VI $_{K\alpha}$ (**f**) transitions. This spectrum contains a total of $\sim$5,000 counts per resolution element in the continuum at 21 Å, enough to detect O VII columns of $N_{\rm O\,VII} \geq 8 \times 10^{14}$ cm$^{-2}$ at a significance level $\geq 3\sigma$. For each labelled ion in the six panels, the three vertical lines from left to right are the rest frame wavelengths (thin line) and the expected wavelengths at $z = 0.011$ ($cz = 3{,}300$ km s$^{-1}$, medium thickness line) and $z = 0.027$ ($cz = 8{,}090$ km s$^{-1}$, thick line). Mkn 421 was also observed with the HST GHRS on 1 February 1995. We retrieved this GHRS spectrum of Mkn 421 from the public HST archive, and re-analysed the data. The $3\sigma$ H I column upper limit of putative H I Ly$\alpha$ at the average redshifts of the two X-ray systems are $N_{\rm H\,I} < 4.7 \times 10^{12}$ cm$^{-2}$ and $N_{\rm H\,I} < 8.5 \times 10^{12}$ cm$^{-2}$ (assuming a temperature of $\log[T({\rm K})] = 6.1$ for both systems). Following our 'target of opportunity observation' request Mkn 421 was observed by FUSE on 19–21 January 2003 with a total exposure time of 62 ks. From this spectrum we derived $3\sigma$ O VI column upper limits of putative O VI$_{2s\rightarrow 2p}$ at the redshift of the H I Ly$\alpha$ absorber and at the average redshifts of the two X-ray systems, of $N_{\rm O\,VI}(z = 0.010160) < 1.4 \times 10^{13}$ cm$^{-2}$, $N_{\rm O\,VI}(z = 0.011) < 1.6 \times 10^{13}$ cm$^{-2}$ and $N_{\rm O\,VI}(z = 0.027) < 1.4 \times 10^{13}$ cm$^{-2}$ respectively. Error bars, $\pm 1\sigma$

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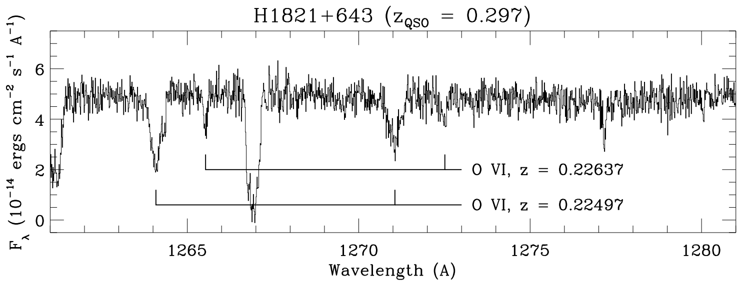
- Only a handful of X-ray sources are bright enough for Chandra and XMM
- Claimed detections are controversial (at best; Nicastro et al. 2005)
- This science awaits new technology (e.g. XEUS)




- ▲ $O^{+5}$ exists in to $T \approx 10^6$K
  - ○ This shows a Li-like doublet at 1031,1037Å
  - ○ UV spectroscopy show many detections
  - ○ Both the corresponding Ly$\alpha$ lines are generally narrow, suggesting photoionization (Tripp et al. 2008)
- ◇ WHIM in emission?
  - ▲ Emphasis of proposed NASA missions
  - ▲ None have yet to be funded